Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.8-P-x-c-x-^m-a+b-x^n-^p

Nasser M. Abbasi

May 13, 2020 Compiled on May 13, 2020 at 2:08pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [594]. This is test number [29].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (594)	% 0. (0)
Mathematica	% 100. (594)	% 0. (0)
Maple	% 97.14 (577)	% 2.86 (17)
Maxima	% 34.68 (206)	% 65.32 (388)
Fricas	% 56.9 (338)	% 43.1 (256)
Sympy	% 76.43 (454)	% 23.57 (140)
Giac	% 70.37 (418)	% 29.63 (176)

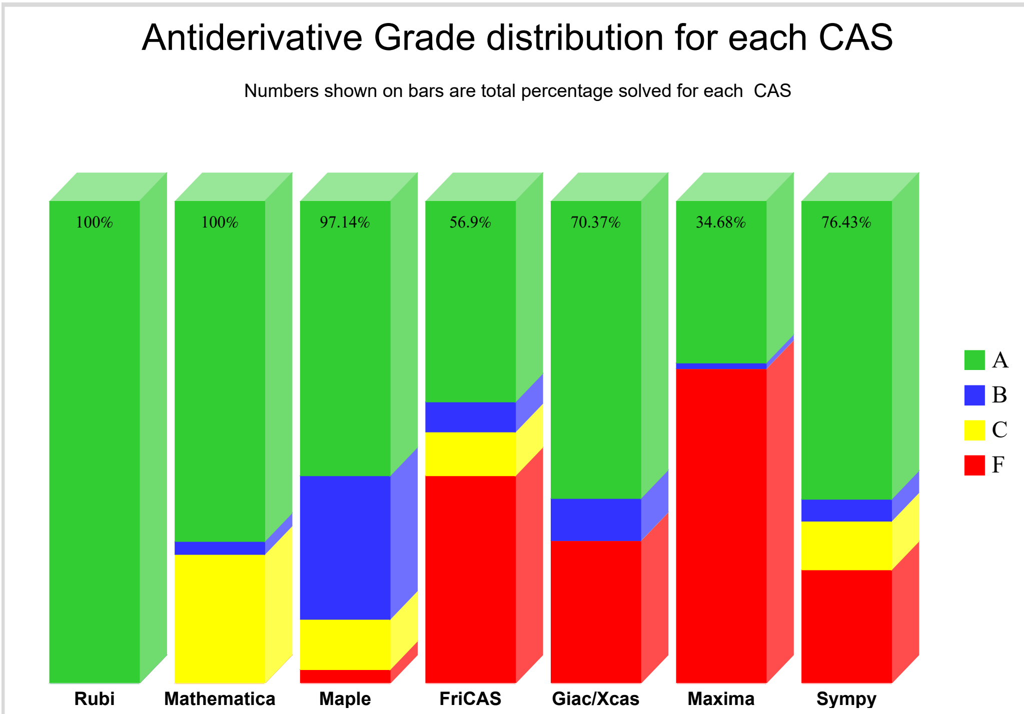
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

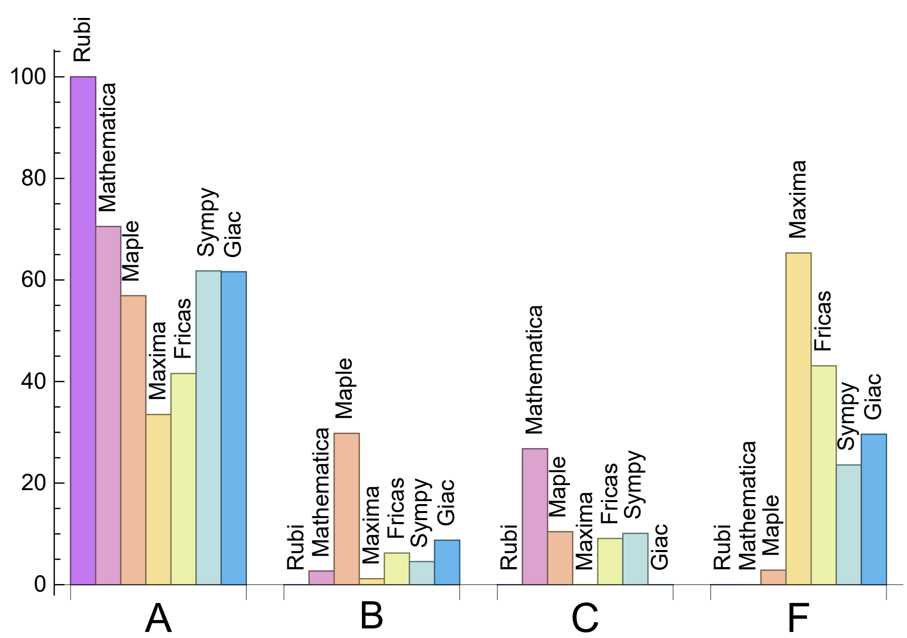
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	70.54	2.69	26.77	0.
Maple	56.9	29.8	10.44	2.86
Maxima	33.5	1.18	0.	65.32
Fricas	41.58	6.23	9.09	43.1
Sympy	61.78	4.55	10.1	23.57
Giac	61.62	8.75	0.	29.63

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	245.1	1.	221.5	1.
Mathematica	0.16	170.81	0.91	154.	0.95
Maple	0.01	388.7	1.44	289.	1.27
Maxima	1.12	139.43	1.33	105.5	1.28
Fricas	4.23	2531.22	11.34	402.5	2.93
Sympy	9.88	221.46	1.25	129.	0.94
Giac	1.09	304.82	1.7	266.5	1.44

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{41, 567, 590\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

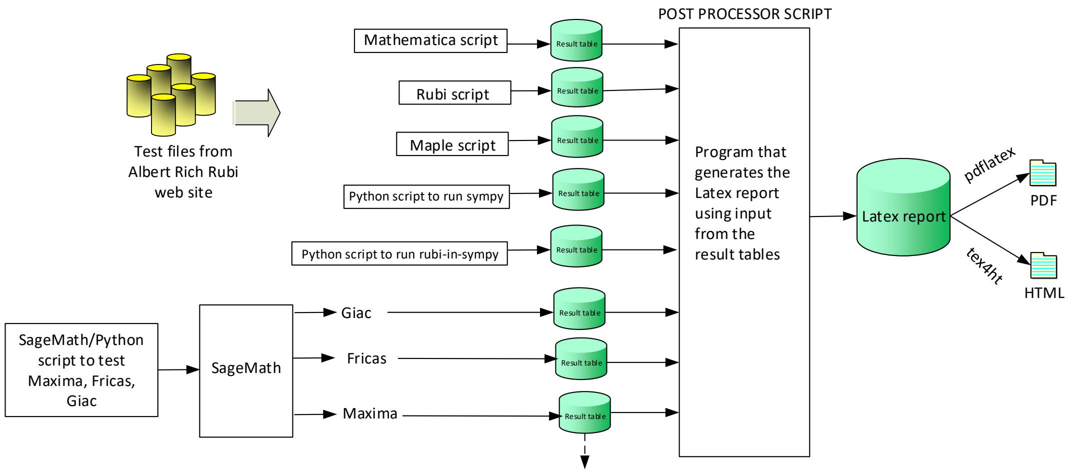
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594 }

B grade: { 21, 32, 33, 34, 35, 36, 41, 44, 45, 46, 47, 369, 370, 371, 372, 557 }

C grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 124, 161, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 567, 590 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 76, 77, 78, 80, 93, 94, 97, 98, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 157, 159, 160, 161, 162, 163, 165, 166, 167, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 195, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 422, 423, 424, 425, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 593 }

B grade: { 6, 20, 21, 29, 30, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 123, 125, 127, 129, 149, 150, 156, 158, 164, 168, 169, 171, 172, 186, 187, 188, 190, 191, 192, 193, 194, 196, 197, 199, 200, 221, 222, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 370, 371, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 426, 427, 428, 429, 440, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 557 }

C grade: { 210, 213, 214, 220, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 592, 594 }

F grade: { 474, 475, 476, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 13, 14, 15, 16, 17, 18, 19, 27, 28, 39, 42, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 124, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 180, 181, 182, 183, 184, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 251, 252, 253, 254, 255, 256, 257, 258, 259, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 589, 592, 593

B grade: { 3, 6, 161, 179, 185, 557, 594 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 47, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 221, 222, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 369, 370, 371, 372, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 180, 181, 182, 183, 184, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 571, 572, 573, 574, 575, 579, 580, 585, 589, 593, 594 }

B grade: { 40, 41, 44, 45, 46, 47, 152, 155, 156, 160, 163, 164, 168, 179, 185, 221, 222, 283, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 557, 569, 570, 577, 578, 591, 592 }

C grade: { 7, 8, 9, 10, 11, 12, 24, 25, 26, 57, 58, 70, 71, 72, 73, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 414, 415, 416, 417, 419, 421, 422, 423, 424, 425, 428 }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 154, 157, 158, 162, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 220, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 418, 420, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 586, 587, 588, 590 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 29, 31, 37, 38, 39, 42, 43, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 159, 160, 162, 163, 164, 165, 167, 168, 169, 170, 180, 181, 182, 183, 184, 211, 212, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 294, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 351, 352, 353, 358, 359, 360, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 495, 496, 497, 498, 499, 510, 511, 512, 513, 514, 515, 516, 517, 518, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 552, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581 }

B grade: { 73, 74, 125, 127, 129, 131, 140, 149, 150, 158, 166, 171, 172, 175, 176, 179, 185, 403, 404, 405, 406, 407, 485, 486, 487, 488, 557 }

C grade: { 18, 19, 20, 21, 27, 28, 30, 32, 33, 34, 35, 36, 49, 123, 161, 210, 213, 214, 215, 216, 219, 365, 366, 367, 368, 369, 370, 371, 372, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 534, 535, 536, 537, 538, 539, 547, 548, 549, 550, 582 }

F grade: { 6, 40, 41, 44, 45, 46, 47, 65, 68, 69, 75, 151, 173, 174, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 230, 231, 232, 247, 248, 249, 250, 256, 257, 258, 259, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 361, 362, 363, 364, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 476, 492, 493, 494, 551, 553, 576, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

2.1.7 Giac

A grade: { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 39, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 118, 120, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 170, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 191, 195, 196, 197, 201, 202, 203, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 579, 580 }

B grade: { 3, 6, 30, 32, 33, 34, 44, 45, 115, 117, 119, 121, 123, 125, 127, 129, 131, 149, 150, 151, 161, 169, 171, 172, 173, 174, 179, 186, 187, 188, 192, 193, 194, 198, 199, 200, 204, 205, 206, 254, 369, 370, 371, 485, 486, 557, 577, 578, 591, 592, 593, 594 }

C grade: { }

F grade: { 29, 31, 37, 38, 40, 41, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	53	104	127	223	105
normalized size	1	1.	0.74	0.74	1.44	1.76	3.1	1.46
time (sec)	N/A	0.033	0.071	0.023	0.954	1.262	7.773	1.108

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	155	194	320	433	644	320
normalized size	1	1.	0.96	1.2	1.99	2.69	4.	1.99
time (sec)	N/A	0.105	0.152	0.005	0.931	1.283	50.669	1.107

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	294	495	709	1067	1406	710
normalized size	1	1.	1.07	1.81	2.59	3.89	5.13	2.59
time (sec)	N/A	0.194	0.538	0.006	0.958	1.281	116.817	1.111

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	82	91	173	216	354	174
normalized size	1	1.	0.72	0.8	1.52	1.89	3.11	1.53
time (sec)	N/A	0.071	0.126	0.045	0.962	1.191	15.201	1.08

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	303	447	675	1000	1365	697
normalized size	1	1.	0.95	1.4	2.11	3.12	4.27	2.18
time (sec)	N/A	0.244	0.457	0.007	0.991	1.269	117.588	1.115

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	708	708	678	1417	1836	2993	0	1909
normalized size	1	1.	0.96	2.	2.59	4.23	0.	2.7
time (sec)	N/A	0.625	2.263	0.007	1.053	1.385	0.	1.181

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	4590	76	216
normalized size	1	1.	0.77	1.16	0.	28.51	0.47	1.34
time (sec)	N/A	0.111	0.055	0.044	0.	7.861	0.634	1.115

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	180	238	0	4961	105	252
normalized size	1	1.	0.95	1.26	0.	26.25	0.56	1.33
time (sec)	N/A	0.139	0.151	0.003	0.	7.973	1.103	1.109

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	205	272	0	5554	146	279
normalized size	1	1.	0.95	1.27	0.	25.83	0.68	1.3
time (sec)	N/A	0.187	0.154	0.004	0.	8.437	1.904	1.124

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	306	0	5960	185	312
normalized size	1	1.	0.95	1.27	0.	24.83	0.77	1.3
time (sec)	N/A	0.223	0.192	0.004	0.	7.257	3.36	1.103

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	186	0	4590	76	197
normalized size	1	1.	0.78	1.16	0.	28.51	0.47	1.22
time (sec)	N/A	0.121	0.058	0.048	0.	6.319	1.243	1.113

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	125	188	0	4563	78	155
normalized size	1	1.	0.78	1.17	0.	28.34	0.48	0.96
time (sec)	N/A	0.1	0.049	0.004	0.	6.228	1.211	1.079

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	58	26	22
normalized size	1	1.	1.	0.89	1.16	3.05	1.37	1.16
time (sec)	N/A	0.015	0.005	0.022	1.446	0.954	0.104	1.079

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	58	26	22
normalized size	1	1.	1.	0.89	1.16	3.05	1.37	1.16
time (sec)	N/A	0.013	0.004	0.001	1.432	0.892	0.097	1.159

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	53	17	23
normalized size	1	1.	1.	0.77	1.	2.41	0.77	1.05
time (sec)	N/A	0.012	0.004	0.004	1.439	0.917	0.096	1.192

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	54	17	26
normalized size	1	1.	1.	0.86	1.09	2.45	0.77	1.18
time (sec)	N/A	0.013	0.004	0.004	1.472	1.009	0.092	1.119

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	112	44	45
normalized size	1	1.	1.	0.8	1.05	2.73	1.07	1.1
time (sec)	N/A	0.027	0.008	0.004	1.442	0.948	0.135	1.088

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	35	46	72	54	38
normalized size	1	1.	1.07	1.21	1.59	2.48	1.86	1.31
time (sec)	N/A	0.022	0.01	0.067	1.459	0.903	0.157	1.119

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	34	45	72	53	35
normalized size	1	1.	1.	1.17	1.55	2.48	1.83	1.21
time (sec)	N/A	0.02	0.007	0.002	1.437	1.027	0.213	1.059

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	195	0	323	88	65
normalized size	1	1.	0.9	5.	0.	8.28	2.26	1.67
time (sec)	N/A	0.027	0.017	0.043	0.	1.091	0.431	1.117

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	129	228	0	339	105	78
normalized size	1	1.	3.15	5.56	0.	8.27	2.56	1.9
time (sec)	N/A	0.043	0.049	0.01	0.	1.083	0.477	1.098

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	0	760	26	155
normalized size	1	1.	0.76	0.8	0.	6.44	0.22	1.31
time (sec)	N/A	0.126	0.011	0.041	0.	0.99	0.135	1.079

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	94	0	765	22	155
normalized size	1	1.	0.76	0.8	0.	6.48	0.19	1.31
time (sec)	N/A	0.106	0.012	0.043	0.	1.009	0.156	1.116

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	4590	76	224
normalized size	1	1.	0.77	1.16	0.	28.51	0.47	1.39
time (sec)	N/A	0.166	0.039	0.043	0.	6.199	0.658	1.085

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	122	108	0	2425	75	162
normalized size	1	1.	0.91	0.81	0.	18.1	0.56	1.21
time (sec)	N/A	0.113	0.023	0.043	0.	6.385	0.299	1.102

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	123	111	0	2820	70	128
normalized size	1	1.	0.92	0.83	0.	21.04	0.52	0.96
time (sec)	N/A	0.089	0.03	0.041	0.	6.3	0.414	1.095

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	72	43	57	95	60	50
normalized size	1	1.	1.95	1.16	1.54	2.57	1.62	1.35
time (sec)	N/A	0.058	0.02	0.048	1.468	0.878	0.513	1.082

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	71	45	59	95	60	51
normalized size	1	1.	1.82	1.15	1.51	2.44	1.54	1.31
time (sec)	N/A	0.042	0.018	0.007	1.43	0.999	0.434	1.074

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	76	117	0	387	58	0
normalized size	1	1.	1.58	2.44	0.	8.06	1.21	0.
time (sec)	N/A	0.037	0.019	0.044	0.	1.081	0.491	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	72	84	0	122	85	150
normalized size	1	1.	1.53	1.79	0.	2.6	1.81	3.19
time (sec)	N/A	0.034	0.024	0.047	0.	1.02	0.443	1.098

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	99	122	0	466	58	0
normalized size	1	1.	1.74	2.14	0.	8.18	1.02	0.
time (sec)	N/A	0.069	0.029	0.006	0.	1.053	0.583	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	106	110	0	130	95	132
normalized size	1	1.	2.26	2.34	0.	2.77	2.02	2.81
time (sec)	N/A	0.061	0.039	0.007	0.	1.025	0.468	1.138

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	87	0	138	100	224
normalized size	1	1.	2.92	1.74	0.	2.76	2.	4.48
time (sec)	N/A	0.077	0.052	0.044	0.	1.05	0.536	1.126

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	150	135	0	140	110	219
normalized size	1	1.	2.83	2.55	0.	2.64	2.08	4.13
time (sec)	N/A	0.081	0.059	0.007	0.	1.033	0.554	1.122

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	149	132	0	140	109	123
normalized size	1	1.	2.76	2.44	0.	2.59	2.02	2.28
time (sec)	N/A	0.06	0.047	0.003	0.	0.983	0.523	1.093

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	90	0	138	102	115
normalized size	1	1.	2.77	1.7	0.	2.6	1.92	2.17
time (sec)	N/A	0.058	0.043	0.005	0.	1.044	0.623	1.083

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	95	117	0	463	70	0
normalized size	1	1.	1.56	1.92	0.	7.59	1.15	0.
time (sec)	N/A	0.041	0.017	0.005	0.	1.051	0.669	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	116	122	0	539	73	0
normalized size	1	1.	1.66	1.74	0.	7.7	1.04	0.
time (sec)	N/A	0.072	0.028	0.007	0.	1.087	0.766	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	32	42	107	42	43
normalized size	1	1.	1.25	0.8	1.05	2.68	1.05	1.08
time (sec)	N/A	0.03	0.01	0.005	1.457	1.001	0.123	1.204

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	122	310	0	1058	0	0
normalized size	1	1.	1.74	4.43	0.	15.11	0.	0.
time (sec)	N/A	0.067	0.046	0.004	0.	11.304	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	238	345	0	1129	0	0
normalized size	1	1.	2.7	3.92	0.	12.83	0.	0.
time (sec)	N/A	0.112	0.526	0.006	0.	9.575	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	16	22	7	18
normalized size	1	1.	1.09	1.09	1.45	2.	0.64	1.64
time (sec)	N/A	0.011	0.002	0.039	0.995	0.918	0.069	1.502

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	218	0	51	20	22
normalized size	1	1.	1.	10.38	0.	2.43	0.95	1.05
time (sec)	N/A	0.015	0.003	0.005	0.	1.018	0.312	1.472

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	247	121	0	948	0	362
normalized size	1	1.	3.48	1.7	0.	13.35	0.	5.1
time (sec)	N/A	0.094	0.267	0.005	0.	5.691	0.	1.249

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	288	345	0	972	0	351
normalized size	1	1.	3.79	4.54	0.	12.79	0.	4.62
time (sec)	N/A	0.102	0.177	0.006	0.	5.776	0.	1.102

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	253	340	0	968	0	180
normalized size	1	1.	3.24	4.36	0.	12.41	0.	2.31
time (sec)	N/A	0.108	0.251	0.005	0.	5.751	0.	1.13

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	244	124	0	952	0	169
normalized size	1	1.	3.25	1.65	0.	12.69	0.	2.25
time (sec)	N/A	0.105	0.261	0.007	0.	5.992	0.	1.089

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	36	35	77	24	36
normalized size	1	1.	0.97	1.12	1.09	2.41	0.75	1.12
time (sec)	N/A	0.034	0.012	0.045	1.443	1.28	0.31	1.064

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	87	63	158	323	70
normalized size	1	1.	1.13	1.58	1.15	2.87	5.87	1.27
time (sec)	N/A	0.057	0.033	0.043	1.454	1.344	0.891	1.071

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.006	0.001	0.001	0.978	1.196	0.072	1.079

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	43	101	5	45
normalized size	1	1.	1.	1.1	1.43	3.37	0.17	1.5
time (sec)	N/A	0.03	0.009	0.005	1.446	1.238	0.121	1.067

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	46	15	23
normalized size	1	1.	1.	0.94	1.22	2.56	0.83	1.28
time (sec)	N/A	0.019	0.005	0.004	1.443	1.401	0.112	1.073

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	131	232	117	131
normalized size	1	1.	1.	0.87	1.16	2.05	1.04	1.16
time (sec)	N/A	0.099	0.004	0.001	0.985	1.118	0.087	1.085

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	180	90	100
normalized size	1	1.	1.	0.85	1.14	2.05	1.02	1.14
time (sec)	N/A	0.06	0.003	0.001	0.96	1.108	0.079	1.087

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	120	58	68
normalized size	1	1.	1.	0.85	1.13	2.	0.97	1.13
time (sec)	N/A	0.038	0.002	0.001	0.99	1.08	0.076	1.063

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	23	8	14
normalized size	1	1.	1.	0.92	1.17	1.92	0.67	1.17
time (sec)	N/A	0.014	0.001	0.002	0.949	1.253	0.069	1.063

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	186	0	4590	76	216
normalized size	1	1.	0.77	1.16	0.	28.51	0.47	1.34
time (sec)	N/A	0.098	0.045	0.002	0.	8.105	0.688	1.08

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	180	238	0	4961	105	252
normalized size	1	1.	0.95	1.26	0.	26.25	0.56	1.33
time (sec)	N/A	0.127	0.148	0.004	0.	7.999	1.111	1.084

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	78	1618	0	0	265	0
normalized size	1	1.	0.13	2.77	0.	0.	0.45	0.
time (sec)	N/A	0.464	0.04	0.009	0.	0.	5.448	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	76	1546	0	0	170	0
normalized size	1	1.	0.14	2.78	0.	0.	0.31	0.
time (sec)	N/A	0.327	0.032	0.008	0.	0.	3.5	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	75	1480	0	0	163	0
normalized size	1	1.	0.14	2.82	0.	0.	0.31	0.
time (sec)	N/A	0.239	0.029	0.009	0.	0.	2.861	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	75	1536	0	0	78	0
normalized size	1	1.	0.15	3.13	0.	0.	0.16	0.
time (sec)	N/A	0.158	0.032	0.011	0.	0.	2.872	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	96	1662	0	0	163	0
normalized size	1	1.	0.18	3.18	0.	0.	0.31	0.
time (sec)	N/A	0.249	0.051	0.009	0.	0.	12.033	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	123	1782	0	0	163	0
normalized size	1	1.	0.22	3.22	0.	0.	0.29	0.
time (sec)	N/A	0.317	0.089	0.088	0.	0.	102.473	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	138	1902	0	0	0	0
normalized size	1	1.	0.24	3.27	0.	0.	0.	0.
time (sec)	N/A	0.41	0.117	0.057	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	590	135	1491	0	0	187	0
normalized size	1	1.	0.23	2.53	0.	0.	0.32	0.
time (sec)	N/A	0.556	0.121	0.046	0.	0.	3.088	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	130	1547	0	0	189	0
normalized size	1	1.	0.22	2.6	0.	0.	0.32	0.
time (sec)	N/A	0.433	0.121	0.005	0.	0.	18.522	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	628	170	1673	0	0	0	0
normalized size	1	1.	0.27	2.66	0.	0.	0.	0.
time (sec)	N/A	0.5	0.185	0.006	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	196	1793	0	0	0	0
normalized size	1	1.	0.29	2.65	0.	0.	0.	0.
time (sec)	N/A	0.675	0.253	0.006	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	200	211	0	10904	156	262
normalized size	1	1.	1.08	1.13	0.	58.62	0.84	1.41
time (sec)	N/A	0.178	0.084	0.002	0.	8.11	1.034	1.094

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	214	325	0	15101	245	327
normalized size	1	1.	0.96	1.46	0.	68.02	1.1	1.47
time (sec)	N/A	0.319	0.17	0.005	0.	14.948	2.644	1.092

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	280	277	446	0	18692	325	427
normalized size	1	0.99	0.98	1.58	0.	66.28	1.15	1.51
time (sec)	N/A	0.443	0.244	0.003	0.	57.848	5.48	1.085

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	270	269	444	0	26617	546	383
normalized size	1	0.99	0.99	1.63	0.	97.86	2.01	1.41
time (sec)	N/A	0.49	0.284	0.004	0.	15.671	8.799	1.07

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	439	837	0	0	1314	648
normalized size	1	1.	1.06	2.01	0.	0.	3.16	1.56
time (sec)	N/A	0.7	0.451	0.005	0.	0.	86.1	1.068

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	643	678	1339	0	0	0	1060
normalized size	1	1.	1.05	2.08	0.	0.	0.	1.64
time (sec)	N/A	1.097	0.349	0.007	0.	0.	0.	1.088

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	50	120	44	51
normalized size	1	1.	1.26	0.88	1.16	2.79	1.02	1.19
time (sec)	N/A	0.078	0.012	0.005	1.401	1.556	0.124	1.087

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	54	38	50	122	46	51
normalized size	1	1.	1.17	0.83	1.09	2.65	1.	1.11
time (sec)	N/A	0.084	0.012	0.005	1.416	1.516	0.121	1.08

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	50	122	48	51
normalized size	1	1.	1.	0.86	1.14	2.77	1.09	1.16
time (sec)	N/A	0.043	0.008	0.004	1.426	1.549	0.12	1.071

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	47	407	0	0	92	0
normalized size	1	1.	0.2	1.77	0.	0.	0.4	0.
time (sec)	N/A	0.06	0.02	0.013	0.	0.	1.639	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	43	368	0	0	97	0
normalized size	1	1.	0.17	1.43	0.	0.	0.38	0.
time (sec)	N/A	0.067	0.016	0.016	0.	0.	2.356	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	63	407	0	0	82	0
normalized size	1	1.	0.44	2.83	0.	0.	0.57	0.
time (sec)	N/A	0.027	0.024	0.013	0.	0.	2.362	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	67	370	0	0	99	0
normalized size	1	1.	0.5	2.74	0.	0.	0.73	0.
time (sec)	N/A	0.033	0.026	0.011	0.	0.	1.65	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	90	1003	0	0	122	0
normalized size	1	1.	0.19	2.14	0.	0.	0.26	0.
time (sec)	N/A	0.121	0.056	0.068	0.	0.	3.347	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	481	91	949	0	0	128	0
normalized size	1	1.	0.19	1.97	0.	0.	0.27	0.
time (sec)	N/A	0.139	0.057	0.087	0.	0.	3.507	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	92	952	0	0	112	0
normalized size	1	1.	0.34	3.51	0.	0.	0.41	0.
time (sec)	N/A	0.066	0.04	0.035	0.	0.	3.547	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	93	1012	0	0	129	0
normalized size	1	1.	0.35	3.8	0.	0.	0.48	0.
time (sec)	N/A	0.062	0.043	0.028	0.	0.	3.535	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	520	520	89	1004	0	0	124	0
normalized size	1	1.	0.17	1.93	0.	0.	0.24	0.
time (sec)	N/A	0.213	0.048	0.034	0.	0.	2.167	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	533	89	950	0	0	129	0
normalized size	1	1.	0.17	1.78	0.	0.	0.24	0.
time (sec)	N/A	0.172	0.044	0.036	0.	0.	2.77	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	953	0	0	114	0
normalized size	1	1.	0.35	3.72	0.	0.	0.45	0.
time (sec)	N/A	0.083	0.032	0.019	0.	0.	2.753	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	92	1013	0	0	131	0
normalized size	1	1.	0.37	4.04	0.	0.	0.52	0.
time (sec)	N/A	0.064	0.034	0.012	0.	0.	2.279	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	49	407	0	0	92	0
normalized size	1	1.	0.39	3.2	0.	0.	0.72	0.
time (sec)	N/A	0.019	0.021	0.013	0.	0.	1.597	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	45	368	0	0	97	0
normalized size	1	1.	0.32	2.59	0.	0.	0.68	0.
time (sec)	N/A	0.025	0.016	0.011	0.	0.	2.419	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	63	407	0	0	82	0
normalized size	1	1.	0.24	1.54	0.	0.	0.31	0.
time (sec)	N/A	0.054	0.026	0.008	0.	0.	2.439	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	67	370	0	0	97	0
normalized size	1	1.	0.27	1.5	0.	0.	0.39	0.
time (sec)	N/A	0.054	0.024	0.008	0.	0.	1.669	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	47	407	0	0	92	0
normalized size	1	1.	0.37	3.23	0.	0.	0.73	0.
time (sec)	N/A	0.025	0.015	0.01	0.	0.	2.302	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	43	368	0	0	97	0
normalized size	1	1.	0.3	2.57	0.	0.	0.68	0.
time (sec)	N/A	0.025	0.011	0.009	0.	0.	1.719	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	63	407	0	0	82	0
normalized size	1	1.	0.24	1.55	0.	0.	0.31	0.
time (sec)	N/A	0.05	0.02	0.009	0.	0.	1.689	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	67	370	0	0	97	0
normalized size	1	1.	0.27	1.49	0.	0.	0.39	0.
time (sec)	N/A	0.052	0.019	0.011	0.	0.	2.395	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	1003	0	0	122	0
normalized size	1	1.	0.35	3.92	0.	0.	0.48	0.
time (sec)	N/A	0.045	0.062	0.038	0.	0.	3.327	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	90	949	0	0	128	0
normalized size	1	1.	0.34	3.61	0.	0.	0.49	0.
time (sec)	N/A	0.042	0.062	0.04	0.	0.	3.556	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	91	952	0	0	112	0
normalized size	1	1.	0.18	1.92	0.	0.	0.23	0.
time (sec)	N/A	0.133	0.039	0.014	0.	0.	3.545	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	93	1012	0	0	128	0
normalized size	1	1.	0.19	2.07	0.	0.	0.26	0.
time (sec)	N/A	0.115	0.05	0.017	0.	0.	3.563	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	89	1004	0	0	124	0
normalized size	1	1.	0.37	4.17	0.	0.	0.51	0.
time (sec)	N/A	0.069	0.051	0.03	0.	0.	2.175	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	89	950	0	0	129	0
normalized size	1	1.	0.36	3.83	0.	0.	0.52	0.
time (sec)	N/A	0.061	0.044	0.033	0.	0.	2.77	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	90	953	0	0	114	0
normalized size	1	1.	0.16	1.74	0.	0.	0.21	0.
time (sec)	N/A	0.218	0.032	0.014	0.	0.	2.731	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	92	1013	0	0	129	0
normalized size	1	1.	0.17	1.88	0.	0.	0.24	0.
time (sec)	N/A	0.163	0.039	0.016	0.	0.	2.293	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	75	720	0	0	78	0
normalized size	1	1.	0.15	1.47	0.	0.	0.16	0.
time (sec)	N/A	0.148	0.029	0.003	0.	0.	1.626	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	75	681	0	0	82	0
normalized size	1	1.	0.15	1.35	0.	0.	0.16	0.
time (sec)	N/A	0.149	0.035	0.004	0.	0.	1.722	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	76	683	0	0	73	0
normalized size	1	1.	0.15	1.33	0.	0.	0.14	0.
time (sec)	N/A	0.163	0.025	0.004	0.	0.	1.701	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	78	726	0	0	83	0
normalized size	1	1.	0.15	1.43	0.	0.	0.16	0.
time (sec)	N/A	0.154	0.025	0.005	0.	0.	1.722	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	42	291	0	0	61	0
normalized size	1	1.	0.17	1.18	0.	0.	0.25	0.
time (sec)	N/A	0.081	0.01	0.004	0.	0.	1.385	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	38	267	0	0	65	0
normalized size	1	1.	0.14	0.99	0.	0.	0.24	0.
time (sec)	N/A	0.087	0.009	0.005	0.	0.	1.461	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	58	291	0	0	56	0
normalized size	1	1.	0.21	1.06	0.	0.	0.2	0.
time (sec)	N/A	0.087	0.022	0.004	0.	0.	1.43	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	62	269	0	0	66	0
normalized size	1	1.	0.24	1.03	0.	0.	0.25	0.
time (sec)	N/A	0.078	0.022	0.004	0.	0.	1.448	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	134	101	0	0	126	304
normalized size	1	1.	1.54	1.16	0.	0.	1.45	3.49
time (sec)	N/A	0.065	0.035	0.004	0.	0.	0.77	1.092

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	0	0	124	288
normalized size	1	1.	0.84	0.69	0.	0.	0.57	1.32
time (sec)	N/A	0.171	0.082	0.003	0.	0.	0.759	1.081

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	168	142	0	0	155	343
normalized size	1	1.	1.53	1.29	0.	0.	1.41	3.12
time (sec)	N/A	0.082	0.16	0.004	0.	0.	1.276	1.091

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	0	0	155	321
normalized size	1	1.	0.93	0.78	0.	0.	0.64	1.33
time (sec)	N/A	0.202	0.204	0.004	0.	0.	1.258	1.078

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	193	180	0	0	194	367
normalized size	1	1.	1.42	1.32	0.	0.	1.43	2.7
time (sec)	N/A	0.11	0.146	0.006	0.	0.	2.194	1.089

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	0	0	192	346
normalized size	1	1.	0.94	0.83	0.	0.	0.72	1.3
time (sec)	N/A	0.23	0.207	0.006	0.	0.	2.155	1.083

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	217	177	0	0	231	400
normalized size	1	1.	1.34	1.09	0.	0.	1.43	2.47
time (sec)	N/A	0.13	0.178	0.013	0.	0.	6.646	1.073

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	274	225	0	0	231	378
normalized size	1	1.	0.94	0.77	0.	0.	0.79	1.3
time (sec)	N/A	0.268	0.284	0.014	0.	0.	6.626	1.08

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	42	44	47	119	313	50
normalized size	1	1.	1.75	1.83	1.96	4.96	13.04	2.08
time (sec)	N/A	0.018	0.015	0.004	1.442	1.293	0.459	1.053

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	68	116	0	83	116
normalized size	1	1.	1.01	0.69	1.18	0.	0.85	1.18
time (sec)	N/A	0.067	0.067	0.004	1.536	0.	0.411	1.064

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	187	161	0	0	471	396
normalized size	1	1.	1.61	1.39	0.	0.	4.06	3.41
time (sec)	N/A	0.095	0.051	0.002	0.	0.	5.494	1.083

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	229	280	0	0	466	371
normalized size	1	1.	0.83	1.01	0.	0.	1.68	1.34
time (sec)	N/A	0.197	0.1	0.003	0.	0.	5.469	1.087

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	211	228	0	0	507	440
normalized size	1	1.	1.45	1.56	0.	0.	3.47	3.01
time (sec)	N/A	0.128	0.201	0.003	0.	0.	7.278	1.086

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	305	344	0	0	505	413
normalized size	1	1.	0.99	1.12	0.	0.	1.64	1.34
time (sec)	N/A	0.255	0.33	0.004	0.	0.	7.284	1.095

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	244	286	0	0	563	481
normalized size	1	1.	1.36	1.6	0.	0.	3.15	2.69
time (sec)	N/A	0.167	0.213	0.005	0.	0.	10.206	1.101

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	337	396	0	0	558	454
normalized size	1	1.	0.99	1.16	0.	0.	1.64	1.33
time (sec)	N/A	0.311	0.307	0.006	0.	0.	10.127	1.098

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	276	274	0	0	612	531
normalized size	1	1.	1.31	1.3	0.	0.	2.9	2.52
time (sec)	N/A	0.211	0.258	0.013	0.	0.	16.479	1.098

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	369	394	0	0	610	504
normalized size	1	1.	0.99	1.06	0.	0.	1.64	1.35
time (sec)	N/A	0.379	0.394	0.013	0.	0.	16.342	1.089

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	24	34	55	27	34
normalized size	1	1.	0.96	0.86	1.21	1.96	0.96	1.21
time (sec)	N/A	0.011	0.002	0.04	0.956	1.056	0.06	1.069

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	27	36	66	29	36
normalized size	1	1.	0.97	0.82	1.09	2.	0.88	1.09
time (sec)	N/A	0.014	0.001	0.039	1.055	1.065	0.061	1.044

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	123	58	68
normalized size	1	1.	1.	0.85	1.13	2.05	0.97	1.13
time (sec)	N/A	0.062	0.002	0.039	0.982	1.	0.066	1.055

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	36	66	31	36
normalized size	1	1.	1.	0.82	1.09	2.	0.94	1.09
time (sec)	N/A	0.013	0.001	0.041	0.986	1.053	0.059	1.082

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	123	60	68
normalized size	1	1.	1.	0.85	1.13	2.05	1.	1.13
time (sec)	N/A	0.027	0.002	0.041	0.975	1.056	0.069	1.047

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	72	134	61	72
normalized size	1	1.	1.	0.83	1.11	2.06	0.94	1.11
time (sec)	N/A	0.098	0.003	0.04	1.01	1.087	0.068	1.057

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	103	190	90	103
normalized size	1	1.	1.	0.84	1.12	2.07	0.98	1.12
time (sec)	N/A	0.053	0.003	0.041	0.981	1.123	0.07	1.059

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	27	20	66	29	22
normalized size	1	1.	1.94	1.59	1.18	3.88	1.71	1.29
time (sec)	N/A	0.005	0.001	0.039	1.014	1.055	0.059	1.061

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	68	123	58	68
normalized size	1	1.	1.33	1.13	1.51	2.73	1.29	1.51
time (sec)	N/A	0.019	0.002	0.038	0.993	1.044	0.065	1.074

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	72	134	60	72
normalized size	1	1.	1.3	1.08	1.44	2.68	1.2	1.44
time (sec)	N/A	0.02	0.003	0.042	1.002	1.025	0.066	1.054

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	190	88	103
normalized size	1	1.	1.19	1.	1.34	2.47	1.14	1.34
time (sec)	N/A	0.051	0.003	0.04	1.004	1.096	0.068	1.058

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	72	134	61	72
normalized size	1	1.	1.3	1.08	1.44	2.68	1.22	1.44
time (sec)	N/A	0.023	0.003	0.039	1.044	1.089	0.066	1.056

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	190	90	103
normalized size	1	1.	1.19	1.	1.34	2.47	1.17	1.34
time (sec)	N/A	0.042	0.004	0.039	0.942	0.986	0.068	1.051

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	107	201	92	107
normalized size	1	1.	1.18	0.98	1.3	2.45	1.12	1.3
time (sec)	N/A	0.084	0.004	0.04	0.942	1.071	0.071	1.06

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	138	258	121	142
normalized size	1	1.	1.14	0.94	1.27	2.37	1.11	1.3
time (sec)	N/A	0.08	0.006	0.	0.933	1.154	0.073	1.059

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	203	375	180	208
normalized size	1	1.	1.19	1.	1.34	2.48	1.19	1.38
time (sec)	N/A	0.106	0.004	0.001	0.95	0.924	0.085	1.054

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	220	248	0	0	518	452
normalized size	1	1.	1.42	1.6	0.	0.	3.34	2.92
time (sec)	N/A	0.118	0.139	0.003	0.	0.	15.127	1.084

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	253	326	0	0	583	505
normalized size	1	1.	1.35	1.73	0.	0.	3.1	2.69
time (sec)	N/A	0.155	0.219	0.006	0.	0.	66.184	1.084

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	286	280	0	0	0	555
normalized size	1	1.	1.3	1.27	0.	0.	0.	2.52
time (sec)	N/A	0.189	0.308	0.012	0.	0.	0.	1.087

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	114	166	882	88	131
normalized size	1	1.	0.77	1.13	1.64	8.73	0.87	1.3
time (sec)	N/A	0.096	0.029	0.043	1.477	1.666	0.359	1.111

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	20	54	19	20
normalized size	1	1.	1.	0.73	0.91	2.45	0.86	0.91
time (sec)	N/A	0.013	0.009	0.04	1.424	1.498	0.09	1.082

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	129	198	0	88	155
normalized size	1	1.	0.87	1.05	1.61	0.	0.72	1.26
time (sec)	N/A	0.103	0.049	0.003	1.486	0.	0.454	1.098

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	78	114	166	869	88	131
normalized size	1	1.	0.77	1.13	1.64	8.6	0.87	1.3
time (sec)	N/A	0.077	0.015	0.041	1.455	1.444	0.362	1.084

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	113	226	225	5063	68	177
normalized size	1	1.	0.8	1.6	1.6	35.91	0.48	1.26
time (sec)	N/A	0.098	0.051	0.002	1.456	1.62	0.365	1.108

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	99	129	198	0	85	154
normalized size	1	1.	0.8	1.05	1.61	0.	0.69	1.25
time (sec)	N/A	0.118	0.044	0.022	1.46	0.	0.459	1.11

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	129	241	252	0	292	193
normalized size	1	1.	0.79	1.48	1.55	0.	1.79	1.18
time (sec)	N/A	0.124	0.071	0.003	1.447	0.	3.075	1.113

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	31	10	15
normalized size	1	1.	1.	0.92	1.15	2.38	0.77	1.15
time (sec)	N/A	0.004	0.003	0.039	0.943	1.392	0.076	1.075

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	125	201	1041	51	147
normalized size	1	1.	0.95	1.1	1.76	9.13	0.45	1.29
time (sec)	N/A	0.099	0.031	0.043	1.452	1.61	0.3	1.131

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	65	28	153	86	53	126
normalized size	1	1.	1.81	0.78	4.25	2.39	1.47	3.5
time (sec)	N/A	0.031	0.03	0.042	1.47	1.476	0.306	1.088

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	140	231	0	199	169
normalized size	1	1.	0.94	1.03	1.7	0.	1.46	1.24
time (sec)	N/A	0.118	0.058	0.041	1.455	0.	1.189	1.109

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	125	205	743	70	147
normalized size	1	1.	0.95	1.1	1.8	6.52	0.61	1.29
time (sec)	N/A	0.121	0.028	0.001	1.45	1.647	0.273	1.109

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	148	237	263	5162	148	185
normalized size	1	1.	0.96	1.54	1.71	33.52	0.96	1.2
time (sec)	N/A	0.117	0.092	0.041	1.458	1.951	1.078	1.105

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	140	235	0	189	167
normalized size	1	1.	0.92	1.03	1.73	0.	1.39	1.23
time (sec)	N/A	0.14	0.063	0.046	1.454	0.	1.171	1.103

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	164	252	279	0	580	201
normalized size	1	1.	0.93	1.43	1.59	0.	3.3	1.14
time (sec)	N/A	0.144	0.105	0.003	1.481	0.	6.537	1.122

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.008	0.001	0.	0.954	1.487	0.056	1.05

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	102	103	440	73	95
normalized size	1	1.	0.94	1.92	1.94	8.3	1.38	1.79
time (sec)	N/A	0.042	0.035	0.003	1.437	1.561	0.324	1.062

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	203	171	0	0	187	392
normalized size	1	1.	1.64	1.38	0.	0.	1.51	3.16
time (sec)	N/A	0.087	0.05	0.004	0.	0.	1.029	1.09

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	283	286	0	0	187	365
normalized size	1	1.	1.02	1.03	0.	0.	0.68	1.32
time (sec)	N/A	0.196	0.188	0.003	0.	0.	0.997	1.063

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	249	244	0	0	2394	539
normalized size	1	1.	1.68	1.65	0.	0.	16.18	3.64
time (sec)	N/A	0.203	0.081	0.005	0.	0.	156.777	1.088

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	221	289	0	0	1406	528
normalized size	1	1.	1.28	1.68	0.	0.	8.17	3.07
time (sec)	N/A	0.165	0.314	0.008	0.	0.	169.346	1.085

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	263	328	0	0	0	595
normalized size	1	1.	1.19	1.48	0.	0.	0.	2.69
time (sec)	N/A	0.263	0.492	0.011	0.	0.	0.	1.082

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	313	368	0	0	0	662
normalized size	1	1.	1.18	1.38	0.	0.	0.	2.49
time (sec)	N/A	0.32	0.324	0.012	0.	0.	0.	1.099

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	311	429	0	0	2392	459
normalized size	1	1.	0.97	1.34	0.	0.	7.5	1.44
time (sec)	N/A	0.351	0.318	0.005	0.	0.	158.354	1.104

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	319	482	0	0	1406	493
normalized size	1	1.	0.94	1.41	0.	0.	4.12	1.45
time (sec)	N/A	0.305	0.196	0.009	0.	0.	170.756	1.079

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	366	519	0	0	0	562
normalized size	1	1.	0.93	1.32	0.	0.	0.	1.43
time (sec)	N/A	0.439	0.278	0.01	0.	0.	0.	1.098

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	411	560	0	0	0	629
normalized size	1	1.	0.94	1.28	0.	0.	0.	1.44
time (sec)	N/A	0.531	0.366	0.014	0.	0.	0.	1.093

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	20	41	15	20
normalized size	1	1.	0.82	0.73	1.82	3.73	1.36	1.82
time (sec)	N/A	0.013	0.001	0.002	0.964	1.728	0.076	1.103

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	16	26	8	16
normalized size	1	1.	1.27	0.73	1.45	2.36	0.73	1.45
time (sec)	N/A	0.012	0.001	0.001	0.94	1.682	0.066	1.065

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	19	5	9
normalized size	1	1.	1.	0.89	1.	2.11	0.56	1.
time (sec)	N/A	0.009	0.	0.001	0.936	1.671	0.057	1.05

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.007	0.001	0.	0.943	1.702	0.057	1.053

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	16	5	9
normalized size	1	1.	1.	1.14	1.29	2.29	0.71	1.29
time (sec)	N/A	0.016	0.001	0.001	0.96	1.664	0.08	1.059

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	16	28	10	9
normalized size	1	1.	0.82	0.73	1.45	2.55	0.91	0.82
time (sec)	N/A	0.018	0.001	0.002	0.937	1.661	0.097	1.068

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	23	41	17	9
normalized size	1	1.	0.82	0.73	2.09	3.73	1.55	0.82
time (sec)	N/A	0.019	0.001	0.	0.942	1.695	0.103	1.053

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	256	296	0	0	0	539
normalized size	1	1.	1.55	1.79	0.	0.	0.	3.27
time (sec)	N/A	0.261	0.257	0.045	0.	0.	0.	1.084

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	301	367	0	0	0	807
normalized size	1	1.	1.6	1.95	0.	0.	0.	4.29
time (sec)	N/A	0.327	0.376	0.045	0.	0.	0.	1.09

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	318	393	0	0	0	828
normalized size	1	1.	1.55	1.92	0.	0.	0.	4.04
time (sec)	N/A	0.313	0.351	0.045	0.	0.	0.	1.102

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	342	462	0	0	0	506
normalized size	1	1.	1.01	1.37	0.	0.	0.	1.5
time (sec)	N/A	0.399	0.325	0.003	0.	0.	0.	1.083

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	427	603	0	0	0	759
normalized size	1	1.	1.11	1.57	0.	0.	0.	1.98
time (sec)	N/A	0.565	0.307	0.004	0.	0.	0.	1.097

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	445	627	0	0	0	780
normalized size	1	1.	1.11	1.56	0.	0.	0.	1.94
time (sec)	N/A	0.567	0.309	0.006	0.	0.	0.	1.097

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	257	340	0	0	0	574
normalized size	1	1.	1.4	1.85	0.	0.	0.	3.12
time (sec)	N/A	0.204	0.163	0.008	0.	0.	0.	1.076

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	302	409	0	0	0	848
normalized size	1	1.	1.49	2.01	0.	0.	0.	4.18
time (sec)	N/A	0.274	0.222	0.009	0.	0.	0.	1.093

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	338	431	0	0	0	884
normalized size	1	1.	1.5	1.92	0.	0.	0.	3.93
time (sec)	N/A	0.31	0.24	0.011	0.	0.	0.	1.12

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	359	515	0	0	0	537
normalized size	1	1.	1.02	1.46	0.	0.	0.	1.52
time (sec)	N/A	0.34	0.242	0.009	0.	0.	0.	1.107

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	415	654	0	0	0	795
normalized size	1	1.	1.05	1.66	0.	0.	0.	2.01
time (sec)	N/A	0.492	0.344	0.01	0.	0.	0.	1.096

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	460	675	0	0	0	833
normalized size	1	1.	1.1	1.62	0.	0.	0.	2.
time (sec)	N/A	0.536	0.328	0.011	0.	0.	0.	1.112

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	309	389	0	0	0	656
normalized size	1	1.	1.28	1.61	0.	0.	0.	2.72
time (sec)	N/A	0.339	0.287	0.011	0.	0.	0.	1.129

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	359	472	0	0	0	942
normalized size	1	1.	1.34	1.76	0.	0.	0.	3.51
time (sec)	N/A	0.435	0.352	0.011	0.	0.	0.	1.12

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	380	488	0	0	0	986
normalized size	1	1.	1.33	1.71	0.	0.	0.	3.46
time (sec)	N/A	0.391	0.277	0.012	0.	0.	0.	1.124

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	411	561	0	0	0	620
normalized size	1	1.	1.	1.36	0.	0.	0.	1.5
time (sec)	N/A	0.486	0.364	0.012	0.	0.	0.	1.108

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	473	716	0	0	0	892
normalized size	1	1.	1.02	1.55	0.	0.	0.	1.93
time (sec)	N/A	0.686	0.531	0.011	0.	0.	0.	1.124

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	480	500	731	0	0	0	936
normalized size	1	1.	1.04	1.52	0.	0.	0.	1.95
time (sec)	N/A	0.666	0.429	0.013	0.	0.	0.	1.117

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	360	434	0	0	0	738
normalized size	1	1.	1.23	1.48	0.	0.	0.	2.52
time (sec)	N/A	0.431	0.395	0.013	0.	0.	0.	1.084

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	422	522	0	0	0	1044
normalized size	1	1.	1.27	1.58	0.	0.	0.	3.15
time (sec)	N/A	0.567	0.368	0.015	0.	0.	0.	1.103

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	439	538	0	0	0	1087
normalized size	1	1.	1.26	1.54	0.	0.	0.	3.11
time (sec)	N/A	0.524	0.368	0.015	0.	0.	0.	1.112

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	461	607	0	0	0	703
normalized size	1	1.	1.	1.31	0.	0.	0.	1.52
time (sec)	N/A	0.619	0.462	0.014	0.	0.	0.	1.093

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	516	530	767	0	0	0	992
normalized size	1	1.	1.03	1.49	0.	0.	0.	1.92
time (sec)	N/A	0.85	0.703	0.014	0.	0.	0.	1.107

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	555	783	0	0	0	1035
normalized size	1	1.	1.04	1.47	0.	0.	0.	1.94
time (sec)	N/A	0.824	0.553	0.014	0.	0.	0.	1.109

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	0	61	0
normalized size	1	1.	0.65	0.79	0.	0.	0.5	0.
time (sec)	N/A	0.068	0.047	0.073	0.	0.	1.84	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	0	0	97	0
normalized size	1	1.	0.93	1.03	0.	0.	1.11	0.
time (sec)	N/A	0.062	0.047	0.015	0.	0.	1.962	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	95	0	0	92	0
normalized size	1	1.	0.93	1.07	0.	0.	1.03	0.
time (sec)	N/A	0.06	0.042	0.019	0.	0.	1.965	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	85	101	0	0	66	0
normalized size	1	1.	0.67	0.8	0.	0.	0.52	0.
time (sec)	N/A	0.072	0.045	0.018	0.	0.	1.965	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	131	193	0	0	102	0
normalized size	1	1.	0.51	0.75	0.	0.	0.4	0.
time (sec)	N/A	0.124	0.103	0.005	0.	0.	2.157	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	28	80	16
normalized size	1	1.	1.	0.93	1.14	2.	5.71	1.14
time (sec)	N/A	0.006	0.009	0.043	1.059	1.387	7.557	1.078

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	34	74	104	43
normalized size	1	1.	0.93	0.83	1.17	2.55	3.59	1.48
time (sec)	N/A	0.023	0.037	0.042	1.073	1.35	11.095	1.074

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	27	24	31	69	109	30
normalized size	1	1.	1.08	0.96	1.24	2.76	4.36	1.2
time (sec)	N/A	0.028	0.037	0.043	1.082	1.505	10.461	1.085

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	59	93	133	42
normalized size	1	1.	1.	0.92	1.55	2.45	3.5	1.11
time (sec)	N/A	0.03	0.042	0.042	1.078	1.302	13.776	1.097

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	24	58	14
normalized size	1	1.	1.	0.92	1.17	2.	4.83	1.17
time (sec)	N/A	0.003	0.005	0.004	1.544	1.309	2.615	1.067

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	281	516	0	0	260	0
normalized size	1	1.	0.73	1.34	0.	0.	0.68	0.
time (sec)	N/A	0.424	0.224	0.017	0.	0.	5.734	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	51	173	0	2955	36	136
normalized size	1	1.	0.47	1.59	0.	27.11	0.33	1.25
time (sec)	N/A	0.064	0.01	0.019	0.	8.737	0.372	1.107

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	47	169	0	2493	36	136
normalized size	1	1.	0.43	1.55	0.	22.87	0.33	1.25
time (sec)	N/A	0.041	0.009	0.017	0.	8.558	0.373	1.093

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	187	266	282	429	192	332
normalized size	1	1.	0.9	1.28	1.36	2.06	0.92	1.6
time (sec)	N/A	0.316	0.09	0.005	0.956	1.273	1.167	1.057

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	218	228	351	155	266
normalized size	1	1.	0.91	1.28	1.34	2.06	0.91	1.56
time (sec)	N/A	0.242	0.066	0.004	0.955	1.234	1.137	1.064

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	170	174	267	117	200
normalized size	1	1.	0.9	1.29	1.32	2.02	0.89	1.52
time (sec)	N/A	0.183	0.053	0.003	0.953	1.273	1.082	1.069

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	88	124	123	190	83	136
normalized size	1	1.	0.92	1.29	1.28	1.98	0.86	1.42
time (sec)	N/A	0.14	0.043	0.003	0.949	1.195	1.007	1.069

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	97	104	171	68	107
normalized size	1	1.	0.94	1.21	1.3	2.14	0.85	1.34
time (sec)	N/A	0.12	0.031	0.005	0.958	1.588	4.913	1.058

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	94	104	180	70	128
normalized size	1	1.	0.95	1.16	1.28	2.22	0.86	1.58
time (sec)	N/A	0.116	0.043	0.007	0.966	1.503	11.439	1.059

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	116	126	213	85	170
normalized size	1	1.	0.93	1.22	1.33	2.24	0.89	1.79
time (sec)	N/A	0.129	0.069	0.007	0.948	1.527	51.496	1.055

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	128	161	169	269	0	248
normalized size	1	1.	1.	1.26	1.32	2.1	0.	1.94
time (sec)	N/A	0.162	0.064	0.007	0.953	1.471	0.	1.067

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	164	210	224	355	0	317
normalized size	1	1.	1.	1.28	1.37	2.16	0.	1.93
time (sec)	N/A	0.181	0.062	0.009	0.955	1.513	0.	1.068

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	194	260	281	441	0	387
normalized size	1	1.	0.95	1.27	1.37	2.15	0.	1.89
time (sec)	N/A	0.209	0.144	0.01	0.959	1.667	0.	1.066

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	351	592	0	767	450	613
normalized size	1	1.	1.01	1.7	0.	2.2	1.29	1.76
time (sec)	N/A	0.333	0.079	0.003	0.	1.398	1.874	1.062

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	311	554	0	726	496	595
normalized size	1	1.	0.98	1.75	0.	2.3	1.57	1.88
time (sec)	N/A	0.306	0.098	0.003	0.	1.444	1.451	1.082

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	306	544	0	679	411	541
normalized size	1	1.	0.98	1.74	0.	2.18	1.32	1.73
time (sec)	N/A	0.298	0.093	0.004	0.	1.376	1.822	1.072

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	266	502	0	635	459	521
normalized size	1	1.	0.95	1.8	0.	2.28	1.65	1.87
time (sec)	N/A	0.273	0.092	0.004	0.	1.407	1.383	1.075

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	264	492	0	566	371	467
normalized size	1	1.	0.96	1.8	0.	2.07	1.35	1.7
time (sec)	N/A	0.267	0.091	0.003	0.	1.297	1.719	1.089

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	231	450	0	1289	422	466
normalized size	1	1.	0.94	1.84	0.	5.26	1.72	1.9
time (sec)	N/A	0.216	0.16	0.003	0.	1.519	1.473	1.084

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	442	0	1350	340	414
normalized size	1	1.	0.95	1.84	0.	5.62	1.42	1.72
time (sec)	N/A	0.153	0.147	0.004	0.	1.361	1.89	1.075

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	224	419	0	1260	406	428
normalized size	1	1.	0.99	1.85	0.	5.55	1.79	1.89
time (sec)	N/A	0.193	0.128	0.004	0.	1.444	2.515	1.095

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	218	414	0	1280	326	378
normalized size	1	1.	0.97	1.85	0.	5.71	1.46	1.69
time (sec)	N/A	0.17	0.117	0.005	0.	1.537	3.058	1.076

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	220	412	0	1251	411	417
normalized size	1	1.	0.97	1.81	0.	5.51	1.81	1.84
time (sec)	N/A	0.186	0.111	0.007	0.	1.386	8.047	1.081

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	220	410	0	1312	328	370
normalized size	1	1.	0.98	1.82	0.	5.83	1.46	1.64
time (sec)	N/A	0.17	0.082	0.005	0.	1.308	9.841	1.08

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	231	440	0	1359	432	444
normalized size	1	1.	0.95	1.82	0.	5.62	1.79	1.83
time (sec)	N/A	0.188	0.111	0.007	0.	1.435	30.419	1.075

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	231	441	0	1353	348	401
normalized size	1	1.	0.95	1.81	0.	5.55	1.43	1.64
time (sec)	N/A	0.174	0.112	0.008	0.	1.378	52.712	1.074

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	266	491	0	597	473	508
normalized size	1	1.	0.96	1.77	0.	2.16	1.71	1.83
time (sec)	N/A	0.222	0.112	0.009	0.	1.41	142.622	1.085

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	266	493	0	667	0	456
normalized size	1	1.	0.95	1.76	0.	2.38	0.	1.63
time (sec)	N/A	0.199	0.115	0.008	0.	1.396	0.	1.072

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	308	546	0	711	0	566
normalized size	1	1.	0.98	1.74	0.	2.27	0.	1.81
time (sec)	N/A	0.238	0.099	0.009	0.	1.343	0.	1.078

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	311	548	0	759	0	531
normalized size	1	1.	0.99	1.74	0.	2.41	0.	1.69
time (sec)	N/A	0.229	0.097	0.009	0.	1.384	0.	1.077

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	346	600	0	798	0	640
normalized size	1	1.	0.99	1.71	0.	2.27	0.	1.82
time (sec)	N/A	0.258	0.11	0.01	0.	1.272	0.	1.104

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	205	288	300	656	224	405
normalized size	1	1.	0.93	1.31	1.36	2.98	1.02	1.84
time (sec)	N/A	0.341	0.139	0.012	0.958	1.244	11.509	1.066

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	167	240	243	555	180	335
normalized size	1	1.	0.93	1.33	1.35	3.08	1.	1.86
time (sec)	N/A	0.265	0.111	0.013	0.965	1.259	11.093	1.058

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	192	186	432	138	293
normalized size	1	1.	0.92	1.37	1.33	3.09	0.99	2.09
time (sec)	N/A	0.199	0.098	0.01	0.959	1.305	10.644	1.064

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	142	132	305	97	278
normalized size	1	1.	0.9	1.38	1.28	2.96	0.94	2.7
time (sec)	N/A	0.145	0.058	0.01	0.959	1.135	9.22	1.058

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	125	135	293	95	169
normalized size	1	1.	0.95	1.25	1.35	2.93	0.95	1.69
time (sec)	N/A	0.125	0.122	0.011	0.956	1.435	33.825	1.073

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	132	157	340	0	177
normalized size	1	1.	0.89	1.21	1.44	3.12	0.	1.62
time (sec)	N/A	0.142	0.077	0.014	0.955	1.494	0.	1.059

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	118	167	186	431	0	271
normalized size	1	1.	0.91	1.28	1.43	3.32	0.	2.08
time (sec)	N/A	0.154	0.115	0.015	0.979	1.428	0.	1.07

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	160	229	244	549	0	371
normalized size	1	1.	0.91	1.31	1.39	3.14	0.	2.12
time (sec)	N/A	0.203	0.105	0.018	1.049	1.62	0.	1.067

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	198	282	305	670	0	447
normalized size	1	1.	0.93	1.32	1.43	3.13	0.	2.09
time (sec)	N/A	0.234	0.203	0.02	1.017	1.608	0.	1.074

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	364	622	0	1145	490	609
normalized size	1	1.	0.99	1.69	0.	3.1	1.33	1.65
time (sec)	N/A	0.467	0.353	0.01	0.	1.413	10.508	1.078

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	319	584	0	1057	530	597
normalized size	1	1.	0.95	1.74	0.	3.16	1.58	1.78
time (sec)	N/A	0.705	0.166	0.01	0.	1.298	37.454	1.087

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	315	567	0	992	440	532
normalized size	1	1.	0.96	1.73	0.	3.02	1.34	1.62
time (sec)	N/A	0.369	0.248	0.01	0.	1.366	10.915	1.08

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	282	529	0	2094	484	537
normalized size	1	1.	0.95	1.78	0.	7.03	1.62	1.8
time (sec)	N/A	0.463	0.157	0.009	0.	1.705	35.782	1.084

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	277	514	0	2125	398	471
normalized size	1	1.	0.96	1.78	0.	7.38	1.38	1.64
time (sec)	N/A	0.326	0.157	0.01	0.	1.447	10.133	1.1

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	255	495	0	1935	461	494
normalized size	1	1.	0.94	1.83	0.	7.14	1.7	1.82
time (sec)	N/A	0.289	0.153	0.009	0.	1.482	16.851	1.076

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	251	482	0	1917	376	433
normalized size	1	1.	0.95	1.83	0.	7.26	1.42	1.64
time (sec)	N/A	0.264	0.145	0.009	0.	1.524	6.359	1.072

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	255	474	0	1877	457	477
normalized size	1	1.	0.96	1.79	0.	7.08	1.72	1.8
time (sec)	N/A	0.253	0.153	0.012	0.	1.48	26.608	1.076

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	250	463	0	1952	381	417
normalized size	1	1.	0.96	1.78	0.	7.51	1.47	1.6
time (sec)	N/A	0.246	0.152	0.011	0.	1.463	58.686	1.081

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	255	486	0	1968	473	483
normalized size	1	1.	0.95	1.81	0.	7.32	1.76	1.8
time (sec)	N/A	0.288	0.168	0.013	0.	1.525	138.391	1.082

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	253	477	0	1991	0	429
normalized size	1	1.	0.94	1.77	0.	7.37	0.	1.59
time (sec)	N/A	0.272	0.162	0.011	0.	1.559	0.	1.081

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	281	529	0	2159	0	522
normalized size	1	1.	0.95	1.78	0.	7.27	0.	1.76
time (sec)	N/A	0.384	0.179	0.015	0.	1.465	0.	1.117

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	280	520	0	2182	0	468
normalized size	1	1.	0.94	1.75	0.	7.35	0.	1.58
time (sec)	N/A	0.37	0.178	0.014	0.	1.555	0.	1.087

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	319	575	0	1034	0	590
normalized size	1	1.	0.96	1.72	0.	3.1	0.	1.77
time (sec)	N/A	0.457	0.181	0.016	0.	1.372	0.	1.101

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	317	566	0	1099	0	528
normalized size	1	1.	0.95	1.69	0.	3.28	0.	1.58
time (sec)	N/A	0.434	0.187	0.017	0.	1.384	0.	1.086

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	370	631	0	1188	0	651
normalized size	1	1.	0.99	1.68	0.	3.17	0.	1.74
time (sec)	N/A	0.534	0.36	0.016	0.	1.361	0.	1.081

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	246	361	371	883	0	471
normalized size	1	1.	0.92	1.36	1.39	3.32	0.	1.77
time (sec)	N/A	0.436	0.143	0.015	0.954	1.234	0.	1.078

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	208	313	315	765	0	402
normalized size	1	1.	0.92	1.38	1.39	3.38	0.	1.78
time (sec)	N/A	0.331	0.12	0.013	0.963	1.346	0.	1.097

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	170	266	258	635	0	319
normalized size	1	1.	0.91	1.43	1.39	3.41	0.	1.72
time (sec)	N/A	0.269	0.099	0.012	0.95	1.294	0.	1.079

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	145	213	198	470	0	197
normalized size	1	1.	0.99	1.46	1.36	3.22	0.	1.35
time (sec)	N/A	0.201	0.074	0.013	0.953	1.26	0.	1.088

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	105	156	147	329	0	135
normalized size	1	1.	0.96	1.43	1.35	3.02	0.	1.24
time (sec)	N/A	0.152	0.054	0.01	0.951	1.165	0.	1.078

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	104	147	174	374	0	173
normalized size	1	1.	0.91	1.29	1.53	3.28	0.	1.52
time (sec)	N/A	0.155	0.106	0.015	0.953	1.368	0.	1.07

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	121	163	194	497	0	234
normalized size	1	1.	0.9	1.22	1.45	3.71	0.	1.75
time (sec)	N/A	0.171	0.091	0.014	0.962	1.4	0.	1.097

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	149	213	246	652	0	255
normalized size	1	1.	0.91	1.31	1.51	4.	0.	1.56
time (sec)	N/A	0.2	0.106	0.017	0.98	1.31	0.	1.073

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	200	293	313	837	0	437
normalized size	1	1.	0.92	1.34	1.44	3.84	0.	2.
time (sec)	N/A	0.264	0.137	0.02	0.981	1.443	0.	1.101

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	238	349	378	984	0	513
normalized size	1	1.	0.92	1.35	1.47	3.81	0.	1.99
time (sec)	N/A	0.304	0.18	0.019	0.988	1.64	0.	1.077

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	411	706	0	1589	0	675
normalized size	1	1.	0.99	1.7	0.	3.82	0.	1.62
time (sec)	N/A	0.742	0.482	0.013	0.	1.326	0.	1.091

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	380	668	0	1496	0	663
normalized size	1	1.	0.99	1.74	0.	3.9	0.	1.73
time (sec)	N/A	1.05	0.402	0.013	0.	1.352	0.	1.092

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	362	651	0	1424	0	598
normalized size	1	1.	0.97	1.74	0.	3.8	0.	1.59
time (sec)	N/A	0.606	0.374	0.012	0.	1.443	0.	1.088

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	329	611	0	2915	0	601
normalized size	1	1.	0.95	1.77	0.	8.45	0.	1.74
time (sec)	N/A	0.763	0.224	0.013	0.	1.481	0.	1.092

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	323	596	0	2986	0	539
normalized size	1	1.	0.96	1.77	0.	8.89	0.	1.6
time (sec)	N/A	0.509	0.313	0.012	0.	1.461	0.	1.108

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	300	574	0	2723	0	558
normalized size	1	1.	0.95	1.82	0.	8.62	0.	1.77
time (sec)	N/A	0.505	0.206	0.011	0.	1.535	0.	1.093

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	294	561	0	2716	0	495
normalized size	1	1.	0.96	1.83	0.	8.85	0.	1.61
time (sec)	N/A	0.412	0.202	0.011	0.	1.608	0.	1.092

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	284	550	0	2538	0	522
normalized size	1	1.	0.94	1.83	0.	8.43	0.	1.73
time (sec)	N/A	0.368	0.2	0.011	0.	1.493	0.	1.094

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	279	539	0	2587	418	463
normalized size	1	1.	0.96	1.85	0.	8.86	1.43	1.59
time (sec)	N/A	0.307	0.19	0.011	0.	1.499	170.751	1.097

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	286	547	0	2591	0	525
normalized size	1	1.	0.94	1.81	0.	8.55	0.	1.73
time (sec)	N/A	0.339	0.223	0.014	0.	1.505	0.	1.112

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	283	539	0	2618	0	486
normalized size	1	1.	0.94	1.79	0.	8.7	0.	1.61
time (sec)	N/A	0.329	0.213	0.013	0.	1.844	0.	1.084

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	303	574	0	2773	0	547
normalized size	1	1.	0.96	1.81	0.	8.75	0.	1.73
time (sec)	N/A	0.37	0.232	0.013	0.	2.072	0.	1.101

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	299	566	0	2785	0	491
normalized size	1	1.	0.95	1.79	0.	8.81	0.	1.55
time (sec)	N/A	0.368	0.233	0.017	0.	1.83	0.	1.1

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	328	611	0	3031	0	586
normalized size	1	1.	0.96	1.78	0.	8.84	0.	1.71
time (sec)	N/A	0.569	0.238	0.02	0.	1.761	0.	1.099

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	324	603	0	3013	0	532
normalized size	1	1.	0.95	1.77	0.	8.84	0.	1.56
time (sec)	N/A	0.547	0.236	0.017	0.	1.777	0.	1.098

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	366	659	0	1472	0	656
normalized size	1	1.	0.96	1.73	0.	3.86	0.	1.72
time (sec)	N/A	0.714	0.392	0.019	0.	1.619	0.	1.098

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	376	651	0	1543	0	594
normalized size	1	1.	0.99	1.71	0.	4.06	0.	1.56
time (sec)	N/A	0.669	0.429	0.018	0.	1.625	0.	1.08

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	419	716	0	1638	0	717
normalized size	1	1.	0.99	1.69	0.	3.86	0.	1.69
time (sec)	N/A	0.853	0.466	0.022	0.	1.686	0.	1.105

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	45	59	140	53	61
normalized size	1	1.	1.09	0.83	1.09	2.59	0.98	1.13
time (sec)	N/A	0.072	0.014	0.005	1.415	1.555	0.129	1.05

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	73	24	34
normalized size	1	1.	1.	0.83	1.07	2.43	0.8	1.13
time (sec)	N/A	0.04	0.004	0.004	1.423	1.515	0.09	1.075

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	38	50	117	44	51
normalized size	1	1.	1.2	0.86	1.14	2.66	1.	1.16
time (sec)	N/A	0.06	0.009	0.003	1.412	1.499	0.123	1.061

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	50	35	46	112	42	47
normalized size	1	1.	1.22	0.85	1.12	2.73	1.02	1.15
time (sec)	N/A	0.042	0.007	0.004	1.412	1.546	0.131	1.058

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	37	49	126	46	51
normalized size	1	1.	1.26	0.88	1.17	3.	1.1	1.21
time (sec)	N/A	0.049	0.008	0.006	1.423	1.493	0.175	1.088

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	60	44	58	144	49	61
normalized size	1	1.	1.22	0.9	1.18	2.94	1.	1.24
time (sec)	N/A	0.05	0.015	0.008	1.472	1.534	0.195	1.065

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	38	85	27	39
normalized size	1	1.	1.	0.84	1.19	2.66	0.84	1.22
time (sec)	N/A	0.033	0.004	0.007	1.401	1.423	0.109	1.047

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	47	35	46	112	42	47
normalized size	1	1.	1.15	0.85	1.12	2.73	1.02	1.15
time (sec)	N/A	0.041	0.007	0.004	1.407	1.548	0.138	1.068

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	53	33	43	108	41	45
normalized size	1	1.	1.36	0.85	1.1	2.77	1.05	1.15
time (sec)	N/A	0.04	0.01	0.005	1.408	1.501	0.128	1.056

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	58	112	49	61
normalized size	1	1.	1.	0.8	1.05	2.04	0.89	1.11
time (sec)	N/A	0.055	0.003	0.002	0.928	1.295	0.064	1.064

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	58	112	49	61
normalized size	1	1.	1.	0.8	1.05	2.04	0.89	1.11
time (sec)	N/A	0.038	0.002	0.001	0.946	1.284	0.064	1.067

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	54	104	46	57
normalized size	1	1.	1.	0.82	1.08	2.08	0.92	1.14
time (sec)	N/A	0.025	0.002	0.002	0.95	1.283	0.065	1.049

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	51	103	44	55
normalized size	1	1.	1.	0.85	1.11	2.24	0.96	1.2
time (sec)	N/A	0.025	0.004	0.003	0.949	1.491	0.272	1.083

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	51	113	41	55
normalized size	1	1.	1.	0.89	1.16	2.57	0.93	1.25
time (sec)	N/A	0.035	0.005	0.004	0.943	1.439	0.296	1.067

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	51	111	42	55
normalized size	1	1.	1.	0.89	1.16	2.52	0.95	1.25
time (sec)	N/A	0.034	0.004	0.005	0.937	1.504	0.366	1.062

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	107	198	92	111
normalized size	1	1.	1.18	0.98	1.3	2.41	1.12	1.35
time (sec)	N/A	0.059	0.004	0.002	0.939	1.221	0.074	1.08

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	107	196	94	111
normalized size	1	1.	1.18	0.98	1.3	2.39	1.15	1.35
time (sec)	N/A	0.051	0.003	0.	0.933	1.316	0.071	1.06

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	103	185	88	107
normalized size	1	1.	1.19	1.	1.34	2.4	1.14	1.39
time (sec)	N/A	0.061	0.003	0.001	0.939	1.362	0.098	1.063

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	184	88	105
normalized size	1	1.	1.	0.85	1.14	2.09	1.	1.19
time (sec)	N/A	0.052	0.008	0.001	0.949	1.495	0.353	1.051

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	74	99	204	82	104
normalized size	1	1.	1.	0.89	1.19	2.46	0.99	1.25
time (sec)	N/A	0.063	0.008	0.005	0.948	1.534	0.34	1.052

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	75	100	198	85	105
normalized size	1	1.	1.	0.89	1.19	2.36	1.01	1.25
time (sec)	N/A	0.064	0.008	0.006	0.943	1.466	0.395	1.053

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	155	288	138	161
normalized size	1	1.	1.26	1.05	1.41	2.62	1.25	1.46
time (sec)	N/A	0.079	0.004	0.001	0.941	1.307	0.075	1.059

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	155	285	138	161
normalized size	1	1.	1.26	1.05	1.41	2.59	1.25	1.46
time (sec)	N/A	0.069	0.004	0.001	0.919	1.268	0.076	1.082

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	134	113	151	274	134	157
normalized size	1	1.	1.28	1.08	1.44	2.61	1.28	1.5
time (sec)	N/A	0.097	0.004	0.001	0.934	1.284	0.075	1.083

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	110	147	265	131	154
normalized size	1	1.	1.	0.87	1.16	2.09	1.03	1.21
time (sec)	N/A	0.074	0.009	0.001	0.953	1.492	0.376	1.073

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	125	110	147	305	128	154
normalized size	1	1.	1.	0.88	1.18	2.44	1.02	1.23
time (sec)	N/A	0.092	0.008	0.006	0.94	1.453	0.399	1.058

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	111	149	308	129	155
normalized size	1	1.	1.	0.88	1.18	2.44	1.02	1.23
time (sec)	N/A	0.086	0.009	0.006	0.936	1.467	0.461	1.055

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	204	373	184	211
normalized size	1	1.	1.31	1.1	1.48	2.7	1.33	1.53
time (sec)	N/A	0.099	0.005	0.001	0.949	1.304	0.087	1.062

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	204	371	185	211
normalized size	1	1.	1.31	1.1	1.48	2.69	1.34	1.53
time (sec)	N/A	0.093	0.004	0.002	0.94	1.301	0.087	1.051

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	173	148	198	355	178	205
normalized size	1	1.	1.33	1.14	1.52	2.73	1.37	1.58
time (sec)	N/A	0.145	0.004	0.001	0.935	1.283	0.084	1.064

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	145	194	347	175	203
normalized size	1	1.	1.	0.87	1.17	2.09	1.05	1.22
time (sec)	N/A	0.109	0.009	0.003	0.936	1.468	0.456	1.061

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	145	194	433	168	203
normalized size	1	1.	1.	0.9	1.2	2.67	1.04	1.25
time (sec)	N/A	0.133	0.009	0.006	0.942	1.49	0.473	1.046

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	147	197	413	173	205
normalized size	1	1.	1.	0.89	1.19	2.49	1.04	1.23
time (sec)	N/A	0.125	0.009	0.006	0.949	1.527	0.558	1.055

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	191	231	0	10954	178	302
normalized size	1	1.	0.93	1.13	0.	53.43	0.87	1.47
time (sec)	N/A	0.261	0.105	0.003	0.	9.928	1.597	1.081

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	184	221	0	9513	150	285
normalized size	1	1.	0.95	1.15	0.	49.29	0.78	1.48
time (sec)	N/A	0.248	0.089	0.003	0.	9.578	1.501	1.078

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	200	209	0	9754	160	257
normalized size	1	1.	1.09	1.14	0.	53.3	0.87	1.4
time (sec)	N/A	0.226	0.056	0.003	0.	9.561	1.653	1.075

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	176	200	0	10437	160	246
normalized size	1	1.	0.99	1.13	0.	58.97	0.9	1.39
time (sec)	N/A	0.132	0.092	0.003	0.	9.515	1.272	1.073

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	176	207	0	10657	0	267
normalized size	1	1.	0.96	1.12	0.	57.92	0.	1.45
time (sec)	N/A	0.206	0.089	0.005	0.	10.685	0.	1.09

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	184	216	0	10066	0	279
normalized size	1	1.	0.96	1.12	0.	52.43	0.	1.45
time (sec)	N/A	0.214	0.239	0.007	0.	9.231	0.	1.083

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	192	225	0	9917	0	285
normalized size	1	1.	0.95	1.11	0.	48.85	0.	1.4
time (sec)	N/A	0.194	0.178	0.005	0.	8.22	0.	1.078

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	174	219	0	4988	109	258
normalized size	1	1.	0.92	1.15	0.	26.25	0.57	1.36
time (sec)	N/A	0.166	0.129	0.009	0.	6.236	2.422	1.074

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	186	228	0	5293	124	273
normalized size	1	1.	0.93	1.14	0.	26.46	0.62	1.36
time (sec)	N/A	0.153	0.174	0.007	0.	6.774	1.667	1.103

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	189	253	0	5011	116	266
normalized size	1	1.	0.95	1.27	0.	25.18	0.58	1.34
time (sec)	N/A	0.132	0.179	0.002	0.	6.246	1.284	1.079

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	199	274	0	12189	0	311
normalized size	1	1.	0.9	1.23	0.	54.91	0.	1.4
time (sec)	N/A	0.313	0.157	0.01	0.	7.164	0.	1.095

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	213	275	0	11634	0	328
normalized size	1	1.	0.92	1.19	0.	50.36	0.	1.42
time (sec)	N/A	0.343	0.262	0.012	0.	7.342	0.	1.1

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	221	276	0	12122	0	344
normalized size	1	1.	0.91	1.14	0.	50.09	0.	1.42
time (sec)	N/A	0.345	0.169	0.013	0.	9.802	0.	1.094

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	225	289	0	12670	0	373
normalized size	1	1.	0.86	1.1	0.	48.36	0.	1.42
time (sec)	N/A	0.404	0.175	0.013	0.	10.179	0.	1.078

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	198	255	0	5122	148	288
normalized size	1	1.	0.92	1.19	0.	23.82	0.69	1.34
time (sec)	N/A	0.197	0.179	0.01	0.	9.744	5.124	1.113

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	214	256	0	5632	170	306
normalized size	1	1.	0.9	1.07	0.	23.56	0.71	1.28
time (sec)	N/A	0.204	0.282	0.01	0.	9.66	3.385	1.103

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	213	308	0	5612	163	301
normalized size	1	1.	0.95	1.37	0.	24.94	0.72	1.34
time (sec)	N/A	0.188	0.286	0.003	0.	8.387	2.117	1.127

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	229	331	0	13407	0	359
normalized size	1	1.	0.89	1.29	0.	52.17	0.	1.4
time (sec)	N/A	0.414	0.178	0.013	0.	10.692	0.	1.116

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	248	334	0	13300	0	377
normalized size	1	1.	0.93	1.25	0.	49.81	0.	1.41
time (sec)	N/A	0.462	0.221	0.017	0.	10.085	0.	1.117

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	253	337	0	13346	0	390
normalized size	1	1.	0.92	1.22	0.	48.36	0.	1.41
time (sec)	N/A	0.5	0.21	0.016	0.	9.171	0.	1.093

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	255	351	0	14256	0	421
normalized size	1	1.	0.86	1.18	0.	47.84	0.	1.41
time (sec)	N/A	0.589	0.37	0.017	0.	12.219	0.	1.085

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	230	275	0	5952	201	333
normalized size	1	1.	0.93	1.11	0.	24.	0.81	1.34
time (sec)	N/A	0.242	0.233	0.01	0.	8.785	13.595	1.075

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	241	278	0	6597	214	346
normalized size	1	1.	0.89	1.03	0.	24.43	0.79	1.28
time (sec)	N/A	0.254	0.346	0.012	0.	8.837	7.049	1.088

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	239	360	0	6020	202	333
normalized size	1	1.	0.96	1.44	0.	24.08	0.81	1.33
time (sec)	N/A	0.222	0.232	0.004	0.	6.224	4.049	1.084

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	259	394	0	15555	0	409
normalized size	1	1.	0.89	1.35	0.	53.45	0.	1.41
time (sec)	N/A	0.517	0.228	0.016	0.	7.407	0.	1.094

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	279	397	0	15244	0	427
normalized size	1	1.	0.93	1.32	0.	50.64	0.	1.42
time (sec)	N/A	0.601	0.292	0.018	0.	7.531	0.	1.073

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	284	400	0	15485	0	441
normalized size	1	1.	0.92	1.29	0.	49.95	0.	1.42
time (sec)	N/A	0.656	0.286	0.019	0.	7.404	0.	1.071

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	284	415	0	15115	0	459
normalized size	1	1.	0.84	1.22	0.	44.46	0.	1.35
time (sec)	N/A	0.773	0.478	0.019	0.	12.297	0.	1.069

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	35	80	54	36
normalized size	1	1.	1.97	1.	1.21	2.76	1.86	1.24
time (sec)	N/A	0.058	0.014	0.007	1.419	0.992	0.287	1.062

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	35	80	54	36
normalized size	1	1.	1.97	1.	1.21	2.76	1.86	1.24
time (sec)	N/A	0.035	0.005	0.006	1.423	1.005	0.286	1.077

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	38	81	54	39
normalized size	1	1.	1.87	0.94	1.23	2.61	1.74	1.26
time (sec)	N/A	0.056	0.013	0.006	1.432	1.024	0.287	1.057

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	38	81	54	39
normalized size	1	1.	1.87	0.94	1.23	2.61	1.74	1.26
time (sec)	N/A	0.037	0.006	0.005	1.417	0.941	0.289	1.057

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	87	0	139	100	235
normalized size	1	1.	2.92	1.74	0.	2.78	2.	4.7
time (sec)	N/A	0.087	0.041	0.004	0.	1.082	0.414	1.111

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	149	135	0	142	110	223
normalized size	1	1.	2.81	2.55	0.	2.68	2.08	4.21
time (sec)	N/A	0.093	0.069	0.006	0.	1.101	0.436	1.094

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	148	132	0	142	109	131
normalized size	1	1.	2.74	2.44	0.	2.63	2.02	2.43
time (sec)	N/A	0.077	0.042	0.003	0.	1.122	0.421	1.068

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	90	0	139	102	122
normalized size	1	1.	2.77	1.7	0.	2.62	1.92	2.3
time (sec)	N/A	0.076	0.053	0.005	0.	1.032	0.431	1.092

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	239	90	117
normalized size	1	1.	1.	0.82	1.1	2.46	0.93	1.21
time (sec)	N/A	0.121	0.028	0.001	0.954	0.844	0.071	1.095

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	234	90	117
normalized size	1	1.	1.	0.82	1.1	2.41	0.93	1.21
time (sec)	N/A	0.101	0.028	0.001	0.936	0.875	0.072	1.066

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	231	90	117
normalized size	1	1.	1.	0.82	1.1	2.38	0.93	1.21
time (sec)	N/A	0.091	0.024	0.001	0.941	0.839	0.071	1.321

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	107	228	90	117
normalized size	1	1.	1.	0.82	1.1	2.35	0.93	1.21
time (sec)	N/A	0.08	0.017	0.002	0.951	0.893	0.071	1.054

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	103	217	87	113
normalized size	1	1.	1.	0.84	1.12	2.36	0.95	1.23
time (sec)	N/A	0.073	0.013	0.	0.933	0.847	0.07	1.067

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	81	100	192	85	112
normalized size	1	1.	1.	0.92	1.14	2.18	0.97	1.27
time (sec)	N/A	0.058	0.029	0.003	0.944	0.993	0.34	1.057

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	100	212	82	112
normalized size	1	1.	1.	0.94	1.16	2.47	0.95	1.3
time (sec)	N/A	0.066	0.041	0.004	0.948	1.02	0.348	1.071

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	78	100	205	82	108
normalized size	1	1.	0.91	0.91	1.16	2.38	0.95	1.26
time (sec)	N/A	0.072	0.06	0.005	0.94	1.113	0.456	1.066

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	76	101	205	82	107
normalized size	1	1.	0.88	0.88	1.17	2.38	0.95	1.24
time (sec)	N/A	0.071	0.061	0.006	0.943	1.246	0.732	1.052

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	76	101	197	82	104
normalized size	1	1.	0.9	0.88	1.17	2.29	0.95	1.21
time (sec)	N/A	0.073	0.061	0.005	1.205	1.235	2.885	1.076

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	204	414	167	216
normalized size	1	1.	1.	0.93	1.25	2.54	1.02	1.33
time (sec)	N/A	0.21	0.04	0.001	0.956	1.117	0.095	1.074

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	204	409	167	216
normalized size	1	1.	1.	0.93	1.25	2.51	1.02	1.33
time (sec)	N/A	0.159	0.028	0.001	0.936	1.084	0.14	1.066

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	150	152	204	405	167	216
normalized size	1	1.	0.95	0.96	1.29	2.56	1.06	1.37
time (sec)	N/A	0.126	0.076	0.001	0.959	0.862	0.1	1.073

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	163	152	204	400	167	216
normalized size	1	1.	1.03	0.96	1.29	2.53	1.06	1.37
time (sec)	N/A	0.129	0.024	0.001	0.947	0.862	0.102	1.052

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	125	149	200	387	163	212
normalized size	1	1.	0.82	0.97	1.31	2.53	1.07	1.39
time (sec)	N/A	0.127	0.074	0.002	0.947	0.9	0.087	1.057

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	154	153	197	351	162	211
normalized size	1	1.	1.03	1.03	1.32	2.36	1.09	1.42
time (sec)	N/A	0.106	0.044	0.003	0.934	0.938	0.473	1.068

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	152	152	197	379	156	209
normalized size	1	1.	1.03	1.03	1.34	2.58	1.06	1.42
time (sec)	N/A	0.128	0.061	0.006	0.958	0.992	0.5	1.055

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	127	150	197	383	156	207
normalized size	1	1.	0.86	1.02	1.34	2.61	1.06	1.41
time (sec)	N/A	0.128	0.071	0.006	0.939	1.	0.551	1.074

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	149	198	375	156	207
normalized size	1	1.	0.81	0.98	1.3	2.47	1.03	1.36
time (sec)	N/A	0.118	0.081	0.005	0.926	0.991	0.873	1.07

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	125	149	198	371	155	205
normalized size	1	1.	0.82	0.98	1.3	2.44	1.02	1.35
time (sec)	N/A	0.117	0.073	0.006	0.948	0.991	2.775	1.064

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	293	591	246	315
normalized size	1	1.	1.	1.	1.31	2.65	1.1	1.41
time (sec)	N/A	0.293	0.051	0.001	0.957	0.873	0.1	1.068

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	224	293	586	246	315
normalized size	1	1.	1.	1.	1.31	2.63	1.1	1.41
time (sec)	N/A	0.228	0.047	0.002	0.932	0.87	0.101	1.077

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	293	583	246	315
normalized size	1	1.	1.05	1.06	1.38	2.75	1.16	1.49
time (sec)	N/A	0.179	0.048	0.001	0.943	0.828	0.099	1.079

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	224	293	578	246	315
normalized size	1	1.	1.05	1.06	1.38	2.73	1.16	1.49
time (sec)	N/A	0.177	0.033	0.	0.955	0.877	0.1	1.078

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	170	221	289	564	243	311
normalized size	1	1.	0.82	1.07	1.4	2.72	1.17	1.5
time (sec)	N/A	0.177	0.101	0.001	0.948	0.833	0.095	1.065

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	214	224	286	497	240	308
normalized size	1	1.	1.07	1.12	1.43	2.48	1.2	1.54
time (sec)	N/A	0.146	0.078	0.004	0.961	0.997	0.605	1.069

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	172	224	286	570	236	308
normalized size	1	1.	0.87	1.13	1.44	2.88	1.19	1.56
time (sec)	N/A	0.182	0.137	0.007	0.946	1.021	0.633	1.061

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	174	222	286	551	236	305
normalized size	1	1.	0.88	1.12	1.44	2.78	1.19	1.54
time (sec)	N/A	0.197	0.117	0.008	0.951	1.011	0.716	1.063

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	172	220	286	551	235	304
normalized size	1	1.	0.82	1.05	1.37	2.64	1.12	1.45
time (sec)	N/A	0.18	0.126	0.006	0.947	1.006	1.061	1.062

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	170	220	286	528	233	302
normalized size	1	1.	0.81	1.05	1.37	2.53	1.11	1.44
time (sec)	N/A	0.177	0.121	0.007	0.948	0.977	2.979	1.074

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	334	533	0	0	874	513
normalized size	1	1.	1.01	1.61	0.	0.	2.64	1.55
time (sec)	N/A	1.07	0.438	0.006	0.	0.	29.166	1.078

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	299	505	0	0	842	477
normalized size	1	1.	0.96	1.61	0.	0.	2.69	1.52
time (sec)	N/A	0.988	0.238	0.005	0.	0.	29.526	1.095

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	290	483	0	0	789	450
normalized size	1	1.	0.99	1.64	0.	0.	2.68	1.53
time (sec)	N/A	0.976	0.262	0.003	0.	0.	63.182	1.083

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	272	455	0	0	811	428
normalized size	1	1.	0.99	1.65	0.	0.	2.95	1.56
time (sec)	N/A	0.921	0.373	0.003	0.	0.	33.016	1.083

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	257	254	429	0	0	804	397
normalized size	1	0.99	0.98	1.66	0.	0.	3.1	1.53
time (sec)	N/A	0.373	0.294	0.003	0.	0.	28.682	1.08

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	256	258	426	0	0	0	409
normalized size	1	0.99	1.	1.65	0.	0.	0.	1.59
time (sec)	N/A	0.471	0.212	0.006	0.	0.	0.	1.063

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	257	423	0	0	0	404
normalized size	1	1.	1.02	1.67	0.	0.	0.	1.6
time (sec)	N/A	0.454	0.28	0.007	0.	0.	0.	1.092

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	258	257	423	0	0	0	393
normalized size	1	0.99	0.99	1.63	0.	0.	0.	1.51
time (sec)	N/A	0.38	0.321	0.007	0.	0.	0.	1.082

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	264	442	0	0	0	423
normalized size	1	0.99	0.96	1.6	0.	0.	0.	1.53
time (sec)	N/A	0.436	0.44	0.006	0.	0.	0.	1.096

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	334	562	0	0	0	512
normalized size	1	1.	0.99	1.67	0.	0.	0.	1.52
time (sec)	N/A	0.717	0.398	0.011	0.	0.	0.	1.091

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	294	533	0	0	0	475
normalized size	1	1.	0.95	1.71	0.	0.	0.	1.53
time (sec)	N/A	0.64	0.212	0.011	0.	0.	0.	1.086

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	288	280	506	0	27702	0	444
normalized size	1	0.99	0.97	1.74	0.	95.52	0.	1.53
time (sec)	N/A	0.498	0.198	0.01	0.	11.473	0.	1.092

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	285	502	0	27753	0	451
normalized size	1	1.	0.99	1.74	0.	96.03	0.	1.56
time (sec)	N/A	0.505	0.196	0.01	0.	13.028	0.	1.088

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	268	462	0	27752	0	429
normalized size	1	1.	0.97	1.67	0.	100.55	0.	1.55
time (sec)	N/A	0.37	0.176	0.008	0.	12.37	0.	1.084

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	287	269	507	0	28260	0	452
normalized size	1	0.99	0.93	1.75	0.	97.79	0.	1.56
time (sec)	N/A	0.559	0.198	0.012	0.	156.896	0.	1.086

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	285	517	0	0	0	473
normalized size	1	1.	0.95	1.72	0.	0.	0.	1.57
time (sec)	N/A	0.593	0.299	0.014	0.	0.	0.	1.102

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	304	292	527	0	28467	0	483
normalized size	1	0.99	0.95	1.72	0.	93.03	0.	1.58
time (sec)	N/A	0.577	0.488	0.013	0.	121.35	0.	1.078

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	336	303	561	0	0	0	520
normalized size	1	0.99	0.9	1.66	0.	0.	0.	1.54
time (sec)	N/A	0.727	0.454	0.014	0.	0.	0.	1.096

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	342	619	0	30286	0	541
normalized size	1	1.	0.99	1.79	0.	87.79	0.	1.57
time (sec)	N/A	0.891	0.305	0.011	0.	34.187	0.	1.084

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	315	515	0	29083	0	512
normalized size	1	1.	0.97	1.58	0.	89.49	0.	1.58
time (sec)	N/A	0.642	0.253	0.012	0.	28.726	0.	1.09

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	287	490	0	15354	0	454
normalized size	1	1.	0.97	1.65	0.	51.7	0.	1.53
time (sec)	N/A	0.43	0.234	0.011	0.	19.229	0.	1.122

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	297	498	0	15553	0	481
normalized size	1	1.	0.92	1.54	0.	48.15	0.	1.49
time (sec)	N/A	0.482	0.3	0.011	0.	22.23	0.	1.091

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	295	506	0	15406	0	467
normalized size	1	1.	0.94	1.62	0.	49.22	0.	1.49
time (sec)	N/A	0.429	0.238	0.008	0.	20.815	0.	1.09

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	345	311	618	0	0	0	529
normalized size	1	0.99	0.9	1.78	0.	0.	0.	1.52
time (sec)	N/A	0.723	0.274	0.017	0.	0.	0.	1.098

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	336	622	0	0	0	556
normalized size	1	1.	0.93	1.72	0.	0.	0.	1.54
time (sec)	N/A	0.83	0.57	0.016	0.	0.	0.	1.092

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	357	337	626	0	31293	0	568
normalized size	1	0.99	0.94	1.74	0.	86.92	0.	1.58
time (sec)	N/A	0.814	0.606	0.017	0.	139.296	0.	1.115

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	392	352	680	0	0	0	612
normalized size	1	0.99	0.89	1.72	0.	0.	0.	1.55
time (sec)	N/A	1.006	0.591	0.017	0.	0.	0.	1.096

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	583	583	132	793	0	0	129	0
normalized size	1	1.	0.23	1.36	0.	0.	0.22	0.
time (sec)	N/A	0.733	0.129	0.016	0.	0.	3.132	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	121	773	0	0	107	0
normalized size	1	1.	0.22	1.38	0.	0.	0.19	0.
time (sec)	N/A	0.484	0.079	0.007	0.	0.	2.87	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	114	753	0	0	107	0
normalized size	1	1.	0.21	1.4	0.	0.	0.2	0.
time (sec)	N/A	0.323	0.062	0.006	0.	0.	2.689	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	107	735	0	0	105	0
normalized size	1	1.	0.21	1.44	0.	0.	0.21	0.
time (sec)	N/A	0.173	0.046	0.004	0.	0.	1.981	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	128	740	0	0	105	0
normalized size	1	1.	0.25	1.43	0.	0.	0.2	0.
time (sec)	N/A	0.205	0.141	0.007	0.	0.	3.335	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	126	759	0	0	107	0
normalized size	1	1.	0.23	1.39	0.	0.	0.2	0.
time (sec)	N/A	0.343	0.124	0.007	0.	0.	2.948	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	569	131	778	0	0	112	0
normalized size	1	1.	0.23	1.37	0.	0.	0.2	0.
time (sec)	N/A	0.464	0.194	0.009	0.	0.	2.966	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	134	836	0	0	129	0
normalized size	1	1.	0.23	1.41	0.	0.	0.22	0.
time (sec)	N/A	0.641	0.111	0.02	0.	0.	27.031	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	574	127	817	0	0	129	0
normalized size	1	1.	0.22	1.42	0.	0.	0.22	0.
time (sec)	N/A	0.469	0.103	0.009	0.	0.	18.138	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	118	800	0	0	129	0
normalized size	1	1.	0.22	1.48	0.	0.	0.24	0.
time (sec)	N/A	0.327	0.095	0.007	0.	0.	13.504	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	107	779	0	0	109	0
normalized size	1	1.	0.2	1.49	0.	0.	0.21	0.
time (sec)	N/A	0.262	0.088	0.007	0.	0.	11.561	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	561	108	782	0	0	109	0
normalized size	1	1.	0.19	1.39	0.	0.	0.19	0.
time (sec)	N/A	0.321	0.064	0.005	0.	0.	11.124	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	532	109	785	0	0	107	0
normalized size	1	1.	0.2	1.48	0.	0.	0.2	0.
time (sec)	N/A	0.246	0.047	0.004	0.	0.	11.264	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	579	119	810	0	0	265	0
normalized size	1	1.	0.21	1.4	0.	0.	0.46	0.
time (sec)	N/A	0.4	0.103	0.007	0.	0.	22.444	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	607	121	825	0	0	267	0
normalized size	1	1.	0.2	1.36	0.	0.	0.44	0.
time (sec)	N/A	0.56	0.098	0.008	0.	0.	34.63	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	733	733	172	1674	0	0	238	0
normalized size	1	1.	0.23	2.28	0.	0.	0.32	0.
time (sec)	N/A	1.91	0.299	0.019	0.	0.	5.106	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	681	158	1197	0	0	223	0
normalized size	1	1.	0.23	1.76	0.	0.	0.33	0.
time (sec)	N/A	1.422	0.254	0.008	0.	0.	4.524	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	667	667	143	1311	0	0	223	0
normalized size	1	1.	0.21	1.97	0.	0.	0.33	0.
time (sec)	N/A	1.047	0.194	0.006	0.	0.	4.307	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	639	135	1557	0	0	194	0
normalized size	1	1.	0.21	2.44	0.	0.	0.3	0.
time (sec)	N/A	0.719	0.148	0.005	0.	0.	3.902	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	620	620	185	1118	0	0	235	0
normalized size	1	1.	0.3	1.8	0.	0.	0.38	0.
time (sec)	N/A	0.547	0.317	0.008	0.	0.	9.255	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	211	1248	0	0	236	0
normalized size	1	1.	0.33	1.96	0.	0.	0.37	0.
time (sec)	N/A	0.65	0.244	0.009	0.	0.	5.507	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	640	218	1529	0	0	255	0
normalized size	1	1.	0.34	2.39	0.	0.	0.4	0.
time (sec)	N/A	0.765	0.411	0.009	0.	0.	6.011	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	637	637	254	1114	0	0	265	0
normalized size	1	1.	0.4	1.75	0.	0.	0.42	0.
time (sec)	N/A	0.844	0.387	0.01	0.	0.	6.899	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	253	1286	0	0	274	0
normalized size	1	1.	0.36	1.85	0.	0.	0.39	0.
time (sec)	N/A	1.084	0.387	0.01	0.	0.	7.315	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	652	180	1571	0	0	240	0
normalized size	1	1.	0.28	2.41	0.	0.	0.37	0.
time (sec)	N/A	0.787	0.26	0.009	0.	0.	6.551	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	659	211	1180	0	0	304	0
normalized size	1	1.	0.32	1.79	0.	0.	0.46	0.
time (sec)	N/A	0.98	0.425	0.008	0.	0.	9.384	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	711	213	1376	0	0	308	0
normalized size	1	1.	0.3	1.94	0.	0.	0.43	0.
time (sec)	N/A	1.123	0.452	0.01	0.	0.	10.029	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	743	192	1679	0	0	304	0
normalized size	1	1.	0.26	2.26	0.	0.	0.41	0.
time (sec)	N/A	1.331	0.191	0.009	0.	0.	9.386	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	791	179	1764	0	0	512	0
normalized size	1	1.	0.23	2.23	0.	0.	0.65	0.
time (sec)	N/A	2.111	0.527	0.017	0.	0.	10.768	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	742	742	162	1269	0	0	525	0
normalized size	1	1.	0.22	1.71	0.	0.	0.71	0.
time (sec)	N/A	1.552	0.35	0.008	0.	0.	9.442	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	723	148	1383	0	0	525	0
normalized size	1	1.	0.2	1.91	0.	0.	0.73	0.
time (sec)	N/A	1.239	0.272	0.006	0.	0.	8.804	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	139	1629	0	0	444	0
normalized size	1	1.	0.2	2.35	0.	0.	0.64	0.
time (sec)	N/A	0.899	0.177	0.006	0.	0.	7.325	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	215	1188	0	0	473	0
normalized size	1	1.	0.32	1.76	0.	0.	0.7	0.
time (sec)	N/A	0.707	0.439	0.009	0.	0.	21.367	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	224	1317	0	0	474	0
normalized size	1	1.	0.32	1.9	0.	0.	0.68	0.
time (sec)	N/A	0.791	0.34	0.009	0.	0.	10.297	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	232	1613	0	0	462	0
normalized size	1	1.	0.33	2.32	0.	0.	0.67	0.
time (sec)	N/A	0.888	0.356	0.009	0.	0.	10.501	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	243	1193	0	0	484	0
normalized size	1	1.	0.35	1.72	0.	0.	0.7	0.
time (sec)	N/A	0.957	0.612	0.01	0.	0.	11.966	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	741	741	246	1342	0	0	495	0
normalized size	1	1.	0.33	1.81	0.	0.	0.67	0.
time (sec)	N/A	1.24	0.603	0.01	0.	0.	12.167	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	191	1606	0	0	476	0
normalized size	1	1.	0.28	2.33	0.	0.	0.69	0.
time (sec)	N/A	0.921	0.244	0.009	0.	0.	11.759	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	240	1196	0	0	524	0
normalized size	1	1.	0.35	1.73	0.	0.	0.76	0.
time (sec)	N/A	1.004	0.507	0.01	0.	0.	15.539	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	240	1375	0	0	536	0
normalized size	1	1.	0.32	1.84	0.	0.	0.72	0.
time (sec)	N/A	1.284	0.781	0.009	0.	0.	16.457	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	705	705	202	1663	0	0	527	0
normalized size	1	1.	0.29	2.36	0.	0.	0.75	0.
time (sec)	N/A	1.008	0.461	0.01	0.	0.	15.513	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	226	1273	0	0	573	0
normalized size	1	1.	0.32	1.78	0.	0.	0.8	0.
time (sec)	N/A	1.123	0.645	0.025	0.	0.	23.72	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	227	1470	0	0	576	0
normalized size	1	1.	0.3	1.92	0.	0.	0.75	0.
time (sec)	N/A	1.33	0.507	0.008	0.	0.	24.308	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	796	796	194	1773	0	0	541	0
normalized size	1	1.	0.24	2.23	0.	0.	0.68	0.
time (sec)	N/A	1.534	0.4	0.025	0.	0.	22.843	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	120	114	0	0	0	112	0
normalized size	1	1.18	1.12	0.	0.	0.	1.1	0.
time (sec)	N/A	0.076	0.062	0.264	0.	0.	91.678	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	114	0
normalized size	1	1.17	1.08	0.	0.	0.	1.07	0.
time (sec)	N/A	0.091	0.062	0.227	0.	0.	145.318	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	0	0
normalized size	1	1.17	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.076	0.234	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	73	142	63	76
normalized size	1	1.	1.	0.81	1.07	2.09	0.93	1.12
time (sec)	N/A	0.044	0.005	0.001	0.917	1.381	0.063	1.088

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	77	155	66	80
normalized size	1	1.	1.	0.79	1.05	2.12	0.9	1.1
time (sec)	N/A	0.064	0.003	0.	0.919	1.433	0.066	1.058

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	138	258	121	142
normalized size	1	1.	1.14	0.94	1.27	2.37	1.11	1.3
time (sec)	N/A	0.074	0.004	0.	0.915	1.489	0.075	1.085

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	142	273	124	146
normalized size	1	1.	1.13	0.93	1.25	2.39	1.09	1.28
time (sec)	N/A	0.084	0.004	0.001	0.917	1.438	0.079	1.083

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	203	375	180	208
normalized size	1	1.	1.19	1.	1.34	2.48	1.19	1.38
time (sec)	N/A	0.108	0.005	0.	0.911	1.382	0.088	1.063

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	207	390	184	212
normalized size	1	1.	1.19	0.99	1.33	2.5	1.18	1.36
time (sec)	N/A	0.113	0.005	0.001	0.916	1.441	0.084	1.078

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	267	490	241	274
normalized size	1	1.	1.22	1.03	1.38	2.54	1.25	1.42
time (sec)	N/A	0.156	0.006	0.001	0.931	1.481	0.09	1.106

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	271	504	245	278
normalized size	1	1.	1.22	1.02	1.37	2.55	1.24	1.4
time (sec)	N/A	0.15	0.006	0.001	0.92	1.498	0.095	1.088

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	214	177	0	0	952	419
normalized size	1	1.	1.61	1.33	0.	0.	7.16	3.15
time (sec)	N/A	0.124	0.066	0.004	0.	0.	13.139	1.075

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	221	208	0	0	887	443
normalized size	1	1.	1.36	1.28	0.	0.	5.48	2.73
time (sec)	N/A	0.203	0.084	0.004	0.	0.	12.548	1.076

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	296	294	0	0	950	392
normalized size	1	1.	1.01	1.	0.	0.	3.24	1.34
time (sec)	N/A	0.222	0.215	0.004	0.	0.	13.21	1.081

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	311	325	0	0	886	416
normalized size	1	1.	0.97	1.01	0.	0.	2.76	1.3
time (sec)	N/A	0.334	0.173	0.004	0.	0.	12.174	1.101

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	315	362	0	0	517	427
normalized size	1	1.	0.99	1.14	0.	0.	1.63	1.34
time (sec)	N/A	0.27	0.34	0.005	0.	0.	16.141	1.088

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	294	334	0	0	508	409
normalized size	1	1.	0.95	1.08	0.	0.	1.64	1.32
time (sec)	N/A	0.274	0.269	0.009	0.	0.	31.183	1.093

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	347	432	0	0	578	478
normalized size	1	1.	0.99	1.23	0.	0.	1.65	1.36
time (sec)	N/A	0.318	0.328	0.005	0.	0.	68.514	1.101

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	329	373	0	0	0	456
normalized size	1	1.	0.97	1.1	0.	0.	0.	1.34
time (sec)	N/A	0.328	0.35	0.012	0.	0.	0.	1.096

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	379	400	0	0	0	528
normalized size	1	1.	0.99	1.05	0.	0.	0.	1.38
time (sec)	N/A	0.406	0.409	0.014	0.	0.	0.	1.081

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	366	403	0	0	0	513
normalized size	1	1.	0.96	1.06	0.	0.	0.	1.35
time (sec)	N/A	0.402	0.421	0.012	0.	0.	0.	1.092

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	202	390	0	0	252	0
normalized size	1	1.	0.48	0.93	0.	0.	0.6	0.
time (sec)	N/A	0.381	0.636	0.032	0.	0.	7.096	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	215	380	0	0	212	0
normalized size	1	1.	0.55	0.96	0.	0.	0.54	0.
time (sec)	N/A	0.334	0.63	0.008	0.	0.	6.297	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	182	361	0	0	212	0
normalized size	1	1.	0.49	0.98	0.	0.	0.57	0.
time (sec)	N/A	0.298	0.721	0.01	0.	0.	6.005	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	211	337	0	0	158	0
normalized size	1	1.	0.6	0.95	0.	0.	0.45	0.
time (sec)	N/A	0.273	0.184	0.013	0.	0.	4.281	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	171	313	0	0	156	0
normalized size	1	1.	0.52	0.95	0.	0.	0.47	0.
time (sec)	N/A	0.194	0.128	0.007	0.	0.	4.143	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	208	339	0	0	204	0
normalized size	1	1.	0.6	0.98	0.	0.	0.59	0.
time (sec)	N/A	0.255	0.357	0.015	0.	0.	8.392	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	208	339	0	0	206	0
normalized size	1	1.	0.61	0.99	0.	0.	0.6	0.
time (sec)	N/A	0.263	0.391	0.012	0.	0.	5.704	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	204	360	0	0	230	0
normalized size	1	1.	0.6	1.05	0.	0.	0.67	0.
time (sec)	N/A	0.262	0.233	0.016	0.	0.	5.271	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	205	362	0	0	235	0
normalized size	1	1.	0.57	1.01	0.	0.	0.66	0.
time (sec)	N/A	0.295	0.301	0.013	0.	0.	5.414	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	175	385	0	0	211	0
normalized size	1	1.	0.53	1.17	0.	0.	0.64	0.
time (sec)	N/A	0.281	0.234	0.016	0.	0.	5.735	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	179	404	0	0	216	0
normalized size	1	1.	0.5	1.12	0.	0.	0.6	0.
time (sec)	N/A	0.324	0.234	0.015	0.	0.	6.044	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	145	361	0	0	189	0
normalized size	1	1.	0.41	1.03	0.	0.	0.54	0.
time (sec)	N/A	0.316	0.243	0.017	0.	0.	5.702	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	145	385	0	0	192	0
normalized size	1	1.	0.39	1.03	0.	0.	0.51	0.
time (sec)	N/A	0.362	0.24	0.018	0.	0.	6.313	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	146	408	0	0	246	0
normalized size	1	1.	0.36	1.02	0.	0.	0.62	0.
time (sec)	N/A	0.386	0.165	0.019	0.	0.	7.904	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	148	429	0	0	246	0
normalized size	1	1.	0.35	1.01	0.	0.	0.58	0.
time (sec)	N/A	0.433	0.167	0.018	0.	0.	8.464	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	476	225	462	0	0	462	0
normalized size	1	1.	0.47	0.97	0.	0.	0.97	0.
time (sec)	N/A	0.44	0.865	0.039	0.	0.	18.042	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	238	434	0	0	398	0
normalized size	1	1.	0.53	0.96	0.	0.	0.88	0.
time (sec)	N/A	0.408	0.747	0.007	0.	0.	15.031	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	205	413	0	0	398	0
normalized size	1	1.	0.48	0.97	0.	0.	0.93	0.
time (sec)	N/A	0.353	0.863	0.01	0.	0.	14.209	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	196	392	0	0	396	0
normalized size	1	1.	0.48	0.96	0.	0.	0.97	0.
time (sec)	N/A	0.324	0.701	0.015	0.	0.	10.174	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	175	368	0	0	394	0
normalized size	1	1.	0.46	0.96	0.	0.	1.03	0.
time (sec)	N/A	0.255	0.553	0.006	0.	0.	9.642	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	224	411	0	0	405	0
normalized size	1	1.	0.56	1.02	0.	0.	1.	0.
time (sec)	N/A	0.353	0.512	0.019	0.	0.	26.863	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	222	411	0	0	406	0
normalized size	1	1.	0.55	1.02	0.	0.	1.	0.
time (sec)	N/A	0.348	0.523	0.013	0.	0.	11.26	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	194	409	0	0	377	0
normalized size	1	1.	0.48	1.01	0.	0.	0.93	0.
time (sec)	N/A	0.343	0.362	0.02	0.	0.	9.062	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	194	408	0	0	381	0
normalized size	1	1.	0.48	1.	0.	0.	0.93	0.
time (sec)	N/A	0.336	0.358	0.014	0.	0.	9.128	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	163	409	0	0	379	0
normalized size	1	1.	0.42	1.06	0.	0.	0.98	0.
time (sec)	N/A	0.347	0.219	0.021	0.	0.	10.707	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	165	409	0	0	386	0
normalized size	1	1.	0.43	1.06	0.	0.	1.	0.
time (sec)	N/A	0.349	0.223	0.015	0.	0.	11.021	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	163	408	0	0	406	0
normalized size	1	1.	0.42	1.04	0.	0.	1.04	0.
time (sec)	N/A	0.342	0.202	0.02	0.	0.	9.893	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	164	411	0	0	415	0
normalized size	1	1.	0.4	1.	0.	0.	1.01	0.
time (sec)	N/A	0.394	0.221	0.015	0.	0.	11.456	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	174	416	0	0	444	0
normalized size	1	1.	0.46	1.1	0.	0.	1.18	0.
time (sec)	N/A	0.371	0.294	0.023	0.	0.	12.967	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	174	437	0	0	449	0
normalized size	1	1.	0.43	1.08	0.	0.	1.11	0.
time (sec)	N/A	0.425	0.314	0.018	0.	0.	13.571	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	171	417	0	0	398	0
normalized size	1	1.	0.43	1.05	0.	0.	1.	0.
time (sec)	N/A	0.42	0.326	0.019	0.	0.	15.602	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	172	441	0	0	401	0
normalized size	1	1.	0.41	1.04	0.	0.	0.95	0.
time (sec)	N/A	0.462	0.381	0.02	0.	0.	15.493	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	149	462	0	0	403	0
normalized size	1	1.	0.33	1.03	0.	0.	0.9	0.
time (sec)	N/A	0.49	0.191	0.023	0.	0.	20.177	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	151	483	0	0	403	0
normalized size	1	1.	0.32	1.02	0.	0.	0.85	0.
time (sec)	N/A	0.549	0.184	0.02	0.	0.	21.79	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	212	335	0	0	177	0
normalized size	1	1.	0.59	0.93	0.	0.	0.49	0.
time (sec)	N/A	0.3	0.131	0.018	0.	0.	6.138	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	212	325	0	0	156	0
normalized size	1	1.	0.63	0.97	0.	0.	0.46	0.
time (sec)	N/A	0.265	0.145	0.007	0.	0.	5.023	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	193	248	0	0	156	0
normalized size	1	1.	0.63	0.81	0.	0.	0.51	0.
time (sec)	N/A	0.222	0.169	0.006	0.	0.	4.724	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	160	229	0	0	129	0
normalized size	1	1.	0.54	0.77	0.	0.	0.43	0.
time (sec)	N/A	0.198	0.095	0.005	0.	0.	3.56	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	150	208	0	0	128	0
normalized size	1	1.	0.54	0.75	0.	0.	0.46	0.
time (sec)	N/A	0.142	0.094	0.004	0.	0.	2.627	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	159	222	0	0	126	0
normalized size	1	1.	0.56	0.78	0.	0.	0.44	0.
time (sec)	N/A	0.18	0.227	0.013	0.	0.	4.07	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	157	299	0	0	128	0
normalized size	1	1.	0.51	0.97	0.	0.	0.41	0.
time (sec)	N/A	0.224	0.227	0.009	0.	0.	3.31	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	148	293	0	0	126	0
normalized size	1	1.	0.49	0.98	0.	0.	0.42	0.
time (sec)	N/A	0.216	0.14	0.015	0.	0.	3.096	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	149	316	0	0	131	0
normalized size	1	1.	0.46	0.98	0.	0.	0.41	0.
time (sec)	N/A	0.258	0.158	0.01	0.	0.	3.441	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	147	335	0	0	158	0
normalized size	1	1.	0.42	0.97	0.	0.	0.46	0.
time (sec)	N/A	0.281	0.147	0.017	0.	0.	4.537	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	134	354	0	0	163	0
normalized size	1	1.	0.36	0.94	0.	0.	0.43	0.
time (sec)	N/A	0.327	0.208	0.01	0.	0.	4.883	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	220	378	0	0	202	0
normalized size	1	1.	0.6	1.04	0.	0.	0.55	0.
time (sec)	N/A	0.523	0.173	0.027	0.	0.	34.669	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	176	358	0	0	172	0
normalized size	1	1.	0.51	1.04	0.	0.	0.5	0.
time (sec)	N/A	0.367	0.152	0.016	0.	0.	25.754	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	166	340	0	0	172	0
normalized size	1	1.	0.53	1.08	0.	0.	0.55	0.
time (sec)	N/A	0.268	0.146	0.008	0.	0.	19.986	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	181	331	0	0	156	0
normalized size	1	1.	0.61	1.11	0.	0.	0.53	0.
time (sec)	N/A	0.2	0.129	0.007	0.	0.	17.232	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	165	331	0	0	156	0
normalized size	1	1.	0.5	0.99	0.	0.	0.47	0.
time (sec)	N/A	0.251	0.196	0.009	0.	0.	15.565	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	116	250	0	0	133	0
normalized size	1	1.	0.38	0.83	0.	0.	0.44	0.
time (sec)	N/A	0.193	0.077	0.011	0.	0.	14.639	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	116	250	0	0	131	0
normalized size	1	1.	0.42	0.91	0.	0.	0.48	0.
time (sec)	N/A	0.117	0.054	0.006	0.	0.	13.983	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	125	336	0	0	289	0
normalized size	1	1.	0.39	1.04	0.	0.	0.89	0.
time (sec)	N/A	0.292	0.127	0.017	0.	0.	24.447	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	123	355	0	0	291	0
normalized size	1	1.	0.36	1.03	0.	0.	0.85	0.
time (sec)	N/A	0.383	0.114	0.01	0.	0.	34.144	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	140	363	0	0	316	0
normalized size	1	1.	0.38	0.99	0.	0.	0.86	0.
time (sec)	N/A	0.478	0.106	0.016	0.	0.	42.437	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	136	383	0	0	321	0
normalized size	1	1.	0.35	0.99	0.	0.	0.83	0.
time (sec)	N/A	0.606	0.119	0.011	0.	0.	41.397	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	0.214	0.314	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	170	147	0	0	0	141	0
normalized size	1	1.19	1.03	0.	0.	0.	0.99	0.
time (sec)	N/A	0.131	0.111	0.25	0.	0.	70.572	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	145	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.115	0.222	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	18	5	9
normalized size	1	1.	1.	0.88	1.	2.25	0.62	1.12
time (sec)	N/A	0.01	0.001	0.	0.916	1.346	0.06	1.054

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	12
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.2
time (sec)	N/A	0.015	0.001	0.001	0.928	1.346	0.07	1.059

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	26	8	12
normalized size	1	1.	1.	0.9	1.1	2.6	0.8	1.2
time (sec)	N/A	0.013	0.001	0.001	0.921	1.359	0.068	1.052

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	18	23	53	15	20
normalized size	1	1.	2.1	1.8	2.3	5.3	1.5	2.
time (sec)	N/A	0.01	0.003	0.006	0.895	1.353	0.093	1.048

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	22	58	24	22
normalized size	1	1.	1.	0.71	0.92	2.42	1.	0.92
time (sec)	N/A	0.024	0.006	0.003	1.372	1.355	0.114	1.052

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	128	46	53
normalized size	1	1.	1.	0.78	1.02	2.56	0.92	1.06
time (sec)	N/A	0.049	0.017	0.007	1.388	1.433	0.162	1.083

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	51	128	46	53
normalized size	1	1.	0.92	0.78	1.02	2.56	0.92	1.06
time (sec)	N/A	0.048	0.013	0.007	1.387	1.405	0.165	1.049

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	62	157	56	65
normalized size	1	1.	0.93	0.78	1.03	2.62	0.93	1.08
time (sec)	N/A	0.051	0.013	0.007	1.374	1.435	0.181	1.046

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	62	157	56	65
normalized size	1	1.	0.87	0.78	1.03	2.62	0.93	1.08
time (sec)	N/A	0.049	0.011	0.006	1.384	1.434	0.18	1.072

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	126	48	47
normalized size	1	1.	1.	0.78	1.02	2.52	0.96	0.94
time (sec)	N/A	0.026	0.006	0.005	1.373	1.375	0.138	1.044

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	51	124	48	53
normalized size	1	1.	1.	0.78	1.02	2.48	0.96	1.06
time (sec)	N/A	0.043	0.01	0.004	1.367	1.415	0.146	1.062

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	100	85	113	342	105	116
normalized size	1	1.	0.91	0.77	1.03	3.11	0.95	1.05
time (sec)	N/A	0.118	0.087	0.01	1.405	1.497	0.359	1.072

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	85	113	342	105	116
normalized size	1	1.	0.88	0.77	1.03	3.11	0.95	1.05
time (sec)	N/A	0.117	0.077	0.009	1.381	1.438	0.351	1.074

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	122	68	82	259	70	85
normalized size	1	1.	1.51	0.84	1.01	3.2	0.86	1.05
time (sec)	N/A	0.069	0.431	0.01	1.375	1.527	0.201	1.061

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	100	338	82	103
normalized size	1	1.	0.91	0.79	1.09	3.67	0.89	1.12
time (sec)	N/A	0.116	0.03	0.01	1.386	1.654	0.285	1.056

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	119	115	142	736	124	150
normalized size	1	1.	0.8	0.78	0.96	4.97	0.84	1.01
time (sec)	N/A	0.172	0.057	0.016	1.374	1.683	0.503	1.07

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	121	115	142	736	124	150
normalized size	1	1.	0.83	0.79	0.97	5.04	0.85	1.03
time (sec)	N/A	0.173	0.082	0.013	1.381	1.506	0.501	1.062

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	128	539	116	143
normalized size	1	1.	0.78	0.78	0.9	3.8	0.82	1.01
time (sec)	N/A	0.148	0.063	0.013	1.376	1.462	0.475	1.065

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	111	128	539	116	143
normalized size	1	1.	0.78	0.78	0.9	3.8	0.82	1.01
time (sec)	N/A	0.145	0.062	0.013	1.377	1.461	0.469	1.06

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	103	102	117	377	110	120
normalized size	1	1.	0.91	0.9	1.04	3.34	0.97	1.06
time (sec)	N/A	0.082	0.05	0.013	1.381	1.427	0.384	1.059

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	111	102	128	531	119	134
normalized size	1	1.	0.85	0.78	0.98	4.05	0.91	1.02
time (sec)	N/A	0.148	0.064	0.013	1.395	1.443	0.468	1.05

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	91	76	101	257	102	93
normalized size	1	1.	0.92	0.77	1.02	2.6	1.03	0.94
time (sec)	N/A	0.063	0.014	0.007	1.375	1.444	0.328	1.063

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	130	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	0.341	0.384	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	108	130	0	660	1251	529
normalized size	1	1.	1.29	1.55	0.	7.86	14.89	6.3
time (sec)	N/A	0.056	0.163	0.015	0.	1.545	6.769	1.069

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	120	87	0	347	552	265
normalized size	1	1.	1.97	1.43	0.	5.69	9.05	4.34
time (sec)	N/A	0.04	0.112	0.012	0.	1.537	3.252	1.074

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	45	0	128	163	88
normalized size	1	1.	1.02	1.1	0.	3.12	3.98	2.15
time (sec)	N/A	0.023	0.096	0.011	0.	1.485	1.509	1.056

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	36	15	16
normalized size	1	1.	1.	1.08	0.	3.	1.25	1.33
time (sec)	N/A	0.003	0.001	0.041	0.	1.501	0.058	1.041

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	65	0
normalized size	1	1.	1.	0.	0.	0.	1.55	0.
time (sec)	N/A	0.03	0.056	0.339	0.	0.	9.521	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	313	0
normalized size	1	1.	1.	0.	0.	0.	7.11	0.
time (sec)	N/A	0.029	0.075	0.36	0.	0.	22.753	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	63	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.074	0.355	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	206	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.374	0.464	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	153	0	0
normalized size	1	1.	1.	0.	0.	3.4	0.	0.
time (sec)	N/A	0.395	0.215	0.487	0.	1.895	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	178	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.243	0.437	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	204	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.273	0.385	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	147	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.338	0.432	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	47	0	0
normalized size	1	1.	1.	0.88	1.12	1.96	0.	0.
time (sec)	N/A	0.053	0.148	0.009	1.105	1.618	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.168	0.387	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	128	0	308
normalized size	1	1.	1.	0.	0.	4.57	0.	11.
time (sec)	N/A	0.101	0.307	0.817	0.	2.001	0.	1.119

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	138	104	312	0	320
normalized size	1	1.	1.02	3.07	2.31	6.93	0.	7.11
time (sec)	N/A	0.157	0.355	0.51	1.365	2.007	0.	1.146

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	80	115	0	155
normalized size	1	1.	1.	1.68	2.58	3.71	0.	5.
time (sec)	N/A	0.205	0.524	0.153	1.257	1.798	0.	1.171

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	136	124	223	0	209
normalized size	1	1.	0.91	3.02	2.76	4.96	0.	4.64
time (sec)	N/A	0.553	0.767	0.429	1.367	1.734	0.	1.189

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [124] had the largest ratio of [0.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	20	0.05
2	A	2	1	1.	22	0.045
3	A	2	1	1.	22	0.045
4	A	2	1	1.	25	0.04
5	A	2	1	1.	27	0.037
6	A	2	1	1.	27	0.037
7	A	6	6	1.	15	0.4

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	7	7	1.	15	0.467
9	A	8	7	1.	15	0.467
10	A	9	7	1.	15	0.467
11	A	6	6	1.	15	0.4
12	A	6	6	1.	16	0.375
13	A	3	3	1.	11	0.273
14	A	3	3	1.	15	0.2
15	A	3	3	1.	13	0.231
16	A	3	3	1.	13	0.231
17	A	6	6	1.	15	0.4
18	A	3	3	1.	19	0.158
19	A	3	3	1.	21	0.143
20	A	3	3	1.	31	0.097
21	A	3	3	1.	36	0.083
22	A	12	10	1.	35	0.286
23	A	11	9	1.	33	0.273
24	A	10	8	1.	36	0.222
25	A	10	10	1.	19	0.526
26	A	9	9	1.	18	0.5
27	A	4	4	1.	27	0.148
28	A	4	4	1.	28	0.143
29	A	4	4	1.	24	0.167
30	A	4	4	1.	24	0.167
31	A	4	4	1.	26	0.154
32	A	4	4	1.	26	0.154
33	A	4	4	1.	28	0.143
34	A	4	4	1.	30	0.133
35	A	4	4	1.	29	0.138
36	A	4	4	1.	29	0.138
37	A	4	4	1.	29	0.138
38	A	4	4	1.	32	0.125
39	A	6	6	1.	13	0.462
40	A	4	4	1.	49	0.082
41	A	4	4	1.	57	0.07
42	A	2	2	1.	31	0.065
43	A	2	2	1.	42	0.048
44	A	4	4	1.	42	0.095
45	A	4	4	1.	45	0.089
46	A	4	4	1.	45	0.089
47	A	4	4	1.	44	0.091
48	A	3	3	1.	20	0.15
49	A	6	6	1.	20	0.3
50	A	2	2	1.	16	0.125
51	A	5	5	1.	20	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
52	A	3	3	1.	18	0.167
53	A	2	1	1.	30	0.033
54	A	2	1	1.	30	0.033
55	A	2	1	1.	28	0.036
56	A	2	1	1.	30	0.033
57	A	7	7	1.	30	0.233
58	A	8	8	1.	30	0.267
59	A	7	5	1.	32	0.156
60	A	6	5	1.	32	0.156
61	A	5	5	1.	32	0.156
62	A	4	4	1.	32	0.125
63	A	5	5	1.	32	0.156
64	A	6	5	1.	32	0.156
65	A	7	5	1.	32	0.156
66	A	7	6	1.	32	0.188
67	A	6	6	1.	32	0.188
68	A	5	5	1.	32	0.156
69	A	6	6	1.	32	0.188
70	A	8	8	1.	17	0.471
71	A	10	9	1.	17	0.529
72	A	10	9	0.99	17	0.529
73	A	10	9	0.99	22	0.409
74	A	10	9	1.	22	0.409
75	A	10	9	1.	22	0.409
76	A	9	8	1.	17	0.471
77	A	9	8	1.	19	0.421
78	A	8	7	1.	18	0.389
79	A	3	3	1.	18	0.167
80	A	3	3	1.	22	0.136
81	A	1	1	1.	20	0.05
82	A	1	1	1.	20	0.05
83	A	3	3	1.	33	0.091
84	A	3	3	1.	35	0.086
85	A	1	1	1.	36	0.028
86	A	1	1	1.	36	0.028
87	A	3	3	1.	30	0.1
88	A	3	3	1.	32	0.094
89	A	1	1	1.	33	0.03
90	A	1	1	1.	33	0.03
91	A	1	1	1.	20	0.05
92	A	1	1	1.	24	0.042
93	A	3	3	1.	22	0.136
94	A	3	3	1.	22	0.136
95	A	1	1	1.	20	0.05

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	1	1	1.	20	0.05
97	A	3	3	1.	18	0.167
98	A	3	3	1.	22	0.136
99	A	1	1	1.	35	0.029
100	A	1	1	1.	37	0.027
101	A	3	3	1.	38	0.079
102	A	3	3	1.	38	0.079
103	A	1	1	1.	32	0.031
104	A	1	1	1.	34	0.029
105	A	3	3	1.	35	0.086
106	A	3	3	1.	35	0.086
107	A	3	3	1.	17	0.176
108	A	3	3	1.	18	0.167
109	A	3	3	1.	19	0.158
110	A	3	3	1.	20	0.15
111	A	3	3	1.	15	0.2
112	A	3	3	1.	17	0.176
113	A	3	3	1.	15	0.2
114	A	3	3	1.	17	0.176
115	A	7	5	1.	16	0.312
116	A	13	9	1.	15	0.6
117	A	8	6	1.	16	0.375
118	A	14	10	1.	15	0.667
119	A	9	6	1.	16	0.375
120	A	15	10	1.	15	0.667
121	A	10	6	1.	16	0.375
122	A	16	10	1.	15	0.667
123	A	7	5	1.	15	0.333
124	A	13	9	1.	13	0.692
125	A	7	5	1.	21	0.238
126	A	13	9	1.	20	0.45
127	A	8	6	1.	21	0.286
128	A	14	10	1.	20	0.5
129	A	9	6	1.	21	0.286
130	A	15	10	1.	20	0.5
131	A	10	6	1.	21	0.286
132	A	16	10	1.	20	0.5
133	A	3	2	1.	11	0.182
134	A	3	2	1.	12	0.167
135	A	2	1	1.	15	0.067
136	A	3	2	1.	14	0.143
137	A	2	1	1.	17	0.059
138	A	3	2	1.	19	0.105
139	A	2	1	1.	20	0.05

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	2	2	1.	14	0.143
141	A	4	3	1.	17	0.176
142	A	4	3	1.	19	0.158
143	A	3	2	1.	20	0.1
144	A	4	3	1.	21	0.143
145	A	3	2	1.	22	0.091
146	A	4	3	1.	24	0.125
147	A	3	2	1.	25	0.08
148	A	3	2	1.	25	0.08
149	A	8	6	1.	26	0.231
150	A	9	7	1.	26	0.269
151	A	10	7	1.	26	0.269
152	A	10	7	1.	11	0.636
153	A	3	3	1.	12	0.25
154	A	13	9	1.	15	0.6
155	A	10	7	1.	14	0.5
156	A	9	6	1.	17	0.353
157	A	14	10	1.	19	0.526
158	A	13	9	1.	20	0.45
159	A	2	2	1.	14	0.143
160	A	12	8	1.	17	0.471
161	A	5	5	1.	19	0.263
162	A	15	11	1.	20	0.55
163	A	13	9	1.	21	0.429
164	A	12	8	1.	22	0.364
165	A	16	12	1.	24	0.5
166	A	15	11	1.	25	0.44
167	A	2	2	1.	19	0.105
168	A	11	8	1.	17	0.471
169	A	9	7	1.	20	0.35
170	A	15	11	1.	19	0.579
171	A	11	8	1.	31	0.258
172	A	8	6	1.	31	0.194
173	A	9	7	1.	31	0.226
174	A	10	8	1.	31	0.258
175	A	17	12	1.	30	0.4
176	A	14	10	1.	30	0.333
177	A	15	11	1.	30	0.367
178	A	16	12	1.	30	0.4
179	A	2	2	1.	21	0.095
180	A	2	2	1.	21	0.095
181	A	2	1	1.	19	0.053
182	A	2	2	1.	19	0.105
183	A	2	2	1.	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	2	2	1.	21	0.095
185	A	2	2	1.	21	0.095
186	A	13	9	1.	36	0.25
187	A	13	9	1.	41	0.22
188	A	13	9	1.	46	0.196
189	A	19	13	1.	35	0.371
190	A	19	13	1.	40	0.325
191	A	19	13	1.	45	0.289
192	A	8	6	1.	36	0.167
193	A	8	6	1.	41	0.146
194	A	10	8	1.	46	0.174
195	A	14	10	1.	35	0.286
196	A	14	10	1.	40	0.25
197	A	16	12	1.	45	0.267
198	A	9	7	1.	36	0.194
199	A	9	7	1.	41	0.171
200	A	9	7	1.	46	0.152
201	A	15	11	1.	35	0.314
202	A	15	11	1.	40	0.275
203	A	15	11	1.	45	0.244
204	A	10	8	1.	36	0.222
205	A	10	8	1.	41	0.195
206	A	10	8	1.	46	0.174
207	A	16	12	1.	35	0.343
208	A	16	12	1.	40	0.3
209	A	16	12	1.	45	0.267
210	A	6	5	1.	17	0.294
211	A	7	6	1.	18	0.333
212	A	7	6	1.	19	0.316
213	A	6	5	1.	20	0.25
214	A	8	7	1.	22	0.318
215	A	1	1	1.	23	0.043
216	A	1	1	1.	26	0.038
217	A	1	1	1.	28	0.036
218	A	1	1	1.	31	0.032
219	A	1	1	1.	15	0.067
220	A	12	10	1.	42	0.238
221	A	3	2	1.	11	0.182
222	A	3	2	1.	15	0.133
223	A	3	2	1.	30	0.067
224	A	3	2	1.	30	0.067
225	A	3	2	1.	30	0.067
226	A	3	2	1.	30	0.067
227	A	3	2	1.	30	0.067

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
228	A	3	2	1.	30	0.067
229	A	3	2	1.	30	0.067
230	A	3	2	1.	30	0.067
231	A	3	2	1.	30	0.067
232	A	3	2	1.	30	0.067
233	A	9	8	1.	30	0.267
234	A	9	8	1.	30	0.267
235	A	9	8	1.	30	0.267
236	A	9	8	1.	30	0.267
237	A	9	8	1.	30	0.267
238	A	9	8	1.	28	0.286
239	A	8	7	1.	27	0.259
240	A	8	7	1.	30	0.233
241	A	8	7	1.	30	0.233
242	A	8	7	1.	30	0.233
243	A	8	7	1.	30	0.233
244	A	8	7	1.	30	0.233
245	A	8	7	1.	30	0.233
246	A	8	7	1.	30	0.233
247	A	8	7	1.	30	0.233
248	A	8	7	1.	30	0.233
249	A	8	7	1.	30	0.233
250	A	8	7	1.	30	0.233
251	A	3	2	1.	30	0.067
252	A	3	2	1.	30	0.067
253	A	3	2	1.	30	0.067
254	A	3	2	1.	30	0.067
255	A	3	2	1.	30	0.067
256	A	3	2	1.	30	0.067
257	A	3	2	1.	30	0.067
258	A	3	2	1.	30	0.067
259	A	3	2	1.	30	0.067
260	A	9	8	1.	30	0.267
261	A	12	10	1.	30	0.333
262	A	9	8	1.	30	0.267
263	A	11	10	1.	30	0.333
264	A	9	8	1.	30	0.267
265	A	10	9	1.	28	0.321
266	A	9	9	1.	27	0.333
267	A	9	8	1.	30	0.267
268	A	9	8	1.	30	0.267
269	A	9	8	1.	30	0.267
270	A	9	8	1.	30	0.267
271	A	9	8	1.	30	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	9	8	1.	30	0.267
273	A	9	8	1.	30	0.267
274	A	9	8	1.	30	0.267
275	A	9	8	1.	30	0.267
276	A	3	2	1.	30	0.067
277	A	3	2	1.	30	0.067
278	A	3	2	1.	30	0.067
279	A	3	2	1.	30	0.067
280	A	3	2	1.	30	0.067
281	A	3	2	1.	30	0.067
282	A	3	2	1.	30	0.067
283	A	3	2	1.	30	0.067
284	A	3	2	1.	30	0.067
285	A	3	2	1.	30	0.067
286	A	10	9	1.	30	0.3
287	A	14	10	1.	30	0.333
288	A	10	9	1.	30	0.3
289	A	13	10	1.	30	0.333
290	A	10	9	1.	30	0.3
291	A	12	10	1.	30	0.333
292	A	10	10	1.	30	0.333
293	A	10	10	1.	28	0.357
294	A	9	9	1.	27	0.333
295	A	9	9	1.	30	0.3
296	A	9	9	1.	30	0.3
297	A	10	9	1.	30	0.3
298	A	10	9	1.	30	0.3
299	A	10	8	1.	30	0.267
300	A	10	8	1.	30	0.267
301	A	10	8	1.	30	0.267
302	A	10	8	1.	30	0.267
303	A	10	8	1.	30	0.267
304	A	8	7	1.	16	0.438
305	A	5	4	1.	16	0.25
306	A	8	7	1.	16	0.438
307	A	6	6	1.	14	0.429
308	A	6	5	1.	16	0.312
309	A	6	5	1.	16	0.312
310	A	3	2	1.	16	0.125
311	A	6	6	1.	14	0.429
312	A	6	6	1.	16	0.375
313	A	2	1	1.	21	0.048
314	A	2	1	1.	19	0.053
315	A	2	1	1.	18	0.056

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	2	1	1.	21	0.048
317	A	2	1	1.	21	0.048
318	A	2	1	1.	21	0.048
319	A	3	2	1.	23	0.087
320	A	3	2	1.	21	0.095
321	A	3	2	1.	20	0.1
322	A	2	1	1.	23	0.043
323	A	2	1	1.	23	0.043
324	A	2	1	1.	23	0.043
325	A	3	2	1.	23	0.087
326	A	3	2	1.	21	0.095
327	A	3	2	1.	20	0.1
328	A	2	1	1.	23	0.043
329	A	2	1	1.	23	0.043
330	A	2	1	1.	23	0.043
331	A	3	2	1.	23	0.087
332	A	3	2	1.	21	0.095
333	A	3	2	1.	20	0.1
334	A	2	1	1.	23	0.043
335	A	2	1	1.	23	0.043
336	A	2	1	1.	23	0.043
337	A	10	9	1.	23	0.391
338	A	10	9	1.	23	0.391
339	A	10	9	1.	21	0.429
340	A	8	8	1.	20	0.4
341	A	10	9	1.	23	0.391
342	A	10	9	1.	23	0.391
343	A	10	9	1.	23	0.391
344	A	7	7	1.	23	0.304
345	A	7	7	1.	21	0.333
346	A	7	7	1.	20	0.35
347	A	11	10	1.	23	0.435
348	A	11	10	1.	23	0.435
349	A	11	10	1.	23	0.435
350	A	11	10	1.	23	0.435
351	A	8	8	1.	23	0.348
352	A	8	8	1.	21	0.381
353	A	8	8	1.	20	0.4
354	A	12	10	1.	23	0.435
355	A	12	10	1.	23	0.435
356	A	12	10	1.	23	0.435
357	A	12	10	1.	23	0.435
358	A	9	8	1.	23	0.348
359	A	9	9	1.	21	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	9	8	1.	20	0.4
361	A	13	10	1.	23	0.435
362	A	13	10	1.	23	0.435
363	A	13	10	1.	23	0.435
364	A	13	10	1.	23	0.435
365	A	5	5	1.	20	0.25
366	A	4	4	1.	18	0.222
367	A	5	5	1.	20	0.25
368	A	4	4	1.	18	0.222
369	A	4	4	1.	27	0.148
370	A	4	4	1.	29	0.138
371	A	4	4	1.	28	0.143
372	A	4	4	1.	28	0.143
373	A	2	1	1.	36	0.028
374	A	2	1	1.	36	0.028
375	A	2	1	1.	36	0.028
376	A	2	1	1.	34	0.029
377	A	2	1	1.	33	0.03
378	A	2	1	1.	36	0.028
379	A	2	1	1.	36	0.028
380	A	2	1	1.	36	0.028
381	A	2	1	1.	36	0.028
382	A	2	1	1.	36	0.028
383	A	2	1	1.	38	0.026
384	A	2	1	1.	38	0.026
385	A	3	2	1.	38	0.053
386	A	3	2	1.	36	0.056
387	A	3	2	1.	35	0.057
388	A	3	2	1.	38	0.053
389	A	3	2	1.	38	0.053
390	A	3	2	1.	38	0.053
391	A	2	1	1.	38	0.026
392	A	2	1	1.	38	0.026
393	A	2	1	1.	38	0.026
394	A	2	1	1.	38	0.026
395	A	3	2	1.	38	0.053
396	A	3	2	1.	36	0.056
397	A	3	2	1.	35	0.057
398	A	3	2	1.	38	0.053
399	A	3	2	1.	38	0.053
400	A	3	2	1.	38	0.053
401	A	2	1	1.	38	0.026
402	A	2	1	1.	38	0.026
403	A	13	10	1.	38	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
404	A	13	10	1.	38	0.263
405	A	13	10	1.	38	0.263
406	A	13	10	1.	36	0.278
407	A	10	9	0.99	35	0.257
408	A	10	9	0.99	38	0.237
409	A	10	9	1.	38	0.237
410	A	10	9	0.99	38	0.237
411	A	10	9	0.99	38	0.237
412	A	11	10	1.	38	0.263
413	A	11	10	1.	38	0.263
414	A	11	10	0.99	38	0.263
415	A	11	10	1.	36	0.278
416	A	9	9	1.	35	0.257
417	A	11	10	0.99	38	0.263
418	A	11	10	1.	38	0.263
419	A	11	10	0.99	38	0.263
420	A	11	10	0.99	38	0.263
421	A	12	11	1.	38	0.29
422	A	10	10	1.	38	0.263
423	A	8	8	1.	38	0.21
424	A	8	8	1.	36	0.222
425	A	8	8	1.	35	0.229
426	A	12	10	0.99	38	0.263
427	A	12	10	1.	38	0.263
428	A	12	10	0.99	38	0.263
429	A	12	10	0.99	38	0.263
430	A	10	7	1.	25	0.28
431	A	8	7	1.	25	0.28
432	A	6	6	1.	23	0.261
433	A	5	5	1.	22	0.227
434	A	7	7	1.	25	0.28
435	A	8	8	1.	25	0.32
436	A	9	8	1.	25	0.32
437	A	8	7	1.	25	0.28
438	A	7	7	1.	25	0.28
439	A	6	6	1.	25	0.24
440	A	4	4	1.	25	0.16
441	A	6	6	1.	23	0.261
442	A	4	4	1.	22	0.182
443	A	10	10	1.	25	0.4
444	A	11	11	1.	25	0.44
445	A	13	9	1.	35	0.257
446	A	11	9	1.	35	0.257
447	A	9	8	1.	33	0.242

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	8	7	1.	32	0.219
449	A	11	11	1.	35	0.314
450	A	11	11	1.	35	0.314
451	A	10	9	1.	35	0.257
452	A	11	9	1.	35	0.257
453	A	12	9	1.	35	0.257
454	A	10	10	1.	35	0.286
455	A	11	10	1.	35	0.286
456	A	12	10	1.	35	0.286
457	A	13	10	1.	35	0.286
458	A	14	9	1.	35	0.257
459	A	12	9	1.	35	0.257
460	A	10	8	1.	33	0.242
461	A	9	7	1.	32	0.219
462	A	12	11	1.	35	0.314
463	A	12	11	1.	35	0.314
464	A	11	9	1.	35	0.257
465	A	12	9	1.	35	0.257
466	A	13	9	1.	35	0.257
467	A	11	11	1.	35	0.314
468	A	12	11	1.	35	0.314
469	A	13	11	1.	35	0.314
470	A	11	10	1.	35	0.286
471	A	12	10	1.	35	0.286
472	A	13	10	1.	35	0.286
473	A	14	10	1.	35	0.286
474	A	8	7	1.18	20	0.35
475	A	7	4	1.17	21	0.19
476	A	7	4	1.17	23	0.174
477	A	2	1	1.	23	0.043
478	A	2	1	1.	26	0.038
479	A	3	2	1.	25	0.08
480	A	3	2	1.	28	0.071
481	A	3	2	1.	25	0.08
482	A	3	2	1.	28	0.071
483	A	3	2	1.	25	0.08
484	A	3	2	1.	28	0.071
485	A	9	7	1.	26	0.269
486	A	12	9	1.	29	0.31
487	A	15	11	1.	25	0.44
488	A	18	13	1.	28	0.464
489	A	14	10	1.	25	0.4
490	A	14	10	1.	28	0.357
491	A	15	11	1.	25	0.44

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
492	A	15	11	1.	28	0.393
493	A	16	11	1.	25	0.44
494	A	16	11	1.	28	0.393
495	A	14	12	1.	30	0.4
496	A	13	11	1.	30	0.367
497	A	12	11	1.	30	0.367
498	A	12	11	1.	28	0.393
499	A	11	10	1.	27	0.37
500	A	14	13	1.	30	0.433
501	A	14	13	1.	30	0.433
502	A	14	13	1.	30	0.433
503	A	15	14	1.	30	0.467
504	A	13	13	1.	30	0.433
505	A	14	14	1.	30	0.467
506	A	12	12	1.	30	0.4
507	A	13	12	1.	30	0.4
508	A	14	13	1.	30	0.433
509	A	15	13	1.	30	0.433
510	A	16	12	1.	30	0.4
511	A	15	11	1.	30	0.367
512	A	14	11	1.	30	0.367
513	A	14	11	1.	28	0.393
514	A	13	10	1.	27	0.37
515	A	16	13	1.	30	0.433
516	A	16	14	1.	30	0.467
517	A	16	15	1.	30	0.5
518	A	16	14	1.	30	0.467
519	A	15	15	1.	30	0.5
520	A	15	15	1.	30	0.5
521	A	15	15	1.	30	0.5
522	A	16	16	1.	30	0.533
523	A	14	13	1.	30	0.433
524	A	15	14	1.	30	0.467
525	A	13	12	1.	30	0.4
526	A	14	12	1.	30	0.4
527	A	15	13	1.	30	0.433
528	A	16	13	1.	30	0.433
529	A	12	10	1.	30	0.333
530	A	11	9	1.	30	0.3
531	A	10	9	1.	30	0.3
532	A	10	9	1.	28	0.321
533	A	9	8	1.	27	0.296
534	A	12	11	1.	30	0.367
535	A	13	12	1.	30	0.4

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
536	A	11	10	1.	30	0.333
537	A	12	10	1.	30	0.333
538	A	13	11	1.	30	0.367
539	A	14	11	1.	30	0.367
540	A	12	11	1.	30	0.367
541	A	11	10	1.	30	0.333
542	A	10	9	1.	30	0.3
543	A	9	8	1.	30	0.267
544	A	10	9	1.	30	0.3
545	A	7	6	1.	28	0.214
546	A	4	4	1.	27	0.148
547	A	11	10	1.	30	0.333
548	A	13	12	1.	30	0.4
549	A	15	12	1.	30	0.4
550	A	17	13	1.	30	0.433
551	A	14	4	1.	30	0.133
552	A	12	8	1.19	25	0.32
553	A	13	7	1.	28	0.25
554	A	2	2	1.	22	0.091
555	A	2	2	1.	35	0.057
556	A	2	2	1.	35	0.057
557	A	2	2	1.	22	0.091
558	A	3	3	1.	25	0.12
559	A	6	5	1.	15	0.333
560	A	6	5	1.	15	0.333
561	A	7	6	1.	20	0.3
562	A	7	6	1.	20	0.3
563	A	7	7	1.	17	0.412
564	A	7	6	1.	25	0.24
565	A	11	6	1.	35	0.171
566	A	11	6	1.	35	0.171
567	A	8	5	1.	22	0.227
568	A	11	7	1.	25	0.28
569	A	17	7	1.	15	0.467
570	A	17	7	1.	15	0.467
571	A	14	7	1.	20	0.35
572	A	14	7	1.	20	0.35
573	A	15	9	1.	17	0.529
574	A	14	7	1.	25	0.28
575	A	13	7	1.	18	0.389
576	A	7	4	1.	36	0.111
577	A	4	3	1.	19	0.158
578	A	4	3	1.	19	0.158
579	A	4	2	1.	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
580	A	1	0	1.	9	0.
581	A	3	3	1.	19	0.158
582	A	3	3	1.	19	0.158
583	A	3	3	1.	19	0.158
584	A	13	4	1.	38	0.105
585	A	2	2	1.	58	0.034
586	A	10	3	1.	30	0.1
587	A	13	4	1.	36	0.111
588	A	4	4	1.	35	0.114
589	A	1	1	1.	46	0.022
590	A	10	8	1.	24	0.333
591	A	1	1	1.	48	0.021
592	A	1	1	1.	45	0.022
593	A	1	1	1.	69	0.014
594	A	1	1	1.	86	0.012

Chapter 3

Listing of integrals

3.1 $\int \frac{c+dx+ex^2}{\sqrt{a+bx}} dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^
(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rubi [A] time = 0.0330131, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)^
(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{b^2c-abd+a^2e}{b^2\sqrt{a+bx}}+\frac{(bd-2ae)\sqrt{a+bx}}{b^2}+\frac{e(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2\left(b^2c-abd+a^2e\right)\sqrt{a+bx}}{b^3}+\frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0710323, size = 53, normalized size = 0.74

$$\frac{2\sqrt{a+bx}\left(8a^2e-2ab(5d+2ex)+b^2(15c+x(5d+3ex))\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(5*d + 3*e*
x))))/(15*b^3)
```

Maple [A] time = 0.023, size = 53, normalized size = 0.7

$$\frac{6\,ex^2b^2-8\,abex+10\,b^2dx+16\,a^2e-20\,abd+30\,b^2c}{15\,b^3}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*e*x^2-4*a*b*e*x+5*b^2*d*x+8*a^2*e-10*a*b*d+15*b^2
*c)/b^3
```

Maxima [A] time = 0.954447, size = 104, normalized size = 1.44

$$\frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

Fricas [A] time = 1.26161, size = 127, normalized size = 1.76

$$\frac{2\left(3\,b^2ex^2+15\,b^2c-10\,abd+8\,a^2e+\left(5\,b^2d-4\,abe\right)x\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*a*b*e)*x)*
sqrt(b*x + a)/b^3
```

Sympy [A] time = 7.77256, size = 223, normalized size = 3.1

$$\begin{cases} -\frac{\frac{2ac}{\sqrt{a+bx}}+\frac{2ad\left(-\frac{a}{\sqrt{a+bx}}-\sqrt{a+bx}\right)}{b}+\frac{2ae\left(\frac{a^2}{\sqrt{a+bx}}+2a\sqrt{a+bx}-\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2}+2c\left(-\frac{a}{\sqrt{a+bx}}-\sqrt{a+bx}\right)+\frac{2d\left(\frac{a^2}{\sqrt{a+bx}}+2a\sqrt{a+bx}-\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2e\left(-\frac{a^3}{\sqrt{a+bx}}-3a^2\sqrt{a+bx}+a(a+bx)^{\frac{3}{2}}-\frac{(a+bx)^{\frac{5}{2}}}{5}\right.}{b^2}}{b} \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c/sqrt(a + b*x) + 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b + 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 + 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 2*d*(a**2/sqrt(a + b*x) + 2*a
*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 2*e*(-a**3/sqrt(a + b*x) - 3*a**2*
sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2)/b, Ne(b, 0))
, ((c*x + d*x**2/2 + e*x**3/3)/sqrt(a), True))
```

Giac [A] time = 1.1084, size = 105, normalized size = 1.46

$$\frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

3.2 $\int \frac{\left(c+dx+ex^2\right)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde+b^2\left(-\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b\right.}{b^5}$$

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*
(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2
))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)
```

Rubi [A] time = 0.105497, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde+b^2\left(-\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^2*
(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(7/2
))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2\right)^2}{\sqrt{a+bx}}\,dx=\int\left(\frac{\left(b^2c-abd+a^2e\right)^2}{b^4\sqrt{a+bx}}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)\sqrt{a+bx}}{b^4}+\frac{\left(-6abde+6a^2e^2+b^2\left(d^2+2\right.\right.}{b^4}\right.$$

$$=\frac{2\left(b^2c-abd+a^2e\right)^2\sqrt{a+bx}}{b^5}+\frac{4(bd-2ae)\left(b^2c-abd+a^2e\right)(a+bx)^{3/2}}{3b^5}-\frac{2\left(6abde-6a^2e^2-b^2\right.}{5}$$

Mathematica [A] time = 0.15182, size = 155, normalized size = 0.96

$$\frac{2\sqrt{a+bx}\left(24a^2b^2\left(2e\left(7c+ex^2\right)+7d^2+6dex\right)-32a^3be(9d+2ex)+128a^4e^2-4ab^3\left(21c(5d+2ex)+x\left(21d^2+27dex+\right.\right.\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2*b^2*(7*d^
2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d + 2*e*x) + x*(21*d^2
+ 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42*c*x*(5*d + 3*e*x) + x^2*(63*d
^2 + 90*d*e*x + 35*e^2*x^2))))/(315*b^5)
```

Maple [A] time = 0.005, size = 194, normalized size = 1.2

$$\frac{70\,e^{2}x^{4}b^{4}-80\,ab^{3}e^{2}x^{3}+180\,b^{4}dex^{3}+96\,a^{2}b^{2}e^{2}x^{2}-216\,ab^{3}dex^{2}+252\,b^{4}cex^{2}+126\,b^{4}d^{2}x^{2}-128\,a^{3}be^{2}x+288\,a^{2}b^{2}d\cdots}{315\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*e^2*x^4-40*a*b^3*e^2*x^3+90*b^4*d*e*x^3+48*a^2*
b^2*e^2*x^2-108*a*b^3*d*e*x^2+126*b^4*c*e*x^2+63*b^4*d^2*x^2-64*a^3*b*e^2*x
+144*a^2*b^2*d*e*x-168*a*b^3*c*e*x-84*a*b^3*d^2*x+210*b^4*c*d*x+128*a^4*e^2
-288*a^3*b*d*e+336*a^2*b^2*c*e+168*a^2*b^2*d^2-420*a*b^3*c*d+315*b^4*c^2)/b
^5
```

Maxima [A] time = 0.931278, size = 320, normalized size = 1.99

$$\frac{2\left(315\,\sqrt{bx+a}c^{2}+42\,c\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^{2}\right)e}{b^{2}}\right)+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^{2}\right)d^{2}}{b^{2}}+\cdots\right.}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 42*c*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e
/b^2) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a
)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*
x + a)*a^4)*e^2/b^4)/b
```

Fricas [A] time = 1.28292, size = 433, normalized size = 2.69

$$\frac{2\left(35\,b^{4}e^{2}x^{4}+315\,b^{4}c^{2}-420\,ab^{3}cd+168\,a^{2}b^{2}d^{2}+128\,a^{4}e^{2}+10\left(9\,b^{4}de-4\,ab^{3}e^{2}\right)x^{3}+3\left(21\,b^{4}d^{2}+16\,a^{2}b^{2}e^{2}+6\left(7\,b\cdots\right.\right.\right.}{315\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*e^2*x^4 + 315*b^4*c^2 - 420*a*b^3*c*d + 168*a^2*b^2*d^2 + 128
*a^4*e^2 + 10*(9*b^4*d*e - 4*a*b^3*e^2)*x^3 + 3*(21*b^4*d^2 + 16*a^2*b^2*e^
```

```
2 + 6*(7*b^4*c - 6*a*b^3*d)*e)*x^2 + 48*(7*a^2*b^2*c - 6*a^3*b*d)*e + 2*(10
5*b^4*c*d - 42*a*b^3*d^2 - 32*a^3*b*e^2 - 12*(7*a*b^3*c - 6*a^2*b^2*d)*e)*x
)*sqrt(b*x + a)/b^5
```

Sympy [A] time = 50.6685, size = 644, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**2/sqrt(a + b*x) + 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a +
b*x))/b + 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2
)/3)/b**2 + 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(
3/2)/3)/b**2 + 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
 b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 2*a*e**2*(a**4/sqrt(a + b*x) + 4*
a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a
+ b*x)**(7/2)/7)/b**4 + 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 4*c*d*(
a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 4*c*e*(-a*
*3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(
5/2)/5)/b**2 + 2*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3
*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*
x)**(7/2)/7)/b**3 + 2*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10
*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7
 - (a + b*x)**(9/2)/9)/b**4)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + d*e*x**4/2
 + e**2*x**5/5 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

Giac [A] time = 1.1067, size = 320, normalized size = 1.99

$$\frac{2\left(315\sqrt{bx+a}c^2 + \frac{210\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)cd}{b} + \frac{21\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}a^2\right)d^2}{b^2} + \frac{42\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}a^2\right)ce}{b^2} + \frac{18\cdots}{\cdots}\right.}{315b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c*
d/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
d^2/b^2 + 42*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*c*e/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*e^2/b^4)/b
```

3.3 $\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=274

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde+b^2\left(-\left(6ce+d^2\right)\right)\right)}{7b^7}-$$

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*
d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(5/2))/(5*b^7) - (2*(b*d - 2*a
*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7)
- (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) +
 (6*e^2*(b*d - 2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))
/(13*b^7)
```

Rubi [A] time = 0.193835, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {698}

$$-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde+b^2\left(-\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde+b^2\left(-\left(6ce+d^2\right)\right)\right)}{7b^7}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c -
 a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*b*
d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(5/2))/(5*b^7) - (2*(b*d - 2*a
*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b^7)
- (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b^7) +
 (6*e^2*(b*d - 2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(13/2))
/(13*b^7)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2\right)^3}{\sqrt{a+bx}}\,dx=\int \left(\frac{\left(b^2c-abd+a^2e\right)^3}{b^6\sqrt{a+bx}}+\frac{3(bd-2ae)\left(b^2c-abd+a^2e\right)^2\sqrt{a+bx}}{b^6}+\frac{3\left(b^2c-abd+a^2e\right)\left(b^2d^2\right.}{}\right.$$

$$=\frac{2\left(b^2c-abd+a^2e\right)^3\sqrt{a+bx}}{b^7}+\frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)^2(a+bx)^{3/2}}{b^7}-\frac{6\left(b^2c-abd+a^2\right.}{}$$

Mathematica [A] time = 0.538488, size = 294, normalized size = 1.07

$$\frac{2\sqrt{a+bx}(c+x(d+ex))^3}{b}-\frac{4(a+bx)^{3/2}\left(8a^2b^3\left(78de\left(33c+25ex^2\right)+4e^2x\left(429c+175ex^2\right)+1716d^2ex+429d^3\right)-6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + x*(d + e*x))^3)/b - (4*(a + b*x)^(3/2)*(-2560*a^5*e^3
 + 640*a^4*b*e^2*(13*d + 6*e*x) - 64*a^3*b^2*e*(143*d^2 + 195*d*e*x + e*(14
3*c + 75*e*x^2)) + 8*a^2*b^3*(429*d^3 + 1716*d^2*e*x + 78*d*e*(33*c + 25*e*
x^2) + 4*e^2*x*(429*c + 175*e*x^2)) + b^5*(3003*c^2*(5*d + 6*e*x) + 286*c*x
*(63*d^2 + 135*d*e*x + 70*e^2*x^2) + 5*x^2*(1287*d^3 + 4004*d^2*e*x + 4095*
d*e^2*x^2 + 1386*e^3*x^3)) - 4*a*b^4*(3003*c^2*e + 429*c*(7*d^2 + 18*d*e*x
+ 10*e^2*x^2) + x*(1287*d^3 + 4290*d^2*e*x + 4550*d*e^2*x^2 + 1575*e^3*x^3)
)))/(15015*b^7)
```

Maple [A] time = 0.006, size = 495, normalized size = 1.8

$$2310\,e^3x^6b^6 - 2520\,ab^5e^3x^5 + 8190\,b^6de^2x^5 + 2800\,a^2b^4e^3x^4 - 9100\,ab^5de^2x^4 + 10010\,b^6ce^2x^4 + 10010\,b^6d^2ex^4 - 3200\,a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x)
```

```
[Out] 2/15015*(b*x+a)^(1/2)*(1155*b^6*e^3*x^6-1260*a*b^5*e^3*x^5+4095*b^6*d*e^2*x
^5+1400*a^2*b^4*e^3*x^4-4550*a*b^5*d*e^2*x^4+5005*b^6*c*e^2*x^4+5005*b^6*d^
2*e*x^4-1600*a^3*b^3*e^3*x^3+5200*a^2*b^4*d*e^2*x^3-5720*a*b^5*c*e^2*x^3-57
20*a*b^5*d^2*e*x^3+12870*b^6*c*d*e*x^3+2145*b^6*d^3*x^3+1920*a^4*b^2*e^3*x^
2-6240*a^3*b^3*d*e^2*x^2+6864*a^2*b^4*c*e^2*x^2+6864*a^2*b^4*d^2*e*x^2-1544
4*a*b^5*c*d*e*x^2-2574*a*b^5*d^3*x^2+9009*b^6*c^2*e*x^2+9009*b^6*c*d^2*x^2-
2560*a^5*b*e^3*x+8320*a^4*b^2*d*e^2*x-9152*a^3*b^3*c*e^2*x-9152*a^3*b^3*d^2
*e*x+20592*a^2*b^4*c*d*e*x+3432*a^2*b^4*d^3*x-12012*a*b^5*c^2*e*x-12012*a*b
^5*c*d^2*x+15015*b^6*c^2*d*x+5120*a^6*e^3-16640*a^5*b*d*e^2+18304*a^4*b^2*c
*e^2+18304*a^4*b^2*d^2*e-41184*a^3*b^3*c*d*e-6864*a^3*b^3*d^3+24024*a^2*b^4
*c^2*e+24024*a^2*b^4*c*d^2-30030*a*b^5*c^2*d+15015*b^6*c^3)/b^7
```

Maxima [B] time = 0.957792, size = 709, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 3003*c^2*(5*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2) + 143*c*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 3
5*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2)
 - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^
3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4) + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 143*(35
*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b
*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 65*(63*(b*x + a)^(11
/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2
)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 5*(23
```

```
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8
580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a
^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

Fricas [A] time = 1.2812, size = 1067, normalized size = 3.89

$$\frac{2\left(1155\,b^6e^3x^6+15015\,b^6c^3-30030\,ab^5c^2d+24024\,a^2b^4cd^2-6864\,a^3b^3d^3+5120\,a^6e^3+315\left(13\,b^6de^2-4\,ab^5e^3\right)x^5 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*e^3*x^6 + 15015*b^6*c^3 - 30030*a*b^5*c^2*d + 24024*a^2*b
^4*c*d^2 - 6864*a^3*b^3*d^3 + 5120*a^6*e^3 + 315*(13*b^6*d*e^2 - 4*a*b^5*e^
3)*x^5 + 35*(143*b^6*d^2*e + 40*a^2*b^4*e^3 + 13*(11*b^6*c - 10*a*b^5*d)*e^
2)*x^4 + 5*(429*b^6*d^3 - 320*a^3*b^3*e^3 - 104*(11*a*b^5*c - 10*a^2*b^4*d)
*e^2 + 286*(9*b^6*c*d - 4*a*b^5*d^2)*e)*x^3 + 1664*(11*a^4*b^2*c - 10*a^5*b
*d)*e^2 + 3*(3003*b^6*c*d^2 - 858*a*b^5*d^3 + 640*a^4*b^2*e^3 + 208*(11*a^2
*b^4*c - 10*a^3*b^3*d)*e^2 + 143*(21*b^6*c^2 - 36*a*b^5*c*d + 16*a^2*b^4*d^
2)*e)*x^2 + 1144*(21*a^2*b^4*c^2 - 36*a^3*b^3*c*d + 16*a^4*b^2*d^2)*e + (15
015*b^6*c^2*d - 12012*a*b^5*c*d^2 + 3432*a^2*b^4*d^3 - 2560*a^5*b*e^3 - 832
*(11*a^3*b^3*c - 10*a^4*b^2*d)*e^2 - 572*(21*a*b^5*c^2 - 36*a^2*b^4*c*d + 1
6*a^3*b^3*d^2)*e)*x)*sqrt(b*x + a)/b^7
```

Sympy [A] time = 116.817, size = 1406, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**3/sqrt(a + b*x) + 6*a*c**2*d*(-a/sqrt(a + b*x) - sqrt(a
 + b*x))/b + 6*a*c**2*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)
**(3/2)/3)/b**2 + 6*a*c*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a +
 b*x)**(3/2)/3)/b**2 + 12*a*c*d*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*
x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 2*a*d**3*(-a**3/sqrt(a
 + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b
**3 + 6*a*c*e**2*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b
*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 6*a*d**2*e
*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a
*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 6*a*d*e**2*(-a**5/sqrt(a +
 b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x
)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 + 2*a*e**3*(a*
*6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*
(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a
 + b*x)**(11/2)/11)/b**6 + 2*c**3*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 6*c*
*2*d*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 6*c*
*2*e*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a
+ b*x)**(5/2)/5)/b**2 + 6*c*d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x
) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 12*c*d*e*(a**4/sqrt(a +
 b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/
2)/5 - (a + b*x)**(7/2)/7)/b**3 + 2*d**3*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(
```

```
a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7
/2)/7)/b**3 + 6*c*e**2*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**
3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (
a + b*x)**(9/2)/9)/b**4 + 6*d**2*e*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b
*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)*
*(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 6*d*e**2*(a**6/sqrt(a + b*x) + 6*a**5
*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 15*a**
2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/b**5
+ 2*e**3*(-a**7/sqrt(a + b*x) - 7*a**6*sqrt(a + b*x) + 7*a**5*(a + b*x)**(3
/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(a + b*x)**(7/2) - 7*a**2*(a + b*x)*
*(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a + b*x)**(13/2)/13)/b**6)/b, Ne(b,
0)), ((c**3*x + 3*c**2*d*x**2/2 + d*e**2*x**6/2 + e**3*x**7/7 + x**5*(3*c*e
**2 + 3*d**2*e)/5 + x**4*(6*c*d*e + d**3)/4 + x**3*(3*c**2*e + 3*c*d**2)/3)
/sqrt(a), True))
```

Giac [B] time = 1.11144, size = 710, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*c*d^2/b^2 + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c^2*e/b^2 + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 2574*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*c*d*e/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b
^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 65*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2
/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(
9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
+ a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

3.4 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{a+bx}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a
*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2)
)/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rubi [A] time = 0.0707695, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1850}

$$\frac{2\sqrt{a+bx}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*a
*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/2)
)/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx &= \int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3}+\frac{(be-3af)(a+bx)^{3/2}}{b^3}+\frac{f(\right.\\ &= \frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx}}{b^4}+\frac{2\left(b^2d-2abe+3a^2f\right)(a+bx)^{3/2}}{3b^4}+\frac{2(be-3af)(a}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.125911, size = 82, normalized size = 0.72

$$\frac{2\sqrt{a+bx}\left(8a^2b(7e+3fx)-48a^3f-2ab^2(35d+x(14e+9fx))+b^3(105c+x(35d+3x(7e+5fx)))\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35*d + x*(14
*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x)))))/(105*b^4)
```

Maple [A] time = 0.045, size = 91, normalized size = 0.8

$$-\frac{-30\, f x^3 b^3 + 36\, a b^2 f x^2 - 42\, b^3 e x^2 - 48\, a^2 b f x + 56\, a b^2 e x - 70\, b^3 d x + 96\, a^3 f - 112\, a^2 b e + 140\, a b^2 d - 210\, b^3 c}{105\, b^4}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-15*b^3*f*x^3+18*a*b^2*f*x^2-21*b^3*e*x^2-24*a^2*b*f*
x+28*a*b^2*e*x-35*b^3*d*x+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)/b^4
```

Maxima [A] time = 0.961995, size = 173, normalized size = 1.52

$$\frac{2\left(105\,\sqrt{bx+a}c + \frac{35\left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}a\right)d}{b} + \frac{7\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)e}{b^2} + \frac{3\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)f}{b^3}\right.}{105\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

Fricas [A] time = 1.19074, size = 216, normalized size = 1.89

$$\frac{2\left(15\, b^3 f x^3 + 105\, b^3 c - 70\, a b^2 d + 56\, a^2 b e - 48\, a^3 f + 3\left(7\, b^3 e - 6\, a b^2 f\right)x^2 + \left(35\, b^3 d - 28\, a b^2 e + 24\, a^2 b f\right)x\right)\sqrt{bx+a}}{105\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 3*(7
*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x)*sqrt(b*x
+ a)/b^4
```

Sympy [A] time = 15.2014, size = 354, normalized size = 3.11

$$\begin{cases} -\frac{\frac{2ac}{\sqrt{a+bx}} + \frac{2ad\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right)}{b} + \frac{2ae\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b^2} + \frac{2af\left(-\frac{a^3}{\sqrt{a+bx}} - 3a^2\sqrt{a+bx} + a(a+bx)^{\frac{3}{2}} - \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^3} + 2c\left(-\frac{a}{\sqrt{a+bx}} - \sqrt{a+bx}\right) + \frac{2d\left(\frac{a^2}{\sqrt{a+bx}} + 2a\sqrt{a+bx} - \frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b} +}{b} \\ \frac{cx + \frac{dx^2}{2} + \frac{ex^3}{3} + \frac{fx^4}{4}}{\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c/sqrt(a + b*x) + 2*a*d*(-a/sqrt(a + b*x) - sqrt(a + b*x))
/b + 2*a*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b*
*2 + 2*a*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2)
 - (a + b*x)**(5/2)/5)/b**3 + 2*c*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 2*d*
(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 2*e*(-a**
3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5
/2)/5)/b**2 + 2*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a +
b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0
)), ((c*x + d*x**2/2 + e*x**3/3 + f*x**4/4)/sqrt(a), True))
```

Giac [A] time = 1.08042, size = 174, normalized size = 1.53

$$\frac{2\left(105\sqrt{bx+a}c + \frac{35\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}a^2\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\right.}{b^3}\right.}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b +
 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2
+ 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35
*sqrt(b*x + a)*a^3)*f/b^3)/b
```

3.5 $\int \frac{\left(c+dx+ex^2+fx^3\right)^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=320

$$\frac{2(a+bx)^{5/2}\left(6a^2b^2\left(2df+e^2\right)-20a^3bef+15a^4f^2-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)}{5b^7}+\frac{4(a+bx)^{7/2}\left(10a^2bef-10a^3f^2-\right.}{7b^7}$$

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*(b^2*d - 2
*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/(3*b
^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*
f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f -
 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*
b^7) - (2*(10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9
*b^7) + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*x)^(1
3/2))/(13*b^7)
```

Rubi [A] time = 0.243653, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1850}

$$\frac{2(a+bx)^{5/2}\left(6a^2b^2\left(2df+e^2\right)-20a^3bef+15a^4f^2-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)}{5b^7}+\frac{4(a+bx)^{7/2}\left(10a^2bef-10a^3f^2-\right.}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*(b^2*d - 2
*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/(3*b
^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*
f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e*f -
 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/(7*
b^7) - (2*(10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/(9
*b^7) + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a + b*x)^(1
3/2))/(13*b^7)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)^2}{\sqrt{a+bx}}\,dx=\int \left(\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)^2}{b^6\sqrt{a+bx}}+\frac{2\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+b}}{b^6}\right.$$

$$=\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)^2\sqrt{a+bx}}{b^7}+\frac{4\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be-a^3f\right)(}{3b^7}$$

Mathematica [A] time = 0.457357, size = 303, normalized size = 0.95

$$\frac{2\left(\frac{1}{5}(a+bx)^{5/2}\left(6a^2b^2\left(2df+e^2\right)-20a^3bef+15a^4f^2-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)+\frac{2}{7}(a+bx)^{7/2}\left(10a^2bef-10a^3f^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

```
[Out] (2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x] + (2*(b^2*d - 2*a*b
*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/3 + ((b^
4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e + c*f) + 6*a^2*b
^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/5 + (2*(10*a^2*b*e*f - 10*a^3*f^2 + b^3*
(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2))/7 - ((10*a*b*e*f - 15
*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/2))/9 + (2*f*(b*e - 3*a*f)*(a +
b*x)^(11/2))/11 + (f^2*(a + b*x)^(13/2))/13))/b^7
```

Maple [A] time = 0.007, size = 447, normalized size = 1.4

$$6930 f^2x^6b^6 - 7560\, ab^5f^2x^5 + 16380\, b^6efx^5 + 8400\, a^2b^4f^2x^4 - 18200\, ab^5efx^4 + 20020\, b^6dfx^4 + 10010\, b^6e^2x^4 - 960$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/45045*(b*x+a)^(1/2)*(3465*b^6*f^2*x^6-3780*a*b^5*f^2*x^5+8190*b^6*e*f*x^5
+4200*a^2*b^4*f^2*x^4-9100*a*b^5*e*f*x^4+10010*b^6*d*f*x^4+5005*b^6*e^2*x^4
-4800*a^3*b^3*f^2*x^3+10400*a^2*b^4*e*f*x^3-11440*a*b^5*d*f*x^3-5720*a*b^5*
e^2*x^3+12870*b^6*c*f*x^3+12870*b^6*d*e*x^3+5760*a^4*b^2*f^2*x^2-12480*a^3*
b^3*e*f*x^2+13728*a^2*b^4*d*f*x^2+6864*a^2*b^4*e^2*x^2-15444*a*b^5*c*f*x^2-
15444*a*b^5*d*e*x^2+18018*b^6*c*e*x^2+9009*b^6*d^2*x^2-7680*a^5*b*f^2*x+166
40*a^4*b^2*e*f*x-18304*a^3*b^3*d*f*x-9152*a^3*b^3*e^2*x+20592*a^2*b^4*c*f*x
+20592*a^2*b^4*d*e*x-24024*a*b^5*c*e*x-12012*a*b^5*d^2*x+30030*b^6*c*d*x+15
360*a^6*f^2-33280*a^5*b*e*f+36608*a^4*b^2*d*f+18304*a^4*b^2*e^2-41184*a^3*b
^3*c*f-41184*a^3*b^3*d*e+48048*a^2*b^4*c*e+24024*a^2*b^4*d^2-60060*a*b^5*c*
d+45045*b^6*c^2)/b^7
```

Maxima [A] time = 0.991374, size = 675, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 858*c*(35*((b*x + a)^(3/2) - 3*sqrt(b*x
+ a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 3003*(3*(b*x + a)^(5/2) - 10*(b*x
 + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*x + a)^(9/2) - 1
80*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 +
315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f + 45*(b*e - 4*a*
f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2) + 105*(3*a^2*b*e
 - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*x + a))*d/b^4 +
130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2
- 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a
^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*
x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 60
```

```
06*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

Fricas [A] time = 1.26904, size = 1000, normalized size = 3.12

$$\frac{2\left(3465\,b^6f^2x^6+45045\,b^6c^2-60060\,ab^5cd+24024\,a^2b^4d^2+18304\,a^4b^2e^2+15360\,a^6f^2+630\left(13\,b^6ef-6\,ab^5f^2\right)x^5+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 24024*a^2*b^4
*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*e*f - 6*a*b^5*f^2)*x
^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(11*b^6*d - 10*a*b^5*e)*f)*x^4
+ 10*(1287*b^6*d*e - 572*a*b^5*e^2 - 480*a^3*b^3*f^2 + 13*(99*b^6*c - 88*a*
b^5*d + 80*a^2*b^4*e)*f)*x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 1920*a^
4*b^2*f^2 + 858*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4*d + 8
0*a^3*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a^3*b^3*
c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a*b^5*d^2 - 4576
*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c - 6*a^2*b^4*d)*e + 104*(99*
a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)*f)*x)*sqrt(b*x + a)/b^7
```

Sympy [A] time = 117.588, size = 1365, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-(2*a*c**2/sqrt(a + b*x) + 4*a*c*d*(-a/sqrt(a + b*x) - sqrt(a +
b*x))/b + 4*a*c*e*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2
)/3)/b**2 + 2*a*d**2*(a**2/sqrt(a + b*x) + 2*a*sqrt(a + b*x) - (a + b*x)**(
3/2)/3)/b**2 + 4*a*c*f*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a +
 b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 4*a*d*e*(-a**3/sqrt(a + b*x) - 3*
a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**3 + 4*a*d*
f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*
a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**4 + 2*a*e**2*(a**4/sqrt(a + b
*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)
/5 - (a + b*x)**(7/2)/7)/b**4 + 4*a*e*f*(-a**5/sqrt(a + b*x) - 5*a**4*sqrt(
a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a + b*x)**(5/2) + 5*a*(a +
b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**5 + 2*a*f**2*(a**6/sqrt(a + b*x) + 6
*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2) + 4*a**3*(a + b*x)**(5/2) - 1
5*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9/2)/3 - (a + b*x)**(11/2)/11)/
b**6 + 2*c**2*(-a/sqrt(a + b*x) - sqrt(a + b*x)) + 4*c*d*(a**2/sqrt(a + b*x
) + 2*a*sqrt(a + b*x) - (a + b*x)**(3/2)/3)/b + 4*c*e*(-a**3/sqrt(a + b*x)
- 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a + b*x)**(5/2)/5)/b**2 + 2*
d**2*(-a**3/sqrt(a + b*x) - 3*a**2*sqrt(a + b*x) + a*(a + b*x)**(3/2) - (a
+ b*x)**(5/2)/5)/b**2 + 4*c*f*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) -
2*a**2*(a + b*x)**(3/2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3
 + 4*d*e*(a**4/sqrt(a + b*x) + 4*a**3*sqrt(a + b*x) - 2*a**2*(a + b*x)**(3/
2) + 4*a*(a + b*x)**(5/2)/5 - (a + b*x)**(7/2)/7)/b**3 + 4*d*f*(-a**5/sqrt(
a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*a**2*(a +
b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 + 2*e**2*(-
```

```
a**5/sqrt(a + b*x) - 5*a**4*sqrt(a + b*x) + 10*a**3*(a + b*x)**(3/2)/3 - 2*
a**2*(a + b*x)**(5/2) + 5*a*(a + b*x)**(7/2)/7 - (a + b*x)**(9/2)/9)/b**4 +
 4*e*f*(a**6/sqrt(a + b*x) + 6*a**5*sqrt(a + b*x) - 5*a**4*(a + b*x)**(3/2)
 + 4*a**3*(a + b*x)**(5/2) - 15*a**2*(a + b*x)**(7/2)/7 + 2*a*(a + b*x)**(9
/2)/3 - (a + b*x)**(11/2)/11)/b**5 + 2*f**2*(-a**7/sqrt(a + b*x) - 7*a**6*s
qrt(a + b*x) + 7*a**5*(a + b*x)**(3/2) - 7*a**4*(a + b*x)**(5/2) + 5*a**3*(
a + b*x)**(7/2) - 7*a**2*(a + b*x)**(9/2)/3 + 7*a*(a + b*x)**(11/2)/11 - (a
 + b*x)**(13/2)/13)/b**6)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + e*f*x**6/3 +
f**2*x**7/7 + x**5*(2*d*f + e**2)/5 + x**4*(2*c*f + 2*d*e)/4 + x**3*(2*c*e
+ d**2)/3)/sqrt(a), True))
```

Giac [A] time = 1.11509, size = 697, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqrt(b*x + a)
*a)*c*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*d^2/b^2 + 6006*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(
b*x + a)*a^2)*c*e/b^2 + 2574*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*f/b^3 + 2574*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)
*d*e/b^3 + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^
(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*f/b^4 + 143*
(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420
*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 130*(63*(b*x + a)^(
11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5
/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*f*e/b^5 + 15*(2
31*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*
a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

3.6 $\int \frac{\left(c+dx+ex^2+fx^3\right)^3}{\sqrt{a+bx}} dx$

Optimal. Leaf size=708

$$-\frac{2(a+bx)^{7/2}\left(-30a^2b^4\left(2cef+d^2f+de^2\right)+20a^3b^3\left(3cf^2+6def+e^3\right)-105a^4b^2f\left(df+e^2\right)+168a^5bef^2-84a^6f^3+1\ldots}{7b^{10}}$$

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2*(b^2*d -
2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2))/b
^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*
f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)
^(5/2))/(5*b^10) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e +
3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) -
 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2
))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2
*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2
*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(9/2))/(3*b^10) -
(6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d
^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*
b^10) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*e*f - 12*a
^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) + (6*f^2*(b*e - 3*a*f)
*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*x)^(19/2))/(19*b^10)
```

Rubi [A] time = 0.625186, antiderivative size = 708, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1850}

$$-\frac{2(a+bx)^{7/2}\left(-30a^2b^4\left(2cef+d^2f+de^2\right)+20a^3b^3\left(3cf^2+6def+e^3\right)-105a^4b^2f\left(df+e^2\right)+168a^5bef^2-84a^6f^3+1\ldots}{7b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2*(b^2*d -
2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2))/b
^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*
f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)
^(5/2))/(5*b^10) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d*e +
3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) -
 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2
))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2
*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2
*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(9/2))/(3*b^10) -
(6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4*(d*e^2 + d
^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(11/2))/(11*
b^10) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*f) + b^3*(e^3
 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*a*b*e*f - 12*a
^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) + (6*f^2*(b*e - 3*a*f)
*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*x)^(19/2))/(19*b^10)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
```

```
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)^3}{\sqrt{a+bx}}\,dx = \int \left(\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)^3}{b^9\sqrt{a+bx}} + \frac{3\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be-a^3f\right)^2\sqrt{a}}{b^9}\right.$$

$$= \frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)^3\sqrt{a+bx}}{b^{10}} + \frac{2\left(b^2d-2abe+3a^2f\right)\left(b^3c-ab^2d+a^2be-a^3\right.}{b^{10}}$$

Mathematica [A] time = 2.26284, size = 678, normalized size = 0.96

$$\frac{2\left(\frac{1}{7}(a+bx)^{7/2}\left(30a^2b^4\left(2cef+d^2f+de^2\right)-20a^3b^3\left(3cf^2+6def+e^3\right)+105a^4b^2f\left(df+e^2\right)-168a^5bef^2+84a^6f^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

```
[Out] (2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x] + (b^2*d - 2*a*b*e
+ 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2) + (3*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b*e*f + 12*a^4*f^2
 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a + b*x)^(5/2))/5 + ((
-168*a^5*b*e*f^2 + 84*a^6*f^3 + b^6*(d^3 + 6*c*d*e + 3*c^2*f) + 105*a^4*b^2
*f*(e^2 + d*f) - 12*a*b^5*(d^2*e + c*e^2 + 2*c*d*f) + 30*a^2*b^4*(d*e^2 + d
^2*f + 2*c*e*f) - 20*a^3*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(7/2))/7
+ ((70*a^4*b*e*f^2 - 42*a^5*f^3 - 35*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c
*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*
e*f + 3*c*f^2))*(a + b*x)^(9/2))/3 + (3*(-56*a^3*b*e*f^2 + 42*a^4*f^3 + 21*
a^2*b^2*f*(e^2 + d*f) + b^4*(d*e^2 + d^2*f + 2*c*e*f) - 2*a*b^3*(e^3 + 6*d*
e*f + 3*c*f^2))*(a + b*x)^(11/2))/11 + ((84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a
*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/13 +
(f*(-8*a*b*e*f + 12*a^2*f^2 + b^2*(e^2 + d*f))*(a + b*x)^(15/2))/5 + (3*f^2
*(b*e - 3*a*f)*(a + b*x)^(17/2))/17 + (f^3*(a + b*x)^(19/2))/19))/b^10
```

Maple [B] time = 0.007, size = 1417, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x)
```

```
[Out] -2/4849845*(b*x+a)^(1/2)*(-255255*b^9*f^3*x^9+270270*a*b^8*f^3*x^8-855855*b
^9*e*f^2*x^8-288288*a^2*b^7*f^3*x^7+912912*a*b^8*e*f^2*x^7-969969*b^9*d*f^2
*x^7-969969*b^9*e^2*f*x^7+310464*a^3*b^6*f^3*x^6-983136*a^2*b^7*e*f^2*x^6+1
044582*a*b^8*d*f^2*x^6+1044582*a*b^8*e^2*f*x^6-1119195*b^9*c*f^2*x^6-223839
0*b^9*d*e*f*x^6-373065*b^9*e^3*x^6-338688*a^4*b^5*f^3*x^5+1072512*a^3*b^6*e
*f^2*x^5-1139544*a^2*b^7*d*f^2*x^5-1139544*a^2*b^7*e^2*f*x^5+1220940*a*b^8*
c*f^2*x^5+2441880*a*b^8*d*e*f*x^5+406980*a*b^8*e^3*x^5-2645370*b^9*c*e*f*x^
5-1322685*b^9*d^2*f*x^5-1322685*b^9*d*e^2*x^5+376320*a^5*b^4*f^3*x^4-119168
0*a^4*b^5*e*f^2*x^4+1266160*a^3*b^6*d*f^2*x^4+1266160*a^3*b^6*e^2*f*x^4-135
```

```
6600*a^2*b^7*c*f^2*x^4-2713200*a^2*b^7*d*e*f*x^4-452200*a^2*b^7*e^3*x^4+293
9300*a*b^8*c*e*f*x^4+1469650*a*b^8*d^2*f*x^4+1469650*a*b^8*d*e^2*x^4-323323
0*b^9*c*d*f*x^4-1616615*b^9*c*e^2*x^4-1616615*b^9*d^2*e*x^4-430080*a^6*b^3*
f^3*x^3+1361920*a^5*b^4*e*f^2*x^3-1447040*a^4*b^5*d*f^2*x^3-1447040*a^4*b^5
*e^2*f*x^3+1550400*a^3*b^6*c*f^2*x^3+3100800*a^3*b^6*d*e*f*x^3+516800*a^3*b
^6*e^3*x^3-3359200*a^2*b^7*c*e*f*x^3-1679600*a^2*b^7*d^2*f*x^3-1679600*a^2*
b^7*d*e^2*x^3+3695120*a*b^8*c*d*f*x^3+1847560*a*b^8*c*e^2*x^3+1847560*a*b^8
*d^2*e*x^3-2078505*b^9*c^2*f*x^3-4157010*b^9*c*d*e*x^3-692835*b^9*d^3*x^3+5
16096*a^7*b^2*f^3*x^2-1634304*a^6*b^3*e*f^2*x^2+1736448*a^5*b^4*d*f^2*x^2+1
736448*a^5*b^4*e^2*f*x^2-1860480*a^4*b^5*c*f^2*x^2-3720960*a^4*b^5*d*e*f*x^
2-620160*a^4*b^5*e^3*x^2+4031040*a^3*b^6*c*e*f*x^2+2015520*a^3*b^6*d^2*f*x^
2+2015520*a^3*b^6*d*e^2*x^2-4434144*a^2*b^7*c*d*f*x^2-2217072*a^2*b^7*c*e^2
*x^2-2217072*a^2*b^7*d^2*e*x^2+2494206*a*b^8*c^2*f*x^2+4988412*a*b^8*c*d*e*
x^2+831402*a*b^8*d^3*x^2-2909907*b^9*c^2*e*x^2-2909907*b^9*c*d^2*x^2-688128
*a^8*b*f^3*x+2179072*a^7*b^2*e*f^2*x-2315264*a^6*b^3*d*f^2*x-2315264*a^6*b^
3*e^2*f*x+2480640*a^5*b^4*c*f^2*x+4961280*a^5*b^4*d*e*f*x+826880*a^5*b^4*e^
3*x-5374720*a^4*b^5*c*e*f*x-2687360*a^4*b^5*d^2*f*x-2687360*a^4*b^5*d*e^2*x
+5912192*a^3*b^6*c*d*f*x+2956096*a^3*b^6*c*e^2*x+2956096*a^3*b^6*d^2*e*x-33
25608*a^2*b^7*c^2*f*x-6651216*a^2*b^7*c*d*e*x-1108536*a^2*b^7*d^3*x+3879876
*a*b^8*c^2*e*x+3879876*a*b^8*c*d^2*x-4849845*b^9*c^2*d*x+1376256*a^9*f^3-43
58144*a^8*b*e*f^2+4630528*a^7*b^2*d*f^2+4630528*a^7*b^2*e^2*f-4961280*a^6*b
^3*c*f^2-9922560*a^6*b^3*d*e*f-1653760*a^6*b^3*e^3+10749440*a^5*b^4*c*e*f+5
374720*a^5*b^4*d^2*f+5374720*a^5*b^4*d*e^2-11824384*a^4*b^5*c*d*f-5912192*a
^4*b^5*c*e^2-5912192*a^4*b^5*d^2*e+6651216*a^3*b^6*c^2*f+13302432*a^3*b^6*c
*d*e+2217072*a^3*b^6*d^3-7759752*a^2*b^7*c^2*e-7759752*a^2*b^7*c*d^2+969969
0*a*b^8*c^2*d-4849845*b^9*c^3)/b^10
```

Maxima [B] time = 1.05296, size = 1836, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^(3/2) - 3*
sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqr
t(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f
+ 45*(b*e - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2)
+ 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*
x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
 + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11
/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6) +
 138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
- 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(b*x + a)^(11/2)*f + 385*(b*e -
 5*a*f)*(b*x + a)^(9/2) - 990*(2*a*b*e - 5*a^2*f)*(b*x + a)^(7/2) + 1386*(3
*a^2*b*e - 5*a^3*f)*(b*x + a)^(5/2) - 1155*(4*a^3*b*e - 5*a^4*f)*(b*x + a)^
(3/2) + 3465*(a^4*b*e - a^5*f)*sqrt(b*x + a))*d^2/b^5 + 1615*(231*(b*x + a)
^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x +
a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*s
qrt(b*x + a)*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13
```

```
/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x
 + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6
435*sqrt(b*x + a)*a^7)*e^2*f/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x
+ a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 +
 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)
^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*e*f^2/b
^8 + 323*(3003*(b*x + a)^(15/2)*f^2 + 3465*(2*b*e*f - 7*a*f^2)*(b*x + a)^(1
3/2) + 4095*(b^2*e^2 - 12*a*b*e*f + 21*a^2*f^2)*(b*x + a)^(11/2) - 25025*(a
*b^2*e^2 - 6*a^2*b*e*f + 7*a^3*f^2)*(b*x + a)^(9/2) + 32175*(2*a^2*b^2*e^2
- 8*a^3*b*e*f + 7*a^4*f^2)*(b*x + a)^(7/2) - 9009*(10*a^3*b^2*e^2 - 30*a^4*
b*e*f + 21*a^5*f^2)*(b*x + a)^(5/2) + 15015*(5*a^4*b^2*e^2 - 12*a^5*b*e*f +
 7*a^6*f^2)*(b*x + a)^(3/2) - 45045*(a^5*b^2*e^2 - 2*a^6*b*e*f + a^7*f^2)*s
qrt(b*x + a))*d/b^7 + 21*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*
a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b
*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*
a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqr
t(b*x + a)*a^9)*f^3/b^9)/b
```

Fricas [A] time = 1.38497, size = 2993, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/4849845*(255255*b^9*f^3*x^9 + 4849845*b^9*c^3 - 9699690*a*b^8*c^2*d + 775
9752*a^2*b^7*c*d^2 - 2217072*a^3*b^6*d^3 + 1653760*a^6*b^3*e^3 - 1376256*a^
9*f^3 + 45045*(19*b^9*e*f^2 - 6*a*b^8*f^3)*x^8 + 3003*(323*b^9*e^2*f + 96*a
^2*b^7*f^3 + 19*(17*b^9*d - 16*a*b^8*e)*f^2)*x^7 + 231*(1615*b^9*e^3 - 1344
*a^3*b^6*f^3 + 19*(255*b^9*c - 238*a*b^8*d + 224*a^2*b^7*e)*f^2 + 646*(15*b
^9*d*e - 7*a*b^8*e^2)*f)*x^6 + 63*(20995*b^9*d*e^2 - 6460*a*b^8*e^3 + 5376*
a^4*b^5*f^3 - 76*(255*a*b^8*c - 238*a^2*b^7*d + 224*a^3*b^6*e)*f^2 + 323*(6
5*b^9*d^2 + 56*a^2*b^7*e^2 + 10*(13*b^9*c - 12*a*b^8*d)*e)*f)*x^5 + 35*(461
89*b^9*d^2*e + 12920*a^2*b^7*e^3 - 10752*a^5*b^4*f^3 + 4199*(11*b^9*c - 10*
a*b^8*d)*e^2 + 152*(255*a^2*b^7*c - 238*a^3*b^6*d + 224*a^4*b^5*e)*f^2 + 64
6*(143*b^9*c*d - 65*a*b^8*d^2 - 56*a^3*b^6*e^2 - 10*(13*a*b^8*c - 12*a^2*b^
7*d)*e)*f)*x^4 + 5*(138567*b^9*d^3 - 103360*a^3*b^6*e^3 + 86016*a^6*b^3*f^3
 - 33592*(11*a*b^8*c - 10*a^2*b^7*d)*e^2 - 1216*(255*a^3*b^6*c - 238*a^4*b^
5*d + 224*a^5*b^4*e)*f^2 + 92378*(9*b^9*c*d - 4*a*b^8*d^2)*e + 323*(1287*b^
9*c^2 - 2288*a*b^8*c*d + 1040*a^2*b^7*d^2 + 896*a^4*b^5*e^2 + 160*(13*a^2*b
^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a^4*b^5*c - 10*a^5*b^4*d)*e^2 +
 19456*(255*a^6*b^3*c - 238*a^7*b^2*d + 224*a^8*b*e)*f^2 + 3*(969969*b^9*c*
d^2 - 277134*a*b^8*d^3 + 206720*a^4*b^5*e^3 - 172032*a^7*b^2*f^3 + 67184*(1
1*a^2*b^7*c - 10*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c - 238*a^5*b^4*d + 224
*a^6*b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d + 16*a^2*b^7*d^2)*e - 64
6*(1287*a*b^8*c^2 - 2288*a^2*b^7*c*d + 1040*a^3*b^6*d^2 + 896*a^5*b^4*e^2 +
 160*(13*a^3*b^6*c - 12*a^4*b^5*d)*e)*f)*x^2 + 369512*(21*a^2*b^7*c^2 - 36*
a^3*b^6*c*d + 16*a^4*b^5*d^2)*e - 5168*(1287*a^3*b^6*c^2 - 2288*a^4*b^5*c*d
 + 1040*a^5*b^4*d^2 + 896*a^7*b^2*e^2 + 160*(13*a^5*b^4*c - 12*a^6*b^3*d)*e
)*f + (4849845*b^9*c^2*d - 3879876*a*b^8*c*d^2 + 1108536*a^2*b^7*d^3 - 8268
80*a^5*b^4*e^3 + 688128*a^8*b*f^3 - 268736*(11*a^3*b^6*c - 10*a^4*b^5*d)*e^
2 - 9728*(255*a^5*b^4*c - 238*a^6*b^3*d + 224*a^7*b^2*e)*f^2 - 184756*(21*a
*b^8*c^2 - 36*a^2*b^7*c*d + 16*a^3*b^6*d^2)*e + 2584*(1287*a^2*b^7*c^2 - 22
88*a^3*b^6*c*d + 1040*a^4*b^5*d^2 + 896*a^6*b^3*e^2 + 160*(13*a^4*b^5*c - 1
2*a^5*b^4*d)*e)*f)*x)*sqrt(b*x + a)/b^10
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18098, size = 1909, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/4849845*(4849845*sqrt(b*x + a)*c^3 + 4849845*((b*x + a)^(3/2) - 3*sqrt(b*
x + a)*a)*c^2*d/b + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*c*d^2/b^2 + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 138567*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 41
5701*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)*c^2*f/b^3 + 831402*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(
5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*d*e/b^3 + 92378*(
35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*d*f/b^4 + 46189*(35*(b*x + a
)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(
3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 20995*(63*(b*x + a)^(11/2) -
385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3
+ 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d^2*f/b^5 + 46189*(35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 41990*(63*(b*x + a)^(1
1/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*c*f*e/b^5 + 4845
*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2
 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/
2)*a^5 + 3003*sqrt(b*x + a)*a^6)*c*f^2/b^6 + 20995*(63*(b*x + a)^(11/2) - 3
85*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 +
 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 9690*(231*(b
*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*
(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 +
 3003*sqrt(b*x + a)*a^6)*d*f*e/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x
 + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 3
2175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2
)*a^6 - 6435*sqrt(b*x + a)*a^7)*d*f^2/b^7 + 1615*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*e^3/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*f*e^2/b^7 + 133*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
```

```
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*f^2*e/b^8 + 21*(121
55*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a
^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*
(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)
*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*f^3/b^9)/b
```

3.7 $\int \frac{c+dx}{a+bx^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rubi [A] time = 0.111138, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1860, 31, 634, 617, 204, 628}

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+bx^3}\,dx = \frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}} + \frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\ldots}$$

$$= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\text{Subst}\left(\int \frac{\ldots}{-3\ldots}\right.}{a^{2/3}b^{2/3}\ldots}$$

$$= -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\ldots\right)}{6a^{2/3}b^{2/3}}$$

Mathematica [A] time = 0.0553796, size = 124, normalized size = 0.77

$$\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [A] time = 0.044, size = 186, normalized size = 1.2

$$\frac{c}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{3b}\ln\left(x+\sqrt[3]{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*c/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*c/b/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*c/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))-1/3*d/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*d/b/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*d*3^(1/2)/b/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.86126, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
```

```
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

Sympy [A] time = 0.633847, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Giac [A] time = 1.11479, size = 216, normalized size = 1.34

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(d*(-a/b)^(1/3) + c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*s
qrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^
(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.8 $\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(}{3a(}$$

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rubi [A] time = 0.139398, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(}{3a(}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx = \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}} + \frac{\left(2c+\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \frac{(2}{}$$

$$= \frac{x(c+dx)}{3a\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right.}{}$$

Mathematica [A] time = 0.150732, size = 180, normalized size = 0.95

$$\frac{\frac{\left(a^{2/3}d-2\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^2)

Maple [A] time = 0.003, size = 238, normalized size = 1.3

$$\frac{cx}{3a\left(bx^3+a\right)}+\frac{2c}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2c\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x^3+a)^2,x)

[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9*c/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9*c/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 7.97279, size = 4961, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)

```
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

Sympy [A] time = 1.10274, size = 105, normalized size = 0.56

$$\mathrm{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Giac [A] time = 1.10932, size = 252, normalized size = 1.33

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}}+\frac{dx^{2}+cx}{3\,\left(bx^{3}+a\right)a}+\frac{\sqrt{3}\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}bc-\left(-ab^{2}\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{2}}+\frac{\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1
/3*(d*x^2 + c*x)/((b*x^3 + a)*a) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*
b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^
2) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

3.9 $\int \frac{c+dx}{\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=215

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3))
- ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rubi [A] time = 0.187046, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x))/(6*a*(a + b*x^3)^2) + (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3))
- ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^(1/
3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)*d)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx = \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} - \frac{\int \frac{-5c-4dx}{\left(a+bx^3\right)^2}\,dx}{6a}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}} + \frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\ldots}}{54a^{8/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\ldots\right)}{54a^{8/3}b^{2/3}}$$

$$= \frac{x(c+dx)}{6a\left(a+bx^3\right)^2} + \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)} - \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right)}{27a^{8/3}b^{2/3}}$$

Mathematica [A] time = 0.153778, size = 205, normalized size = 0.95

$$\frac{\frac{\left(2a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{9a^2x(c+dx)}{\left(a+bx^3\right)^2} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{3ax(5c+4d\ldots}{a+bx^3}}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

```
[Out] ((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (2
*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1/3)
 + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)
```

Maple [A] time = 0.004, size = 272, normalized size = 1.3

$$\frac{cx}{6a\left(bx^3+a\right)^2}+\frac{5cx}{18a^2\left(bx^3+a\right)}+\frac{5c}{27ba^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5c}{54ba^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5c\sqrt{3}}{27ba^2}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*c/a*x/(b*x^3+a)^2+5/18*c/a^2*x/(b*x^3+a)+5/27*c/a^2/b/(1/b*a)^(2/3)*ln(
x+(1/b*a)^(1/3))-5/54*c/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))+5/27*c/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))+1/6*d/a*x^2/(b*x^3+a)^2+2/9*d/a^2*x^2/(b*x^3+a)-2/27*d/a^2/b/(1/b*
a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/27*d/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3
)*x+(1/b*a)^(2/3))+2/27*d/a^2*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.43691, size = 5554, normalized size = 25.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 + 2
*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*
b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3
```

```
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(
-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*
d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3
 + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(
1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(1/3)*(a^2*b^2*x
^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*
d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*
(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*
a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))) + ((a^2*b^2*x^6
+ 2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a
^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*s
qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)
/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c
^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)
^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*
b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(
a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2
+ 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) -
20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2)
 + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
- 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((
125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2
*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)
```

Sympy [A] time = 1.90385, size = 146, normalized size = 0.68

$$\mathrm{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{8acx + 7\ldots}{18a^4 + \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*b*d
*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)
```

Giac [A] time = 1.12425, size = 279, normalized size = 1.3

$$-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3}+\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bc-2\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2}+\frac{4\,bdx^5+5\,b}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3
+ 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18*(4*b*d*x^5 + 5*b*c*x
^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2) + 1/54*(5*(-a*b^2)^(1/3)*a*b^
3*c + 2*(-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^4*b^4)
```

3.10 $\int \frac{c+dx}{\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=240

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{\cdots}}\right.}{81\sqrt{3}a^{11/3}b^{2/3}}$$

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2
) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3
)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3
)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243
*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rubi [A] time = 0.223105, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{\cdots}}\right.}{81\sqrt{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] (x*(c + d*x))/(9*a*(a + b*x^3)^3) + (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2
) + (2*x*(10*c + 7*d*x))/(81*a^3*(a + b*x^3)) - (2*(20*b^(1/3)*c + 7*a^(1/3
)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3
)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(243
*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^3\right)^4}\,dx = \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} - \frac{\int \frac{-8c-7dx}{\left(a+bx^3\right)^3}\,dx}{9a}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{\int \frac{40c+28dx}{\left(a+bx^3\right)^2}\,dx}{54a^2}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}c-28\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(80\sqrt[3]{b}c-28\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{486a^{11/3}\sqrt[3]{b}} + \cdots$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \cdots$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \cdots$$

$$= \frac{x(c+dx)}{9a\left(a+bx^3\right)^3} + \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} + \cdots$$

Mathematica [A] time = 0.192324, size = 229, normalized size = 0.95

$$\frac{\frac{2\left(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{54a^3x(c+dx)}{\left(a+bx^3\right)^3}+\frac{9a^2x(8c+7dx)}{\left(a+bx^3\right)^2}-\frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(1 -\right.}{b^{2/3}}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

```
[Out] ((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2
 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c +
 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(20
*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-
20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/b^(2/3))/(486*a^4)
```

Maple [A] time = 0.004, size = 306, normalized size = 1.3

$$\frac{cx}{9\,a\left(bx^3+a\right)^3}+\frac{4\,cx}{27\,a^2\left(bx^3+a\right)^2}+\frac{20\,cx}{81\,a^3\left(bx^3+a\right)}+\frac{40\,c}{243\,a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{20\,c}{243\,a^3b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] 1/9*c/a*x/(b*x^3+a)^3+4/27*c/a^2*x/(b*x^3+a)^2+20/81*c/a^3*x/(b*x^3+a)+40/2
43*c/a^3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-20/243*c/a^3/b/(1/b*a)^(2/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+40/243*c/a^3/b/(1/b*a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/9*d/a*x^2/(b*x^3+a)^3+7/54*d/a^2
*x^2/(b*x^3+a)^2+14/81*d/a^3*x^2/(b*x^3+a)-14/243*d/a^3/b/(1/b*a)^(1/3)*ln(
x+(1/b*a)^(1/3))+7/243*d/a^3/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))+14/243*d/a^3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^
(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.25727, size = 5960, normalized size = 24.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^2*d*x^8 + 240*b^2*c*x^7 + 462*a*b*d*x^5 + 624*a*b*c*x^4 + 402*
a^2*d*x^2 + 492*a^2*c*x - 2*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^
6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*
c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*
b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 + 7840*a*c*d^2
 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*
x^3 + a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3
 + a^6)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*s
qrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)
/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 +
 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8
*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) +
1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x
+ 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqr
t(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(
a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 3
43*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b
 + 8960*c*d)/(a^7*b))) + ((a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3))) - 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)*sq
rt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*
b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^
7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b
^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1
)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2
))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3
)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 + 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt
(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8
000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)
/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^
11*b^2))^(1/3)))*a^8*b*d + 1600*a^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*
```

```
((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/
(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*
d)/(a^7*b))))/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [A] time = 3.36028, size = 185, normalized size = 0.77

$$\mathrm{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 194400ta^4bc^2 + 78}{1372ad^3 + 32000bc^3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x + 67*
a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2*d*x
**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*x**9)
```

Giac [A] time = 1.10275, size = 312, normalized size = 1.3

$$-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{2\sqrt{3}\left(20\left(-ab^2\right)^{\frac{1}{3}}bc - 7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2} + \frac{28\,b^2dx^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^
4 + 2/243*sqrt(3)*(20*(-a*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) + 1/162*(28*b^2*d*x^8 +
 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2*d*x^2 + 82*a^2*c*x)/(
(b*x^3 + a)^3*a^3) + 1/243*(20*(-a*b^2)^(1/3)*a*b^3*c + 7*(-a*b^2)^(2/3)*a*
b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^4)
```

3.11 $\int \frac{a+bx}{d+ex^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))
```

Rubi [A] time = 0.121466, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1860, 31, 634, 617, 204, 628}

$$-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(d + e*x^3),x]
```

```
[Out] -(((b*d^(1/3) + a*e^(1/3))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) - a*e^(1/3))*Log[d^(1/3) + e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3))
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx}{d+ex^3}\,dx = \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}} + \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}+2a\sqrt[3]{e}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{1}{2}\left(\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2}}}{6d^{2/3}e^{2/3}}$$

$$= \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{d^{2/3}e^{2/3}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} + \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x\right.}{6d^{2/3}e^{2/3}}$$

Mathematica [A] time = 0.0579368, size = 125, normalized size = 0.78

$$\frac{-\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\right)-2\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Maple [A] time = 0.048, size = 186, normalized size = 1.2

$$\frac{a}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{a}{6e}\ln\left(x^2-\sqrt[3]{\frac{d}{e}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{a\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{b}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(e*x^3+d),x)
```

```
[Out] 1/3*a/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6*a/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1
/3)*x-1))-1/3*b/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(
2/(d/e)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.31883, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3
*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^
3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(
2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)
/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2
))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^
2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)
 - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^
3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^
(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2)
- (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1
/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a
^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d + a^3*e)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 2*a^2*d*e
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*
e))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*
```

```
d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3
*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*
e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3
*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d -
 a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d
+ a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2
*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^
2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d + 2*(b^3*d
+ a^3*e)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*
b*d^2*e + 2*a^2*d*e)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3)
 + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^
2*d*e + 16*a*b)/(d*e)))
```

Sympy [A] time = 1.24309, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3d^2e^2 + 9tabde - a^3e + b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e + 3ta^2de + 2ab^2d}{a^3e + b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t*
log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)))
)
```

Giac [A] time = 1.11335, size = 197, normalized size = 1.22

$$\frac{\sqrt{3}\left(\left(-de^2\right)^{\frac{1}{3}} ae - \left(-de^2\right)^{\frac{2}{3}} b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{3d} - \frac{\left(-de^{(-1)}\right)^{\frac{1}{3}}\left(\left(-de^{(-1)}\right)^{\frac{1}{3}} b + a\right)\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d} + \frac{\left((-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-d*e^2)^(1/3)*a*e - (-d*e^2)^(2/3)*b)*arctan(1/3*sqrt(3)*(2*x
 + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-2)/d - 1/3*(-d*e^(-1))^(1/3)*(
(-d*e^(-1))^(1/3)*b + a)*log(abs(x - (-d*e^(-1))^(1/3)))/d + 1/6*((-d*e^2)^
(2/3)*b*d*e^2 + (-d*e^2)^(1/3)*a*d*e^3)*e^(-4)*log(x^2 + (-d*e^(-1))^(1/3)*
x + (-d*e^(-1))^(2/3))/d^2
```

3.12 $\int \frac{a+bx}{d-ex^3}\,dx$

Optimal. Leaf size=161

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(2/3))
```

Rubi [A] time = 0.100357, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1861, 31, 634, 617, 204, 628}

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}-\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(d - e*x^3),x]
```

```
[Out] -(((b*d^(1/3) - a*e^(1/3))*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))
])/(Sqrt[3]*d^(2/3)*e^(2/3))) - ((b*d^(1/3) + a*e^(1/3))*Log[d^(1/3) - e^(1
/3)*x])/(3*d^(2/3)*e^(2/3)) + ((b*d^(1/3) + a*e^(1/3))*Log[d^(2/3) + d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(2/3))
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{3d^{2/3}} - \frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\sqrt[3]{e}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}\sqrt[3]{e}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} - \frac{1}{2}\left(-\frac{a}{\sqrt[3]{d}}+\frac{b}{\sqrt[3]{e}}\right)\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\int \cdots}{6d^{2/3}e^{2/3}}$$

$$= -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\text{Subst}\left(\int \cdots\right)}{d^{2/3}e^{2/\cdots}}$$

$$= -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{\cdots}\right)}{6d^{2/3}e^{2/3}}$$

Mathematica [A] time = 0.0489133, size = 125, normalized size = 0.78

$$\frac{-\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\right)-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{6d^{2/3}e^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(d - e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Maple [A] time = 0.004, size = 188, normalized size = 1.2

$$-\frac{a}{3e}\ln\left(x-\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a}{6e}\ln\left(x^2+\sqrt[3]{\frac{d}{e}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}} + \frac{a\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}+1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}} - \frac{b}{3e}\ln\left(x-\sqrt[3]{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-e*x^3+d),x)
```

[Out] -1/3*a/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6*a/e/(d/e)^(2/3)*ln(x^2+(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x+1))-1/3*b/e/(d/e)^(1/3)*ln(x-(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2+(d/e)^(1/3)*x+(d/e)^(2/3))-1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x+1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [C] time = 6.22848, size = 4563, normalized size = 28.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="fricas")

[Out] -1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d - (b^3*d - a^3*e)*x) + 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e))) + 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^

```
(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d +
 a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(9*(I
*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*
e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) -
1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3) + 1)*
(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) +
a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d - 2*(b^3*d - a^3*e)*x - 1/12*
sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*
d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3
*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*b*d^2*e + 6*a^2*d*e
)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d
- a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e
)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e - 144*a*b)/(d*e
)))
```

Sympy [A] time = 1.21121, size = 78, normalized size = 0.48

$$-\operatorname{RootSum}\left(27t^3d^2e^2 - 9tabde - a^3e - b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e - 3ta^2de - 2ab^2d}{a^3e - b^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d))
))
```

Giac [A] time = 1.07933, size = 155, normalized size = 0.96

$$-\frac{\sqrt{3}\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} - ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{(-2)}}{3d} - \frac{\left(bd^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + a\right)e^{\left(-\frac{1}{3}\right)}\log\left(\left|-d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + x\right|\right)}{3d^{\frac{2}{3}}} + \frac{\left(bd^{\frac{2}{3}}e^{\frac{4}{3}} + ad^{\frac{1}{3}}e^{\frac{5}{3}}\right)e^{(-2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*d^(2/3)*e^(4/3) - a*d^(1/3)*e^(5/3))*arctan(1/3*sqrt(3)*(d^
(1/3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-2)/d - 1/3*(b*d^(1/3)*e^(-1/3) +
 a)*e^(-1/3)*log(abs(-d^(1/3)*e^(-1/3) + x))/d^(2/3) + 1/6*(b*d^(2/3)*e^(4/
3) + a*d^(1/3)*e^(5/3))*e^(-2)*log(d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2
/3))/d
```

3.13 $\int \frac{1+x}{1+x^3} dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.0152534, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1586, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{1+x^3} dx &= \int \frac{1}{1-x+x^2} dx \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right)\right) \\
&= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0048812, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 + x^3),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.022, size = 17, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^3+1),x)
```

```
[Out] 2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.44645, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Fricas [A] time = 0.954347, size = 58, normalized size = 3.05

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Sympy [A] time = 0.103765, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.07871, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

3.14 $\int \frac{1-x}{1-x^3} \, dx$

Optimal. Leaf size=19

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.0133185, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1586, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 - x^3),x]

[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1-x}{1-x^3} \, dx &= \int \frac{1}{1+x+x^2} \, dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+2x\right)\right) \\ &= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0043036, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^3+1),x)
```

```
[Out] 2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.43222, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 0.892496, size = 58, normalized size = 3.05

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Sympy [A] time = 0.097308, size = 26, normalized size = 1.37

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**3+1),x)
```

```
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.15932, size = 22, normalized size = 1.16

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

3.15 $\int \frac{1+x}{1-x^3} \, dx$

Optimal. Leaf size=22

$$\frac{1}{3} \log \left(x^2+x+1\right)-\frac{2}{3} \log (1-x)$$

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rubi [A] time = 0.0121661, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1861, 31, 628}

$$\frac{1}{3} \log \left(x^2+x+1\right)-\frac{2}{3} \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 - x^3),x]

[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{1-x^3} \, dx &= -\left(\frac{1}{3} \int \frac{-1-2x}{1+x+x^2} \, dx\right)+\frac{2}{3} \int \frac{1}{1-x} \, dx \\ &= -\frac{2}{3} \log (1-x)+\frac{1}{3} \log \left(1+x+x^2\right) \end{aligned}$$

Mathematica [A] time = 0.003773, size = 22, normalized size = 1.

$$\frac{1}{3} \log \left(x^2+x+1\right)-\frac{2}{3} \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 - x^3),x]
```

```
[Out] (-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Maple [A] time = 0.004, size = 17, normalized size = 0.8

$$-\frac{2\ln(-1+x)}{3}+\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^3+1),x)
```

```
[Out] -2/3*ln(-1+x)+1/3*ln(x^2+x+1)
```

Maxima [A] time = 1.43893, size = 22, normalized size = 1.

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Fricas [A] time = 0.917206, size = 53, normalized size = 2.41

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Sympy [A] time = 0.09598, size = 17, normalized size = 0.77

$$-\frac{2\log(x-1)}{3}+\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

<u>Giac</u> [A] time = 1.19189, size = 23, normalized size = 1.05

$$\frac{1}{3}\log\left(x^2 + x + 1\right) - \frac{2}{3}\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))
```

3.16 $\int \frac{1-x}{1+x^3} \, dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0126704, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1860, 31, 628}

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + x^3),x]
```

```
[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{1+x^3}\,dx&=\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\&=\frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0036034, size = 22, normalized size = 1.

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + x^3),x]

[Out] (2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Maple [A] time = 0.004, size = 19, normalized size = 0.9

$$\frac{2\ln(1+x)}{3}-\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^3+1),x)

[Out] 2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.47228, size = 24, normalized size = 1.09

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="maxima")

[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Fricas [A] time = 1.00901, size = 54, normalized size = 2.45

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="fricas")

[Out] -1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] time = 0.091979, size = 17, normalized size = 0.77

$$\frac{2\log(x+1)}{3}-\frac{\log\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**3+1),x)

[Out] 2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

Giac [A] time = 1.11873, size = 26, normalized size = 1.18

$$-\frac{1}{3}\log\left(x^2 - x + 1\right) + \frac{2}{3}\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

3.17 $\int \frac{3-x}{1-x^3} \, dx$

Optimal. Leaf size=41

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Rubi [A] time = 0.0272263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1861, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x}{1-x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-7-2x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+2\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)-4\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1-x)+\frac{1}{3}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0075016, size = 41, normalized size = 1.

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)/(1 - x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2]
/3
```

Maple [A] time = 0.004, size = 33, normalized size = 0.8

$$-\frac{2\ln(-1+x)}{3}+\frac{\ln\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)/(-x^3+1),x)
```

```
[Out] -2/3*ln(-1+x)+1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.4424, size = 43, normalized size = 1.05

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

Fricas [A] time = 0.948494, size = 112, normalized size = 2.73

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
x - 1)
```

Sympy [A] time = 0.135038, size = 44, normalized size = 1.07

$$-\frac{2\log(x-1)}{3}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x**3+1),x)
```

```
[Out] -2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

Giac [A] time = 1.0882, size = 45, normalized size = 1.1

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log(
abs(x - 1))
```

3.18 $\int \frac{c+dx}{c^3+d^3x^3}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi [A] time = 0.0216336, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1586, 617, 204}

$$-\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (-2*ArcTan[(c - 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{c^3+d^3x^3}\,dx &= \int \frac{1}{c^2-cdx+d^2x^2}\,dx\\ &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2dx}{c}\right)}{cd}\\ &= -\frac{2\tan^{-1}\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}\end{aligned}$$

Mathematica [A] time = 0.0101002, size = 31, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{2dx-c}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
```

```
[Out] (2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A] time = 0.067, size = 35, normalized size = 1.2

$$\frac{2\sqrt{3}}{3cd}\arctan\left(\frac{\left(2d^2x-cd\right)\sqrt{3}}{3cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(d^3*x^3+c^3),x)
```

```
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x-c*d)*3^(1/2)/c/d)
```

Maxima [A] time = 1.45929, size = 46, normalized size = 1.59

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2d^2x-cd)}{3cd}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

Fricas [A] time = 0.903478, size = 72, normalized size = 2.48

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx-c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Sympy [C] time = 0.156581, size = 54, normalized size = 1.86

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c + s
qrt(3)*I*c)/(2*d))/3)/(c*d)
```

Giac [A] time = 1.11921, size = 38, normalized size = 1.31

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

3.19 $\int \frac{c-dx}{c^3-d^3x^3}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi [A] time = 0.0203174, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1586, 617, 204}

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c-dx}{c^3-d^3x^3}\,dx &= \int \frac{1}{c^2+cdx+d^2x^2}\,dx\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2dx}{c}\right)}{cd}\\
&= \frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}
\end{aligned}$$

Mathematica [A] time = 0.0074607, size = 29, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
```

```
[Out] (2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Maple [A] time = 0.002, size = 34, normalized size = 1.2

$$\frac{2\sqrt{3}}{3cd}\arctan\left(\frac{\left(2d^2x+cd\right)\sqrt{3}}{3cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(-d^3*x^3+c^3),x)
```

```
[Out] 2/3*3^(1/2)/c/d*arctan(1/3*(2*d^2*x+c*d)*3^(1/2)/c/d)
```

Maxima [A] time = 1.43697, size = 45, normalized size = 1.55

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2d^2x+cd)}{3cd}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)
```

Fricas [A] time = 1.02726, size = 72, normalized size = 2.48

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx+c)}{3c}\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Sympy [C] time = 0.213179, size = 53, normalized size = 1.83

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sqr
t(3)*I*c)/(2*d))/3)/(c*d)
```

Giac [A] time = 1.05908, size = 35, normalized size = 1.21

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

3.20 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=39

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3))
```

Rubi [A] time = 0.0266326, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1586, 617, 204}

$$-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3))
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=\int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}B}-\frac{\sqrt[3]{a}x}{B}+\frac{\sqrt[3]{b}x^2}{B}}\,dx$$
$$=\frac{(2B)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$
$$=-\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Mathematica [A] time = 0.0165907, size = 35, normalized size = 0.9

$$-\frac{2B\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

Maple [B] time = 0.043, size = 195, normalized size = 5.

$$\frac{B}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{6}\sqrt[3]{a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*B/b^(2/3)*a^(1/
3)/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B/b^(2/3)*a^(1/3
)/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*B/b^(
1/3)/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b^(1/3)/(1/b*a)^(1/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B/b^(1/3)*3^(1/2)/(1/b*a)^(1/3)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.09054, size = 323, normalized size = 8.28

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3-3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x^2+ab^{\frac{1}{3}}x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right),\ \frac{2\,\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,b^{\frac{1}{3}}x-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(1
/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)/(
b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1/3)
)/a^(1/3)]
```

Sympy [C] time = 0.430803, size = 88, normalized size = 2.26

$$\frac{B\left(-\frac{\sqrt{3}i\log\left(x+\frac{-B\sqrt[3]{a}-\sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-B\sqrt[3]{a}+\sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))/
3)/a**(1/3)
```

Giac [A] time = 1.11651, size = 65, normalized size = 1.67

$$\frac{2\,\sqrt{3}Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,b^{\frac{2}{3}}x-a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\,\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\,\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/sq
rt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

3.21 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

Optimal. Leaf size=41

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

```
[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)
)
```

Rubi [A] time = 0.0431953, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1586, 617, 204}

$$\frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] (2*B*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)
)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx = \int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB}+\frac{\sqrt[3]{a}x}{B}+\frac{\sqrt[3]{-b}x^2}{B}}\,dx$$
$$= -\frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$
$$= \frac{2B\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Mathematica [B] time = 0.0490526, size = 129, normalized size = 3.15

$$\frac{\sqrt[3]{-b}B\left(\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)+2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

```
[Out] ((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])))/(6*a^(1/3)*b^(2/3))
```

Maple [B] time = 0.01, size = 228, normalized size = 5.6

$$\frac{B\sqrt[3]{-1}}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B\sqrt[3]{-1}}{6}\sqrt[3]{a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B\sqrt[3]{-1}\sqrt{3}}{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*(-1)^(1/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*B/b^
(2/3)*(-1)^(1/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3)
)+1/3*B/b^(2/3)*(-1)^(1/3)*a^(1/3)/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))+1/3*B/b^(2/3)*(-1)^(1/3)*(-b)^(1/3)/(1/b*a)^(1/3)*l
n(x+(1/b*a)^(1/3))-1/6*B/b^(2/3)*(-1)^(1/3)*(-b)^(1/3)/(1/b*a)^(1/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*B/b^(2/3)*(-1)^(1/3)*(-b)^(1/3)*3^(1/2)
/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08293, size = 339, normalized size = 8.27

$$\left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\,a^{\frac{2}{3}}\,(-b)^{\frac{1}{3}}\,x-3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}\,(-b)^{\frac{2}{3}}\,x^2-a\,(-b)^{\frac{1}{3}}\,x-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-a}{bx^3+a}\right),\ \frac{2\sqrt{\frac{1}{3}}B\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-b)^{\frac{1}{3}}\right.}{a^{\frac{1}{3}}}\right.}{a^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fri
cas")
```

```
[Out] [sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x - 3*sqr
t(1/3)*(2*a^(2/3)*(-b)^(2/3)*x^2 - a*(-b)^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3
)) - a)/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*(-b)^(1/3)*x + a^(1
/3))/a^(1/3))/a^(1/3)]
```

Sympy [C] time = 0.47654, size = 105, normalized size = 2.56

$$-\frac{B\left(-\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}-\frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+\frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

```
[Out] -B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**(
2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I*a
**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

Giac [A] time = 1.09758, size = 78, normalized size = 1.9

$$\frac{2\sqrt{3}Bb\arctan\left(-\frac{\sqrt{3}\left(2\,(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}\,(-b)^{\frac{2}{3}}}\,(-b)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="gia
c")
```

```
[Out] 2/3*sqrt(3)*B*b*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-b)^(1/3))/s
qrt(a^(2/3)*(-b)^(2/3)))/(sqrt(a^(2/3)*(-b)^(2/3))*(-b)^(2/3))
```

3.22 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=118

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

```
[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^
(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.126368, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {260, 1593, 1871, 12, 292, 31, 634, 617, 204, 628}

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^
(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)dx=-\left(C\int\frac{x^2}{a+bx^3}\,dx\right)+\int\frac{Bx+Cx^2}{a+bx^3}\,dx$$
$$=-\frac{C\log\left(a+bx^3\right)}{3b}+\int\frac{x(B+Cx)}{a+bx^3}\,dx$$
$$=-\frac{C\log\left(a+bx^3\right)}{3b}+C\int\frac{x^2}{a+bx^3}\,dx+\int\frac{Bx}{a+bx^3}\,dx$$
$$=B\int\frac{x}{a+bx^3}\,dx$$
$$=-\frac{B\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{B\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$
$$=-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\int-\frac{\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{2/3}}+\frac{B\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}$$
$$=-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{B\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}\right.}{\sqrt[3]{a}b^{2/3}}$$
$$=-\frac{B\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

Mathematica [A] time = 0.011431, size = 90, normalized size = 0.76

$$\frac{B\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
+ b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*
b^(2/3))
```

Maple [A] time = 0.041, size = 94, normalized size = 0.8

$$-\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x)
```

```
[Out] -1/3*B/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b/(1/b*a)^(1/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2
)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.99017, size = 760, normalized size = 6.44

$$\left[\frac{3\sqrt{\frac{1}{3}}Bab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}B\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx\cdots\right.}{6ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b^2)^
(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt
(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*
B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*l
og(b*x - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] time = 0.134989, size = 26, normalized size = 0.22

$$B\,\mathrm{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

```
[Out] B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Giac [A] time = 1.07905, size = 155, normalized size = 1.31

$$-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}B\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}B\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - 1/3*sqrt(3)*(-a*b^2)^(2/
3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*(-
a*b^2)^(2/3)*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

3.23 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=118

$$-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}$$

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^
(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rubi [A] time = 0.106268, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {260, 1871, 12, 200, 31, 634, 617, 204, 628}

$$-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^
(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx &= -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{A+Cx^2}{a+bx^3}\,dx\\
&= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{A}{a+bx^3}\,dx\\
&= A\int \frac{1}{a+bx^3}\,dx\\
&= \frac{A\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}+\frac{A\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}}\\
&= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{A\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}}-\frac{A\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}}\\
&= \frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}\\
&= -\frac{A\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0124255, size = 90, normalized size = 0.76

$$
-\frac{A\left(\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
+ b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*
b^(1/3))
```

Maple [A] time = 0.043, size = 94, normalized size = 0.8

$$\frac{A}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{A}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{A\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x)
```

```
[Out] 1/3*A/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*A/b/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*A/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.00941, size = 765, normalized size = 6.48

$$\left[\frac{3\sqrt{\frac{1}{3}}Aab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(a^2b\right)^{\frac{2}{3}}A\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2l\right.\right.}{6a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*A*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*A*log(a*b*x^2 - (a^2
```

```
*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*A*log(a*b*x + (a^2*b)^(2/3
)))/(a^2*b), 1/6*(6*sqrt(1/3)*A*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^
(2/3)*A*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*
A*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] time = 0.155791, size = 22, normalized size = 0.19

$$A\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

```
[Out] A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Giac [A] time = 1.11605, size = 155, normalized size = 1.31

$$-\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}A\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/
3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*
b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

3.24 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx$

Optimal. Leaf size=161

$$-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rubi [A] time = 0.166297, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {260, 1871, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] -(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx &= -\left(C\int \frac{x^2}{a+bx^3}\,dx\right)+\int \frac{A+Bx+Cx^2}{a+bx^3}\,dx\\
&= -\frac{C\log\left(a+bx^3\right)}{3b}+C\int \frac{x^2}{a+bx^3}\,dx+\int \frac{A+Bx}{a+bx^3}\,dx\\
&= \frac{\int \frac{\sqrt[3]{a}\left(2A\sqrt[3]{b}+\sqrt[3]{a}B\right)+\sqrt[3]{b}\left(-A\sqrt[3]{b}+\sqrt[3]{a}B\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&= \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}+\frac{1}{2}\left(\frac{A}{\sqrt[3]{a}}+\frac{B}{\sqrt[3]{b}}\right.\\
&= \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}+\frac{\left(A\sqrt[3]{b}\right.}{}\\
&= -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\mathrm{l}}{}
\end{aligned}$$

Mathematica [A] time = 0.0388091, size = 124, normalized size = 0.77

$$\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [A] time = 0.043, size = 186, normalized size = 1.2

$$\frac{A}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{A}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{A\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\Bigg(x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x)
```

```
[Out] 1/3*A/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*A/b/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*A/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))-1/3*B/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*B*3^(1/2)/b/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.19941, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3
*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^
3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)
/(a^2*b^2))^(1/3)))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^
2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^
```

```
3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2)
- (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A
^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a + A^3*b)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*B*a^2*b + 2*A^2*a*b
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*
a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3
*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*
b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3
*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a -
 A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a
+ A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^
2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b - 2*A*B^2*a + 2*(B^3*a
+ A^3*b)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*
B*a^2*b + 2*A^2*a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3)
 + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^
2*a*b + 16*A*B)/(a*b)))
```

Sympy [A] time = 0.657534, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tABab - A^3b + B^3a, \left(t \mapsto t\log\left(x + \frac{9t^2Ba^2b + 3tA^2ab + 2AB^2a}{A^3b + B^3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t*
log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a)))
)
```

Giac [A] time = 1.08512, size = 224, normalized size = 1.39

$$-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ab - \left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Aab^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*b*(-a/b)^(1/3) + A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*A*b - (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*A*a*b^3 + (
-a*b^2)^(2/3)*B*a*b^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.25 $\int \frac{bx+cx^2}{d+ex^3} dx$

Optimal. Leaf size=134

$$\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(1/3)*e^
(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + (b*Log[d^(2/3)
 - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/3)*e^(2/3)) + (c*Log[d + e*x^3
])/(3*e)
```

Rubi [A] time = 0.113145, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1593, 1871, 12, 292, 31, 634, 617, 204, 628, 260}

$$\frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}}-\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}}+\frac{c\log\left(d+ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] -((b*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(1/3)*e^
(2/3))) - (b*Log[d^(1/3) + e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + (b*Log[d^(2/3)
 - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(1/3)*e^(2/3)) + (c*Log[d + e*x^3
])/(3*e)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{bx+cx^2}{d+ex^3}\,dx &= \int \frac{x(b+cx)}{d+ex^3}\,dx \\
&= c\int \frac{x^2}{d+ex^3}\,dx + \int \frac{bx}{d+ex^3}\,dx \\
&= \frac{c\log\left(d+ex^3\right)}{3e} + b\int \frac{x}{d+ex^3}\,dx \\
&= \frac{c\log\left(d+ex^3\right)}{3e} - \frac{b\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3\sqrt[3]{d}\sqrt[3]{e}} + \frac{b\int \frac{\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3\sqrt[3]{d}\sqrt[3]{e}} \\
&= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e} + \frac{b\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6\sqrt[3]{d}e^{2/3}} + \frac{b\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{e}} \\
&= -\frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{\sqrt[3]{d}e^{2/3}} \\
&= -\frac{b\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} - \frac{b\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c\log\left(d+ex^3\right)}{3e}
\end{aligned}$$

Mathematica [A] time = 0.0230979, size = 122, normalized size = 0.91

$$\frac{b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-2\sqrt{3}b\sqrt[3]{e}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2c\sqrt[3]{d}\log\left(d+ex^3\right)}{6\sqrt[3]{d}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

```
[Out] (-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^(
1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)
```

Maple [A] time = 0.043, size = 108, normalized size = 0.8

$$-\frac{b}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}+\frac{b}{6e}\ln\left(x^2-\sqrt[3]{\frac{d}{e}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}+\frac{b\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{d}{e}}}+\frac{c\ln\left(ex^3+d\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(e*x^3+d),x)
```

```
[Out] -1/3*b/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6*b/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/
3)*x+(d/e)^(2/3))+1/3*b*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x-1))+1/3*c*ln(e*x^3+d)/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.38466, size = 2425, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*e*sqrt(((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d
*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3
) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1
/3) - 2*c/e)*c*e + 4*c^2)/e^2)*arctan(1/8*sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1
```

```
/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/
e)^2*d*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*
(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e - 8*b^2*e*x + 4*b^2*e*sqrt(-(
(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)
/(d*e^3))^(1/3) - 2*c/e)^2*d*e^2*x - 4*b^2*e*x^2 + 4*c^2*d*x - 4*b*c*d + 2*
(2*c*d*e*x - b*d*e)*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) +
1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e))/(b^2*e)) + 4*c^2*d)*sqrt(((3*
(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d
*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^
3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2)/
b^3) + 2*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d
 - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*e*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e
^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d*e^
2 + (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^
3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + b^2*e*x + c^2*d) - ((3*(I*sqrt(3) + 1)
*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) -
2*c/e)*e + 6*c)*log(-1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^
2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d*e^2*x + b^2*e*x^2 - c
^2*d*x + b*c*d - 1/2*(2*c*d*e*x - b*d*e)*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3
+ 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)))/e
```

Sympy [A] time = 0.299123, size = 75, normalized size = 0.56

$$\mathrm{RootSum}\left(27t^3de^3 - 27t^2cde^2 + 9tc^2de + b^3e - c^3d, \left(t \mapsto t\log\left(x + \frac{9t^2de^2 - 6tcde + c^2d}{b^2e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(e*x**3+d),x)
```

```
[Out] RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**3
*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)))
)
```

Giac [A] time = 1.10152, size = 162, normalized size = 1.21

$$-\frac{\sqrt{3}\left(-de^2\right)^{\frac{2}{3}} b \arctan\left(\frac{\sqrt{3}\left(2x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right) e^{(-2)}}{3d} + \frac{1}{3} ce^{(-1)} \log\left(\left|x^3e + d\right|\right) + \frac{\left(-de^2\right)^{\frac{2}{3}} be^{(-2)} \log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}} x + \left(-de\right)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(-d*e^2)^(2/3)*b*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/
(-d*e^(-1))^(1/3))*e^(-2)/d + 1/3*c*e^(-1)*log(abs(x^3*e + d)) + 1/6*(-d*e^
2)^(2/3)*b*e^(-2)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d - 1/
3*(-d*e^(-1))^(2/3)*b*log(abs(x - (-d*e^(-1))^(1/3)))/d
```

3.26 $\int \frac{a+cx^2}{d-ex^3}\,dx$

Optimal. Leaf size=134

$$\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(1
/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/
(3*e)
```

Rubi [A] time = 0.0891273, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1871, 12, 200, 31, 634, 617, 204, 628, 260}

$$\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (a*ArcTan[(d^(1/3) + 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(1
/3)) - (a*Log[d^(1/3) - e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + (a*Log[d^(2/3) +
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(1/3)) - (c*Log[d - e*x^3])/
(3*e)
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+cx^2}{d-ex^3}\,dx &= c\int \frac{x^2}{d-ex^3}\,dx + \int \frac{a}{d-ex^3}\,dx \\
&= -\frac{c\log\left(d-ex^3\right)}{3e} + a\int \frac{1}{d-ex^3}\,dx \\
&= -\frac{c\log\left(d-ex^3\right)}{3e} + \frac{a\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{3d^{2/3}} + \frac{a\int \frac{2\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}} \\
&= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} - \frac{c\log\left(d-ex^3\right)}{3e} + \frac{a\int \frac{1}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{d}} + \frac{a\int \frac{\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}\sqrt[3]{e}} \\
&= -\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{c\log\left(d-ex^3\right)}{3e} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\right.}{d^{2/3}\sqrt[3]{e}} \\
&= \frac{a\tan^{-1}\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}} - \frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{c\log\left(d-ex^3\right)}{3e}
\end{aligned}
$$

Mathematica [A] time = 0.0299887, size = 123, normalized size = 0.92

$$
\frac{ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right) - 2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right) + 2\sqrt{3}ae^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right) - 2cd^{2/3}\log\left(d-ex^3\right)}{6d^{2/3}e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(d - e*x^3),x]
```

```
[Out] (2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(2
/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x +
e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)
```

Maple [A] time = 0.041, size = 111, normalized size = 0.8

$$-\frac{a}{3e}\ln\left(x-\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{a}{6e}\ln\left(x^2+\sqrt[3]{\frac{d}{e}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{a\sqrt{3}}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}+1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{c\ln\left(ex^3-d\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(-e*x^3+d),x)
```

```
[Out] -1/3*a/e/(d/e)^(2/3)*ln(x-(d/e)^(1/3))+1/6*a/e/(d/e)^(2/3)*ln(x^2+(d/e)^(1/
3)*x+(d/e)^(2/3))+1/3*a/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x+1))-1/3*c/e*ln(e*x^3-d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.29991, size = 2820, normalized size = 21.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(1/3)*e*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2
*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*
(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/
3) + 2*c/e)*c*e + 4*c^2)/e^2)*arctan(-1/8*(2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqr
t(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2
*c/e)*a*d*e^2 - 2*a*c*d*e)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^
3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^
(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3
))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3
/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d^2*e^
2 + 4*a^2*e^2*x^2 - 4*a*c*d*e*x + 4*c^2*d^2 + 2*(a*d*e^2*x - 2*c*d^2*e)*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2
*e^3))^(1/3) + 2*c/e))/(a^2*e^2)) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)
```

```
*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d
^2*e^2 - 8*a*c*d*e*x + 4*c^2*d^2 + 4*(a*d*e^2*x - c*d^2*e)*((1/2)^(1/3)*(I*
sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3)
+ 2*c/e))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*
d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)
*c*e + 4*c^2)/e^2))/(a^3*e)) - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^
3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e*log(-1/2*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^
3))^(1/3) + 2*c/e)*d*e + a*e*x + c*d) + (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/
e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e - 6*c)*
log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^
3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*d^2*e^2 + a^2*e^2*x^2 - a*c*d*e*x + c^2*
d^2 + 1/2*(a*d*e^2*x - 2*c*d^2*e)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a
^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)))/e
```

Sympy [A] time = 0.413526, size = 70, normalized size = 0.52

$$-\operatorname{RootSum}\left(27t^3d^2e^3 - 27t^2cd^2e^2 + 9tc^2d^2e - a^3e^2 - c^3d^2, \left(t \mapsto t\log\left(x + \frac{-3tde + cd}{ae}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(-e*x**3+d),x)
```

```
[Out] -RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a**
3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))
```

Giac [A] time = 1.09519, size = 128, normalized size = 0.96

$$-\frac{1}{3}\,ce^{(-1)}\log\left(\left|x^3e - d\right|\right) + \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)} + 2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{\left(-\frac{1}{3}\right)}}{3d^{\frac{2}{3}}} + \frac{ae^{\left(-\frac{1}{3}\right)}\log\left(d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)} + x^2 + d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d^{\frac{2}{3}}} - \frac{ae^{\left(-\frac{1}{3}\right)}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*c*e^(-1)*log(abs(x^3*e - d)) + 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(d^(1/
3)*e^(-1/3) + 2*x)*e^(1/3)/d^(1/3))*e^(-1/3)/d^(2/3) + 1/6*a*e^(-1/3)*log(d
^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) - 1/3*a*e^(-1/3)*log(ab
s(-d^(1/3)*e^(-1/3) + x))/d^(2/3)
```

3.27 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$

Optimal. Leaf size=37

$$\frac{\log(a+bx)}{b}-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}$$

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rubi [A] time = 0.0576785, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1868, 31, 617, 204}

$$\frac{\log(a+bx)}{b}-\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{ax}{b}+x^2}\,dx}{b^2} + \frac{\int \frac{1}{\frac{a}{b}+x}\,dx}{b}$$
$$= \frac{\log(a+bx)}{b} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2bx}{a}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b} + \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.0202841, size = 72, normalized size = 1.95

$$\frac{-\log\left(a^2-abx+b^2x^2\right)+\log\left(a^3+b^3x^3\right)+2\log(a+bx)+2\sqrt{3}\tan^{-1}\left(\frac{2bx-a}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Log[a^2 - a*
b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

Maple [A] time = 0.048, size = 43, normalized size = 1.2

$$\frac{2\sqrt{3}}{3b}\arctan\left(\frac{\left(2b^2x-ab\right)\sqrt{3}}{3ab}\right)+\frac{\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x)
```

```
[Out] 2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x-a*b)*3^(1/2)/a/b)+ln(b*x+a)/b
```

Maxima [A] time = 1.46757, size = 57, normalized size = 1.54

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2b^2x-ab)}{3ab}\right)}{3b}+\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x + a)/b
```

Fricas [A] time = 0.878056, size = 95, normalized size = 2.57

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)+3\log(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a) + 3*log(b*x + a))/b
```

Sympy [C] time = 0.513175, size = 60, normalized size = 1.62

$$\frac{-\frac{\sqrt{3}i\log\left(x+\frac{-a-\sqrt{3}ia}{2b}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

```
[Out] (-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x + (-a + s
qrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

Giac [A] time = 1.08222, size = 50, normalized size = 1.35

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx-a)}{3a}\right)}{3b}+\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + log(abs(b*x + a))/b
```

3.28 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3} \, dx$

Optimal. Leaf size=39

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

```
[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rubi [A] time = 0.0421376, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1868, 31, 617, 204}

$$\frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{ax}{b}+x^2}\,dx}{b^2} - \frac{\int \frac{1}{-\frac{a}{b}+x}\,dx}{b}$$
$$= -\frac{\log(a-bx)}{b} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2bx}{a}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b} - \frac{\log(a-bx)}{b}$$

Mathematica [A] time = 0.0178963, size = 71, normalized size = 1.82

$$\frac{\log\left(a^2+abx+b^2x^2\right)-\log\left(a^3-b^3x^3\right)-2\log(a-bx)+2\sqrt{3}\tan^{-1}\left(\frac{a+2bx}{\sqrt{3}a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*b
*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

Maple [A] time = 0.007, size = 45, normalized size = 1.2

$$\frac{2\sqrt{3}}{3b}\arctan\left(\frac{\left(2b^2x+ab\right)\sqrt{3}}{3ab}\right)-\frac{\ln(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x)
```

```
[Out] 2/3*3^(1/2)/b*arctan(1/3*(2*b^2*x+a*b)*3^(1/2)/a/b)-1/b*ln(b*x-a)
```

Maxima [A] time = 1.42968, size = 59, normalized size = 1.51

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2b^2x+ab\right)}{3ab}\right)}{3b}-\frac{\log(bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x - a)/b
```

Fricas [A] time = 0.998721, size = 95, normalized size = 2.44

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx+a)}{3a}\right)-3\log(bx-a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a) - 3*log(b*x - a))/b
```

Sympy [C] time = 0.433734, size = 60, normalized size = 1.54

$$-\frac{\frac{\sqrt{3}i\log\left(x+\frac{a-\sqrt{3}ia}{2b}\right)}{3}-\frac{\sqrt{3}i\log\left(x+\frac{a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
```

```
[Out] -(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x + (a + sqr
t(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

Giac [A] time = 1.07399, size = 51, normalized size = 1.31

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2bx+a)}{3a}\right)}{3b}-\frac{\log\left(|bx-a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - log(abs(b*x - a))/b
```

3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx$

Optimal. Leaf size=48

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1/
3)*x])/b^(1/3)
```

Rubi [A] time = 0.0369016, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1863, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1/
3)*x])/b^(1/3)
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \frac{(2C)\int \frac{1}{\frac{4}{b^{2/3}}-\frac{2x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}} + \frac{C\int \frac{1}{\frac{2}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}$$
$$= \frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$
$$= -\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.018627, size = 76, normalized size = 1.58

$$\frac{C\left(-\log\left(b^{2/3}x^2-2\sqrt[3]{b}x+4\right)+\log\left(bx^3+8\right)+2\log\left(\sqrt[3]{b}x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}x-1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Log
[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

Maple [B] time = 0.044, size = 117, normalized size = 2.4

$$\frac{C\sqrt[3]{8}}{3b}\ln\left(x+\sqrt[3]{8}\sqrt[3]{b^{-1}}\right)\left(b^{-1}\right)^{-\frac{2}{3}} - \frac{C\sqrt[3]{8}}{6b}\ln\left(x^2-\sqrt[3]{8}\sqrt[3]{b^{-1}}x+8^{\frac{2}{3}}\left(b^{-1}\right)^{\frac{2}{3}}\right)\left(b^{-1}\right)^{-\frac{2}{3}} + \frac{C\sqrt[3]{8}\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4}\frac{1}{\sqrt[3]{b^{-1}}} - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x)
```

```
[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(1/3)/(1/b)
^(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3*C/b*8^(1/3)/(1
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3*C/b
^(1/3)*ln(b*x^3+8)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08095, size = 387, normalized size = 8.06

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{bx^3+6\sqrt{\frac{1}{3}}\left(bx^2+b^{\frac{2}{3}}x-2b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-6b^{\frac{1}{3}}x-4}{bx^3+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)+Cb^{\frac{2}{3}}\log}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3)*
x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/3)
*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/3)*
x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

Sympy [A] time = 0.49057, size = 58, normalized size = 1.21

$$\operatorname{RootSum}\left(3t^3b^{\frac{5}{3}}-3t^2Cb^{\frac{4}{3}}+tC^2b-C^3b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{b}-C}{C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.30 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

```
[Out] -(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]) + (C*Log[a^(1/3)
 + 2*x])/4
```

Rubi [A] time = 0.0343279, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1863, 31, 617, 204}

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] -(C*ArcTan[(a^(1/3) - 4*x)/(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]) + (C*Log[a^(1/3)
 + 2*x])/4
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = \frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{a}}{2}+x}\,dx + \frac{1}{8}\left(\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{4}-\frac{\sqrt[3]{a}x}{2}+x^2}\,dx$$
$$= \frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right) + \frac{1}{2}C\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{4x}{\sqrt[3]{a}}\right)$$
$$= -\frac{C\tan^{-1}\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}} + \frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)$$

Mathematica [A] time = 0.023975, size = 72, normalized size = 1.53

$$\frac{1}{12}C\left(-\log\left(a^{2/3}-2\sqrt[3]{a}x+4x^2\right)+\log\left(a+8x^3\right)+2\log\left(\sqrt[3]{a}+2x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x] -
 Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

Maple [B] time = 0.047, size = 84, normalized size = 1.8

$$\frac{C8^{\frac{2}{3}}}{24}\ln\left(x+\frac{8^{\frac{2}{3}}}{8}\sqrt[3]{a}\right)-\frac{C8^{\frac{2}{3}}}{48}\ln\left(x^2-\frac{8^{\frac{2}{3}}x}{8}\sqrt[3]{a}+\frac{\sqrt[3]{8}}{8}a^{\frac{2}{3}}\right)+\frac{C8^{\frac{2}{3}}\sqrt{3}}{24}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{8}x}{\sqrt[3]{a}}-1\right)\right)+\frac{C\ln\left(8x^3+a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x)
```

```
[Out] 1/24*C*8^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*C*8^(2/3)*ln(x^2-1/8*8^(2/3)*
a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*C*8^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
*8^(1/3)/a^(1/3)*x-1))+1/12*C*ln(8*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.01958, size = 122, normalized size = 2.6

$$\frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}a^{\frac{2}{3}}x-\sqrt{3}a}{3a}\right)+\frac{1}{4}C\log\left(2x+a^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a) + 1/4*C*log(2
*x + a^(1/3))
```

Sympy [C] time = 0.442604, size = 85, normalized size = 1.81

$$C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2}+x\right)}{4}-\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}-\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}+\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}+\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

```
[Out] C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a**
(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))/(
4*C))/12)
```

Giac [B] time = 1.09818, size = 150, normalized size = 3.19

$$\frac{\sqrt{3}\left(\sqrt{3}i|a|+a\right)C\arctan\left(\frac{\sqrt{3}\left(4x+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{12a}+\frac{\left(\sqrt{3}i|a|+3a\right)C\log\left(x^2+\frac{1}{2}(-a)^{\frac{1}{3}}x+\frac{1}{4}(-a)^{\frac{2}{3}}\right)}{24a}-\frac{\left(C(-a)^{\frac{2}{3}}+2Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*i*abs(a) + a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/3))
/(-a)^(1/3))/a + 1/24*(sqrt(3)*i*abs(a) + 3*a)*C*log(x^2 + 1/2*(-a)^(1/3)*x
 + 1/4*(-a)^(2/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(abs
(x - 1/2*(-a)^(1/3)))/a
```

3.31 $\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 +
(-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi [A] time = 0.0689679, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1864, 31, 617, 204}

$$\frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 +
(-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 1864

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = -\frac{(2C)\int \frac{1}{\frac{4}{(-b)^{2/3}}-\frac{2x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}} - \frac{C\int \frac{1}{\frac{2}{\sqrt[3]{-b}}+x}\,dx}{\sqrt[3]{-b}}$$
$$= -\frac{C\log\left(2+\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$
$$= \frac{2C\tan^{-1}\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}} - \frac{C\log\left(2+\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Mathematica [A] time = 0.0291527, size = 99, normalized size = 1.74

$$\frac{C\left(-b^{2/3}\log\left(b^{2/3}x^2+2\sqrt[3]{b}x+4\right)+2b^{2/3}\log\left(2-\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}x+1}{\sqrt{3}}\right)+(-b)^{2/3}\log\left(8-bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3)*Log[2 -
b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[8
- b*x^3]))/(3*b)
```

Maple [B] time = 0.006, size = 122, normalized size = 2.1

$$\frac{C\sqrt[3]{8}}{3\,b}\ln\left(x-\sqrt[3]{8}\sqrt[3]{b^{-1}}\right)\left(b^{-1}\right)^{-\frac{2}{3}}-\frac{C\sqrt[3]{8}}{6\,b}\ln\left(x^2+\sqrt[3]{8}\sqrt[3]{b^{-1}}x+8^{\frac{2}{3}}\left(b^{-1}\right)^{\frac{2}{3}}\right)\left(b^{-1}\right)^{-\frac{2}{3}}-\frac{C\sqrt[3]{8}\sqrt{3}}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{8^{\frac{2}{3}}x}{4}\frac{1}{\sqrt[3]{b^{-1}}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x)
```

```
[Out] 1/3*C/b*8^(1/3)/(1/b)^(2/3)*ln(x-8^(1/3)*(1/b)^(1/3))-1/6*C/b*8^(1/3)/(1/b)
^(2/3)*ln(x^2+8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))-1/3*C/b*8^(1/3)/(1
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x+1))+1/3*C*(
-b)^(2/3)/b*ln(b*x^3-8)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.05274, size = 466, normalized size = 8.18

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{bx^3-6\sqrt{\frac{1}{3}}\left(bx^2-(-b)^{\frac{2}{3}}x+2\,(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+6\,(-b)^{\frac{1}{3}}x+4}{bx^3-8}\right)+C\,(-b)^{\frac{2}{3}}\log\left(bx-2\,(-b)^{\frac{2}{3}}\right)}{b},-\frac{2\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^3 - 6*sqrt(1/3)*(b*x^2 - (-b)^(
2/3)*x + 2*(-b)^(1/3))*sqrt((-b)^(1/3)/b) + 6*(-b)^(1/3)*x + 4)/(b*x^3 - 8)
) + C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(
1/3)/b)*arctan(sqrt(1/3)*((-b)^(2/3)*x - (-b)^(1/3))*sqrt(-(-b)^(1/3)/b)) -
 C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

Sympy [A] time = 0.582586, size = 58, normalized size = 1.02

$$\operatorname{RootSum}\left(3t^3b^2-3t^2Cb\,(-b)^{\frac{2}{3}}+tC^2\,(-b)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(-\frac{3t}{C}+x+\frac{(-b)^{\frac{2}{3}}}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

```
[Out] RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**3
*b, Lambda(_t, _t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.32 $\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$

Optimal. Leaf size=47

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3)
+ 2*x])/4
```

Rubi [A] time = 0.061084, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1864, 31, 617, 204}

$$\frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3)
+ 2*x])/4
```

Rule 1864

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C, 0
]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = -\left(\frac{1}{4}C\int \frac{1}{\frac{\sqrt[3]{-a}}{2} + x}\,dx\right) - \frac{1}{8}\left(\sqrt[3]{-a}C\right)\int \frac{1}{\frac{1}{4}(-a)^{2/3} - \frac{1}{2}\sqrt[3]{-a}x + x^2}\,dx$$
$$= -\frac{1}{4}C\log\left(\sqrt[3]{-a} + 2x\right) - \frac{1}{2}C\,\mathrm{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{4x}{\sqrt[3]{-a}}\right)$$
$$= \frac{C\tan^{-1}\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}C\log\left(\sqrt[3]{-a} + 2x\right)$$

Mathematica [B] time = 0.0390344, size = 106, normalized size = 2.26

$$\frac{C\left((-a)^{2/3}\log\left(a^{2/3} + 2\sqrt[3]{a}x + 4x^2\right) - a^{2/3}\log\left(8x^3 - a\right) - 2(-a)^{2/3}\log\left(\sqrt[3]{a} - 2x\right) + 2\sqrt{3}(-a)^{2/3}\tan^{-1}\left(\frac{\frac{4x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{12a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*(-a)^(2/3)
*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3)*x + 4*x^2] - a^(2/
3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

Maple [B] time = 0.007, size = 110, normalized size = 2.3

$$-\frac{C8^{\frac{2}{3}}}{24}(-a)^{\frac{2}{3}}\ln\left(x - \frac{8^{\frac{2}{3}}}{8}\sqrt[3]{a}\right)a^{-\frac{2}{3}} + \frac{C8^{\frac{2}{3}}}{48}(-a)^{\frac{2}{3}}\ln\left(x^2 + \frac{8^{\frac{2}{3}}x}{8}\sqrt[3]{a} + \frac{\sqrt[3]{8}}{8}a^{\frac{2}{3}}\right)a^{-\frac{2}{3}} + \frac{C8^{\frac{2}{3}}\sqrt{3}}{24}(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{8}x}{\sqrt[3]{a}} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x)
```

```
[Out] -1/24*C*(-a)^(2/3)*8^(2/3)/a^(2/3)*ln(x-1/8*8^(2/3)*a^(1/3))+1/48*C*(-a)^(2
/3)*8^(2/3)/a^(2/3)*ln(x^2+1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*
C*(-a)^(2/3)*8^(2/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*
x+1))-1/12*C*ln(8*x^3-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.02475, size = 130, normalized size = 2.77

$$\frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}(-a)^{\frac{2}{3}}x+\sqrt{3}a}{3a}\right)-\frac{1}{4}C\log\left(2x+(-a)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a) - 1/4*C*lo
g(2*x + (-a)^(1/3))
```

Sympy [C] time = 0.467872, size = 95, normalized size = 2.02

$$-C\left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}}+x\right)}{4}+\frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}-\frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}}+x\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
```

```
[Out] -C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3)) - sqrt(
3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a)**(2/3)) + sqrt(3)*
I*a/(4*(-a)**(2/3)) + x)/12)
```

Giac [B] time = 1.13767, size = 132, normalized size = 2.81

$$\frac{\sqrt{3}\left(\sqrt{3}i|a|-a\right)C\arctan\left(\frac{\sqrt{3}\left(4x+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{12a}+\frac{\left(\sqrt{3}i|a|-3a\right)C\log\left(x^2+\frac{1}{2}a^{\frac{1}{3}}x+\frac{1}{4}a^{\frac{2}{3}}\right)}{24a}-\frac{\left(2C(-a)^{\frac{2}{3}}+Ca^{\frac{2}{3}}\right)\log\left(\left|x-\frac{1}{2}a^{\frac{1}{3}}\right|\right)}{12a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*i*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^
(1/3))/a + 1/24*(sqrt(3)*i*abs(a) - 3*a)*C*log(x^2 + 1/2*a^(1/3)*x + 1/4*a^
(2/3))/a - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(abs(x - 1/2*a^(1/3)))/a^(2
/3)
```

3.33 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(
1/3) + x])/b
```

Rubi [A] time = 0.0765061, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1867, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(
1/3) + x])/b
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}+\frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$
$$=\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$
$$=-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.0519632, size = 146, normalized size = 2.92

$$\frac{C\left(-b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)+2b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\left(\frac{a}{b}\right)^{2/3}\tan^{-1}\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(2/3)*b^(2/3)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b*x^3]))/(
3*a^(2/3)*b)
```

Maple [A] time = 0.044, size = 87, normalized size = 1.7

$$\frac{2C}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\frac{C}{3b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(1/b*a)^(2/3)*C+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(1/b*a)^(1/3))/b-1/3*C/b*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2
/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.04965, size = 138, normalized size = 2.76

$$\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+3C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 3*
C*log(x + (a/b)^(1/3)))/b
```

Sympy [C] time = 0.536165, size = 100, normalized size = 2.

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [B] time = 1.12556, size = 224, normalized size = 4.48

$$\frac{\sqrt{3}\left(ab^2+\sqrt{3}\sqrt{a^2b^4}i\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}+\frac{\left(3ab^2+\sqrt{3}\sqrt{a^2l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(a*b^2 + sqrt(3)*sqrt(a^2*b^4)*i)*C*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(-a/b)^(2/3) + 2*(a*b^2)^(2
/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/6*(3*a*b^2 + sqr
t(3)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

3.34 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a/
b))^(1/3) + x])/b
```

Rubi [A] time = 0.0808304, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1867, 31, 617, 204}

$$\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```


```
[Out] (2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a/
b))^(1/3) + x])/b
```


Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b} - \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.0587242, size = 150, normalized size = 2.83

$$\frac{C\left(b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}\log\left(a-bx^3\right)-2b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\tan^{-1}\left(\right.\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(2/3)*
b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*
x^3]))/(3*a^(2/3)*b)
```

Maple [B] time = 0.007, size = 135, normalized size = 2.6

$$-\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(-1/b*a)^(2/3)*C+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x-(1/b*a)^(1/3))+1/3*C*(-1/b*a)^(2
/3)/b/(1/b*a)^(2/3)*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*(-1/b*a)^(2
/3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))-1/3*C
/b*ln(b*x^3-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.03322, size = 140, normalized size = 2.64

$$\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-3C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) - 3
*C*log(x + (-a/b)^(1/3)))/b
```

Sympy [C] time = 0.553843, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [B] time = 1.12249, size = 219, normalized size = 4.13

$$-\frac{\sqrt{3}\left(ab^2-\sqrt{3}\sqrt{a^2b^4}i\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}-\frac{\left(3ab^2-\sqrt{3}\sqrt{a^2b^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(a*b^2 - sqrt(3)*sqrt(a^2*b^4)*i)*C*arctan(1/3*sqrt(3)*(2*x +
(a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(a/b)^(2/3) + 2*(-a*b^2)^(2/
3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2) - 1/6*(3*a*b^2 - sqrt(3
)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3)
```

3.35 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a
/b))^(1/3) - x])/b
```

Rubi [A] time = 0.0598418, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1869, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a
/b))^(1/3) - x])/b
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b} - \frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b} + \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$= -\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.0465669, size = 149, normalized size = 2.76

$$\frac{C\left(-b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}\log\left(a+bx^3\right)+2b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\left(-\frac{a}{b}\right)^{2/3}\text{ta}\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (-(a/b))^(2/3)
*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b
*x^3]))/(3*a^(2/3)*b)
```

Maple [B] time = 0.003, size = 132, normalized size = 2.4

$$\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(-1/b*a)^(2/3)*C+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/3*C*(-1/b*a)^(2/
3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*(-1/b*a)^(2/
3)/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/
b*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.982751, size = 140, normalized size = 2.59

$$\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+3C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 3
*C*log(x - (-a/b)^(1/3)))/b
```

Sympy [C] time = 0.523065, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [A] time = 1.09283, size = 123, normalized size = 2.28

$$-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b^2*(-a/b)^(2/3) + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b^2)
```

3.36 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1
/3) - x])/b
```

Rubi [A] time = 0.0582328, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1869, 31, 617, 204}

$$\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(1
/3) - x])/b
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} + \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$
$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$
$$= \frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.0430835, size = 147, normalized size = 2.77

$$\frac{C\left(b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}\log\left(a-bx^3\right)-2b^{2/3}\left(\frac{a}{b}\right)^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}b^{2/3}\left(\frac{a}{b}\right)^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.\right.}{3a^{2/3}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (a/b)^(2/3)*b^(2/3)*L
og[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))/(3
*a^(2/3)*b)
```

Maple [A] time = 0.005, size = 90, normalized size = 1.7

$$-\frac{2C}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)+\frac{C}{3b}\ln\left(x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(1/b*a)^(2/3)*C+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(1/b*a)^(1/3))+1/3*C/b*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+
2/3*C*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))/b*3^(1/2)-1/3*C/b*ln(b*x^3-
a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.0441, size = 138, normalized size = 2.6

$$\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-3C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) - 3*
C*log(x - (a/b)^(1/3)))/b
```

Sympy [C] time = 0.622831, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [A] time = 1.08276, size = 115, normalized size = 2.17

$$\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*(
C*b^2*(a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3))
)/(a*b^2)
```

3.37 $\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=61

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3))
+ (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.0412275, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1863, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3))
+ (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{\left(\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx}{b^{2/3}} + \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx}{\sqrt[3]{b}}$$

$$= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.0173002, size = 95, normalized size = 1.56

$$\frac{C\left(-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3)
+ b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b*x
^3]))/(3*b^(1/3))
```

Maple [B] time = 0.005, size = 117, normalized size = 1.9

$$\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/3*C*a^(2/3)/b/(1/b*a)^(
2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.05073, size = 463, normalized size = 7.59

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3-3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-a}{bx^3+a}\right)+Cb^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b}, \frac{2\sqrt{\frac{1}{3}}Cb^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab\right.}{ab^{\frac{1}{3}}}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqr
t(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*b^(2/3)*x - a*b^(1/3))*sqrt(-1/b^(2/3)) -
 a)/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)*C*
b^(2/3)*arctan(sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x - a*b^(1/3))/(a*b^(1/3))) + C
*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

Sympy [A] time = 0.668899, size = 70, normalized size = 1.15

$$\operatorname{RootSum}\left(3t^3b^{\frac{5}{3}}-3t^2Cb^{\frac{4}{3}}+tC^2b-C^3b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{a}\sqrt[3]{b}-C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1/3))/(2*C*b**(1/3)))
))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.38 $\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(-b)^(
1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi [A] time = 0.0718664, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1866, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(-b)^(
1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 1866

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, -Di
st[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{\left(\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}} - \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}}$$
$$= \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}} + \frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{-b}}$$
$$= -\frac{2C\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}} + \frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Mathematica [A] time = 0.0283125, size = 116, normalized size = 1.66

$$-\frac{C\left(-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+(-b)^{2/3}\log\left(a+bx^3\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] -(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(
2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3]))/(3*b)
```

Maple [B] time = 0.007, size = 122, normalized size = 1.7

$$-\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{2C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] -2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/3*C*a^(2/3)/b/(1/b*a)^
(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*C*(-b)^(2/3)/b*ln(b*x^
3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08685, size = 539, normalized size = 7.7

$$\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,bx^3+3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-a}{bx^3+a}\right)-C\,(-b)^{\frac{2}{3}}\log\left(bx+a^{\frac{1}{3}}\,(-b)^{\frac{2}{3}}\right)}{b},-\frac{2\,\sqrt{\frac{1}{3}}Cb\sqrt{-\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x -
3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*(-b)^(2/3)*x + a*(-b)^(1/3))*sqrt((-
b)^(1/3)/b) - a)/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3)))
/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a^(2/3)*(-b)^
(2/3)*x + a*(-b)^(1/3))*sqrt(-(-b)^(1/3)/b)/a) + C*(-b)^(2/3)*log(b*x + a^(
1/3)*(-b)^(2/3)))/b]
```

Sympy [A] time = 0.765978, size = 73, normalized size = 1.04

$$-\operatorname{RootSum}\left(3t^3b^2-3t^2Cb\,(-b)^{\frac{2}{3}}+tC^2\,(-b)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(\frac{3t\sqrt[3]{a}}{2C}-\frac{\sqrt[3]{a}\,(-b)^{\frac{2}{3}}}{2b}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] -RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**
3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(-b)**(2/3)/(2*b) + x
)))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.39 $\int \frac{-3+x^2}{-1+x^3}\,dx$

Optimal. Leaf size=40

$$\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])
/6
```

Rubi [A] time = 0.0301783, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1875, 31, 634, 618, 204, 628}

$$\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - (2*Log[1 - x])/3 + (5*Log[1 + x + x^2])
/6
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x^2}{-1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{-7-5x}{1+x+x^2}\,dx\right)+\frac{2}{3}\int \frac{1}{1-x}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{5}{6}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= -\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)-3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{2}{3}\log(1-x)+\frac{5}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0098046, size = 50, normalized size = 1.25

$$\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(1-x^3\right)-\log(1-x)+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^2)/(-1 + x^3),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]/2 + Log[1
 - x^3]/3
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$-\frac{2\,\ln(-1+x)}{3}+\frac{5\,\ln\left(x^2+x+1\right)}{6}+\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3)/(x^3-1),x)
```

```
[Out] -2/3*ln(-1+x)+5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.45667, size = 42, normalized size = 1.05

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

Fricas [A] time = 1.00098, size = 107, normalized size = 2.68

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
1)
```

Sympy [A] time = 0.123212, size = 42, normalized size = 1.05

$$-\frac{2\log\left(x-1\right)}{3}+\frac{5\log\left(x^2+x+1\right)}{6}+\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3)/(x**3-1),x)
```

```
[Out] -2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)
```

Giac [A] time = 1.20438, size = 43, normalized size = 1.08

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^3-1),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs(
x - 1))
```

3.40 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.0665512, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {1863, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b*
x^3),x]
```

```
[Out] (-2*(B/a^(1/3) + C/b^(1/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/Sqrt[3] + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}dx}{\sqrt[3]{b}} + \frac{\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}dx}{b^{2/3}}$$
$$= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \left(2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)$$
$$= -\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} + \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Mathematica [A] time = 0.0464287, size = 122, normalized size = 1.74

$$\frac{\sqrt[3]{a}C\left(-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\log\left(a+bx^3\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(
a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1/3)*b^(1/3))
```

Maple [B] time = 0.004, size = 310, normalized size = 4.4

$$\frac{B}{3}\sqrt[3]{a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{6}\sqrt[3]{a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{-\frac{2}{3}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 1/3*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/3*C*a^(2/3)/b/(1/
b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^
(1/3)*x+(1/b*a)^(2/3))+1/3*B/b^(2/3)*a^(1/3)/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*B/b^(1/3)/(1/b*a)^(1/3)*ln(x+(1/
b*a)^(1/3))+1/6*B/b^(1/3)/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))+1/3*B/b^(1/3)*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.3042, size = 1058, normalized size = 15.11

$$\left[\frac{\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{C^{2}ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^{2}a^{\frac{1}{3}}b}{ab}}\,\log\left(-\frac{C^{3}a^{2}+B^{3}ab-2\left(C^{3}ab+B^{3}b^{2}\right)x^{3}+3\left(C^{3}a+B^{3}b\right)a^{\frac{2}{3}}b^{\frac{1}{3}}x-3\sqrt{\frac{1}{3}}\left(\left(2\,B^{2}bx^{2}+C^{2}ax+BCa\right)a^{\frac{2}{3}}b^{\frac{2}{3}}+\left(2\,C^{2}abx^{2}-BCab\ldots\right.\right.}{bx^{3}+a}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)
/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a + B^
3*b)*a^(2/3)*b^(1/3)*x - 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^(2/
3)*b^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) - (2*B*C*a*b*x^2
 - B^2*a*b*x + C^2*a^2)*b^(1/3))*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/
3) + B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2
/3)))/b, (2*sqrt(1/3)*b*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a
^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*b^(2/3) - (2*B*C
*b*x + B^2*b)*a^(1/3) + (2*B^2*b*x - C^2*a)*b^(1/3))*sqrt((C^2*a*b^(1/3) +
2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*b^(2/3)*
log(b*x + a^(1/3)*b^(2/3)))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/(
b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+
a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.41 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rubi [A] time = 0.112343, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.07, Rules used = {1866, 31, 617, 204}

$$\frac{2\left(\sqrt[3]{a}(-b)^{2/3}C+bB\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^2
)/(a + b*x^3),x]
```

```
[Out] (2*(b*B + a^(1/3)*(-b)^(2/3)*C)*ArcTan[(a^(1/3) + 2*(-b)^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

Rule 1866

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, -Di
st[C/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2),
x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /; Fr
eeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = -\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}\,dx}{\sqrt[3]{-b}} + \frac{\left(\sqrt[3]{-b}B - \sqrt[3]{a}C\right)\int \frac{1}{\frac{a^{2/3}}{(-b)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+x^2}\,dx}{(-b)^{2/3}}$$

$$= \frac{C\log\left(\sqrt[3]{a} - \sqrt[3]{-b}x\right)}{\sqrt[3]{-b}} - \left(2\left(\frac{B}{\sqrt[3]{a}} + \frac{bC}{(-b)^{4/3}}\right)\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x,\right.$$

$$= \frac{2\left(\frac{B}{\sqrt[3]{a}} + \frac{bC}{(-b)^{4/3}}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} + \frac{C\log\left(\sqrt[3]{a} - \sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

Mathematica [B] time = 0.526485, size = 238, normalized size = 2.7

$$\frac{\frac{\left(2\sqrt[3]{a}b\sqrt[3]{-b}C+b^{5/3}B+(-b)^{5/3}B\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2b\left(2\sqrt[3]{a}\sqrt[3]{-b}C+\left(b^{2/3}-(-b)^{2/3}\right)B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt[3]{a}(-b)^{2/3}\sqrt[3]{-b^2}C\log\left(a+bx^3\right)}{\sqrt[3]{-b^2}} + 2\sqrt{3}\sqrt[3]{b}\left(2\sqrt[3]{a}\right.}{6\sqrt[3]{a}b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)
*C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*(((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(1/3)*C)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-(-b)^(2/3) + b^(2/3))*
B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(1/3)*x] + ((-b)^(5/3)*B + b^(5
/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2] - 2*a^(1/3)*(-b)^(2/3)*(-b^2)^(1/3)*C*Log[a + b*x^3])/(-b^2)^(1/3))/(
6*a^(1/3)*b)
```

Maple [B] time = 0.006, size = 345, normalized size = 3.9

$$\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\sqrt[3]{a}\sqrt[3]{-b}\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{2C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\sqrt[3]{a}\sqrt[3]{-b}\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{C}{3b}a^{\frac{2}{3}}\ln\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*x
^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^(1/3)*(-b)^(1/3)*B-2/3*C*a^(2/3)/
b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3
)*x+(1/b*a)^(2/3))*a^(1/3)*(-b)^(1/3)*B+1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x^
2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))*a^(1/3)*(-b)^(1/3)*B-2/3*C*a^(2/3)/b/(1/b*a)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*B*(-b)^(2/3)/b/(1
/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-1/6*B*(-b)^(2/3)/b/(1/b*a)^(1/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*B*(-b)^(2/3)*3^(1/2)/b/(1/b*a)^(1/3)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*C*(-b)^(2/3)/b*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.57486, size = 1129, normalized size = 12.83

$$\left[\frac{\sqrt{\frac{1}{3}}b\sqrt{\frac{C^2a(-b)^{\frac{1}{3}}-2\,BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3b^2\right)x^3-3\left(C^3a+B^3b\right)a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x+3\sqrt{\frac{1}{3}}\left(\left(2\,B^2bx^2+C^2ax+BCa\right)a^{\frac{2}{3}}(-b)^{\frac{2}{3}}+\left(2\,C^2a\right.\right.}{bx^3+a}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(1/3)*b*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/
3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 - 3*(C^3*a
 + B^3*b)*a^(2/3)*(-b)^(1/3)*x + 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*
a)*a^(2/3)*(-b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) + (2*
B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*(-b)^(1/3))*sqrt((C^2*a*(-b)^(1/3) - 2*B
*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) - C*(-b)^(2/3)*
log(b*x + a^(1/3)*(-b)^(2/3)))/b, -(2*sqrt(1/3)*b*sqrt(-(C^2*a*(-b)^(1/3) -
 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*
x + B*C)*a^(2/3)*(-b)^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) - (2*B^2*b*x - C^
2*a)*(-b)^(1/3))*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a
^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/
3)))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-b)**(2/3)*
C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2
)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.42 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} dx$

Optimal. Leaf size=11

$$\frac{\log(B-Cx)}{C}$$

```
[Out] Log[B - C*x]/C
```

Rubi [A] time = 0.0105686, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1586, 31}

$$\frac{\log(B-Cx)}{C}$$

Antiderivative was successfully verified.

```
[In] Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

```
[Out] Log[B - C*x]/C
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3} dx &= \int \frac{1}{-B+Cx} dx \\ &= \frac{\log(B-Cx)}{C} \end{aligned}$$

Mathematica [A] time = 0.0015161, size = 12, normalized size = 1.09

$$\frac{\log(Cx-B)}{C}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

```
[Out] Log[-B + C*x]/C
```

Maple [A] time = 0.039, size = 12, normalized size = 1.1

$$\frac{\ln(-Cx+B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x)
```

```
[Out] ln(-C*x+B)/C
```

Maxima [A] time = 0.99526, size = 16, normalized size = 1.45

$$\frac{\log(Cx-B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="maxima")
```

```
[Out] log(C*x - B)/C
```

Fricas [A] time = 0.918183, size = 22, normalized size = 2.

$$\frac{\log(Cx-B)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
```

```
[Out] log(C*x - B)/C
```

Sympy [A] time = 0.068666, size = 7, normalized size = 0.64

$$\frac{\log(-B+Cx)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
```

```
[Out] log(-B + C*x)/C
```

Giac [A] time = 1.50249, size = 18, normalized size = 1.64

$$\frac{\log(|Cx-B|)}{C}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")

[Out] log(abs(C*x - B))/C
```

3.43 $\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=21

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] time = 0.0145199, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1586, 31}

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx &= \int \frac{1}{\frac{\sqrt[3]{a}}{C}+\frac{\sqrt[3]{b}x}{C}}\,dx\\ &= \frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.0028395, size = 21, normalized size = 1.

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

```
[Out] (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Maple [B] time = 0.005, size = 218, normalized size = 10.4

$$\frac{C}{3b}a^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{6b}a^{\frac{2}{3}}\ln\left(x^{2}-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C\sqrt{3}}{3b}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3}\sqrt[3]{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

```
[Out] 1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*C*a^(2/3)/b/(1/b*a)^(
2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*C*a^(2/3)/b/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C*a^(1/3)/b^(2/3)/(1/b*
a)^(1/3)*ln(x+(1/b*a)^(1/3))-1/6*C*a^(1/3)/b^(2/3)/(1/b*a)^(1/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*C*a^(1/3)/b^(2/3)*3^(1/2)/(1/b*a)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b^(1/3)*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.01843, size = 51, normalized size = 2.43

$$\frac{C\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="fricas")
```

```
[Out] C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

Sympy [A] time = 0.311516, size = 20, normalized size = 0.95

$$\frac{C\log\left(\sqrt[3]{a}b^{\frac{2}{3}}+bx\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

```
[Out] C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

Giac [A] time = 1.47152, size = 22, normalized size = 1.05

$$\frac{C\log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algori
thm="giac")
```

```
[Out] C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

3.44 $\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=71

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(C\sqrt[3]{\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rubi [A] time = 0.0941567, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1867, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(C\sqrt[3]{\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}$$

Antiderivative was successfully verified.

```
[In] Int[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) + (C*Log[(a/b)^(1/3) + x])/b
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b}+\frac{\left(B+\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}+\frac{C}{b}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)$$

$$= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a}+\frac{C}{b}\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.267454, size = 247, normalized size = 3.48

$$\frac{\sqrt[3]{b}\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}+B\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}}+B\right)-a^{2/3}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1
/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*(-(a
^(2/3)*B) + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(1/3)
+ b^(1/3)*x] + b^(1/3)*(a^(2/3)*B - a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b
)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log[a +
b*x^3])/(6*a*b)
```

Maple [A] time = 0.005, size = 121, normalized size = 1.7

$$\frac{2C}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\frac{C}{3b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)+\frac{2B\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((1/b*a)^(1/3)*B+2*(1/b*a)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(1/b*a)^(1/3))/b-1/3*C/b*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2
/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/3*B*3^(1/2)/b/(1
/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.69123, size = 948, normalized size = 13.35

$$\left[\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}+B^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}+C^{2}a}{a}}\log\left(-\frac{C^{3}a^{2}+B^{3}ab-2\left(C^{3}ab+B^{3}b^{2}\right)x^{3}+3\left(C^{3}ab+B^{3}b^{2}\right)x\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^{2}-B^{2}abx+C\cdots\right.}{\cdots}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"fricas")
```

```
[Out] [(C*log(x + (a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/
b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 +
3*(C^3*a*b + B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(a/b)^(2/3) - (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*
b*(a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*C*b*(a/b
)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x - C^2*a
 + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqrt(
(2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*b)) + C*l
og(x + (a/b)^(1/3)))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [B] time = 1.2488, size = 362, normalized size = 5.1

$$-\frac{\left(Cb^{2}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(ab^{2}\right)^{\frac{1}{3}}Bb+2\left(ab^{2}\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{2}}+\frac{\sqrt{3}\left(\left(9\left(-a^{2}b^{4}\right)^{\frac{1}{3}}ab^{2}-27^{\frac{5}{6}}\left(-a^{2}b^{4}\right)^{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm=
"giac")
```

```
[Out] -1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) + (a*b^2)^(1/3)*B*b + 2*(a*b^
2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/54*sqrt(3)*
((9*(-a^2*b^4)^(1/3)*a*b^2 - 27^(5/6)*(-a^2*b^4)^(5/6))*B + 18*(a^2*b^3 - s
qrt(3)*sqrt(a^4*b^6)*i)*C)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^2*b^4) + 1/108*((27*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*b^4)^(
5/6))*B + 18*(3*a^2*b^3 - sqrt(3)*sqrt(a^4*b^6)*i)*C)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.45 $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=76

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

```
[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rubi [A] time = 0.102125, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {1867, 31, 617, 204}

$$\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{C\int\frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b}-\frac{\left(B+\sqrt[3]{-\frac{a}{b}}C\right)\int\frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$

$$=-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{\left(2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$

$$=\frac{2\left(\frac{B}{\sqrt[3]{-\frac{a}{b}}}+C\right)\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.177006, size = 288, normalized size = 3.79

$$-\frac{\left(-a^{2/3}B-\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}-2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{2/3}}-\frac{\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}B\sqrt[3]{-\frac{a}{b}}+2\sqrt[3]{a}\sqrt[3]{b}C\left(-\frac{a}{b}\right)^{2/3}\right)}{3ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3)
,x]
```

```
[Out] -(((a^(2/3)*B - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)
*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b
^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B + 2*a^(1/3)*(-(a/
b))^(2/3)*b^(1/3)*C)*Log[a^(1/3) - b^(1/3)*x])/(3*a*b^(2/3)) - ((-(a^(2/3)*
B) - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/3)*C)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(2/3)) - (C*Log[a -
 b*x^3])/(3*b)
```

Maple [B] time = 0.006, size = 345, normalized size = 4.5

$$-\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{6b}\ln\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1/b*a)^(1/3)*B+2*(-1/b*a)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x-(1/b*a)^(1/3))-1/3/b/(1/b*a)^(2/
3)*ln(x-(1/b*a)^(1/3))*(-1/b*a)^(1/3)*B+1/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3
)*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/6/b/(1/b*a)^(2/3)*ln(x^2+(1/b*a)^
(1/3)*x+(1/b*a)^(2/3))*(-1/b*a)^(1/3)*B+2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))+1/3/b/(1/b*a)^(2/3)*3^(
1/2)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))*(-1/b*a)^(1/3)*B-1/3*B/b/(1/
b*a)^(1/3)*ln(x-(1/b*a)^(1/3))+1/6*B/b/(1/b*a)^(1/3)*ln(x^2+(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))-1/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*(1+2/(1/b*a)^(1/3)
*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.77571, size = 972, normalized size = 12.79

$$\left[-\frac{C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-B^3ab+2\left(C^3ab-B^3b^2\right)x^3-3\left(C^3ab-B^3b^2\right)x\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^2-B^2a\right.}{}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [-(C*log(x + (-a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(-a/b)^(2/3) + B^2*b*(
-a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3
 - 3*(C^3*a*b - B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(-a/b)^(2/3) - (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3)
+ B^2*b*(-a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B
*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2
*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(-a/b
)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)/(C^3*
a - B^3*b)) + C*log(x + (-a/b)^(1/3)))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [B] time = 1.10247, size = 351, normalized size = 4.62

$$-\frac{\left(Cb^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}Bb+2\left(-ab^{2}\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{2}}+\frac{\sqrt{3}\left(\left(9\left(-a^{2}b^{4}\right)^{\frac{1}{3}}ab^{2}+27^{\frac{5}{6}}\left(-a^{2}b^{4}\right)^{\frac{5}{6}}\right)B-\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) + (-a*b^2)^(1/3)*B*b + 2*(-a*b^
2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2) + 1/54*sqrt(3)*((
9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-a^2*b^4)^(5/6))*B - 18*(a^2*b^3 + sqr
t(3)*sqrt(a^4*b^6)*i)*C)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*b^4) + 1/108*((27*(-a^2*b^4)^(1/3)*a*b^2 - 27^(5/6)*(-a^2*b^4)^(5/6)
)*B - 18*(3*a^2*b^3 + sqrt(3)*sqrt(a^4*b^6)*i)*C)*log(x^2 + x*(a/b)^(1/3) +
 (a/b)^(2/3))/(a^2*b^4)
```

3.46 $\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

Optimal. Leaf size=78

$$\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rubi [A] time = 0.107623, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {1869, 31, 617, 204}

$$\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

```
[Out] (2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt
[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b}+\frac{\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$
$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{\left(2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{\sqrt[3]{-\frac{a}{b}}b}$$
$$=\frac{2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.251451, size = 253, normalized size = 3.24

$$\frac{\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(B-2C\sqrt[3]{-\frac{a}{b}}\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt[3]{b}\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\left(B-2C\sqrt[3]{-\frac{a}{b}}\right)\right)\log\left(\sqrt[3]{a}\right.}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x
^3),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-(a/b))^(1/3)*b^(1/3)*(-B + 2*(-(a
/b))^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 2*b^(1/
3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*(B - 2*(-(a/b))^(1/3)*C))*Lo
g[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3
)*(B - 2*(-(a/b))^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
+ 2*a*C*Log[a + b*x^3])/(6*a*b)
```

Maple [B] time = 0.005, size = 340, normalized size = 4.4

$$\frac{2C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\sqrt[3]{-\frac{a}{b}}\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{C}{3b}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{6b}\ln\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-1/b*a)^(1/3)*B+2*(-1/b*a)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x)
```

```
[Out] 2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/3/b/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))*(-1/b*a)^(1/3)*B-1/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(
1/3)*x+(1/b*a)^(2/3))*(-1/b*a)^(1/3)*B+2/3*C*(-1/b*a)^(2/3)/b/(1/b*a)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3/b/(1/b*a)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*(-1/b*a)^(1/3)*B-1/3*B/b/(1/b
*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*B/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+
(1/b*a)^(2/3))+1/3*B*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^
(1/3)*x-1))+1/3*C/b*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.75111, size = 968, normalized size = 12.41

$$\left[\frac{C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+C^{2}a}{a}}\log\left(-\frac{C^{3}a^{2}+B^{3}ab-2\left(C^{3}ab+B^{3}b^{2}\right)x^{3}+3\left(C^{3}ab+B^{3}b^{2}\right)x\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2\,BCabx^{2}-B^{2}\cdots\right.}{\cdots}\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="fricas")
```

```
[Out] [(C*log(x - (-a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*(
-a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3
 + 3*(C^3*a*b + B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*
a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(-a/b)^(2/3) + (2*C
^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3)
 - B^2*b*(-a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*
C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b
*x - C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(-a/b)^
(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)/(C^3*a +
 B^3*b)) + C*log(x - (-a/b)^(1/3)))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [A] time = 1.12964, size = 180, normalized size = 2.31

$$-\frac{2\sqrt{3}\left(Cab+\left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x\right.\right.}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorit
hm="giac")
```

```
[Out] -2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) -
 (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b^2)
```

3.47 $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

Optimal. Leaf size=75

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rubi [A] time = 0.104694, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1869, 31, 617, 204}

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-C\sqrt[3]{\frac{a}{b}}\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*(a/b)^(2/3)*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]]
)/(Sqrt[3]*a) - (C*Log[(a/b)^(1/3) - x])/b
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3} C + Bx + Cx^2}{a - bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b} - \frac{\left(B - \sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$= -\frac{C\log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b} + \left(2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a} - \frac{C}{b}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)$$

$$= -\frac{2\left(\frac{\left(\frac{a}{b}\right)^{2/3}B}{a} - \frac{C}{b}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{C\log\left(\sqrt[3]{\frac{a}{b}} - x\right)}{b}$$

Mathematica [B] time = 0.261048, size = 244, normalized size = 3.25

$$\frac{\sqrt[3]{b}\left(a^{2/3}B + \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}} - B\right)\right)\log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) - 2\sqrt[3]{b}\left(a^{2/3}B + \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\left(2C\sqrt[3]{\frac{a}{b}} - B\right)\right)\log\left(\sqrt[3]{a} - \sqrt[3]{b}x\right.}{6ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B - 2*(a/b)^(
1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*(a^(2/3)*B
 + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(1/3
)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)
*C))*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*C*Log[a - b*x^3])
/(6*a*b)
```

Maple [A] time = 0.007, size = 124, normalized size = 1.7

$$-\frac{2C}{3b}\ln\left(x - \sqrt[3]{\frac{a}{b}}\right) + \frac{C}{3b}\ln\left(x^2 + \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + \frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1 + 2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right) - \frac{2B\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1 + 2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1/b*a)^(1/3)*B+2*(1/b*a)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(1/b*a)^(1/3))+1/3*C/b*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+
2/3*C*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))/b*3^(1/2)-2/3*B*3^(1/2)/b/(
1/b*a)^(1/3)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))-1/3*C/b*ln(b*x^3-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.99173, size = 952, normalized size = 12.69

$$\left[-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\sqrt{\frac{1}{3}}\sqrt{\frac{2BCb\left(\frac{a}{b}\right)^{\frac{2}{3}}-B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}-C^2a}{a}}\log\left(-\frac{C^3a^2-B^3ab+2\left(C^3ab-B^3b^2\right)x^3-3\left(C^3ab-B^3b^2\right)x\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(2BCabx^2-B^2abx-C^2\cdots\right.}{\cdots}\right)}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="fricas")
```

```
[Out] [-(C*log(x - (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a/
b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3 -
3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a*b*
x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) + (2*C^2*a*
b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b
*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C*b*(a/
b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*x + C^2
*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(a/b)^(1/3))*sqr
t(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B^3*b)) +
C*log(x - (a/b)^(1/3)))/b]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialDivisionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

```
[Out] Exception raised: PolynomialDivisionFailed
```

Giac [A] time = 1.08892, size = 169, normalized size = 2.25

$$\frac{2\sqrt{3}\left(Cab-\left(ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-\left(ab^2\right)^{\frac{1}{3}}Bb+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\cdots\right.}{3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorith
m="giac")
```

```
[Out] 2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3)
)/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) - (a*b^
2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*
b^2)
```

3.48 $\int \frac{a+ax+cx^2}{1-x^3} \, dx$

Optimal. Leaf size=32

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

```
[Out] -((2*a + c)*Log[1 - x])/3 + ((a - c)*Log[1 + x + x^2])/3
```

Rubi [A] time = 0.0337987, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1875, 31, 628}

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x + c*x^2)/(1 - x^3),x]

[Out] -((2*a + c)*Log[1 - x])/3 + ((a - c)*Log[1 + x + x^2])/3
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ax+cx^2}{1-x^3} \, dx &= \frac{1}{3}\int \frac{a-c+(2a-2c)x}{1+x+x^2} \, dx+\frac{1}{3}(2a+c)\int \frac{1}{1-x} \, dx\\ &= -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0121035, size = 31, normalized size = 0.97

$$\frac{1}{3}\left((a-c)\log\left(x^2+x+1\right)-(2a+c)\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x + c*x^2)/(1 - x^3),x]

[Out] (-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

Maple [A] time = 0.045, size = 36, normalized size = 1.1

$$-\frac{\ln\left(-1+x\right)c}{3}-\frac{2\ln\left(-1+x\right)a}{3}+\frac{\ln\left(x^2+x+1\right)a}{3}-\frac{\ln\left(x^2+x+1\right)c}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a*x+a)/(-x^3+1),x)

[Out] -1/3*ln(-1+x)*c-2/3*ln(-1+x)*a+1/3*ln(x^2+x+1)*a-1/3*ln(x^2+x+1)*c
```

Maxima [A] time = 1.44268, size = 35, normalized size = 1.09

$$\frac{1}{3}\left(a-c\right)\log\left(x^2+x+1\right)-\frac{1}{3}\left(2\,a+c\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")

[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Fricas [A] time = 1.27978, size = 77, normalized size = 2.41

$$\frac{1}{3}\left(a-c\right)\log\left(x^2+x+1\right)-\frac{1}{3}\left(2\,a+c\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")

[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Sympy [A] time = 0.3104, size = 24, normalized size = 0.75

$$\frac{\left(a-c\right)\log\left(x^2+x+1\right)}{3}-\frac{\left(2a+c\right)\log\left(x-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a*x+a)/(-x**3+1),x)

[Out] (a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

Giac [A] time = 1.064, size = 36, normalized size = 1.12

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(abs(x - 1))
```

3.49 $\int \frac{a+bx+cx^2}{1-x^3}\,dx$

Optimal. Leaf size=55

$$\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)+\frac{(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 +
((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rubi [A] time = 0.0567216, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1875, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2+x+1\right)(a+b-2c)-\frac{1}{3}\log(1-x)(a+b+c)+\frac{(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] ((a - b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a + b + c)*Log[1 - x])/3 +
((a + b - 2*c)*Log[1 + x + x^2])/6
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{1-x^3}\,dx &= \frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{1+x+x^2}\,dx+\frac{1}{3}(a+b+c)\int \frac{1}{1-x}\,dx\\ &= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{2}(a-b)\int \frac{1}{1+x+x^2}\,dx+\frac{1}{6}(a+b-2c)\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)+(-a+b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.\\ &= \frac{(a-b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}(a+b+c)\log(1-x)+\frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0333609, size = 62, normalized size = 1.13

$$\frac{1}{6}\left((a+b)\log\left(x^2+x+1\right)-2(a+b)\log(1-x)+2\sqrt{3}(a-b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2c\log\left(1-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
```

```
[Out] (2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 - x] + (a +
b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

Maple [A] time = 0.043, size = 87, normalized size = 1.6

$$-\frac{\ln(-1+x)c}{3}-\frac{\ln(-1+x)b}{3}-\frac{\ln(-1+x)a}{3}+\frac{\ln\left(x^2+x+1\right)a}{6}+\frac{\ln\left(x^2+x+1\right)b}{6}-\frac{\ln\left(x^2+x+1\right)c}{3}+\frac{a\sqrt{3}}{3}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-x^3+1),x)
```

```
[Out] -1/3*ln(-1+x)*c-1/3*ln(-1+x)*b-1/3*ln(-1+x)*a+1/6*ln(x^2+x+1)*a+1/6*ln(x^2+
x+1)*b-1/3*ln(x^2+x+1)*c+1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*a-1/3*3^(1
/2)*arctan(1/3*(1+2*x)*3^(1/2))*b
```

Maxima [A] time = 1.45421, size = 63, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Fricas [A] time = 1.34398, size = 158, normalized size = 2.87

$$\frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(x
^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Sympy [C] time = 0.891344, size = 323, normalized size = 5.87

$$-\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3}-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i\,(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

```
[Out] -(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c**2 - 2*b*c
*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - sqr
t(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a
 - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/
6) + 9*b*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) - (-a/
6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 +
 c/3 + sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 +
 sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)**2)/(a
**3 - b**3))
```

Giac [A] time = 1.07136, size = 70, normalized size = 1.27

$$\frac{1}{3}\left(\sqrt{3}a-\sqrt{3}b\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*
c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(abs(x - 1))
```

3.50 $\int \frac{1+x+x^2}{1-x^3} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0060494, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 - x^3),x]
```

```
[Out] -Log[1 - x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{1-x^3} dx &= \int \frac{1}{1-x} dx \\ &= -\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0013289, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 - x^3),x]
```

```
[Out] -Log[1 - x]
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(-x^3+1),x)
```

```
[Out] -ln(-1+x)
```

Maxima [A] time = 0.978024, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 1.19605, size = 18, normalized size = 2.25

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.072375, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(-x**3+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A] time = 1.07858, size = 9, normalized size = 1.12

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

3.51 $\int \frac{1-x+3x^2}{1-x^3} \, dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rubi [A] time = 0.0302431, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1871, 1586, 618, 204, 260}

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+3x^2}{1-x^3}\,dx &= 3\int \frac{x^2}{1-x^3}\,dx + \int \frac{1-x}{1-x^3}\,dx\\ &= -\log\left(1-x^3\right) + \int \frac{1}{1+x+x^2}\,dx\\ &= -\log\left(1-x^3\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)\\ &= \frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)\end{aligned}$$

Mathematica [A] time = 0.008519, size = 30, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
```

```
[Out] (2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Maple [A] time = 0.005, size = 33, normalized size = 1.1

$$-\ln(-1+x) - \ln\left(x^2+x+1\right) + \frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(-x^3+1),x)
```

```
[Out] -ln(-1+x)-ln(x^2+x+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.44615, size = 43, normalized size = 1.43

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Fricas [A] time = 1.23767, size = 101, normalized size = 3.37

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Sympy [A] time = 0.121207, size = 5, normalized size = 0.17

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(-x**3+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A] time = 1.06748, size = 45, normalized size = 1.5

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\log\left(x^2+x+1\right)-\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(abs(x -
1))
```

3.52 $\int \frac{1+x+4x^2}{1-x^3} \, dx$

Optimal. Leaf size=18

$$-\log \left(x^2+x+1\right)-2 \log (1-x)$$

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rubi [A] time = 0.0188959, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1875, 31, 628}

$$-\log \left(x^2+x+1\right)-2 \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(1 - x^3),x]
```

```
[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{1-x^3} \, dx &= \frac{1}{3} \int \frac{-3-6x}{1+x+x^2} \, dx+2 \int \frac{1}{1-x} \, dx\\ &= -2 \log (1-x)-\log \left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0050965, size = 18, normalized size = 1.

$$-\log \left(x^2+x+1\right)-2 \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(1 - x^3),x]

[Out] -2*Log[1 - x] - Log[1 + x + x^2]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$-2\ln(-1+x)-\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(-x^3+1),x)

[Out] -2*ln(-1+x)-ln(x^2+x+1)
```

Maxima [A] time = 1.44261, size = 22, normalized size = 1.22

$$-\log\left(x^2+x+1\right)-2\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="maxima")

[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Fricas [A] time = 1.4008, size = 46, normalized size = 2.56

$$-\log\left(x^2+x+1\right)-2\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")

[Out] -log(x^2 + x + 1) - 2*log(x - 1)
```

Sympy [A] time = 0.112162, size = 15, normalized size = 0.83

$$-2\log\left(x-1\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(-x**3+1),x)

[Out] -2*log(x - 1) - log(x**2 + x + 1)
```

Giac [A] time = 1.07339, size = 23, normalized size = 1.28

$$-\log\left(x^2+x+1\right)-2\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -log(x^2 + x + 1) - 2*log(abs(x - 1))
```

3.53 $\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx$

Optimal. Leaf size=113

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rubi [A] time = 0.0986718, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1850}

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx = \int (a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7+4ab^3cx^9+$$
$$=a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}$$

Mathematica [A] time = 0.0039217, size = 113, normalized size = 1.

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14
```

Maple [A] time = 0.001, size = 98, normalized size = 0.9

$$a^{4}cx+\frac{a^{4}dx^{2}}{2}+a^{3}bcx^{4}+\frac{4\,a^{3}bdx^{5}}{5}+\frac{6\,a^{2}b^{2}cx^{7}}{7}+\frac{3\,a^{2}b^{2}dx^{8}}{4}+\frac{2\,ab^{3}cx^{10}}{5}+\frac{4\,ab^{3}dx^{11}}{11}+\frac{b^{4}cx^{13}}{13}+\frac{b^{4}dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^2
*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*x^
14
```

Maxima [A] time = 0.985302, size = 131, normalized size = 1.16

$$\frac{1}{14}\,b^{4}dx^{14}+\frac{1}{13}\,b^{4}cx^{13}+\frac{4}{11}\,ab^{3}dx^{11}+\frac{2}{5}\,ab^{3}cx^{10}+\frac{3}{4}\,a^{2}b^{2}dx^{8}+\frac{6}{7}\,a^{2}b^{2}cx^{7}+\frac{4}{5}\,a^{3}bdx^{5}+a^{3}bcx^{4}+\frac{1}{2}\,a^{4}dx^{2}+a^{4}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

Fricas [A] time = 1.11782, size = 232, normalized size = 2.05

$$\frac{1}{14}x^{14}db^{4}+\frac{1}{13}x^{13}cb^{4}+\frac{4}{11}x^{11}db^{3}a+\frac{2}{5}x^{10}cb^{3}a+\frac{3}{4}x^{8}db^{2}a^{2}+\frac{6}{7}x^{7}cb^{2}a^{2}+\frac{4}{5}x^{5}dba^{3}+x^{4}cba^{3}+\frac{1}{2}x^{2}da^{4}+xca^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 4/11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a +
3/4*x^8*d*b^2*a^2 + 6/7*x^7*c*b^2*a^2 + 4/5*x^5*d*b*a^3 + x^4*c*b*a^3 + 1/2
*x^2*d*a^4 + x*c*a^4
```

Sympy [A] time = 0.087344, size = 117, normalized size = 1.04

$$a^{4}cx+\frac{a^{4}dx^{2}}{2}+a^{3}bcx^{4}+\frac{4a^{3}bdx^{5}}{5}+\frac{6a^{2}b^{2}cx^{7}}{7}+\frac{3a^{2}b^{2}dx^{8}}{4}+\frac{2ab^{3}cx^{10}}{5}+\frac{4ab^{3}dx^{11}}{11}+\frac{b^{4}cx^{13}}{13}+\frac{b^{4}dx^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2*
c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11
+ b**4*c*x**13/13 + b**4*d*x**14/14
```

Giac [A] time = 1.08494, size = 131, normalized size = 1.16

$$\frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/2
*a^4*d*x^2 + a^4*c*x
```

3.54 $\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx$

Optimal. Leaf size=88

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Rubi [A] time = 0.0602554, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1850}

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx &= \int \left(a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b^3cx^9+b^3dx^{10}\right. \\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\end{aligned}$$

Mathematica [A] time = 0.0026354, size = 88, normalized size = 1.

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Maple [A] time = 0.001, size = 75, normalized size = 0.9

$$a^3cx+\frac{a^3dx^2}{2}+\frac{3\,a^2bcx^4}{4}+\frac{3\,a^2bdx^5}{5}+\frac{3\,ab^2cx^7}{7}+\frac{3\,ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*a
*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Maxima [A] time = 0.960131, size = 100, normalized size = 1.14

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A] time = 1.10836, size = 180, normalized size = 2.05

$$\frac{1}{11}x^{11}db^3+\frac{1}{10}x^{10}cb^3+\frac{3}{8}x^8db^2a+\frac{3}{7}x^7cb^2a+\frac{3}{5}x^5dba^2+\frac{3}{4}x^4cba^2+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 3/8*x^8*d*b^2*a + 3/7*x^7*c*b^2*a + 3/5
*x^5*d*b*a^2 + 3/4*x^4*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.078548, size = 90, normalized size = 1.02

$$a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**2
*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11
```

Giac [A] time = 1.08721, size = 100, normalized size = 1.14

$$\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/5
*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.55 $\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx$

Optimal. Leaf size=60

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Rubi [A] time = 0.0380882, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1850}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx &= \int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right)\,dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8\end{aligned}$$

Mathematica [A] time = 0.0021509, size = 60, normalized size = 1.

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8
```

Maple [A] time = 0.001, size = 51, normalized size = 0.9

$$a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2\,abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x
^8
```

Maxima [A] time = 0.990225, size = 68, normalized size = 1.13

$$\frac{1}{8}\,b^2dx^8 + \frac{1}{7}\,b^2cx^7 + \frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4 + \frac{1}{2}\,a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

Fricas [A] time = 1.08037, size = 120, normalized size = 2.

$$\frac{1}{8}x^8db^2 + \frac{1}{7}x^7cb^2 + \frac{2}{5}x^5dba + \frac{1}{2}x^4cba + \frac{1}{2}x^2da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d*b^2 + 1/7*x^7*c*b^2 + 2/5*x^5*d*b*a + 1/2*x^4*c*b*a + 1/2*x^2*d*a
^2 + x*c*a^2
```

Sympy [A] time = 0.076342, size = 58, normalized size = 0.97

$$a^2cx + \frac{a^2dx^2}{2} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c*x**7/7 +
b**2*d*x**8/8
```

Giac [A] time = 1.06269, size = 68, normalized size = 1.13

$$\frac{1}{8}\,b^2dx^8 + \frac{1}{7}\,b^2cx^7 + \frac{2}{5}\,abdx^5 + \frac{1}{2}\,abcx^4 + \frac{1}{2}\,a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*x
^2 + a^2*c*x
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} dx$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

```
[Out] c*x + (d*x^2)/2
```

Rubi [A] time = 0.0138278, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1586}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} dx &= \int (c + dx) dx \\ &= cx + \frac{dx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0007036, size = 12, normalized size = 1.

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A] time = 0.949419, size = 14, normalized size = 1.17

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A] time = 1.25331, size = 23, normalized size = 1.92

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A] time = 0.069176, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A] time = 1.06273, size = 14, normalized size = 1.17

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rubi [A] time = 0.0979056, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1586, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^2}\,dx=\int \frac{c+dx}{a+bx^3}\,dx$$

$$=\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$=\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}+\frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right)\int$$

$$=\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}+\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}\right.}{}$$

$$=-\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2}\right.}{6a^{2}}$$

Mathematica [A] time = 0.0450009, size = 124, normalized size = 0.77

$$\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)-2\sqrt{3}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

```
[Out] (-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Maple [A] time = 0.002, size = 186, normalized size = 1.2

$$\frac{c}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*c/b/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*c/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))-1/3*d/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*d/b/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*d*3^(1/2)/b/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.10478, size = 4590, normalized size = 28.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d
^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*
d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)
/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/12*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2
))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^
2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)
 - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*
c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(-1/4*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^
(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/2)^(1/3)*(I*sqrt(3
```

```
) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a
*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3 + a*d^3)*x + 3/4*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a^2*b*d + 2*a*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*
b))) + 1/12*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^
3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c
^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) - 3*sqrt(1/3)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^
3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*
log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 -
 a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3
+ a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^
2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 - 2*a*c*d^2 + 2*(b*c^3
+ a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*
a^2*b*d + 2*a*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3)
 + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^
2*a*b + 16*c*d)/(a*b)))
```

Sympy [A] time = 0.687793, size = 76, normalized size = 0.47

$$\operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)))
)
```

Giac [A] time = 1.08031, size = 216, normalized size = 1.34

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c + \left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(d*(-a/b)^(1/3) + c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*s
qrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-
```

```
a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^
(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=189

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+$$

```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```

Rubi [A] time = 0.126884, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1586, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```


```
[Out] (x*(c + d*x))/(3*a*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3)) + ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1
/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5
/3)*b^(2/3))
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^3}\,dx=\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$$

$$=\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$=\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$=\frac{x(c+dx)}{3a\left(a+bx^3\right)}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}}+$$

$$=\frac{x(c+dx)}{3a\left(a+bx^3\right)}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/}\right.}{18a^{5/3}b^{2/3}}$$

$$=\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{(2\sqrt[3]{}}{}$$

Mathematica [A] time = 0.147975, size = 180, normalized size = 0.95

$$\frac{\frac{\left(a^{2/3}d-2\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6ax(c+dx)}{a+bx^3}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

```
[Out] ((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b^(
1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(1/3
)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
18*a^2)
```

Maple [A] time = 0.004, size = 238, normalized size = 1.3

$$\frac{cx}{3a\left(bx^3+a\right)}+\frac{2c}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2c\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9*c/a/b/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9*c/a/b/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9
*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1
/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.99868, size = 4961, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
 a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*
(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^
5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^
5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3)
- 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/
3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*
a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3 + a^2)*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt(1/3)*(a*b*x^
3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^
3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a
^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2)
)^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2
)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b +
32*c*d)/(a^3*b))) + ((a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
+ 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((
1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)
/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*
d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^
(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/
(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*
(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(
a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1
)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/
2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))))/(a*b*x^3 + a^2)
```

Sympy [A] time = 1.11114, size = 105, normalized size = 0.56

$$\operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Giac [A] time = 1.08426, size = 252, normalized size = 1.33

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{dx^2+cx}{3\left(bx^3+a\right)a}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1
/3*(d*x^2 + c*x)/((b*x^3 + a)*a) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*
b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^
2) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

3.59 $\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) dx$

Optimal. Leaf size=585

$$\frac{54\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\left(1729\sqrt[3]{bc}-935\left(1-\sqrt{3}\right)\sqrt[3]{ad}\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-\right.}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/323323 + (30*a*(247*c
*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2*(19*c*x + 17*d*x^2)*(a + b*x^
3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt
[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.463832, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1852, 1853, 1878, 218, 1877}

$$\frac{54\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\left(1729\sqrt[3]{bc}-935\left(1-\sqrt{3}\right)\sqrt[3]{ad}\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{bx}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{323323b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```


```
[Out] (810*a^3*d*Sqrt[a + b*x^3])/(1729*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)) + (54*a^2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*x^3])/323323 + (30*a*(247*c
*x + 187*d*x^2)*(a + b*x^3)^(3/2))/46189 + (2*(19*c*x + 17*d*x^2)*(a + b*x^
3)^(5/2))/323 - (405*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*d*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(2/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*c - 935*(1 - Sqrt
[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(323323*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 1852

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) dx = \int (c+dx)\left(a+bx^3\right)^{5/2} dx$$
$$= \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2} + \frac{1}{2}(15a)\int \left(\frac{2c}{17}+\frac{2dx}{19}\right)\left(a+bx^3\right)^{3/2} dx$$
$$= \frac{30a\left(247cx+187dx^2\right)\left(a+bx^3\right)^{3/2}}{46189} + \frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2} + \frac{1}{4}$$
$$= \frac{54a^2\left(1729cx+935dx^2\right)\sqrt{a+bx^3}}{323323} + \frac{30a\left(247cx+187dx^2\right)\left(a+bx^3\right)^{3/2}}{46189} +$$
$$= \frac{54a^2\left(1729cx+935dx^2\right)\sqrt{a+bx^3}}{323323} + \frac{30a\left(247cx+187dx^2\right)\left(a+bx^3\right)^{3/2}}{46189} +$$
$$= \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{54a^2\left(1729cx+935dx^2\right)\sqrt{a+bx^3}}{323323} + \frac{30}{}$$

Mathematica [C] time = 0.0398446, size = 78, normalized size = 0.13

$$\frac{a^2x\sqrt{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{5}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{5}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-5/2, 1/3, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[-5/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3
)/a])
```

Maple [B] time = 0.009, size = 1618, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] b*d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*
x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-
b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2
)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)
-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)
/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(
1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b
```

```
*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/
2))))+b*c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b
*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2)))+a*d*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*
x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(
-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b
^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)
^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(
-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(
1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*c*(
2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*
(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4 + bcx^3 + adx + ac\right)\left(bx^3 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dx^7 + b^2cx^6 + 2\,abdx^4 + 2\,abcx^3 + a^2dx + a^2c\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas"
)
```

```
[Out] integral((b^2*d*x^7 + b^2*c*x^6 + 2*a*b*d*x^4 + 2*a*b*c*x^3 + a^2*d*x + a^2
*c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 5.44847, size = 265, normalized size = 0.45

$$\frac{a^{\frac{5}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{2a^{\frac{3}{2}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{3}{2}}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(-\ldots\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**(3/2)*b*c*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(3/
2)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(8/3)) + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*gamma(8/3
)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

3.60 $\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)dx$

Optimal. Leaf size=556

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2*(13*c*x + 11*d*x^2)
*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*d*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - S
qrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.327315, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1852, 1853, 1878, 218, 1877}

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (54*a^2*d*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 + (18*a*(91*c*x + 55*d*x^2)*Sqrt[a + b*x^3])/5005 + (2*(13*c*x + 11*d*x^2)
*(a + b*x^3)^(3/2))/143 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*d*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(91*b^(1/3)*c - 55*(1 - S
qrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1852

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
```

```
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \int (c+dx)\left(a+bx^3\right)^{3/2}\,dx$$

$$= \frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2} + \frac{1}{2}(9a)\int\left(\frac{2c}{11}+\frac{2dx}{13}\right)\sqrt{a+bx^3}\,dx$$

$$= \frac{18a\left(91cx+55dx^2\right)\sqrt{a+bx^3}}{5005} + \frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2} + \frac{1}{4}\left(27a^2\right)$$

$$= \frac{18a\left(91cx+55dx^2\right)\sqrt{a+bx^3}}{5005} + \frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2} + \frac{\left(27a^2d\right)\int}{}$$

$$= \frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{18a\left(91cx+55dx^2\right)\sqrt{a+bx^3}}{5005} + \frac{2}{143}\left(13cx+\right.$$

Mathematica [C] time = 0.0317119, size = 76, normalized size = 0.14

$$\frac{ax\sqrt{a+bx^3}\left(2c\,_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/
a])
```

Maple [B] time = 0.008, size = 1546, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

```
[Out] b*d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3
^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+b*c*(2/11*x^4*(b*x^3+a)^(1/2)+6/5
5/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)))+a*d*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(
-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(
(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*c*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2
)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1
```

```
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4 + bcx^3 + adx + ac\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^4 + bcx^3 + adx + ac\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas"
)
```

```
[Out] integral((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.49993, size = 170, normalized size = 0.31

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\\ \frac{8}{3}\end{matrix}\right.}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d
*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bdx^4 + bcx^3 + adx + ac\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

3.61 $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=525

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} -$$

```
[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4
/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*c -
 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.238771, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1586, 1853, 1878, 218, 1877}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (6*a*d*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4
/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*c -
 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(35*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\,dx = \int (c + dx)\sqrt{a + bx^3}\,dx$$

$$= \frac{2}{35}\left(7cx + 5dx^2\right)\sqrt{a + bx^3} + \frac{1}{2}(3a)\int \frac{\frac{2c}{5} + \frac{2dx}{7}}{\sqrt{a + bx^3}}\,dx$$

$$= \frac{2}{35}\left(7cx + 5dx^2\right)\sqrt{a + bx^3} + \frac{(3ad)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7\sqrt[3]{b}} + \frac{1}{35}\left(3a\left(7c - \frac{5\left(1 - \sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right.\right.$$

$$= \frac{6ad\sqrt{a + bx^3}}{7b^{2/3}\left(\left(1 + \sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)} + \frac{2}{35}\left(7cx + 5dx^2\right)\sqrt{a + bx^3} - \frac{3\sqrt[4]{3}\sqrt{2 - \sqrt{3}}a^{4/3}d\left(\sqrt[3]{a} + \right.}{}$$

Mathematica [C] time = 0.0294621, size = 75, normalized size = 0.14

$$\frac{x\sqrt{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] + d
*x*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/a]
)
```

Maple [B] time = 0.009, size = 1480, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x)
```

```
[Out] b*d*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(
1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))))+b*c*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*I*a*d*3^(1/2)/b*(-b^2*a)^(1/3
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)))-2/3*I*a*c*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x
-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
```

```
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3 + a}(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*x^3 + a)*(d*x + c), x)
```

Sympy [A] time = 2.86097, size = 163, normalized size = 0.31

$$\frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((1/2, 4/3),
(7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + b*d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

3.62 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=490

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\ldots}{\ldots}$$

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.157961, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\cdots\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}\cdots}$$

Mathematica [C] time = 0.0315473, size = 75, normalized size = 0.15

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.011, size = 1536, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x)
```

```
[Out] b*d*(-2/3/b*x^2/((x^3+1/b*a)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^
2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),
(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))))+b*c*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2
*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+a*d*(2/3*x^2/a/((x^3+1/b*
a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*
a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3
)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*
a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/
(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*c*(2/3*x
/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.87228, size = 78, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4+bcx^3+adx+ac}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

3.63 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=522

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\ldots$$

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1
/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.248594, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1586, 1855, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}d\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 - Sqrt[3])*a^(1
/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{c+dx}{\left(a+bx^3\right)^{3/2}}\,dx$$

$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{c}{2}+\frac{dx}{2}}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\cdots}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Mathematica [C] time = 0.0510459, size = 96, normalized size = 0.18

$$\frac{x\left(2c\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3dx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+4c\right)}{6a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(4*c + 2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3
)/a)] + 3*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3
)/a)]))/(6*a*Sqrt[a + b*x^3])
```

Maple [B] time = 0.009, size = 1662, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x)
```

```
[Out] b*d*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+8/27/b*x^2/a/((x^3+1/b*a)*b
)^(1/2)+8/81*I/b^2/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a
)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+b*c*(-2/9*x
/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+4/27/b*x/a/((x^3+1/b*a)*b)^(1/2)-4/81*I/
b^2/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(
1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+a*d*(2
/9*x^2/a/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+10/27*x^2/a^2/((x^3+1/b*a)*b)^(1
/2)+10/81*I/a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(
1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*c*(2/9*x/a/b
^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+14/27*x/a^2/((x^3+1/b*a)*b)^(1/2)-14/81*I/
a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
```

```
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(
1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*x^3 + a)*(d*x + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 12.033, size = 163, normalized size = 0.31

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{5}{2} \\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{5}{2} \\ \frac{5}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{4}{3}, \frac{5}{2} \\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix}\frac{5}{3}, \frac{5}{2} \\ \frac{8}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 5/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamm
a(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

3.64 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{7/2}} \, dx$

Optimal. Leaf size=554

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}d+7\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\ldots}{27a^{\ldots}}$$

```
[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27*a^2*Sqrt[
a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^
2*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.316835, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1586, 1855, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}d+7\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{10\ldots}{27a^2b^{2/3}\left(\left(1\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```


```
[Out] (2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + (2*x*(7*c + 5*d*x))/(27*a^2*Sqrt[
a + b*x^3]) - (10*d*Sqrt[a + b*x^3])/(27*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*c + 5*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^
2*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\left(a + bx^3\right)^{7/2}} dx = \int \frac{c + dx}{\left(a + bx^3\right)^{5/2}} dx$$

$$= \frac{2x(c + dx)}{9a\left(a + bx^3\right)^{3/2}} - \frac{2\int \frac{-\frac{7c}{2} - \frac{5dx}{2}}{\left(a+bx^3\right)^{3/2}} dx}{9a}$$

$$= \frac{2x(c + dx)}{9a\left(a + bx^3\right)^{3/2}} + \frac{2x(7c + 5dx)}{27a^2\sqrt{a + bx^3}} + \frac{4\int \frac{\frac{7c}{4} - \frac{5dx}{4}}{\sqrt{a+bx^3}} dx}{27a^2}$$

$$= \frac{2x(c + dx)}{9a\left(a + bx^3\right)^{3/2}} + \frac{2x(7c + 5dx)}{27a^2\sqrt{a + bx^3}} - \frac{(5d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a+bx^3}} dx}{27a^2\sqrt[3]{b}} + \frac{\left(7c + \frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}}{27a^2}$$

$$= \frac{2x(c + dx)}{9a\left(a + bx^3\right)^{3/2}} + \frac{2x(7c + 5dx)}{27a^2\sqrt{a + bx^3}} - \frac{10d\sqrt{a + bx^3}}{27a^2b^{2/3}\left(\left(1 + \sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}x\right)} + \frac{5\sqrt{2 - \sqrt{3}}d\left(\sqrt[3]{a} + \right.}{}$$

Mathematica [C] time = 0.0890753, size = 123, normalized size = 0.22

$$\frac{14cx\left(a + bx^3\right)\sqrt{\frac{bx^3}{a} + 1}\,{}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 4cx\left(10a + 7bx^3\right) + 27dx^2\left(a + bx^3\right)\sqrt{\frac{bx^3}{a} + 1}\,{}_2F_1\left(\frac{2}{3}, \frac{5}{2}; \frac{5}{3}; -\frac{bx^3}{a}\right)}{54a^2\left(a + bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (4*c*x*(10*a + 7*b*x^3) + 14*c*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*d*x^2*(a + b*x^3)*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(54*a^2*(a + b*x^3)^(3
/2))
```

Maple [B] time = 0.088, size = 1782, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x)
```

```
[Out] b*d*(-2/15*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+8/135*x^2/a/b^3*(b*x^3+a)^
(1/2)/(x^3+1/b*a)^2+8/81/b*x^2/a^2/((x^3+1/b*a)*b)^(1/2)+8/243*I/b^2/a^2*3^
(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)
```

```
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+b*c*(-2/15*x/b^4*(b*x^3+a)^(1/2)/(
x^3+1/b*a)^3+4/135*x/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+28/405/b*x/a^2/((x
^3+1/b*a)*b)^(1/2)-28/1215*I/b^2/a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b
^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2)))+a*d*(2/15*x^2/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+
22/135*x^2/a^2/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+22/81*x^2/a^3/((x^3+1/b*a)
*b)^(1/2)+22/243*I/a^3*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b
^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2
*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(
1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b
^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/
3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+a*c*(2/
15*x/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+26/135*x/a^2/b^2*(b*x^3+a)^(1/2)/(
x^3+1/b*a)^2+182/405*x/a^3/((x^3+1/b*a)*b)^(1/2)-182/1215*I/a^3*3^(1/2)/b*(
-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\left(bx^3 + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}(dx + c)}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*x^3 + a)*(d*x + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a
^3), x)
```

Sympy [A] time = 102.473, size = 163, normalized size = 0.29

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{7}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 7/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + b*d*x**5*gamm
a(5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamm
a(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\left(bx^3 + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

3.65 $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{9/2}} dx$

Optimal. Leaf size=581

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(55\left(1-\sqrt{3}\right)\sqrt[3]{a}d+91\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} -$$

```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/(135*a^2*(
a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt[a + b*x^3]) - (22*d
*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (1
1*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rubi [A] time = 0.409681, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1586, 1855, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(55\left(1-\sqrt{3}\right)\sqrt[3]{a}d+91\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{}{81a^3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```


```
[Out] (2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + (2*x*(13*c + 11*d*x))/(135*a^2*(
a + b*x^3)^(3/2)) + (2*x*(91*c + 55*d*x))/(405*a^3*Sqrt[a + b*x^3]) - (22*d
*Sqrt[a + b*x^3])/(81*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (1
1*Sqrt[2 - Sqrt[3]]*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(27*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 +
Sqrt[3]]*(91*b^(1/3)*c + 55*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}} dx = \int \frac{c + dx}{(a + bx^3)^{7/2}} dx$$

$$= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} - \frac{2\int \frac{-\frac{13c}{2} - \frac{11dx}{2}}{(a+bx^3)^{5/2}} dx}{15a}$$

$$= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{4\int \frac{\frac{91c}{4} + \frac{55dx}{4}}{(a+bx^3)^{3/2}} dx}{135a^2}$$

$$= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{2x(91c + 55dx)}{405a^3\sqrt{a + bx^3}} - \frac{8\int \frac{-\frac{91c}{8} + \frac{55dx}{8}}{\sqrt{a+bx^3}} dx}{405a^3}$$

$$= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{2x(91c + 55dx)}{405a^3\sqrt{a + bx^3}} - \frac{(11d)\int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a+bx^3}} dx}{81a^3\sqrt[3]{b}} +$$

$$= \frac{2x(c + dx)}{15a(a + bx^3)^{5/2}} + \frac{2x(13c + 11dx)}{135a^2(a + bx^3)^{3/2}} + \frac{2x(91c + 55dx)}{405a^3\sqrt{a + bx^3}} - \frac{22d\sqrt{a + bx^3}}{81a^3b^{2/3}\left((1 + \sqrt{3})\sqrt[3]{a} + \right.}$$

Mathematica [C] time = 0.116788, size = 138, normalized size = 0.24

$$\frac{4cx\left(157a^2 + 221abx^3 + 91b^2x^6\right) + 182cx\sqrt{\frac{bx^3}{a} + 1}\,(a + bx^3)^2\,{}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 405dx^2\sqrt{\frac{bx^3}{a} + 1}\,(a + bx^3)^2\,{}_2F_1\left(\frac{2}{3},\right.}{810a^3(a + bx^3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

```
[Out] (4*c*x*(157*a^2 + 221*a*b*x^3 + 91*b^2*x^6) + 182*c*x*(a + b*x^3)^2*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 405*d*x^2*(a
+ b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 7/2, 5/3, -((b*x^3)/a
)])/(810*a^3*(a + b*x^3)^(5/2))
```

Maple [B] time = 0.057, size = 1902, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x)
```

```
[Out] b*d*(-2/21*x^2/b^5*(b*x^3+a)^(1/2)/(x^3+1/b*a)^4+8/315*x^2/a/b^4*(b*x^3+a)^
(1/2)/(x^3+1/b*a)^3+88/2835*x^2/a^2/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+88/17
01/b*x^2/a^3/((x^3+1/b*a)*b)^(1/2)+88/5103*I/b^2/a^3*3^(1/2)*(-b^2*a)^(1/3)
*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2
*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
```

```
2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3)))^(1/2))))+b*c*(-2/21*x/b^5*(b*x^3+a)^(1/2)/(x^3+1/b*a)^4+4/315*x
/a/b^4*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+52/2835*x/a^2/b^3*(b*x^3+a)^(1/2)/(x^3
+1/b*a)^2+52/1215/b*x/a^3/((x^3+1/b*a)*b)^(1/2)-52/3645*I/b^2/a^3*3^(1/2)*(
-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+a*d*(2/21*x^2/a/b^4*(
b*x^3+a)^(1/2)/(x^3+1/b*a)^4+34/315*x^2/a^2/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)
^3+374/2835*x^2/a^3/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+374/1701*x^2/a^4/((x^
3+1/b*a)*b)^(1/2)+374/5103*I/a^4*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*
(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2
))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-
b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
))+a*c*(2/21*x/a/b^4*(b*x^3+a)^(1/2)/(x^3+1/b*a)^4+38/315*x/a^2/b^3*(b*x^3+
a)^(1/2)/(x^3+1/b*a)^3+494/2835*x/a^3/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+494
/1215*x/a^4/((x^3+1/b*a)*b)^(1/2)-494/3645*I/a^4*3^(1/2)/b*(-b^2*a)^(1/3)*(
I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a
)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4 + bcx^3 + adx + ac}{\left(bx^3 + a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}(dx+c)}{b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*x^3 + a)*(d*x + c)/(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
+ 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bdx^4+bcx^3+adx+ac}{\left(bx^3+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

3.66 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=590

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*g*x^2*Sqrt
[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d -
 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c - 2*
a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rubi [A] time = 0.555599, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1888, 1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```


```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*f*x*Sqrt[a + b*x^3])/(5*b) + (2*g*x^2*Sqrt
[a + b*x^3])/(7*b) + (2*(7*b*d - 4*a*g)*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*b*d -
 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(5*b*c - 2*
a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{\frac{7bc}{2}+\frac{1}{2}(7bd-4ag)x+\frac{7}{2}bex^2+\frac{7}{2}bfx^3}{\sqrt{a+bx^3}}dx}{7b}$$

$$= \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{\frac{7}{4}b(5bc-2af)+\frac{5}{4}b(7bd-4ag)x+\frac{35}{4}b^2ex^2}{\sqrt{a+bx^3}}dx}{35b^2}$$

$$= \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{\frac{7}{4}b(5bc-2af)+\frac{5}{4}b(7bd-4ag)x}{\sqrt{a+bx^3}}dx}{35b^2} + e\int \frac{x^2}{\sqrt{a+bx^3}}dx$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{(7bd-4ag)\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{7b^{4/3}} + \frac{(7\sqrt[3]{\ldots}}{\ldots}$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2(7bd-4ag)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2\ldots}}{\ldots}$$

Mathematica [C] time = 0.121405, size = 135, normalized size = 0.23

$$\frac{42x\sqrt{\frac{bx^3}{a}+1}(5bc-2af)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+15x^2\sqrt{\frac{bx^3}{a}+1}(7bd-4ag)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4\left(a+bx^3\right)(35e+3x(7f+5\ldots}{210b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

```
[Out] (4*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) + 42*(5*b*c - 2*a*f)*x*Sqrt[1 + (
b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*(7*b*d - 4*a*
g)*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/
(210*b*Sqrt[a + b*x^3])
```

Maple [B] time = 0.046, size = 1491, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] g*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(
1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/
2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))
```

```
^(1/2))))+f*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(
-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/
2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a
)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*I*c*3^(1/2)/b*(-b^2*a)^(1/3)*(I*
(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^
(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.08759, size = 187, normalized size = 0.32

$$e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}+\frac{gx^5\Gamma\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((1/2, 4/3),
 (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gamma(5/
3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.67 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=594

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*e*
Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(b*d - 4*a*g)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^
(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(b*c + 2*a*
f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.432719, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1858, 1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4a\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*e*
Sqrt[a + b*x^3])/(3*a*b) - (2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(b*d - 4*a*g)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^
(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(b*c + 2*a*
f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(5/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{1}{2}b(bc+2af)+\frac{1}{2}b(bd-4ag)x+\frac{3}{2}b^2ex^2}{\sqrt{a+bx^3}}dx}{3ab^2}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{1}{2}b(bc+2af)+\frac{1}{2}b(bd-4ag)x}{\sqrt{a+bx^3}}dx}{3ab^2} - \frac{e\int \frac{x^2}{\sqrt{a+bx^3}}dx}{a}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{(bd-4ag)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{3ab^{4/3}} +$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{3ab\sqrt{a+bx^3}} - \frac{2e\sqrt{a+bx^3}}{3ab} - \frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + -$$

Mathematica [C] time = 0.121357, size = 130, normalized size = 0.22

$$\frac{2x\sqrt{\frac{bx^3}{a}+1}(2af+bc)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3x^2\sqrt{\frac{bx^3}{a}+1}(bd-4ag)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-4a(e+x(f-3gx))+4bcx}{6ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

```
[Out] (4*b*c*x - 4*a*(e + x*(f - 3*g*x)) + 2*(b*c + 2*a*f)*x*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*(b*d - 4*a*g)*x^2*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a
 + b*x^3])
```

Maple [B] time = 0.005, size = 1547, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] g*(-2/3/b*x^2/((x^3+1/b*a)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+f*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^
(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
```

```
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*e/b/(b*x^3+a)^(1/2)+d*(2/
3*x^2/a/((x^3+1/b*a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/
2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))
^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)
-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)
/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(
1/2))))+c*(2/3*x/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(
I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a
)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b
*x^3 + a^2), x)
```

Sympy [A] time = 18.5223, size = 189, normalized size = 0.32

$$e\left(\begin{cases}-\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+ - g$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/3, 3
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**5*gam
ma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gam
ma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.68 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=628

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+7bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(4}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) - (2*
(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(27*a^2*b*
Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(7*b*c + 2*a*f) + (1 - Sqr
t[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.500377, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1858, 1854, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+7bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5b}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) - (2*
(5*b*d + 4*a*g)*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) - (2*(3*a*e - x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(27*a^2*b*
Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*(5*b*d + 4*a*g)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*a^(5/3)*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*(7*b*c + 2*a*f) + (1 - Sqr
t[3])*a^(1/3)*(5*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2\int \frac{-\frac{1}{2}b(7bc+2af)-\frac{1}{2}b(5bd+4ag)x-\frac{3}{2}b^2ex^2}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab^2}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}} + \frac{4\int \frac{\frac{1}{4}b(7bc}{}}{}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}} - \frac{(5bd+4a}{}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(3ae-x(7bc+}{27a^2b}$$

Mathematica [C] time = 0.184539, size = 170, normalized size = 0.27

$$\frac{-4a^2(15e+x(5f+27gx))+10x\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(2af+7bc)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+40abx\left(5c+fx^3\right)+27x^2\left(a+bx^3\right)\sqrt{}}{270a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

```
[Out] (140*b^2*c*x^4 + 40*a*b*x*(5*c + f*x^3) - 4*a^2*(15*e + x*(5*f + 27*g*x)) +
 10*(7*b*c + 2*a*f)*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)] + 27*(5*b*d + 4*a*g)*x^2*(a + b*x^3)*Sqrt[1 + (b*
x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(270*a^2*b*(a + b*x
^3)^(3/2))
```

Maple [B] time = 0.006, size = 1673, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x)
```

```
[Out] g*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+8/27/b*x^2/a/((x^3+1/b*a)*b)^
(1/2)+8/81*I/b^2/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^
(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3
```

```
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(-2/9*x/b^3
*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+4/27/b*x/a/((x^3+1/b*a)*b)^(1/2)-4/81*I/b^2/
a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/9*e/b/(b
*x^3+a)^(3/2)+d*(2/9*x^2/a/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+10/27*x^2/a^2/
((x^3+1/b*a)*b)^(1/2)+10/81*I/a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^
2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*
((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b
*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/
2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(
-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
)))+c*(2/9*x/a/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+14/27*x/a^2/((x^3+1/b*a)*b
)^(1/2)-14/81*I/a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*
a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1
/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b
^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{7/2}}\,dx$

Optimal. Leaf size=676

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(2af+13bc)+5\left(1-\sqrt{3}\right.\right.}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (2
*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^3*b*Sqrt[a + b*x^3])
 - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a^3*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (2*(9*a*e - x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(
135*a^2*b*(a + b*x^3)^(3/2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*a^(
8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c +
2*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rubi [A] time = 0.674968, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1858, 1854, 1855, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(2af+13bc)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4\right.}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (2
*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(405*a^3*b*Sqrt[a + b*x^3])
 - (2*(11*b*d + 4*a*g)*Sqrt[a + b*x^3])/(81*a^3*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (2*(9*a*e - x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(
135*a^2*b*(a + b*x^3)^(3/2)) + (Sqrt[2 - Sqrt[3]]*(11*b*d + 4*a*g)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*a^(
8/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*b^(1/3)*(13*b*c +
2*a*f) + 5*(1 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
```

```
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*a^3*b^(5/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}} - \frac{2\int \frac{-\frac{1}{2}b(13bc+2af)-\frac{1}{2}b(11bd+4ag)x-\frac{9}{2}b^2ex^2}{(a+bx^3)^{5/2}}dx}{15ab^2}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}} - \frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b\left(a+bx^3\right)^{3/2}} + \frac{4\int}{}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}} - \frac{2(9ae-}{}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}} - \frac{2(9ae-}{}$$

$$= \frac{2x\left(bc-af+(bd-ag)x+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}} + \frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}} - \frac{2(11}{81a^3b^{5/3}}$$

Mathematica [C] time = 0.253436, size = 196, normalized size = 0.29

$$\frac{44a^2bx\left(157c+34fx^3\right)-4a^3(297e+x(77f+405gx))+44ab^2x^4\left(221c+14fx^3\right)+154x\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}+1}(2af+1}{8910a^3b\left(a+bx^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

```
[Out] (4004*b^3*c*x^7 + 44*a*b^2*x^4*(221*c + 14*f*x^3) + 44*a^2*b*x*(157*c + 34*
f*x^3) - 4*a^3*(297*e + x*(77*f + 405*g*x)) + 154*(13*b*c + 2*a*f)*x*(a + b
*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ 405*(11*b*d + 4*a*g)*x^2*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[2/3, 7/2, 5/3, -((b*x^3)/a)])/(8910*a^3*b*(a + b*x^3)^(5/2))
```

Maple [B] time = 0.006, size = 1793, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x)
```

```
[Out] g*(-2/15*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+8/135*x^2/a/b^3*(b*x^3+a)^(1
/2)/(x^3+1/b*a)^2+8/81/b*x^2/a^2/((x^3+1/b*a)*b)^(1/2)+8/243*I/b^2/a^2*3^(1
/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3
```

```
^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(
-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(-2/15*x/b^4*(b*x^3+a)^(1/2)/(x^3+
1/b*a)^3+4/135*x/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+28/405/b*x/a^2/((x^3+1
/b*a)*b)^(1/2)-28/1215*I/b^2/a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x
-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)))-2/15*e/b/(b*x^3+a)^(5/2)+d*(2/15*x^2/a/b^3*(b*x^3+a)^(
1/2)/(x^3+1/b*a)^3+22/135*x^2/a^2/b^2*(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+22/81*x
^2/a^3/((x^3+1/b*a)*b)^(1/2)+22/243*I/a^3*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(
1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))))+c*(2/15*x/a/b^3*(b*x^3+a)^(1/2)/(x^3+1/b*a)^3+26/135*x/a^2/b^2*
(b*x^3+a)^(1/2)/(x^3+1/b*a)^2+182/405*x/a^3/((x^3+1/b*a)*b)^(1/2)-182/1215*
I/a^3*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)
^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{b^4x^{12} + 4\,ab^3x^9 + 6\,a^2b^2x^6 + 4\,a^3bx^3 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/(b^4*x^12 + 4*a*
b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

3.70 $\int \frac{(a+bx)^2}{c+dx^3}\,dx$

Optimal. Leaf size=186

$$\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}}+$$

```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1
/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3
) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(2
/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c +
d*x^3])/(3*d)
```

Rubi [A] time = 0.177635, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}-\frac{a\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] -((a*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1
/3))])/(Sqrt[3]*c^(2/3)*d^(2/3))) - (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3
) + d^(1/3)*x])/(3*c^(2/3)*d^(2/3)) + (a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(2
/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(2/3)) + (b^2*Log[c +
d*x^3])/(3*d)
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = b^2\int \frac{x^2}{c+dx^3}\,dx + \int \frac{a^2+2abx}{c+dx^3}\,dx$$

$$= \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{\int \frac{\sqrt[3]{c}\left(2ab\sqrt[3]{c}+2a^2\sqrt[3]{d}\right)+\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}\sqrt[3]{d}} - \frac{\left(2ab\sqrt[3]{c}-a^2\sqrt[3]{d}\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}\sqrt[3]{d}}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d} + \frac{1}{2}\left(a\left(\frac{a}{\sqrt[3]{c}}+\frac{2b}{\sqrt[3]{d}}\right)\right)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x}$$

$$= -\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx\right.}{3d}$$

$$= -\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}\right.}{6c^{2/3}d^{2/}}$$

Mathematica [A] time = 0.0843078, size = 200, normalized size = 1.08

$$-\frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6cd^{2/3}} + \frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}-2abc^{2/3}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3cd^{2/3}} + \frac{\left(a^2\sqrt[3]{c}\sqrt[3]{d}+2abc^{2/3}\right)\tan}{\sqrt{3}cd^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x^3),x]
```

```
[Out] ((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3
))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) - ((-2*a*b*c^(2/3) + a^2*c^(1/3)
*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3)) + (
b^2*Log[c + d*x^3])/(3*d)
```

Maple [A] time = 0.002, size = 211, normalized size = 1.1

$$\frac{a^2}{3d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^2}{6d}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{a^2\sqrt{3}}{3d}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{2ab}{3d}\ln\left(x+\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x^3+c),x)
```

```
[Out] 1/3*a^2/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*a^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^
(1/3)*x+(c/d)^(2/3))+1/3*a^2/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c
/d)^(1/3)*x-1))-2/3*a*b/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/3*a*b/d/(c/d)^(1/
3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+2/3*a*b*3^(1/2)/d/(c/d)^(1/3)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))+1/3*b^2*ln(d*x^3+c)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.10969, size = 10904, normalized size = 58.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3)
 + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^
(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(
3) + 1) - 2*b^2/d)*d*log(2*b^5*c^2 + 7*a^3*b^2*c*d + 1/2*(2*(1/2)^(2/3)*(b^
4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c
 + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*
a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c
 + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*
a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*b*c^2*
d^2 + 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^
```

```
3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d) + (8*a^2*b^3*c*d + a^5*d^2)*x
) - (6*b^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqr
t(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b
*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1
/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b
*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*s
qrt(3) + 1) - 2*b^2/d)*d + 3*sqrt(1/3)*d*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(2
*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2
*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-
I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*
a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)
 + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*
a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)
*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))*log(-2*b^5*c^2 - 7*a^3*b^2*c
*d - 1/2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3)
 + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^
(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*
b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(
3) + 1) - 2*b^2/d)^2*b*c^2*d^2 - 1/2*(4*b^3*c^2*d - a^3*c*d^2)*(2*(1/2)^(2/
3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8
*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^
2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8
*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^
2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d) +
2*(8*a^2*b^3*c*d + a^5*d^2)*x + 3/2*sqrt(1/3)*(2*b^3*c^2*d + a^3*c*d^2 + (2
*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^
6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3)
 + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2
*b^2/d)*b*c^2*d^2)*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(2*(1/2)^(2/3)*(b^4/d^2
- (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3
*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^
3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3
*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^
3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b^2*c*d + (2*(
1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/
d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) +
 (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/
d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) +
 (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b
^2/d)^2*c*d^2)/(c*d^2))) - (6*b^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^
3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^
2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2
)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)*d - 3*sqrt(1/3)*d*sqrt(-(4*b^
4*c + 32*a^3*b*d + 4*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
```

```
/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c +
 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c
^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^
6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c
^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^
6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2)))*log(-
2*b^5*c^2 - 7*a^3*b^2*c*d - 1/2*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*
d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) -
 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c
^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) -
 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c
^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/d)^2*b*c^2*d^2 - 1/2*(4*b^3*c^2*d -
a^3*c*d^2)*(2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(
3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d
)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2
)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d
)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqr
t(3) + 1) - 2*b^2/d) + 2*(8*a^2*b^3*c*d + a^5*d^2)*x - 3/2*sqrt(1/3)*(2*b^3
*c^2*d + a^3*c*d^2 + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c +
 2*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1
/3)*(I*sqrt(3) + 1) - 2*b^2/d)*b*c^2*d^2)*sqrt(-(4*b^4*c + 32*a^3*b*d + 4*(
2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(2*b
^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3
) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3) + (1/2)^(1/3)*(2*b
^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2*a^3*b*d)*b^2/(c*d^3
) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
2*b^2/d)*b^2*c*d + (2*(1/2)^(2/3)*(b^4/d^2 - (b^4*c + 2*a^3*b*d)/(c*d^2))*(
-I*sqrt(3) + 1)/(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2
*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3
) + (1/2)^(1/3)*(2*b^6/d^3 + (8*b^3*c + a^3*d)*a^3/(c^2*d^2) - 3*(b^4*c + 2
*a^3*b*d)*b^2/(c*d^3) + (b^6*c^2 - 2*a^3*b^3*c*d + a^6*d^2)/(c^2*d^3))^(1/3
)*(I*sqrt(3) + 1) - 2*b^2/d)^2*c*d^2)/(c*d^2))))/d
```

Sympy [A] time = 1.03391, size = 156, normalized size = 0.84

$$\operatorname{RootSum}\left(27t^3c^2d^3 - 27t^2b^2c^2d^2 + t\left(18a^3bcd^2 + 9b^4c^2d\right) - a^6d^2 + 2a^3b^3cd - b^6c^2, \left(t \mapsto t\log\left(x + \frac{18t^2bc^2d^2 + 3ta^3cd^2}{a^5d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x**3+c),x)
```

```
[Out] RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**2
 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _t*
log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7*a*
*3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d))))
```

Giac [A] time = 1.09442, size = 262, normalized size = 1.41

$$\frac{b^2\log\left(\left|dx^3+c\right|\right)}{3\,d}-\frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}}+a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}+\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}a^2d-2\left(-cd^2\right)^{\frac{2}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\cdots\right.}{3\left(-\cdots\right.}\right.}{3\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(
1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d) + 1/3*sqrt(3)*((-c*d^2)^(1/3)*a^2*d -
 2*(-c*d^2)^(2/3)*a*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3)
)/(c*d^2) + 1/6*((-c*d^2)^(1/3)*a^2*c*d^3 + 2*(-c*d^2)^(2/3)*a*b*c*d^2)*log
(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^2*d^4)
```

3.71 $\int \frac{(a+bx)^3}{c+dx^3} dx$

Optimal. Leaf size=222

$$\frac{\left(3a^2b\sqrt[3]{c}d^{2/3}+a^3(-d)+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(3a^2b\sqrt[3]{c}d^{2/3}+a^3(-d)+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(-3a^2b\sqrt[3]{c}d^2\right.}{}$$

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*
b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3)) +
((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rubi [A] time = 0.318566, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(3a^2b\sqrt[3]{c}d^{2/3}+a^3(-d)+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(3a^2b\sqrt[3]{c}d^{2/3}+a^3(-d)+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(-3a^2b\sqrt[3]{c}d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2*
d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^2*
b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3)) +
((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \int \left(\frac{b^3}{d}-\frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{d\left(c+dx^3\right)}\right)dx$$

$$= \frac{b^3x}{d}-\frac{\int \frac{b^3c-a^3d-3a^2bdx-3ab^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d}+\left(3ab^2\right)\int \frac{x^2}{c+dx^3}\,dx-\frac{\int \frac{b^3c-a^3d-3a^2bdx}{c+dx^3}\,dx}{d}$$

$$= \frac{b^3x}{d}+\frac{ab^2\log\left(c+dx^3\right)}{d}-\frac{\int \frac{\sqrt[3]{c}\left(-3a^2b\sqrt[3]{c}d+2\sqrt[3]{d}\left(b^3c-a^3d\right)\right)+\sqrt[3]{d}\left(-3a^2b\sqrt[3]{c}d-\sqrt[3]{d}\left(b^3c-a^3d\right)\right)x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d^{4/3}}-\frac{\left(b^3c+3a^2b\sqrt[3]{c}\cdots\right.}{\cdots}$$

$$= \frac{b^3x}{d}-\frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{ab^2\log\left(c+dx^3\right)}{d}-\frac{\left(b^3c-3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\int\cdots}{2\sqrt[3]{c}d\cdots}$$

$$= \frac{b^3x}{d}-\frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\cdots\right.}{6c^{2/3}d^{4/3}}$$

$$= \frac{b^3x}{d}+\frac{\left(b^3c-3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}-\frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(b\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.170185, size = 214, normalized size = 0.96

$$\frac{\left(3a^2b\sqrt[3]{c}d^{2/3}+a^3(-d)+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-2\left(3a^2b\sqrt[3]{c}d^{2/3}+a^3(-d)+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+2\sqrt{3}\left(-3a^2b\right.}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x^3),x]
```

```
[Out] (6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3
*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1/3
)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d^(2
/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c^(2/
3)*d^(1/3)*Log[c + d*x^3])/(6*c^(2/3)*d^(4/3))
```

Maple [A] time = 0.005, size = 325, normalized size = 1.5

$$\frac{b^3x}{d}+\frac{a^3}{3d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b^3c}{3d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^3}{6d}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{b^3c}{6d^2}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x^3+c),x)
```

```
[Out] b^3*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^3-1/3/d^2/(c/d)^(2/3)*ln(x+(c
/d)^(1/3))*b^3*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^3+1/
6/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^3*c+1/3/d/(c/d)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^3-1/3/d^2/(c/d)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^3*c-1/d*a^2*b/(c/d)^(1/3)*l
n(x+(c/d)^(1/3))+1/2/d*a^2*b/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+
1/d*a^2*b*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))+a*b^2
*ln(d*x^3+c)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 14.9484, size = 15101, normalized size = 68.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x - 2*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/
(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^
2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^
4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1
/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*
a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^
2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
)^(1/3)*(I*sqrt(3) + 1))*d*log(-3*a*b^8*c^3 + 15*a^4*b^5*c^2*d + 15*a^7*b^2
*c*d^2 + 3/4*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2
))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d
^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (
b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) -
6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/
(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
 - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3
)*(I*sqrt(3) + 1))^2*a^2*b*c^2*d^3 - 1/2*(b^6*c^3*d - 20*a^3*b^3*c^2*d^2 +
a^6*c*d^3)*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))
*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3
) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^
9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*
a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c
*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) -
 (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*
(I*sqrt(3) + 1)) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)
*x) + (18*a*b^2 + (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(
c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2
/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4
) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/
3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3)*(I*sqrt(3) + 1))*d + 3*sqrt(1/3)*d*sqrt((12*a^2*b^4*c - 48*a^5*b*d -
 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sq
rt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^
9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 -
 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d
 + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) -
 (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c
^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt
(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*
d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*
d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/
(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d
^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*d^2)))*log(3*a*b^8*c^3 - 15*a^4*b^5
*c^2*d - 15*a^7*b^2*c*d^2 - 3/4*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*
c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c +
a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9
*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/
(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*
c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 -
 a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d
^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*a^2*b*c^2*d^3 + 1/2*(b^6*c^3*d - 20
*a^3*b^3*c^2*d^2 + a^6*c*d^3)*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c
+ a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^
```

5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d
^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c
^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c
+ a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a
^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3
)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6
*b^3*c*d^2 - a^9*d^3)*x + 3/4*sqrt(1/3)*(2*b^6*c^3*d + 14*a^3*b^3*c^2*d^2 +
2*a^6*c*d^3 + 3*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c
d^2))(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/
(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4)
- (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3
) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*
b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*
d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^
(1/3)*(I*sqrt(3) + 1))*a^2*b*c^2*d^3)*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12*
(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3
) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^
3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a
^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (
1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^
9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 -
3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3)
+ 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(
c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2
/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4
) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/
3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a
*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2
*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))
^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*d^2))) + (18*a*b^2 + (6*(1/2)^(2/3)*(3*
a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6
/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*
d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*
a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3
b^6/d^3 - 27(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*
c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*d - 3*sqrt(1
/3)*d*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 -
(2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a
^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*
c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2
- a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*
(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*
b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*
d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)
*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3
b^6/d^3 - 27(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*
c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54
*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*
b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2
*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)
/(c*d^2)))*log(3*a*b^8*c^3 - 15*a^4*b^5*c^2*d - 15*a^7*b^2*c*d^2 - 3/4*(6*(
1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) +
1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 -
3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b
^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)
^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^
3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a

```
^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1)
)^2*a^2*b*c^2*d^3 + 1/2*(b^6*c^3*d - 20*a^3*b^3*c^2*d^2 + a^6*c*d^3)*(6*(1/
2)^(2/3)*(3*a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)
/(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*
a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6
*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(
1/3)*(54*a^3*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3
- 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3
*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))
- 2*(b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^9*d^3)*x - 3/4*sqrt(1
/3)*(2*b^6*c^3*d + 14*a^3*b^3*c^2*d^2 + 2*a^6*c*d^3 + 3*(6*(1/2)^(2/3)*(3*a
^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/
d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d
 + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a
^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*
b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c
^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*a^2*b*c^2*d^3
)*sqrt((12*a^2*b^4*c - 48*a^5*b*d - 12*(6*(1/2)^(2/3)*(3*a^2*b^4/d^2 - (2*a
^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6/d^3 - 27*(2*a^2*b
^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*c*d^
2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2 - a^
9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3*b^6/d^3 - 27*(2*a
^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*d + 3*a^6*b^3*
c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*a^6*b^3*c*d^2
- a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))*a*b^2*c*d - (6*(1/2)^(2/3)*(3*
a^2*b^4/d^2 - (2*a^2*b^4*c + a^5*b*d)/(c*d^2))*(-I*sqrt(3) + 1)/(54*a^3*b^6
/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*c^2*
d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d - 24*
a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3) - 6*a*b^2/d + (1/2)^(1/3)*(54*a^3
*b^6/d^3 - 27*(2*a^2*b^4*c + a^5*b*d)*a*b^2/(c*d^3) - (b^9*c^3 - 3*a^3*b^6*
c^2*d + 3*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4) - (b^9*c^3 - 3*a^3*b^6*c^2*d -
 24*a^6*b^3*c*d^2 - a^9*d^3)/(c^2*d^4))^(1/3)*(I*sqrt(3) + 1))^2*c*d^2)/(c*
d^2))))/d
```

Sympy [A] time = 2.64378, size = 245, normalized size = 1.1

$$\frac{b^3x}{d} + \text{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t\left(27a^5bcd^3 + 54a^2b^4c^2d^2\right) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3, \left(t \mapsto t\,\text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x**3+c),x)
```

```
[Out] b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + _t*(27*
a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2 -
3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c**2
*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d + 1
5*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 + 24*a
**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

Giac [A] time = 1.09219, size = 327, normalized size = 1.47

$$\frac{b^3x}{d}+\frac{ab^2\log\left(\left|dx^3+c\right|\right)}{d}-\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}b^3c-\left(-cd^2\right)^{\frac{1}{3}}a^3d+3\left(-cd^2\right)^{\frac{2}{3}}a^2b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2}-\frac{\left(\left(-cd^2\right)^{\frac{1}{3}}b^3c-\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] b^3*x/d + a*b^2*log(abs(d*x^3 + c))/d - 1/3*sqrt(3)*((-c*d^2)^(1/3)*b^3*c -
 (-c*d^2)^(1/3)*a^3*d + 3*(-c*d^2)^(2/3)*a^2*b)*arctan(1/3*sqrt(3)*(2*x + (
-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^2) - 1/6*((-c*d^2)^(1/3)*b^3*c - (-c*d^2)^(
1/3)*a^3*d - 3*(-c*d^2)^(2/3)*a^2*b)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3
))/(c*d^2) - 1/3*(3*a^2*b*d^3*(-c/d)^(1/3) - b^3*c*d^2 + a^3*d^3)*(-c/d)^(1
/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^3)
```

3.72 $\int \frac{(a+bx)^4}{c+dx^3}\,dx$

Optimal. Leaf size=282

$$-\frac{\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{5/3}}+\frac{\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)\log\left(\sqrt[3]{\right.}}{3c^{2/3}d^{5/3}}$$

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^3
*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3)
)])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*(4*
a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) - ((b*c^(1/
3)*(b^3*c - 4*a^3*d) - d^(1/3)*(4*a*b^3*c - a^4*d))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(5/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rubi [A] time = 0.443201, antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}+a^4(-d)+4ab^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}+\frac{\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{\right.}}{3c^{2/3}d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^3
*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3)
)])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*(4*
a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a*b^3
*c - a^4*d - (b*c^(1/3)*(b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x^3])/d
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \int \left(\frac{4ab^3}{d}+\frac{b^4x}{d}-\frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x-6a^2b^2dx^2}{d\left(c+dx^3\right)}\right)dx$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\int \frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x-6a^2b^2dx^2}{c+dx^3}\,dx}{d}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\left(6a^2b^2\right)\int \frac{x^2}{c+dx^3}\,dx-\frac{\int \frac{4ab^3c-a^4d+b\left(b^3c-4a^3d\right)x}{c+dx^3}\,dx}{d}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\frac{2a^2b^2\log\left(c+dx^3\right)}{d}-\frac{\int \frac{\sqrt[3]{c}\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)+2\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)+\sqrt[3]{d}\left(b\sqrt[3]{c}\left(b^3c-4a^3d\right)-\sqrt[3]{d}\left(4ab^3c-a^4d\right)\right)x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}}{3c^{2/3}d^{4/3}}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{2a^2b^2\log\left(c+dx^3\right)}{d}-\frac{\left(b^4c^{4/3}+4ab^3c\right.}{}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}-\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a^3d\right)}{\sqrt[3]{d}}\right)\log\left(c^{2/3}\right.}{6c^{2/3}d^{4/3}}$$

$$= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}}-\frac{\left(4ab^3c-a^4d-\frac{b\sqrt[3]{c}\left(b^3c-4a^3\right.}{\sqrt[3]{d}}\right.}{3c^{2/3}d^{4/}}$$

Mathematica [A] time = 0.244274, size = 277, normalized size = 0.98

$$\frac{-\frac{\left(-4a^3b\sqrt[3]{c}d+a^4d^{4/3}-4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}}+\frac{2\left(-4a^3b\sqrt[3]{c}d+a^4d^{4/3}-4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}}+\frac{2\sqrt{3}\left(-4a^3b\sqrt[3]{c}d+a^4\left(-d^{4/3}\right.\right.}{}}{6d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x^3),x]
```

```
[Out] (24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3) + 4*a*b^3
*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c^(
1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(
1/3)*d + a^4*d^(4/3))*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3) - 4
*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])/(6*d^
(5/3))
```

Maple [A] time = 0.003, size = 446, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x^3+c),x)
```

```
[Out] 1/2*b^4*x^2/d+4*a*b^3*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^4-4/3/d^2/(
c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b^3*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))*a^4+2/3/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a*b
^3*c+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^4-
4/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b^3*c
-4/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))*a^3*b+1/3/d^2/(c/d)^(1/3)*ln(x+(c/d)^(
1/3))*b^4*c+2/3/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^3*b-1/6/d
^2/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^4*c+4/3/d*3^(1/2)/(c/d)^
(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^3*b-1/3/d^2*3^(1/2)/(c/d)^(
1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^4*c+2*a^2*b^2*ln(d*x^3+c)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 57.8475, size = 18692, normalized size = 66.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^4*x^2 + 48*a*b^3*x + 2*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/
d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1
)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^
2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^
2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3
+ a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c
^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9
*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3
*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sq
rt(3) + 1))*d*log(-8*a*b^11*c^4 - 66*a^4*b^8*c^3*d + 48*a^7*b^5*c^2*d^2 + 2
6*a^10*b^2*c*d^3 - 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1
/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c
*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d
 + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3
*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*
d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b
^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
(b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^
12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/
(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*
c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b
^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b
^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
(b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^
12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/
(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*
b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*
a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b
^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))
- (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)*x) + (36*a^2*
b^2 - (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4
*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*
a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*
a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*
b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3
) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7
*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 -
 a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*
a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*d + 3*sqrt(1/3)
*d*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d
- 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d
^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b
^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a
^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^
6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432
*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d
^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
 + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12
*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(
1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(
c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*
d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^
3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2
*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*
b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) -
```

(b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b
^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)
/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^3)))*log(8*a*b^11*c^4 + 66
*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^10*b^2*c*d^3 + 1/4*(b^4*c^3*d^3
- 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7
*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6
/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b
^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12
*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c
^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6
*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2
- 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 + a^8*c*d^4)*(12*a^2*b^2/d -
2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^
2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6
*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*
a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^
4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
+ (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*
d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^12*c^4 + 52*a^3*b^9*c^3*d - 5
2*a^9*b^3*c*d^3 - a^12*d^4)*x + 3/4*sqrt(1/3)*(20*a^2*b^6*c^3*d^2 + 32*a^5*
b^3*c^2*d^3 + 2*a^8*c*d^4 + (b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d -
2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^
2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^
4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^
9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6
*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*
a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^
4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5)
+ (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*
d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)))*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c
*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a
*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6
b^6/d^3 - 18(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4)
- (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (
b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4
)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^
4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 5
2*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6
*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)
)*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*
c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/
d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^
12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*
c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^
2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4
*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9
*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*
c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c
*d^3)/(c*d^3))) + (36*a^2*b^2 - (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d
^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)
/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2
/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2
*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 +
a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^
2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*

```
c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*
d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqr
t(3) + 1))*d - 3*sqrt(1/3)*d*sqrt(-(64*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7
*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 +
19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 -
18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 -
4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5)
)^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d +
 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c
*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^
2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c
*d^2 + (12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^
4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4
*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52
*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3
*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/
3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^
7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3
- a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4
*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^
3)))*log(8*a*b^11*c^4 + 66*a^4*b^8*c^3*d - 48*a^7*b^5*c^2*d^2 - 26*a^10*b^2
*c*d^3 + 1/4*(b^4*c^3*d^3 - 4*a^3*b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*
(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I
*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b
*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a
^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^
9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 1
8*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
+ 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4
*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))
^(1/3)*(I*sqrt(3) + 1))^2 - 1/2*(28*a^2*b^6*c^3*d^2 - 56*a^5*b^3*c^2*d^3 +
a^8*c*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 1
9*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 1
8*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
+ 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4
*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))
^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d +
4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*
d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2
 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)) - 2*(b^12*
c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)*x - 3/4*sqrt(1/3)*(20
*a^2*b^6*c^3*d^2 + 32*a^5*b^3*c^2*d^3 + 2*a^8*c*d^4 + (b^4*c^3*d^3 - 4*a^3*
b*c^2*d^4)*(12*a^2*b^2/d - 2*(1/2)^(2/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 1
9*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 1
8*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4
+ 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4
*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))
^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d +
4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*
d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2
 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3)*(I*sqrt(3) + 1)))*sqrt(-(64
*a*b^7*c^2 - 128*a^4*b^4*c*d + 64*a^7*b*d^2 - 24*(12*a^2*b^2/d - 2*(1/2)^(2
/3)*(36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))
*(-I*sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a
^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3
 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 -
4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3
 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*
```

```
c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4
 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d
^5))^(1/3)*(I*sqrt(3) + 1))*a^2*b^2*c*d^2 + (12*a^2*b^2/d - 2*(1/2)^(2/3)*(
36*a^4*b^4/d^2 - (4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)/(c*d^3))*(-I*
sqrt(3) + 1)/(432*a^6*b^6/d^3 - 18*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*
d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 + 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^
12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9
*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^(1/3) - (1/2)^(1/3)*(432*a^6*b^6/d^3 - 18
*(4*a*b^7*c^2 + 19*a^4*b^4*c*d + 4*a^7*b*d^2)*a^2*b^2/(c*d^4) - (b^12*c^4 +
 52*a^3*b^9*c^3*d - 52*a^9*b^3*c*d^3 - a^12*d^4)/(c^2*d^5) + (b^12*c^4 - 4*
a^3*b^9*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^9*b^3*c*d^3 + a^12*d^4)/(c^2*d^5))^
(1/3)*(I*sqrt(3) + 1))^2*c*d^3)/(c*d^3))))/d
```

Sympy [A] time = 5.47972, size = 325, normalized size = 1.15

$$\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}+\text{RootSum}\Big(27t^3c^2d^5-162t^2a^2b^2c^2d^4+t\left(36a^7bcd^4+171a^4b^4c^2d^3+36ab^7c^3d^2\right)-a^{12}d^4+4a^9b^3cd^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x**3+c),x)
```

```
[Out] 4*a*b**3*x/d + b**4*x**2/(2*d) + RootSum(27*_t**3*c**2*d**5 - 162*_t**2*a**
2*b**2*c**2*d**4 + _t*(36*a**7*b*c*d**4 + 171*a**4*b**4*c**2*d**3 + 36*a*b*
*7*c**3*d**2) - a**12*d**4 + 4*a**9*b**3*c*d**3 - 6*a**6*b**6*c**2*d**2 + 4
*a**3*b**9*c**3*d - b**12*c**4, Lambda(_t, _t*log(x + (36*_t**2*a**3*b*c**2
*d**4 - 9*_t**2*b**4*c**3*d**3 + 3*_t*a**8*c*d**4 - 168*_t*a**5*b**3*c**2*d
**3 + 84*_t*a**2*b**6*c**3*d**2 + 26*a**10*b**2*c*d**3 + 48*a**7*b**5*c**2*
d**2 - 66*a**4*b**8*c**3*d - 8*a*b**11*c**4)/(a**12*d**4 + 52*a**9*b**3*c*d
**3 - 52*a**3*b**9*c**3*d - b**12*c**4))))
```

Giac [A] time = 1.08479, size = 427, normalized size = 1.51

$$\frac{2\,a^2b^2\log\left(\left|dx^3+c\right|\right)}{d}+\frac{b^4dx^2+8\,ab^3dx}{2\,d^2}-\frac{\sqrt{3}\left(4\,\left(-cd^2\right)^{\frac{1}{3}}ab^3cd-\left(-cd^2\right)^{\frac{1}{3}}a^4d^2-\left(-cd^2\right)^{\frac{2}{3}}b^4c+4\,\left(-cd^2\right)^{\frac{2}{3}}a^3bd\right)\text{arct}}{3\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 - 1/3*s
qrt(3)*(4*(-c*d^2)^(1/3)*a*b^3*c*d - (-c*d^2)^(1/3)*a^4*d^2 - (-c*d^2)^(2/3
)*b^4*c + 4*(-c*d^2)^(2/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))
/(-c/d)^(1/3))/(c*d^3) - 1/6*(4*(-c*d^2)^(1/3)*a*b^3*c*d - (-c*d^2)^(1/3)*a
^4*d^2 + (-c*d^2)^(2/3)*b^4*c - 4*(-c*d^2)^(2/3)*a^3*b*d)*log(x^2 + x*(-c/d
)^(1/3) + (-c/d)^(2/3))/(c*d^3) + 1/3*(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5
*(-c/d)^(1/3) + 4*a*b^3*c*d^4 - a^4*d^5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1
/3)))/(c*d^5)
```

3.73 $\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$

Optimal. Leaf size=272

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{6d^{2/3}e^{5/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{3d^{2/3}e^{5/3}}$$

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d^
(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(S
qrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d - 2*
a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((e^(1/3)*(2*b*c*d
- a^2*e) - d^(1/3)*(c^2*d - 2*a*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2])/(6*d^(2/3)*e^(5/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rubi [A] time = 0.490332, antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(2bcd-a^2e\right)-\sqrt[3]{d}\left(c^2d-2abe\right)\right)}{3d^{2/3}e^{5/3}}+\frac{(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d^
(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(S
qrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d - 2*
a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2*e -
 (d^(1/3)*(c^2*d - 2*a*b*e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2])/(6*d^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^2}{d+ex^3}\,dx = \int \left(\frac{2bc}{e}+\frac{c^2x}{e}-\frac{2bcd-a^2e+\left(c^2d-2abe\right)x-\left(b^2+2ac\right)ex^2}{e\left(d+ex^3\right)}\right)dx$$

$$=\frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\int \frac{2bcd-a^2e+\left(c^2d-2abe\right)x-\left(b^2+2ac\right)ex^2}{d+ex^3}\,dx}{e}$$

$$=\frac{2bcx}{e}+\frac{c^2x^2}{2e}-\left(-b^2-2ac\right)\int \frac{x^2}{d+ex^3}\,dx-\frac{\int \frac{2bcd-a^2e+\left(c^2d-2abe\right)x}{d+ex^3}\,dx}{e}$$

$$=\frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(b^2+2ac\right)\log\left(d+ex^3\right)}{3e}-\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}\left(2bcd-a^2e\right)+\sqrt[3]{d}\left(c^2d-2abe\right)\right)+\sqrt[3]{e}\left(-\sqrt[3]{e}\left(2bcd-a^2e\right)+\sqrt[3]{d}\right.}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}}{3d^{2/3}e^{4/3}}$$

$$=\frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}}+\frac{\left(b^2+2ac\right)\log\left(d+ex^3\right)}{3e}-\frac{\left(c^2d\right.}{}$$

$$=\frac{2bcx}{e}+\frac{c^2x^2}{2e}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{4/3}}+\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c^2d-2abe\right)}{\sqrt[3]{e}}\right)\log}{6d^{2/3}e^{4/3}}$$

$$=\frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{5/3}}-\frac{\left(2bcd-a^2e-\frac{\sqrt[3]{d}\left(c\right.}{}\right.}{3}$$

Mathematica [A] time = 0.284016, size = 269, normalized size = 0.99

$$\frac{2e^{2/3}\left(2ac+b^2\right)\log\left(d+ex^3\right)-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae\left(a\sqrt[3]{e}-2b\sqrt[3]{d}\right)-2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{d^{2/3}}+\frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae\left(a\sqrt[3]{e}-2b\sqrt[3]{d}\right)-2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{d^{2/3}}+}{6e^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

```
[Out] (12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))*
(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(
1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^(1/
3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) - 2*b*
c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/(6*e^(
5/3))
```

Maple [B] time = 0.004, size = 444, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x^3+d),x)
```

```
[Out] 1/2*c^2*x^2/e+2*b*c*x/e+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^2-2/3/e^2/(d/
e)^(2/3)*ln(x+(d/e)^(1/3))*b*c*d-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/
e)^(2/3))*a^2+1/3/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b*c*d+1
/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2-2/3/e^
2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b*c*d-2/3/e/(
d/e)^(1/3)*ln(x+(d/e)^(1/3))*a*b+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*c^2*
d+1/3/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*b-1/6/e^2/(d/e)^(1/
3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c^2*d+2/3/e*3^(1/2)/(d/e)^(1/3)*arctan
(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b-1/3/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c^2*d+2/3/e*ln(e*x^3+d)*a*c+1/3/e*ln(e*x^3+d
)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 15.671, size = 26617, normalized size = 97.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^2*x^2 + 24*b*c*x - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*
c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(
2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*
e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a
^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3
)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 +
 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^
5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3
*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*
d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e
^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2
*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e -
 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e*l
og(-4*b*c^5*d^4 - (5*b^4*c^2 - 4*a*b^2*c^3 + 2*a^2*c^4)*d^3*e + 2*(a*b^5 -
2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 + (7*a^4*b^2 - 2*a^5*c)*d*e^3 - 1/4*(c^2
*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/
e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^
2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*
(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^
2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3
 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4
*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(
1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 -
 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 +
a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)
*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4
*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2 - 1/2*
(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2 - 4*(a*b^3 + 3*a^2*b*c)*d^2*e^
3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^
4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2
*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) +
 (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b
*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4
*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d
^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d
*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*
d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^
2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^
4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d
*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)*x) + (6*b^2 + 12
*a*c + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 -
3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^
4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a
^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
 b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^
4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c
^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e +
b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^
2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^
6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*
```

c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e + 3*sqrt(1/3)*e*sqrt(-(32
*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^
2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(
b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2
)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*
b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a
^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*
(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^
2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4
 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3
+ a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^
2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*
(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^
3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 +
 b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3
*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4
) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^
4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4
)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(
I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^
2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b
^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2
*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6
*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c
)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4
 - 12*a*b^2*c)*d*e)/(d*e^3)))*log(4*b*c^5*d^4 + (5*b^4*c^2 - 4*a*b^2*c^3 +
2*a^2*c^4)*d^3*e - 2*(a*b^5 - 2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 - (7*a^4*b
^2 - 2*a^5*c)*d*e^3 + 1/4*(c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e)^2 + 1/2*(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2
- 4*(a*b^3 + 3*a^2*b*c)*d^2*e^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*
a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))
/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*
b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9
*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e
^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4
 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*
e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c
^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^
6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d
*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d
^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e
 - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) -
 2*(c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*
a^4*b*c)*d*e^3)*x + 3/4*sqrt(1/3)*(4*a*b^3*d^2*e^3 + 2*a^4*d*e^4 + 2*(3*b^2
*c^2 - 2*a*c^3)*d^3*e^2 - (c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*

```
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e))*sqrt(-(32*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*
d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d
*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*
c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4
 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^
2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^
3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^
2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*
(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^
2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3
 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4
*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(
1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2
 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*
e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2
*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^
2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^
3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^
6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/
(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(
2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4)
+ (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*
b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^
4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*
d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)
/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4 - 12*a*b^2*c)*d*e)/(d*e^3))) + (6*b^2 + 12
*a*c + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2
+ b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 -
3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^
4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a
^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
 b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^
4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c
^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e +
b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^
2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^
6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*
c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*e - 3*sqrt(1/3)*e*sqrt(-(32
*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^
2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(
b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2
)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*
b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a
^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*
(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))
```

```
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^
2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4
 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3
+ a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^
2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*
(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^
3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 +
 b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3
*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4
) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^
4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e -
b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4
)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(
I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^
2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b
^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2
*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6
*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c
)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4
 - 12*a*b^2*c)*d*e)/(d*e^3)))*log(4*b*c^5*d^4 + (5*b^4*c^2 - 4*a*b^2*c^3 +
2*a^2*c^4)*d^3*e - 2*(a*b^5 - 2*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^2 - (7*a^4*b
^2 - 2*a^5*c)*d*e^3 + 1/4*(c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e)^2 + 1/2*(a^4*d*e^4 + 2*(3*b^2*c^2 + 2*a*c^3)*d^3*e^2
- 4*(a*b^3 + 3*a^2*b*c)*d^2*e^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2 + 2*
a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*e^3))
/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*
b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9
*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e
^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4
 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*
e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c
^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^
6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d
*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d
^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e
 - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)/e) -
 2*(c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*
a^4*b*c)*d*e^3)*x - 3/4*sqrt(1/3)*(4*a*b^3*d^2*e^3 + 2*a^4*d*e^4 + 2*(3*b^2
*c^2 - 2*a*c^3)*d^3*e^2 - (c^2*d^3*e^3 - 2*a*b*d^2*e^4)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e
+ 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e +
 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*
d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2
*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^
5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 -
3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 +
2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2
```

```
 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^
2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6
*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3
*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3)
 - 2*(b^2 + 2*a*c)/e))*sqrt(-(32*b*c^3*d^2 + 32*a^3*b*e^2 + (2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*((b^2 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*
d*e + 2*a^3*b*e^2)/(d*e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d
*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*
c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4
 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^
2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^
3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^
2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*
(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^
2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3
 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4
*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(
1/3) - 2*(b^2 + 2*a*c)/e)^2*d*e^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b^2
 + 2*a*c)^2/e^2 - (2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)/(d*
e^3))/(2*(b^2 + 2*a*c)^3/e^3 - 3*(2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2
*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4) + (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^
2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^
3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^
6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/
(d^2*e^5))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*(b^2 + 2*a*c)^3/e^3 - 3*(
2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e + 2*a^3*b*e^2)*(b^2 + 2*a*c)/(d*e^4)
+ (c^6*d^4 - 2*b^3*c^3*d^3*e + b^6*d^2*e^2 + 9*a^2*b^2*c^2*d^2*e^2 + 6*a^4*
b*c*d*e^3 + a^6*e^4 + 2*(c^3*d^2*e^2 - b^3*d*e^3)*a^3 + 6*(b*c^4*d^3*e - b^
4*c*d^2*e^2)*a)/(d^2*e^5) - (c^6*d^4 - a^6*e^4 + 2*(4*b^3*c^3 - 3*a*b*c^4)*
d^3*e - 2*(4*a^3*b^3 - 3*a^4*b*c)*d*e^3)/(d^2*e^5))^(1/3) - 2*(b^2 + 2*a*c)
/e)*(b^2 + 2*a*c)*d*e^2 + 4*(b^4 - 12*a*b^2*c)*d*e)/(d*e^3))))/e
```

Sympy [B] time = 8.7991, size = 546, normalized size = 2.01

$$\frac{2bcx}{e} + \frac{c^2x^2}{2e} + \operatorname{RootSum}\left(27t^3d^2e^5 + t^2\left(-54acd^2e^4 - 27b^2d^2e^4\right) + t\left(18a^3bde^4 + 27a^2c^2d^2e^3 + 9b^4d^2e^3 + 18bc^3d^3e^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

```
[Out] 2*b*c*x/e + c**2*x**2/(2*e) + RootSum(27*_t**3*d**2*e**5 + _t**2*(-54*a*c*d
**2*e**4 - 27*b**2*d**2*e**4) + _t*(18*a**3*b*d*e**4 + 27*a**2*c**2*d**2*e*
*3 + 9*b**4*d**2*e**3 + 18*b*c**3*d**3*e**2) - a**6*e**4 - 6*a**4*b*c*d*e**
3 + 2*a**3*b**3*d*e**3 - 2*a**3*c**3*d**2*e**2 - 9*a**2*b**2*c**2*d**2*e**2
 + 6*a*b**4*c*d**2*e**2 - 6*a*b*c**4*d**3*e - b**6*d**2*e**2 + 2*b**3*c**3*
d**3*e - c**6*d**4, Lambda(_t, _t*log(x + (18*_t**2*a*b*d**2*e**4 - 9*_t**2
*c**2*d**3*e**3 + 3*_t*a**4*d*e**4 - 36*_t*a**2*b*c*d**2*e**3 - 12*_t*a*b**
3*d**2*e**3 + 12*_t*a*c**3*d**3*e**2 + 18*_t*b**2*c**2*d**3*e**2 - 2*a**5*c
*d*e**3 + 7*a**4*b**2*d*e**3 + 8*a**3*b*c**2*d**2*e**2 - 4*a**2*b**3*c*d**2
*e**2 - 2*a**2*c**4*d**3*e + 2*a*b**5*d**2*e**2 + 4*a*b**2*c**3*d**3*e - 5*
b**4*c**2*d**3*e - 4*b*c**5*d**4)/(a**6*e**4 - 6*a**4*b*c*d*e**3 + 8*a**3*b
**3*d*e**3 + 6*a*b*c**4*d**3*e - 8*b**3*c**3*d**3*e - c**6*d**4))))
```

Giac [A] time = 1.07009, size = 383, normalized size = 1.41

$$\frac{1}{3}\left(b^2+2\,ac\right)e^{(-1)}\log\left(\left|x^3e+d\right|\right)-\frac{\sqrt{3}\left(2\left(-de^2\right)^{\frac{1}{3}}bcde-\left(-de^2\right)^{\frac{2}{3}}c^2d+2\left(-de^2\right)^{\frac{2}{3}}abe-\left(-de^2\right)^{\frac{1}{3}}a^2e^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de\right.\right.}{3\left(-de^{(-1}\right.}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*(b^2 + 2*a*c)*e^(-1)*log(abs(x^3*e + d)) - 1/3*sqrt(3)*(2*(-d*e^2)^(1/3
)*b*c*d*e - (-d*e^2)^(2/3)*c^2*d + 2*(-d*e^2)^(2/3)*a*b*e - (-d*e^2)^(1/3)*
a^2*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^
(-3)/d + 1/3*((-d*e^(-1))^(1/3)*c^2*d*e^4 + 2*b*c*d*e^4 - 2*(-d*e^(-1))^(1/
3)*a*b*e^5 - a^2*e^5)*(-d*e^(-1))^(1/3)*e^(-5)*log(abs(x - (-d*e^(-1))^(1/3
)))/d + 1/2*(c^2*x^2*e + 4*b*c*x*e)*e^(-2) - 1/6*(2*(-d*e^2)^(1/3)*b*c*d*e
+ (-d*e^2)^(2/3)*c^2*d - 2*(-d*e^2)^(2/3)*a*b*e - (-d*e^2)^(1/3)*a^2*e^2)*e
^(-3)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d
```

3.74 $\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$

Optimal. Leaf size=416

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-e\left(b^3d-a^3e\right)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(3\right.}{}$$

```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c
^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2
*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^
2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7
/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) -
((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d +
a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

Rubi [A] time = 0.70029, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-e\left(b^3d-a^3e\right)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] -(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*c
^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c^2
*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^
2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7
/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) -
((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2*c*d +
a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2
/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
```

```
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^3}{d+ex^3}\,dx = \int \left(-\frac{c^3d-b^3e-6abce}{e^2}+\frac{3c\left(b^2+ac\right)x}{e}+\frac{3bc^2x^2}{e}+\frac{c^3x^3}{e}+\frac{c^3d^2-6abcde-e\left(b^3d-a^3e\right)-3\cdots}{\cdots}\right.$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e\left(b^3d-a^3e\right)-3e\left(b^2cd+ac^2d\cdots\right.}{d+ex^3}}{e^2}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\int \frac{c^3d^2-6abcde-e\left(b^3d-a^3e\right)-3e\left(b^2cd+ac^2d\cdots\right.}{d+ex^3}}{e^2}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{\left(bc^2d-ab^2e-a^2ce\right)\log\left(d+ex^3\right)\cdots}{e^2}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e\left(b^3d-a^3e\right)+\cdots\right.}{\cdots}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}+\frac{\left(c^3d^2-6abcde-e\left(b^3d-a^3e\right)+\cdots\right.}{\cdots}$$

$$= -\frac{\left(c^3d-b^3e-6abce\right)x}{e^2}+\frac{3c\left(b^2+ac\right)x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}-\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac^2d^{4/3}e^{2/3}\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.451074, size = 439, normalized size = 1.06

$$-\frac{\frac{2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2-6abcde+3ac^2d^{4/3}e^{2/3}+3b^2cd^{4/3}e^{2/3}-b^3de+c^3d^2\right)}{d^{2/3}}+\frac{4\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(-3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2-6abcde+3ac^2d^{4/3}\cdots\right.}{d^{2/3}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e^(4/3)*x^2
 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3]*(c^3*d^2 - 3*a*c^2
*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1/3)*e^(2/3) + a^3*e) - 3*c*(b^
2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
)/d^(2/3) + (4*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^(2/3)
 - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(1/3) +
 e^(1/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4
/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Lo
g[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 12*e^(1/3)*(-(b*c^2
*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])/(12*e^(7/3))
```

Maple [B] time = 0.005, size = 837, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x^3+d),x)
```

```
[Out] -2/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b*c*d+
b*c^2*x^3/e-1/6/e^3/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c^3*d^2+6
```

```
/e*a*b*c*x-2/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a*b*c*d+1/e^2/(d/e)^(2/3)*ln
(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*b*c*d-1/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/
3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*c^2*d-1/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3
*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^2*c*d+1/e*b^3*x-1/e^2*ln(e*x^3+d)*b*c^2*d-1
/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b^3*d+1/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(
1/3))*c^3*d^2+1/6/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b^3*d-1
/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a^2*b+1/2/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)
*x+(d/e)^(2/3))*a^2*b+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1))*a^3+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^3-1/6/e/(d/e)^(2/3)*
ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^3+3/2/e*x^2*b^2*c-1/e^2*c^3*d*x+3/2/e*x
^2*a*c^2+1/e*ln(e*x^3+d)*a^2*c+1/e*ln(e*x^3+d)*a*b^2+1/4*c^3*x^4/e+1/e*3^(1
/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2*b+1/e^2/(d/e)^(
1/3)*ln(x+(d/e)^(1/3))*b^2*c*d-1/2/e^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/
e)^(2/3))*a*c^2*d-1/2/e^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b^2
*c*d-1/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^
3*d+1/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c^3
*d^2+1/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a*c^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 86.1001, size = 1314, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

```
[Out] b*c**2*x**3/e + c**3*x**4/(4*e) + RootSum(27*_t**3*d**2*e**7 + _t**2*(-81*a
**2*c*d**2*e**6 - 81*a*b**2*d**2*e**6 + 81*b*c**2*d**3*e**5) + _t*(27*a**5*
b*d*e**6 + 54*a**4*c**2*d**2*e**5 - 27*a**3*b**2*c*d**2*e**5 + 54*a**2*b**4
*d**2*e**5 + 27*a**2*b*c**3*d**3*e**4 + 27*a*b**3*c**2*d**3*e**4 - 27*a*c**
5*d**4*e**3 + 27*b**5*c*d**3*e**4 + 54*b**2*c**4*d**4*e**3) - a**9*e**6 - 9
*a**7*b*c*d*e**5 + 3*a**6*b**3*d*e**5 - 3*a**6*c**3*d**2*e**4 - 27*a**5*b**
```

```
2*c**2*d**2*e**4 + 18*a**4*b**4*c*d**2*e**4 - 18*a**4*b*c**4*d**3*e**3 - 3*
a**3*b**6*d**2*e**4 - 21*a**3*b**3*c**3*d**3*e**3 - 3*a**3*c**6*d**4*e**2 +
 27*a**2*b**5*c**2*d**3*e**3 - 27*a**2*b**2*c**5*d**4*e**2 - 9*a*b**7*c*d**
3*e**3 + 18*a*b**4*c**4*d**4*e**2 - 9*a*b*c**7*d**5*e + b**9*d**3*e**3 - 3*
b**6*c**3*d**4*e**2 + 3*b**3*c**6*d**5*e - c**9*d**6, Lambda(_t, _t*log(x +
 (27*_t**2*a**2*b*d**2*e**6 - 27*_t**2*a*c**2*d**3*e**5 - 27*_t**2*b**2*c*d
**3*e**5 + 3*_t*a**6*d*e**6 - 90*_t*a**4*b*c*d**2*e**5 - 60*_t*a**3*b**3*d*
*2*e**5 + 60*_t*a**3*c**3*d**3*e**4 + 270*_t*a**2*b**2*c**2*d**3*e**4 + 90*
_t*a*b**4*c*d**3*e**4 - 90*_t*a*b*c**4*d**4*e**3 + 3*_t*b**6*d**3*e**4 - 60
*_t*b**3*c**3*d**4*e**3 + 3*_t*c**6*d**5*e**2 - 3*a**8*c*d*e**5 + 15*a**7*b
**2*d*e**5 + 30*a**6*b*c**2*d**2*e**4 - 48*a**5*b**3*c*d**2*e**4 - 15*a**5*
c**4*d**3*e**3 + 15*a**4*b**5*d**2*e**4 - 15*a**4*b**2*c**3*d**3*e**3 - 15*
a**3*b**4*c**2*d**3*e**3 - 48*a**3*b*c**5*d**4*e**2 - 30*a**2*b**6*c*d**3*e
**3 + 15*a**2*b**3*c**4*d**4*e**2 + 15*a**2*c**7*d**5*e - 3*a*b**8*d**3*e**
3 - 48*a*b**5*c**3*d**4*e**2 - 30*a*b**2*c**6*d**5*e - 15*b**7*c**2*d**4*e*
*2 - 15*b**4*c**5*d**5*e + 3*b*c**8*d**6)/(a**9*e**6 - 18*a**7*b*c*d*e**5 +
 24*a**6*b**3*d*e**5 + 3*a**6*c**3*d**2*e**4 + 27*a**5*b**2*c**2*d**2*e**4
- 45*a**4*b**4*c*d**2*e**4 + 45*a**4*b*c**4*d**3*e**3 + 3*a**3*b**6*d**2*e*
*4 - 60*a**3*b**3*c**3*d**3*e**3 - 24*a**3*c**6*d**4*e**2 - 27*a**2*b**5*c*
*2*d**3*e**3 + 27*a**2*b**2*c**5*d**4*e**2 - 18*a*b**7*c*d**3*e**3 - 45*a*b
**4*c**4*d**4*e**2 - 18*a*b*c**7*d**5*e - b**9*d**3*e**3 - 24*b**6*c**3*d**
4*e**2 - 3*b**3*c**6*d**5*e + c**9*d**6)))) + x**2*(3*a*c**2 + 3*b**2*c)/(2
*e) + x*(6*a*b*c*e + b**3*e - c**3*d)/e**2
```

Giac [A] time = 1.06767, size = 648, normalized size = 1.56

$$-\left(bc^2d - ab^2e - a^2ce\right)e^{(-2)}\log\left(\left|x^3e + d\right|\right) + \frac{\sqrt{3}\left(\left(-de^2\right)^{\frac{1}{3}}c^3d^2 - \left(-de^2\right)^{\frac{1}{3}}b^3de - 6\left(-de^2\right)^{\frac{1}{3}}abcde + 3\left(-de^2\right)^{\frac{2}{3}}b^2cd + 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -(b*c^2*d - a*b^2*e - a^2*c*e)*e^(-2)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*((-
d*e^2)^(1/3)*c^3*d^2 - (-d*e^2)^(1/3)*b^3*d*e - 6*(-d*e^2)^(1/3)*a*b*c*d*e
+ 3*(-d*e^2)^(2/3)*b^2*c*d + 3*(-d*e^2)^(2/3)*a*c^2*d - 3*(-d*e^2)^(2/3)*a^
2*b*e + (-d*e^2)^(1/3)*a^3*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3)
)/(-d*e^(-1))^(1/3))*e^(-3)/d - 1/3*(c^3*d^2*e^7 - 3*(-d*e^(-1))^(1/3)*b^2*
c*d*e^8 - 3*(-d*e^(-1))^(1/3)*a*c^2*d*e^8 - b^3*d*e^8 - 6*a*b*c*d*e^8 + 3*(
-d*e^(-1))^(1/3)*a^2*b*e^9 + a^3*e^9)*(-d*e^(-1))^(1/3)*e^(-9)*log(abs(x -
(-d*e^(-1))^(1/3)))/d + 1/4*(c^3*x^4*e^3 + 4*b*c^2*x^3*e^3 + 6*b^2*c*x^2*e^
3 + 6*a*c^2*x^2*e^3 - 4*c^3*d*x*e^2 + 4*b^3*x*e^3 + 24*a*b*c*x*e^3)*e^(-4)
+ 1/6*((-d*e^2)^(1/3)*c^3*d^2 - (-d*e^2)^(1/3)*b^3*d*e - 6*(-d*e^2)^(1/3)*a
*b*c*d*e - 3*(-d*e^2)^(2/3)*b^2*c*d - 3*(-d*e^2)^(2/3)*a*c^2*d + 3*(-d*e^2)
^(2/3)*a^2*b*e + (-d*e^2)^(1/3)*a^3*e^2)*e^(-3)*log(x^2 + (-d*e^(-1))^(1/3)
*x + (-d*e^(-1))^(2/3))/d
```

3.75 $\int \frac{\left(a+bx+cx^2\right)^4}{d+ex^3} dx$

Optimal. Leaf size=645

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(\sqrt[3]{e}\left(-12a^2bcde+a^4e^2-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)+\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+\right.\right.}{6d^{2/3}e^{8/3}}$$

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*
d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e
- 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5
)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*
d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^
(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/
3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2*c^2*
d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/3)*(b^4*
d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(1/3
) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((e^(1/3)*(6*b^2*c^2*d^2 + 4*a*c^3*d^
2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/3)*(b^4*d*e + 12*a*b^2*c
*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(2/3) - d^(1/3)*e^(1
/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(8/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rubi [A] time = 1.09664, antiderivative size = 643, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(\frac{\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+12ab^2cde+b^4de\right)}{\sqrt[3]{e}}-12a^2bcde+a^4e^2-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)}{6d^{2/3}e^{7/3}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3*
d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*e
- 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*x^5
)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2*(b*
d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b^2*d^
(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2*e^(1/
3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b^2*c^2*
d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/3)*(b^4*
d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*Log[d^(1/3
) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((6*b^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b
^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*
a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1
/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b*c^2*d*e + 6*
a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^4}{d+ex^3}\,dx = \int\Bigg(-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x}{e^2}-\frac{c\left(c^3d-4b^3e\right.}{e^2}$$

$$=-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d-4b^3e-\right.}{3e^2}$$

$$=-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d-4b^3e-\right.}{3e^2}$$

$$=-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d-4b^3e-\right.}{3e^2}$$

$$=-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d-4b^3e-\right.}{3e^2}$$

$$=-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d-4b^3e-\right.}{3e^2}$$

$$=-\frac{2\left(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce\right)x}{e^2}-\frac{\left(4bc^3d-b^4e-12ab^2ce-6a^2c^2e\right)x^2}{2e^2}-\frac{c\left(c^3d-4b^3e-\right.}{3e^2}$$

Mathematica [A] time = 0.348562, size = 678, normalized size = 1.05

$$-\frac{\frac{5\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-4b\left(3a^2cde^{4/3}+a^3\sqrt[3]{d}e^2+c^3d^{7/3}\right)+6a^2c^2d^{4/3}e+a^4e^{7/3}+6b^2\left(2acd^{4/3}e+c^2d^2\sqrt[3]{e}\right)-4ab^3de^{4/3}+4ac^3d^2\sqrt[3]{e}+b^4d^{4/3}e\right)}{d^{2/3}}+\frac{10\log\left(d+ex^3\right)\left(4c\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

```
[Out] (60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^(
2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)*(
-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^4 +
 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*e^(1
/3))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) + 12*a*
b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^
3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*a*c^3*d
^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3) + a^4*
e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(7/3) + 3*
a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (5*
(4*a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4
/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^
(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^(1/3)*e^(1/3)
*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2
+ 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(30*e^(8/3))
```

Maple [B] time = 0.007, size = 1339, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4/(e*x^3+d),x)
```

```
[Out] -4/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2*b*c*
d-4/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b^2*c
*d-4/3/e^3/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*b*c^3*d^2-1/e^2/(d/e)^(1/3)*ln(x^2
-(d/e)^(1/3)*x+(d/e)^(2/3))*a^2*c^2*d-4/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))
*a*b^3*d+4/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a*c^3*d^2+2/e^3/(d/e)^(2/3)*
ln(x+(d/e)^(1/3))*b^2*c^2*d^2+2/3/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e
)^(2/3))*a*b^3*d-2/3/e^3/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*c^
3*d^2-4/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^2*b*c*d+2/e^2/(d/e)^(2/3)*ln(x^
2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^2*b*c*d-4/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*b^3*d+4/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a*c^3*d^2+2/e^3/(d/e)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^2*c^2*d^2+4/e^2/(d/e)^(1/3)*ln(x+(d/e)^
(1/3))*a*b^2*c*d-2/e^2/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a*b^2*
c*d-2/e^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^2*c
^2*d+4/3/e^3*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b*
c^3*d^2+4/e*x^3*a*b*c^2+6/e*x^2*a*b^2*c-2/e^2*x^2*b*c^3*d-4/3/e/(d/e)^(1/3)
*ln(x+(d/e)^(1/3))*a^3*b+1/3/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*b^4*d+2/3/e/
(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^3*b+1/3/e/(d/e)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^4-1/6/e^2/(d/e)^(1/3)*ln(x^2-(
d/e)^(1/3)*x+(d/e)^(2/3))*b^4*d-4/3/e^2*ln(e*x^3+d)*b^3*c*d-6/e^2*b^2*c^2*d
*x+1/2/e*x^2*b^4+12/e*a^2*b*c*x-4/e^2*a*c^3*d*x-1/e^3/(d/e)^(2/3)*ln(x^2-(d
/e)^(1/3)*x+(d/e)^(2/3))*b^2*c^2*d^2-4/e^2*ln(e*x^3+d)*a*b*c^2*d-1/3/e^2*3^
(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b^4*d+4/3/e*3^(1/
2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a^3*b+1/6*c^4*x^6/e+
1/e*x^4*a*c^3+3/2/e*x^4*b^2*c^2+4/3/e*x^3*b^3*c-1/3/e^2*x^3*c^4*d+3/e*x^2*a
^2*c^2+4/e*a*b^3*x+4/3/e*ln(e*x^3+d)*a^3*c+2/e*ln(e*x^3+d)*a^2*b^2+1/3/e^3*
ln(e*x^3+d)*c^4*d^2+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a^4-1/6/e/(d/e)^(2/
3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a^4+2/3/e^3/(d/e)^(1/3)*ln(x^2-(d/e)^(
1/3)*x+(d/e)^(2/3))*b*c^3*d^2+2/e^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*a^2*c^2*d
+4/5*b*c^3*x^5/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)

[Out] Timed out
```

Giac [A] time = 1.08789, size = 1060, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")

[Out] 1/3*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b^2*e^2 + 4*a^3*c*e^2)*
e^(-3)*log(abs(x^3*e + d)) + 1/3*sqrt(3)*(6*(-d*e^2)^(1/3)*b^2*c^2*d^2*e +
4*(-d*e^2)^(1/3)*a*c^3*d^2*e - 4*(-d*e^2)^(2/3)*b*c^3*d^2 + (-d*e^2)^(2/3)*
b^4*d*e + 12*(-d*e^2)^(2/3)*a*b^2*c*d*e + 6*(-d*e^2)^(2/3)*a^2*c^2*d*e - 4*
(-d*e^2)^(1/3)*a*b^3*d*e^2 - 12*(-d*e^2)^(1/3)*a^2*b*c*d*e^2 - 4*(-d*e^2)^(
2/3)*a^3*b*e^2 + (-d*e^2)^(1/3)*a^4*e^3)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-
1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-4)/d - 1/3*(4*(-d*e^(-1))^(1/3)*b*c^3*d^2
*e^11 + 6*b^2*c^2*d^2*e^11 + 4*a*c^3*d^2*e^11 - (-d*e^(-1))^(1/3)*b^4*d*e^1
2 - 12*(-d*e^(-1))^(1/3)*a*b^2*c*d*e^12 - 6*(-d*e^(-1))^(1/3)*a^2*c^2*d*e^1
2 - 4*a*b^3*d*e^12 - 12*a^2*b*c*d*e^12 + 4*(-d*e^(-1))^(1/3)*a^3*b*e^13 + a
^4*e^13)*(-d*e^(-1))^(1/3)*e^(-13)*log(abs(x - (-d*e^(-1))^(1/3)))/d + 1/30
*(5*c^4*x^6*e^5 + 24*b*c^3*x^5*e^5 + 45*b^2*c^2*x^4*e^5 + 30*a*c^3*x^4*e^5
- 10*c^4*d*x^3*e^4 + 40*b^3*c*x^3*e^5 + 120*a*b*c^2*x^3*e^5 - 60*b*c^3*d*x^
2*e^4 + 15*b^4*x^2*e^5 + 180*a*b^2*c*x^2*e^5 + 90*a^2*c^2*x^2*e^5 - 180*b^2
*c^2*d*x*e^4 - 120*a*c^3*d*x*e^4 + 120*a*b^3*x*e^5 + 360*a^2*b*c*x*e^5)*e^(
-6) + 1/6*(6*(-d*e^2)^(1/3)*b^2*c^2*d^2*e + 4*(-d*e^2)^(1/3)*a*c^3*d^2*e +
4*(-d*e^2)^(2/3)*b*c^3*d^2 - (-d*e^2)^(2/3)*b^4*d*e - 12*(-d*e^2)^(2/3)*a*b
^2*c*d*e - 6*(-d*e^2)^(2/3)*a^2*c^2*d*e - 4*(-d*e^2)^(1/3)*a*b^3*d*e^2 - 12
*(-d*e^2)^(1/3)*a^2*b*c*d*e^2 + 4*(-d*e^2)^(2/3)*a^3*b*e^2 + (-d*e^2)^(1/3)
*a^4*e^3)*e^(-4)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d
```

3.76 $\int \frac{2x^2+x^4}{1+x^3} \, dx$

Optimal. Leaf size=43

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/2
```

Rubi [A] time = 0.078164, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x^2+x^4}{1+x^3}\,dx &= \int \frac{x^2\left(2+x^2\right)}{1+x^3}\,dx\\
&= \int \left(x+\frac{x(-1+2x)}{1+x^3}\right)dx\\
&= \frac{x^2}{2}+\int \frac{x(-1+2x)}{1+x^3}\,dx\\
&= \frac{x^2}{2}+\frac{1}{3}\int \frac{-3+3x}{1-x+x^2}\,dx+\int \frac{1}{1+x}\,dx\\
&= \frac{x^2}{2}+\log(1+x)-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{x^2}{2}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0124251, size = 54, normalized size = 1.26

$$\frac{1}{6}\left(3x^2-\log\left(x^2-x+1\right)+4\log\left(x^3+1\right)+2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x +
x^2] + 4*Log[1 + x^3])/6
```

Maple [A] time = 0.005, size = 38, normalized size = 0.9

$$\frac{x^2}{2}+\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2)/(x^3+1),x)
```

```
[Out] 1/2*x^2+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+ln(1+x)
```

Maxima [A] time = 1.40092, size = 50, normalized size = 1.16

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

Fricas [A] time = 1.55632, size = 120, normalized size = 2.79

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(x + 1)
```

Sympy [A] time = 0.123992, size = 44, normalized size = 1.02

$$\frac{x^2}{2} + \log(x+1) + \frac{\log\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2)/(x**3+1),x)
```

```
[Out] x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

Giac [A] time = 1.08713, size = 51, normalized size = 1.19

$$\frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
+ log(abs(x + 1))
```

3.77 $\int \frac{2x^2+x^4}{1-x^3} \, dx$

Optimal. Leaf size=46

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -x^2/2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^2]/
2
```

Rubi [A] time = 0.0838093, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1593, 1887, 1875, 31, 634, 618, 204, 628}

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^2 + x^4)/(1 - x^3),x]
```


```
[Out] -x^2/2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^2]/
2
```


Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x^2+x^4}{1-x^3}\,dx &= \int \frac{x^2\left(2+x^2\right)}{1-x^3}\,dx \\
&= \int \left(-x+\frac{x(1+2x)}{1-x^3}\right) dx \\
&= -\frac{x^2}{2}+\int \frac{x(1+2x)}{1-x^3}\,dx \\
&= -\frac{x^2}{2}+\frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx+\int \frac{1}{1-x}\,dx \\
&= -\frac{x^2}{2}-\log(1-x)-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx \\
&= -\frac{x^2}{2}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0121212, size = 54, normalized size = 1.17

$$
\frac{1}{6}\left(-3x^2+\log\left(x^2+x+1\right)-4\log\left(1-x^3\right)-2\log(1-x)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x^2 + x^4)/(1 - x^3),x]
```

```
[Out] (-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x +
x^2] - 4*Log[1 - x^3])/6
```

Maple [A] time = 0.005, size = 38, normalized size = 0.8

$$
-\frac{x^2}{2}-\ln(-1+x)-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2)/(-x^3+1),x)
```

```
[Out] -1/2*x^2-ln(-1+x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41591, size = 50, normalized size = 1.09

$$-\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Fricas [A] time = 1.51551, size = 122, normalized size = 2.65

$$-\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(x - 1)
```

Sympy [A] time = 0.120693, size = 46, normalized size = 1.

$$-\frac{x^2}{2}-\log(x-1)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2)/(-x**3+1),x)
```

```
[Out] -x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/3
```

Giac [A] time = 1.0801, size = 51, normalized size = 1.11

$$-\frac{1}{2}x^2-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1)
 - log(abs(x - 1))
```

3.78 $\int \frac{1-x+4x^3}{1+x^3} \, dx$

Optimal. Leaf size=44

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
+ x^2]/3
```

Rubi [A] time = 0.0426502, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1887, 1860, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
+ x^2]/3
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x+4x^3}{1+x^3}\,dx &= \int \left(4-\frac{3+x}{1+x^3}\right)dx\\
&= 4x-\int \frac{3+x}{1+x^3}\,dx\\
&= 4x-\frac{1}{3}\int \frac{7-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx\\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx-2\int \frac{1}{1-x+x^2}\,dx\\
&= 4x-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)+4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= 4x+\frac{4\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0078517, size = 44, normalized size = 1.

$$\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)-\frac{4\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
```

```
[Out] 4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
 + x^2]/3
```

Maple [A] time = 0.004, size = 38, normalized size = 0.9

$$4\,x+\frac{\ln\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)-\frac{2\,\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x+1)/(x^3+1),x)
```

```
[Out] 4*x+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*ln(1+x)
```

Maxima [A] time = 1.42637, size = 50, normalized size = 1.14

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

Fricas [A] time = 1.54927, size = 122, normalized size = 2.77

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(x + 1)
```

Sympy [A] time = 0.119879, size = 48, normalized size = 1.09

$$4x-\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x+1)/(x**3+1),x)
```

```
[Out] 4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3 -
 sqrt(3)/3)/3
```

Giac [A] time = 1.0711, size = 51, normalized size = 1.16

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) - 2
/3*log(abs(x + 1))
```

3.79 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=230

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)}}}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}\right.}}}$$

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4
*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rubi [A] time = 0.0595062, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4
*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx + \int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)}{}$$

Mathematica [C] time = 0.0203391, size = 47, normalized size = 0.2

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right) + \left(1+\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] time = 0.013, size = 407, normalized size = 1.8

$$2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/}{-3/2-i/}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*
3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I
*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*3
^(1/2)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
```

```
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x+\sqrt{3}+1}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 1.63872, size = 92, normalized size = 0.4

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma
(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi
))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.80 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=257

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x}{(-x+\sqrt{}}}}{\sqrt{(-x}}$$

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rubi [A] time = 0.0669402, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1878, 218, 1877}

$$-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E(}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + S
qrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx + \int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-}{}$$

Mathematica [C] time = 0.0157115, size = 43, normalized size = 0.17

$$\left(1+\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [A] time = 0.016, size = 368, normalized size = 1.4

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3
/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))-2/
3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))-2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(
1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}{x^3-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)/(x^3 - 1), x)
```

Sympy [A] time = 2.35611, size = 97, normalized size = 0.38

$$-\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*
gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar
(2*I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.81 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3])
```

Rubi [A] time = 0.0270309, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1879}

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx=\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.023749, size = 63, normalized size = 0.44

$$-\frac{x\sqrt{1-x^3}\left(x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)-2\left(1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] -(x*Sqrt[1 - x^3]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
 x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [B] time = 0.013, size = 407, normalized size = 2.8

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\left(\left(3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2\ldots}{3/2\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*
3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*
I*3^(1/2))*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1
/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+
2*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 2.36226, size = 82, normalized size = 0.57

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)*
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.82 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Rubi [A] time = 0.0327293, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1879}

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx=-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.0255676, size = 67, normalized size = 0.5

$$\frac{x\sqrt{x^3+1}\left(2\left(1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)}{2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [B] time = 0.011, size = 370, normalized size = 2.7

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2
+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))-2/3*I*
3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))
^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(
1/2)))^(1/2))-2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x + sqrt(3) + 1)/(x^3 + 1), x)
```

Sympy [A] time = 1.65042, size = 99, normalized size = 0.73

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*
pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=468

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.120721, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1878, 218, 1877}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx + \int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0556628, size = 90, normalized size = 0.19

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.068, size = 1003, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3
))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2*I*a^(1/3)/b*(-b^
2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b*(
-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.3471, size = 122, normalized size = 0.26

$$\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

3.84 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=481

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\cdots$$

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.138949, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1878, 218, 1877}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx &= \left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx+\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx\\ &= -\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{(1-\sqrt{\cdots}}{(1+\sqrt{\cdots}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}\end{aligned}$$

Mathematica [C] time = 0.0568437, size = 91, normalized size = 0.19

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/
(2*Sqrt[a - b*x^3])
```

Maple [B] time = 0.087, size = 949, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(
-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b
*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^
(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*
3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b
/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2*I*a^(1/3)/b*(b^2*a)^(1/3)*(-I*(x+1/2
/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1
/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1
/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2
)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/
b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a
)^(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/
b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2
)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a
)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(
1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^3+a}b^{\frac{1}{3}}x - \sqrt{-bx^3+a}a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{bx^3-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((sqrt(-b*x^3 + a)*b^(1/3)*x - sqrt(-b*x^3 + a)*a^(1/3)*(sqrt(3) +
1))/(b*x^3 - a), x)
```

Sympy [A] time = 3.50657, size = 128, normalized size = 0.27

$$-\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3
*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

3.85 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=271

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.0657175, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1879}

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```


```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```


Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.0404672, size = 92, normalized size = 0.34

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/
(2*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.035, size = 952, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(
-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*
x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*Elliptic
E(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(
1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*3
^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/
(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2*I*a^(1/3)/b*(b^2*a)^(1/3)*(-I*(x+1/2/
b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/
3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)
*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*
(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)
,(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^
(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*
(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*
(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^
(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2
)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/
```

```
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 3.54667, size = 112, normalized size = 0.41

$$\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*g
amma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*
a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

3.86 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.0618671, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1879}

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```


```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```


Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{}}{(1-\sqrt{}}\right.\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.0434111, size = 93, normalized size = 0.35

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.028, size = 1012, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3
))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2*I*a^(1/3)/b*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b
*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
```

```
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3-a}\,b^{\frac{1}{3}}x + \sqrt{-bx^3-a}\,a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(sqrt(-b*x^3 - a)*b^(1/3)*x + sqrt(-b*x^3 - a)*a^(1/3)*(sqrt(3) +
 1))/(b*x^3 + a), x)
```

Sympy [A] time = 3.53534, size = 129, normalized size = 0.48

$$-\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hype
r((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

3.87 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=520

$$\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(1
/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3
^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.213352, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(b/a)^(1/3)*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(1
/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3
^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(1+\sqrt{3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0484349, size = 89, normalized size = 0.17

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2\left(1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])
)/(2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.034, size = 1004, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a
)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*
I*(b/a)^(1/3)*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/
3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2*I/b*(-b^2*a)^(1
/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

[Out] integral((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)

Sympy [A] time = 2.16748, size = 124, normalized size = 0.24

$$\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)

[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=533

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] (-2*(b/a)^(1/3)*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) -
(2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(
1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.172301, antiderivative size = 533, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1878, 218, 1877}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```


```
[Out] (-2*(b/a)^(1/3)*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) -
(2*Sqrt[2 + Sqrt[3]]*((1 + Sqrt[3])*b^(1/3) - (1 - Sqrt[3])*a^(1/3)*(b/a)^(
1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```


Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}} - \left(-1-\sqrt{3}+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] time = 0.044056, size = 89, normalized size = 0.17

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)-2\left(1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
(b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2
*Sqrt[a - b*x^3])
```

Maple [B] time = 0.036, size = 950, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1
/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)/(-b*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*E
llipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/
3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*Elliptic
F(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(
1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2*I/b*(b^2*a)^(1/3)*(-I*(x+1/2/b
*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2
)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3
)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*
b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*
(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)
,(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^
(1/3)))^(1/2))+2/3*I*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^
(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/
2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(
1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^
2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^3 + a}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{-bx^3 + a}\left(\sqrt{3} + 1\right)}{bx^3 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(-b*x^3 + a)*x*(b/a)^(1/3) - sqrt(-b*x^3 + a)*(sqrt(3) + 1))/
(b*x^3 - a), x)
```

Sympy [A] time = 2.7697, size = 129, normalized size = 0.24

$$-\frac{x^{2}\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*
x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^{3} + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

3.89 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (
b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a + b*x
^3])
```

Rubi [A] time = 0.0831561, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {1879}

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (
b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a + b*x
^3])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.032436, size = 90, normalized size = 0.35

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)-2\left(1+\sqrt{3}\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
(b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2
*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.019, size = 953, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1
/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)/(b*x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*El
lipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3
))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF
(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1
/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2*I/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*
(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)
*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)
))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b
/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(
-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1
/3)))^(1/2))+2/3*I*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1
/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a
```

```
)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 2.75303, size = 114, normalized size = 0.45

$$\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(
a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)
/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

3.90 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

```
[Out] (-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(
2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
(b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3
)*Sqrt[-((1 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a - b*
x^3])
```

Rubi [A] time = 0.0644166, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {1879}

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(
2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
(b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(1/3
)*Sqrt[-((1 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a - b*
x^3])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.0338203, size = 92, normalized size = 0.37

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2\left(1+\sqrt{3}\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])
)/(2*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.012, size = 1013, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1
/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2*I/b*(-b^2*a)^
(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^
```

```
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3 - a}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{-bx^3 - a}\left(\sqrt{3} + 1\right)}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(sqrt(-b*x^3 - a)*x*(b/a)^(1/3) + sqrt(-b*x^3 - a)*(sqrt(3) + 1))
/(b*x^3 + a), x)
```

Sympy [A] time = 2.27895, size = 131, normalized size = 0.52

$$-\frac{ix^2\sqrt[3]{\frac{b}{a}}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2),
(4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.91 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rubi [A] time = 0.0190498, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1877}

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx=\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.0206797, size = 49, normalized size = 0.39

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)+\left(1-\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] time = 0.013, size = 407, normalized size = 3.2

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*
3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I
*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*3
^(1/2)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 1.59724, size = 92, normalized size = 0.72

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/
(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi
))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.92 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3])
```

Rubi [A] time = 0.0252626, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt
[1 - x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx=-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0162588, size = 45, normalized size = 0.32

$$\left(1-\sqrt{3}\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

```
[Out] (1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometric
2F1[1/2, 2/3, 5/3, x^3])/2
```

Maple [B] time = 0.011, size = 368, normalized size = 2.6

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3
/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))-2/
3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))+2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(
1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}{x^3-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^3 - 1), x)
```

Sympy [A] time = 2.41928, size = 97, normalized size = 0.68

$$-\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*
pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar
(2*I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.93 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=264

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x}{(-x-\sqrt{3}}}}{\sqrt{-\frac{}{(-x}}}$$

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.0540904, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E}{\sqrt{-\frac{1-x}{(-x-\sqrt{3}+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*S
qrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\left(\left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)}{}$$

Mathematica [C] time = 0.0261258, size = 63, normalized size = 0.24

$$-\frac{x\sqrt{1-x^3}\left(2\left(\sqrt{3}-1\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

```
[Out] -(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
 x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [A] time = 0.008, size = 407, normalized size = 1.5

$$-2\,\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\left(\left(3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2}{3/2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*
3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*
I*3^(1/2))*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1
/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-
2*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2
```

```
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 2.43886, size = 82, normalized size = 0.31

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - I*x*gamm
a(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + sqrt(3)*I*x*gamma(1
/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.94 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

<u>**Optimal**</u>. Leaf size=247

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}}$$

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

<u>**Rubi**</u> **[A]** time = 0.0543306, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1880, 219, 1879}

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

<u>Rule 1880</u>

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

<u>Rule 219</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

<u>Rule 1879</u>

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\left(\left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx\right)+\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1}{}$$

Mathematica [C] time = 0.0239983, size = 67, normalized size = 0.27

$$\frac{x\sqrt{x^3+1}\left(x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)-2\left(\sqrt{3}-1\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)\right)}{2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
+ x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [A] time = 0.008, size = 370, normalized size = 1.5

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2
+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))-2/3*I*
3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))
^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(
1/2)))^(1/2))+2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
```

```
+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^3 + 1), x)
```

Sympy [A] time = 1.6692, size = 97, normalized size = 0.39

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*
gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.95 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

```
[Out] (-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt
[1 + x^3])
```

Rubi [A] time = 0.0246693, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt
[1 + x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx=-\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.0153862, size = 47, normalized size = 0.37

$$\left(\sqrt{3}-1\right)x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

```
[Out] (-1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] - (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [B] time = 0.01, size = 407, normalized size = 3.2

$$-2\frac{3/2-i/2\sqrt{3}}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+}{-3/2-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-x+3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I
*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2)))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*
I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*
3^(1/2)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I
*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 2.30229, size = 92, normalized size = 0.73

$$-\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] -x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamm
a(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*p
i))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.96 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rubi [A] time = 0.024946, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1877}

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx = \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.0110959, size = 43, normalized size = 0.3

$$\frac{1}{2}x\left(2\left(\sqrt{3}-1\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

```
[Out] (x*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeometr
ic2F1[1/2, 2/3, 5/3, x^3]))/2
```

Maple [B] time = 0.009, size = 368, normalized size = 2.6

$$-\frac{2i}{3}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x+3^(1/2))/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-
3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))+2
/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))-2*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(
-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^
(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}{x^3-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*(x + sqrt(3) - 1)/(x^3 - 1), x)
```

Sympy [A] time = 1.71918, size = 97, normalized size = 0.68

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*g
amma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
2*I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.97 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=263

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+\cdots}{\left(-x-\cdots\right.}}}{\sqrt{-\cdots}}$$

```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/
(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.0502606, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1880, 219, 1879}

$$-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\cdots}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+\cdots\right.}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/
(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx - \int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-}{}$$

Mathematica [C] time = 0.0203051, size = 63, normalized size = 0.24

$$\frac{x\sqrt{1-x^3}\left(2\left(\sqrt{3}-1\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [A] time = 0.009, size = 407, normalized size = 1.6

$$2\frac{-3/2-i/2\sqrt{3}}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\left(\left(3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/2+}{3/2-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x+3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I*3
^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I
*3^(1/2))*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2)
)/(3/2-1/2*I*3^(1/2)))^(1/2)))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/
2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3
/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2
*3^(1/2)*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*
3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)
```

```
,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 1.68934, size = 82, normalized size = 0.31

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)
*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + I*x*gamma(
1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.98 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=248

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}+\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(s\right.}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{}}$$

```
[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*S
qrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.0519657, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{(x-\sqrt{3}+1)^2}}E\left(\sin^{-1}\right.}{\sqrt{-\frac{x+1}{(x-\sqrt{3}+1)^2}}\sqrt{-x^3-}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*S
qrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx - \int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1}{}$$

Mathematica [C] time = 0.0187202, size = 67, normalized size = 0.27

$$-\frac{x\sqrt{x^3+1}\left(x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)-2\left(\sqrt{3}-1\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)\right)}{2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

```
[Out] -(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
 + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [A] time = 0.011, size = 370, normalized size = 1.5

$$\frac{2i}{3}\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-x+3^(1/2))/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+
1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))+2/3*I*3
^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^
(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))-2*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
```

```
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)/(x^3 + 1), x)
```

Sympy [A] time = 2.39549, size = 97, normalized size = 0.39

$$\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(
5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*p
i))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pola
r(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.99 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0447983, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1877}

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0623925, size = 90, normalized size = 0.35

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-2\left(\sqrt{3}-1\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.038, size = 1003, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3
))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2*I*a^(1/3)/b*(-b^
2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b*(
-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-
```

```
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.32725, size = 122, normalized size = 0.48

$$\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b
*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1/3,
 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3} - 1\right)}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

3.100 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.0417543, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] time = 0.0619548, size = 90, normalized size = 0.34

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(\sqrt{3}-1\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/(2*Sqrt[a - b*x^3])
```

Maple [B] time = 0.04, size = 949, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(
-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b
*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^
(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*
3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b
/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))-2*I*a^(1/3)/b*(b^2*a)^(1/3)*(-I*(x+1/2
/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1
/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1
/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2
)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/
b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a
)^(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/
b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2
)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a
)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(
1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^
```

```
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-bx^3+a}b^{\frac{1}{3}}x + \sqrt{-bx^3+a}a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{bx^3-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((sqrt(-b*x^3 + a)*b^(1/3)*x + sqrt(-b*x^3 + a)*a^(1/3)*(sqrt(3) -
1))/(b*x^3 - a), x)
```

Sympy [A] time = 3.55613, size = 128, normalized size = 0.49

$$-\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
, b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

3.101 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=497

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{}$$

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*
Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.132815, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1880, 219, 1879}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*
Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
```

```
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\left(\left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx\right)+\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\left(1-\sqrt{3}\right)\cdots}\right.\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.0393337, size = 91, normalized size = 0.18

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(\sqrt{3}-1\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/(2*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.014, size = 952, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(
-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*
x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*Elliptic
E(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(
1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*3
^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/
(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))-2*I*a^(1/3)/b*(b^2*a)^(1/3)*(-I*(x+1/2/
b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/
3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)
*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*
(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)
,(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^
(1/3)))^(1/2))+2/3*I*a^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*
(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*
(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^
(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2
)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 3.54467, size = 112, normalized size = 0.23

$$\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*g
amma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/6)
*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

3.102 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=488

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4
*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.114932, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1880, 219, 1879}

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4
*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
```

```
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\left(\left(2\sqrt{3}\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx\right)+\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{}}{(1-\sqrt{}}\right.\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.0501872, size = 93, normalized size = 0.19

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(\sqrt[3]{b}x\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-2\left(\sqrt{3}-1\right)\sqrt[3]{a}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/
2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.017, size = 1012, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I/b^(2/3)*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3
))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2*I*a^(1/3)/b*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*I*a^(1/3)*3^(1/2)/b
*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3-a}\,b^{\frac{1}{3}}x - \sqrt{-bx^3-a}\,a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(sqrt(-b*x^3 - a)*b^(1/3)*x - sqrt(-b*x^3 - a)*a^(1/3)*(sqrt(3) -
 1))/(b*x^3 + a), x)
```

Sympy [A] time = 3.56258, size = 128, normalized size = 0.26

$$-\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hype
r((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

3.103 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=241

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(2/
3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + (b
/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*
Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0692954, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1877}

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```


```
[Out] (2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(2/
3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + (b
/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)*
Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0511168, size = 89, normalized size = 0.37

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-2\left(\sqrt{3}-1\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.03, size = 1004, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a
)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*
I*(b/a)^(1/3)*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/
3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2*I/b*(-b^2*a)^(1
/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/
```

```
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.17516, size = 124, normalized size = 0.51

$$\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)
```

```
[Out] x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

3.104 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

```
[Out] (-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - (
b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[(1 - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.060878, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1877}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^(2
/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - (
b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1/3)
*Sqrt[(1 - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7-4\sqrt{3}\right.}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] time = 0.0439258, size = 89, normalized size = 0.36

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(\sqrt{3}-1\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

```
[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
(b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2
*Sqrt[a - b*x^3])
```

Maple [B] time = 0.033, size = 950, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1
/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)/(-b*x^3+a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*E
llipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/
3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*Elliptic
F(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(
1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))-2*I/b*(b^2*a)^(1/3)*(-I*(x+1/2/b
*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2
)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3
)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*
b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*
(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)
,(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^
(1/3)))^(1/2))+2/3*I*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^
(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/
2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(
1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^
```

```
2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^3 + a}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{-bx^3 + a}\left(\sqrt{3} - 1\right)}{bx^3 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(-b*x^3 + a)*x*(b/a)^(1/3) + sqrt(-b*x^3 + a)*(sqrt(3) - 1))/
(b*x^3 - a), x)
```

Sympy [A] time = 2.77021, size = 129, normalized size = 0.52

$$-\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4
/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

3.105 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=549

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),4\sqrt{3}-\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

```
[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(
1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]
) - (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b/
a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.217697, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1880, 219, 1879}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```


```
[Out] (2*(b/a)^(1/3)*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(
1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]
) - (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b/
a)^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]
])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```


Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} - \left(-1+\sqrt{3}+\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.0317337, size = 90, normalized size = 0.16

$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(\sqrt{3}-1\right){}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] -(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
(b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2
*Sqrt[-a + b*x^3])
```

Maple [B] time = 0.014, size = 953, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*(b/a)^(1/3)*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1
/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)/(b*x^3-a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*El
lipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3
))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF
(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1
/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)))-2*I/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*
(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)
*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)
))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b
/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(
-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1
/3)))^(1/2))+2/3*I*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1
/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a
)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{bx^3 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 2.731, size = 114, normalized size = 0.21

$$\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)
```

```
[Out] I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(
a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt
(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

3.106 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=540

$$\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-\cdots}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] (-2*(b/a)^(1/3)*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3
]) + (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b
/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3
]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.16282, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (-2*(b/a)^(1/3)*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(b/a)^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3
]) + (2*Sqrt[2 - Sqrt[3]]*((1 - Sqrt[3])*b^(1/3) - (1 + Sqrt[3])*a^(1/3)*(b
/a)^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3
]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
```

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} + \left(1-\sqrt{3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right.}{\left(1-\sqrt{3}\right.}\right.\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.0393996, size = 92, normalized size = 0.17

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(x\sqrt[3]{\frac{b}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-2\left(\sqrt{3}-1\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[-a - b*x^3])
```

Maple [B] time = 0.016, size = 1013, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1
/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2*I/b*(-b^2*a)^
(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-bx^3 - a}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{-bx^3 - a}\left(\sqrt{3} - 1\right)}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(sqrt(-b*x^3 - a)*x*(b/a)^(1/3) - sqrt(-b*x^3 - a)*(sqrt(3) - 1))
/(b*x^3 + a), x)
```

Sympy [A] time = 2.29273, size = 129, normalized size = 0.24

$$-\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.107 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=490

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.147741, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.029043, size = 75, normalized size = 0.15

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Maple [A] time = 0.003, size = 720, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*d*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*I*c*3^(1/2)/b*(-b^
2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx+c}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 1.62566, size = 78, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

3.108 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=503

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\ldots$$

```
[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.148506, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1878, 218, 1877}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a - b*x^3],x]
```


```
[Out] (2*d*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*
(b^(1/3)*c + (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```


Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
```

```
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx &= -\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}}-\left(-c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx\\ &= \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}\end{aligned}$$

Mathematica [C] time = 0.0354163, size = 75, normalized size = 0.15

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d
*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])
```

Maple [A] time = 0.004, size = 681, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^3+a)^(1/2),x)
```

```
[Out] 2/3*I*d*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+
a)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/
3*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)
*b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1
/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*3^(1/
2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2
*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2/3*I*c*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/
b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/
2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/
3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)
*b/(b^2*a)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b
*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2
),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)
^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3 + a}(dx + c)}{bx^3 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^3 + a)*(d*x + c)/(b*x^3 - a), x)
```

Sympy [A] time = 1.72232, size = 82, normalized size = 0.16

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqr
t(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

3.109 $\int \frac{c+dx}{\sqrt{-a+bx^3}} dx$

Optimal. Leaf size=515

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] (-2*d*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c + (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.163478, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1880, 219, 1879}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (-2*d*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c + (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
```

```
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} - \left(-c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] time = 0.0246097, size = 76, normalized size = 0.15

$$\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{bx^3}{a}\right)\right)}{2\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] + d
*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])
```

Maple [A] time = 0.004, size = 683, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3-a)^(1/2),x)
```

```
[Out] 2/3*I*d*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)*((x-1/b*(b^2*a)^(1/3))/(-3/2/
b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a
)^(1/2)*((-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*EllipticE(1/3
*3^(1/2)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*
b/(b^2*a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(b^2*a)^(1/3)))^(1/2))+1/b*(b^2*a)^(1/3)*EllipticF(1/3*3^(1/2
)*(-I*(x+1/2/b*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*
a)^(1/3))^(1/2),(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(b^2*a)^(1/3)))^(1/2)))+2/3*I*c*3^(1/2)/b*(b^2*a)^(1/3)*(-I*(x+1/2/b
*(b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2
)*((x-1/b*(b^2*a)^(1/3))/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3
)))^(1/2)*(I*(x+1/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*
b/(b^2*a)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(b^2*a)^(1/3))*3^(1/2)*b/(b^2*a)^(1/3))^(1/2),
(-I*3^(1/2)/b*(b^2*a)^(1/3)/(-3/2/b*(b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(b^2*a)^(
1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{bx^3-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^3 - a), x)
```

Sympy [A] time = 1.70066, size = 73, normalized size = 0.14

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3)
) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*gamm
a(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

3.110 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=508

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\sqrt[4]{3}\sqrt{\cdots}$$

```
[Out] (-2*d*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c - (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.153626, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\sqrt[4]{3}\sqrt{2+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a - b*x^3],x]
```


```
[Out] (-2*d*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(b^(1/3)*c - (1 + Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```


Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
```

```
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} + \left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots}\right.\right.}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] time = 0.025052, size = 78, normalized size = 0.15

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a - b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[-a - b*x^3])
```

Maple [A] time = 0.005, size = 726, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^3-a)^(1/2),x)
```

```
[Out] -2/3*I*d*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*I*c*3^(1/2)/b*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^3 - a}(dx + c)}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^3 - a)*(d*x + c)/(b*x^3 + a), x)
```

Sympy [A] time = 1.72232, size = 83, normalized size = 0.16

$$-\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-bx^3 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

3.111 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=246

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x$$

```
[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1
- x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3])
```

Rubi [A] time = 0.0810052, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2d\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(x+1)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[1 + x^3],x]
```

```
[Out] (2*d*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1
- x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = d\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx + \left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(c-\left(1-\sqrt{3}\right.\right.}{}$$

Mathematica [C] time = 0.0102201, size = 42, normalized size = 0.17

$$cx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{1}{2}dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[1 + x^3],x]
```

```
[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (d*x^2*Hypergeometric2F1[1/2,
2/3, 5/3, -x^3])/2
```

Maple [A] time = 0.004, size = 291, normalized size = 1.2

$$2\frac{d\left(3/2-i/2\sqrt{3}\right)}{\sqrt{x^3+1}}\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\left(\left(-3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{1+x}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2}{-3/2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(x^3+1)^(1/2),x)
```

```
[Out] 2*d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*
I*3^(1/2))*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2)))+2*c*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1
/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3
/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{x^3 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx + c}{\sqrt{x^3 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(x^3 + 1), x)
```

Sympy [A] time = 1.38533, size = 61, normalized size = 0.25

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3
)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*g
amma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{x^3 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 + 1), x)
```

3.112 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=271

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1}{}$$

```
[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rubi [A] time = 0.0873074, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1878, 218, 1877}

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{2d\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] (2*d*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -\left(d\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx\right)+\left(c+d-\sqrt{3}d\right)\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(c+d-\right.}{}$$

Mathematica [C] time = 0.0093198, size = 38, normalized size = 0.14

$$cx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+\frac{1}{2}dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[1 - x^3],x]
```

```
[Out] c*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + (d*x^2*Hypergeometric2F1[1/2, 2
/3, 5/3, x^3])/2
```

Maple [A] time = 0.005, size = 267, normalized size = 1.

$$-\frac{2i}{3}d\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i}{2}\sqrt{3}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*(
(-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x
+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))
-2/3*I*c*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3 + 1}(dx + c)}{x^3 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*(d*x + c)/(x^3 - 1), x)
```

Sympy [A] time = 1.4608, size = 65, normalized size = 0.24

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/
(3*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

3.113 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=275

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(}{}$$

```
[Out] (-2*d*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(c + d + Sqrt[3]*d)*(1 - x)*Sqrt
[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 -
 Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3])
```

Rubi [A] time = 0.0873031, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1880, 219, 1879}

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2d\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(c + d + Sqrt[3]*d)*(1 - x)*Sqrt
[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 -
 Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\left(d\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\right)+\left(c+d+\sqrt{3}d\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{2\sqrt{2-\sqrt{3}}\left(c+d+\right.}{}$$

Mathematica [C] time = 0.021755, size = 58, normalized size = 0.21

$$\frac{x\sqrt{1-x^3}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + d*x*Hypergeom
etric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Maple [A] time = 0.004, size = 291, normalized size = 1.1

$$2\,\frac{d\left(-3/2-i/2\sqrt{3}\right)}{\sqrt{x^3-1}}\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2-i/2\sqrt{3}}{3/2-i/2\sqrt{3}}}\sqrt{\frac{x+1/2+i/2\sqrt{3}}{3/2+i/2\sqrt{3}}}\left(\left(3/2-i/2\sqrt{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{-1+x}{-3/2-i/2\sqrt{3}}},\sqrt{\frac{3/}{3/}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(x^3-1)^(1/2),x)
```

```
[Out] 2*d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2
*I*3^(1/2))*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2*c*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/
2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)
/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{x^3-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(x^3 - 1), x)
```

Sympy [A] time = 1.42982, size = 56, normalized size = 0.2

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*d*x**2
*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(x^3 - 1), x)
```

3.114 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=261

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1}{}$$

```
[Out] (-2*d*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 +
 x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
 x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x)*Sq
rt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1
 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])
```

Rubi [A] time = 0.0777453, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1880, 219, 1879}

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2d\sqrt{-x^3-1}}{x-\sqrt{3}+1}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(x+1)\sqrt{}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*d*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*d*(1 +
 x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] +
 x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x)*Sq
rt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1
 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])
```

Rule 1880

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 + Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 - 2*
(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = d\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx + \left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(c-\right.}{}$$

Mathematica [C] time = 0.0220398, size = 62, normalized size = 0.24

$$\frac{x\sqrt{x^3+1}\left(2c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+dx\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)}{2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + d*x*Hypergeo
metric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Maple [A] time = 0.004, size = 269, normalized size = 1.

$$-\frac{2i}{3}d\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i}{2}\sqrt{3}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}}{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)\sqrt{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3
/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))-2/3*
I*c*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/
2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^3 - 1}(dx + c)}{x^3 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(d*x + c)/(x^3 + 1), x)
```

Sympy [A] time = 1.44814, size = 66, normalized size = 0.25

$$-\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] -I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(
4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))
/(3*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\sqrt{-x^3 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

3.115 $\int \frac{c+dx}{a-bx^4} \, dx$

Optimal. Leaf size=87

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqr
t[a]*Sqrt[b])
```

Rubi [A] time = 0.0654591, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1876, 212, 208, 205, 275}

$$\frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqr
t[a]*Sqrt[b])
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a-bx^4}\,dx &= \int \left(\frac{c}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx\\
&= c\int \frac{1}{a-bx^4}\,dx + d\int \frac{x}{a-bx^4}\,dx\\
&= \frac{c\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{2\sqrt{a}} + \frac{c\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{2\sqrt{a}} + \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2\right)\\
&= \frac{c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0353374, size = 134, normalized size = 1.54

$$\frac{-\left(\sqrt[4]{a}d+\sqrt[4]{b}c\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)+\sqrt[4]{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+2\sqrt[4]{b}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt[4]{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}d\log\left(\sqrt[4]{a}+\sqrt[4]{\cdot}\right.}{4a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4),x]
```

```
[Out] (2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d)*Log[a^(1
/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*d*Log[a^(1/
4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*Sqrt[b])
```

Maple [A] time = 0.004, size = 101, normalized size = 1.2

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{d}{4}\ln\left(\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)
^(1/4)/a*arctan(x/(1/b*a)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/
(-a-x^2*(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.770489, size = 126, normalized size = 1.45

$$-\operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2bd^2-16tabc^2d+ad^4-bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2+16t^2a^2bc^2d+8ta^2d^4-4t}{4acd^4+bc^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*
d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a**
2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 +
b*c**5))))
```

Giac [B] time = 1.09158, size = 304, normalized size = 3.49

$$\frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab}-\frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd+\left(-ab^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))
/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/(a*b) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2) + 1/
4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d + (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

3.116 $\int \frac{c+dx}{a+bx^4}\,dx$

Optimal. Leaf size=219

$$-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d}{}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1
/4))
```

Rubi [A] time = 0.170693, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1
/4))
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{a+bx^4}\,dx &= \int \left(\frac{c}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx \\
&= c\int \frac{1}{a+bx^4}\,dx + d\int \frac{x}{a+bx^4}\,dx \\
&= \frac{c\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{c\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right) \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} + \frac{c\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\sqrt{b}} - \frac{c\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\,\text{Subst}\left(\int\right.}{2} \\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c}{}
\end{aligned}
$$

Mathematica [A] time = 0.0819112, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}d+\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}c-2\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{b}c\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x\right.\right.}{8a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/
4)*Sqrt[b])
```

Maple [A] time = 0.003, size = 151, normalized size = 0.7

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
)/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.758778, size = 124, normalized size = 0.57

$$\mathrm{RootSum}\left(256t^4a^3b^2 + 32t^2a^2bd^2 - 16tabc^2d + ad^4 + bc^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3bd^2 - 16t^2a^2bc^2d - 8ta^2d^4 - 4tab\right.}{4acd^4 - bc^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*d
**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a**2
*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4 - b
*c**5))))
```

Giac [A] time = 1.08081, size = 288, normalized size = 1.32

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} c\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}} c\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd - \left(ab^3\right)^{\frac{1}{4}} bc\right)\mathrm{arcta}}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)
```

3.117 $\int \frac{c+dx}{\left(a-bx^4\right)^2} dx$

Optimal. Leaf size=110

$$\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a-bx^4\right)}$$

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7
/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0816042, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x(c+dx)}{4a\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a - b*x^4)) + (3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7
/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(1/4)) + (d*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\left(a-bx^4\right)^2}\,dx &= \frac{x(c+dx)}{4a\left(a-bx^4\right)}-\frac{\int \frac{-3c-2dx}{a-bx^4}\,dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}-\frac{\int \left(-\frac{3c}{a-bx^4}-\frac{2dx}{a-bx^4}\right)dx}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{(3c)\int \frac{1}{a-bx^4}\,dx}{4a}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{(3c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{8a^{3/2}}+\frac{(3c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{8a^{3/2}}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{x(c+dx)}{4a\left(a-bx^4\right)}+\frac{3c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.160481, size = 168, normalized size = 1.53

$$\frac{\frac{4ax(c+dx)}{a-bx^4}-\frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c+2\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c-2\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{6\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{2\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/
b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sq
rt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt
[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [A] time = 0.004, size = 142, normalized size = 1.3

$$-\frac{cx}{4\,a\left(bx^4-a\right)}+\frac{3\,c}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{dx^2}{4\,a\left(bx^4-a\right)}-\frac{d}{8\,a}\ln\left(\left(-a+x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*
a)^(1/4)))+3/8*c/a^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-1/4*d*x^2/a/(b*x
^4-a)-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.27555, size = 155, normalized size = 1.41

$$\operatorname{RootSum}\left(65536t^4a^7b^2 - 2048t^2a^4bd^2 + 1152ta^2bc^2d + 16ad^4 - 81bc^4, \left(t \mapsto t\log\left(x + \frac{32768t^3a^6bd^2 + 4608t^2a^4bc^2d - }{192acd^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*a**2*b*c**
2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (32768*_t**3*a**6*b*d**2
 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 360*
a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) - (c*x + d*x**2)/(-4*a**2 + 4*a
*b*x**4)
```

Giac [B] time = 1.09117, size = 343, normalized size = 3.12

$$\frac{3\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{dx^2 + cx}{4\left(bx^4 - a\right)a} - \frac{\sqrt{2}\left(2\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) - 1/16*sqrt(2)*(2
*sqrt(2)*sqrt(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt
(-a*b)*b*d - 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^2)
```

3.118 $\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rubi [A] time = 0.201599, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(16*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^2}\,dx = \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx}{a+bx^4}\,dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} - \frac{\int \left(-\frac{3c}{a+bx^4} - \frac{2dx}{a+bx^4}\right)dx}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{(3c)\int \frac{1}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{(3c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{(3c)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}} + \frac{(3c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}\sqrt{b}} - \frac{(3c)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-\cdots}}{16\sqrt{2}a^{7/4}\sqrt[4]{b}\cdots}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\cdots\right.}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} - \frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\cdots}\right.}{16\sqrt{2}a^{7/\cdots}}$$

Mathematica [A] time = 0.204034, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4} - \frac{2\left(4\sqrt[4]{a}d+3\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{2\left(3\sqrt{2}\sqrt[4]{b}c-4\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{3\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{\sqrt[4]{b}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/4)
*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (3*Sqr
t[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3
*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))
/(32*a^(7/4))
```

Maple [A] time = 0.004, size = 188, normalized size = 0.8

$$\frac{cx}{4\,a\left(bx^4+a\right)} + \frac{3\,c\sqrt{2}}{32\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{3\,c\cdots}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16*
c/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/16*c/a^2*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*d*x^2/a/(b*x^4+a)
+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.25806, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7b^2 + 2048t^2a^4bd^2 - 1152ta^2bc^2d + 16ad^4 + 81bc^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6bd^2 - 4608t^2a^4bc}{192}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c**
2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d**
2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 + 360
*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 + 4*a
*b*x**4)
```

Giac [A] time = 1.07802, size = 321, normalized size = 1.33

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b} + \frac{dx^2 + cx}{4\left(bx^4 + a\right)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{abb}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b*d
+ 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^2*b^2)
```

3.119 $\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=136

$$\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4))
+ (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.110073, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a - b*x^4))
+ (21*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (21*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(1/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx &= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}-\frac{\int \frac{-7c-6dx}{\left(a-bx^4\right)^2}\,dx}{8a}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{\int \frac{21c+12dx}{a-bx^4}\,dx}{32a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{\int \left(\frac{21c}{a-bx^4}+\frac{12dx}{a-bx^4}\right)dx}{32a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{(21c)\int \frac{1}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{(21c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{64a^{5/2}}+\frac{(21c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{64a^{5/2}}+\frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,\right.}{16a^2}\\
&= \frac{x(c+dx)}{8a\left(a-bx^4\right)^2}+\frac{x(7c+6dx)}{32a^2\left(a-bx^4\right)}+\frac{21c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{21c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.146495, size = 193, normalized size = 1.42

$$\frac{\frac{16a^2x(c+dx)}{\left(a-bx^4\right)^2}+\frac{4ax(7c+6dx)}{a-bx^4}-\frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c+4\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c-4\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{42\sqrt[4]{a}c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{12\sqrt{a}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt{b}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) + (
42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)*c
 + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4)*c
 - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[Sqrt[
a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [A] time = 0.006, size = 180, normalized size = 1.3

$$\frac{cx}{8a\left(bx^{4}-a\right)^{2}}-\frac{7cx}{32a^{2}\left(bx^{4}-a\right)}+\frac{21c}{128a^{3}}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{21c}{64a^{3}}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{dx^{2}}{8a\left(bx^{4}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(1/b*a)^(1/4)*ln(
(x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/
b*a)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*
b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.1943, size = 194, normalized size = 1.43

$$-\operatorname{RootSum}\left(268435456t^{4}a^{11}b^{2}-4718592t^{2}a^{6}bd^{2}-2709504ta^{3}bc^{2}d+20736ad^{4}-194481bc^{4},\left(t\mapsto t\log\left(x+\frac{-671}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*_
t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-671
08864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d**4
 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 453789*b
*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 -
64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Giac [B] time = 1.08903, size = 367, normalized size = 2.7

$$\frac{21\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b}-\frac{21\,\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b}+\frac{3\,\sqrt{2}\left(4\,\sqrt{2}\sqrt{-ab}bd+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1
/4) + sqrt(-a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a
*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^3*b^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d + 7*(-a*b^3)^(1/4)*
b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^2
) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2
)
```

3.120 $\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx$

Optimal. Leaf size=266

$$\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4))
+ (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/
4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rubi [A] time = 0.230498, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)}-\frac{21c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{21c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] (x*(c + d*x))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x))/(32*a^2*(a + b*x^4))
+ (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - (21*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) + (21*c*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(1/4)) - (21*c*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/
4)*b^(1/4)) + (21*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(128*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx = \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{\int \frac{21c+12dx}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{\int \left(\frac{21c}{a+bx^4} + \frac{12dx}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{(21c)\int \frac{1}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{(21c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{64a^{5/2}} + \frac{(21c)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{64a^{5/2}} + \frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\right.}{16a^2}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} + \frac{(21c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{128a^{5/2}\sqrt{b}} + \frac{(21c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+}}{128a^{5/2}\sqrt{b}}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\log}{}$$

$$= \frac{x(c+dx)}{8a\left(a+bx^4\right)^2} + \frac{x(7c+6dx)}{32a^2\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

Mathematica [A] time = 0.207356, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(c+dx)}{\left(a+bx^4\right)^2} + \frac{8a^{3/4}x(7c+6dx)}{a+bx^4} - \frac{6\left(8\sqrt[4]{a}d+7\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(7\sqrt{2}\sqrt[4]{b}c-8\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - \frac{21\sqrt{2}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x\right.}{\sqrt[4]{b}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

Maple [A] time = 0.006, size = 222, normalized size = 0.8

$$\frac{cx}{8a\left(bx^4+a\right)^2} + \frac{7cx}{32a^2\left(bx^4+a\right)} + \frac{21c\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21c\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(1/b*a)^(1/4)*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x+1)+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)
^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 2.15468, size = 192, normalized size = 0.72

$$\operatorname{RootSum}\left(268435456t^4a^{11}b^2 + 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 + 194481bc^4, \left(t \mapsto t\log\left(x + \frac{-6710886}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 - 2709504*_t
*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b*c**2*d - 589824*_t*a**4*d**4
- 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 - 453789*b*
c**5)))) + (11*a*c*x + 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4 + 64
*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Giac [A] time = 1.08334, size = 346, normalized size = 1.3

$$\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\left(ab^3\right)^{\frac{1}{4}}b\right.}{128\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1
/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32*(6*b*
d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

3.121 $\int \frac{c+dx}{\left(a-bx^4\right)^4} dx$

Optimal. Leaf size=162

$$\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(25
6*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt
[b])
```

Rubi [A] time = 0.130489, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1855, 1876, 212, 208, 205, 275}

$$\frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2}+\frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}+\frac{x(c+dx)}{12a\left(a-bx^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a - b*x^4)^4,x]
```



```
[Out] (x*(c + d*x))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a - b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a - b*x^4)) + (77*c*ArcTan[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(25
6*a^(15/4)*b^(1/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt
[b])
```


Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{c+dx}{\left(a-bx^4\right)^4}\,dx = \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} - \frac{\int \frac{-11c-10dx}{\left(a-bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2} + \frac{\int \frac{77c+60dx}{\left(a-bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-120dx}{a-bx^4}\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)} - \frac{\int \left(-\frac{231c}{a-bx^4} - \frac{120dx}{a-bx^4}\right)dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)} + \frac{(77c)\int \frac{1}{a-bx^4}\,dx}{128a^3} + \frac{(5d)\int \frac{x}{a-bx^4}\,dx}{16a^3}$$

$$= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)} + \frac{(77c)\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx}{256a^{7/2}} + \frac{(77c)\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx}{256a^{7/2}} +$$

$$= \frac{x(c+dx)}{12a\left(a-bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a-bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a-bx^4\right)} + \frac{77c\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5c}{}$$

Mathematica [A] time = 0.177661, size = 217, normalized size = 1.34

$$\frac{\frac{128a^3x(c+dx)}{\left(a-bx^4\right)^3} + \frac{16a^2x(11c+10dx)}{\left(a-bx^4\right)^2} + \frac{4ax(77c+60dx)}{a-bx^4} - \frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c+40\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c-40\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{462\sqrt[4]{a}c\tan^{-1}}{\sqrt[4]{b}}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x^
4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/4)
*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*Sqrt[a]*d)*Log[a^(1/4
) - b^(1/4)*x])/Sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[a^(1
```

```
/4) + b^(1/4)*x])/Sqrt[b] + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt
[b])/(1536*a^4)
```

Maple [A] time = 0.013, size = 177, normalized size = 1.1

$$\frac{1}{\left(bx^4-a\right)^3}\left(-\frac{5\,b^2dx^{10}}{32\,a^3}-\frac{77\,b^2cx^9}{384\,a^3}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}-\frac{11\,dx^2}{32\,a}-\frac{51\,cx}{128\,a}\right)+\frac{77\,c}{512\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{77}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x
^5-11/32*d/a*x^2-51/128*c/a*x)/(b*x^4-a)^3+77/512/a^4*c*(1/b*a)^(1/4)*ln((x
+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b
*a)^(1/4))-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 6.64574, size = 231, normalized size = 1.43

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^2-838860800t^2a^8bd^2+485703680ta^4bc^2d+2560000ad^4-35153041bc^4,\left(t\mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*log(
```

```
x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d - 26
21440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**3
)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) - (153*a**2*c*x + 132*a**2*d*
x**2 - 198*a*b*c*x**5 - 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)/
(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Giac [B] time = 1.0728, size = 400, normalized size = 2.47

$$\frac{77\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b}-\frac{77\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b}-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/(a^4*b) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77
*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^4*b^2) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d - 77*(-a*b^3)^
(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x
^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

3.122 $\int \frac{c+dx}{\left(a+bx^4\right)^4} dx$

Optimal. Leaf size=291

$$\frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2}-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{77c\tan}{25}$$

```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b
^(1/4))
```

Rubi [A] time = 0.267577, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)}+\frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2}-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{77c\tan}{25}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^4)^4,x]
```


```
[Out] (x*(c + d*x))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x))/(96*a^2*(a + b*x^4
)^2) + (x*(77*c + 60*d*x))/(384*a^3*(a + b*x^4)) + (5*d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(1/4)) + (77*c*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b
^(1/4))
```


Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+bx^4\right)^4}\,dx = \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} - \frac{\int \frac{-11c-10dx}{\left(a+bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{\int \frac{77c+60dx}{\left(a+bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} - \frac{\int \frac{-231c-120dx}{a+bx^4}\,dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} - \frac{\int \left(-\frac{231c}{a+bx^4} - \frac{120dx}{a+bx^4}\right)dx}{384a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{(77c)\int \frac{1}{a+bx^4}\,dx}{128a^3} + \frac{(5d)\int \frac{x}{a+bx^4}\,dx}{16a^3}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{(77c)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{256a^{7/2}} + \frac{(77c)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{256a^{7/2}} + \ldots$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{(77c)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{512a^{7/2}\sqrt{b}} + \ldots$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x \ldots\right.}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

$$= \frac{x(c+dx)}{12a\left(a+bx^4\right)^3} + \frac{x(11c+10dx)}{96a^2\left(a+bx^4\right)^2} + \frac{x(77c+60dx)}{384a^3\left(a+bx^4\right)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} + \ldots$$

Mathematica [A] time = 0.284213, size = 274, normalized size = 0.94

$$\frac{\frac{256a^{11/4}x(c+dx)}{\left(a+bx^4\right)^3} + \frac{32a^{7/4}x(11c+10dx)}{\left(a+bx^4\right)^2} + \frac{8a^{3/4}x(77c+60dx)}{a+bx^4} - \frac{6\left(80\sqrt[4]{a}d+77\sqrt{2}\sqrt[4]{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\sqrt[4]{a}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{b}} - 231\ldots}{3072a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c + 10*d*x))/
(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^4) - (6*(77*Sqrt[2]*
b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b]
+ (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/Sqrt[b] - (231*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

Maple [A] time = 0.014, size = 225, normalized size = 0.8

$$\frac{1}{\left(bx^4+a\right)^3}\left(\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{77\,b^2cx^9}{384\,a^3}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}+\frac{11\,dx^2}{32\,a}+\frac{51\,cx}{128\,a}\right)+\frac{77\,c\sqrt{2}}{1024\,a^4}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^
5+11/32*d/a*x^2+51/128*c/a*x)/(b*x^4+a)^3+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/
2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1
/2)+(1/b*a)^(1/2)))+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x+1)+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x-1)+5/32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 6.6255, size = 231, normalized size = 0.79

$$\mathrm{RootSum}\left(68719476736t^4a^{15}b^2+838860800t^2a^8bd^2-485703680ta^4bc^2d+2560000ad^4+35153041bc^4,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 485703
680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*log(
x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d - 2
621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*d**
3)/(788480000*a*c*d**4 - 2706784157*b*c**5))) + (153*a**2*c*x + 132*a**2*d
```

```
*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*x**10)
/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Giac [A] time = 1.07999, size = 378, normalized size = 1.3

$$\frac{77\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^{4}b}-\frac{77\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}c\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^{4}b}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\left(ab^{3}\right)^{\frac{1}{4}}\right.}{512\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3
)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b*c)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2) + 1/38
4*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*a^2*d
*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

3.123 $\int \frac{c+dx}{1-x^4} dx$

Optimal. Leaf size=24

$$\frac{1}{2}c\tan^{-1}(x) + \frac{1}{2}c\tanh^{-1}(x) + \frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

```
[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rubi [A] time = 0.0180047, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1876, 212, 206, 203, 275}

$$\frac{1}{2}c\tan^{-1}(x) + \frac{1}{2}c\tanh^{-1}(x) + \frac{1}{2}d\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(1 - x^4),x]

[Out] (c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{1-x^4}\,dx &= \int \left(\frac{c}{1-x^4}+\frac{dx}{1-x^4}\right)dx\\ &= c\int \frac{1}{1-x^4}\,dx + d\int \frac{x}{1-x^4}\,dx\\ &= \frac{1}{2}c\int \frac{1}{1-x^2}\,dx + \frac{1}{2}c\int \frac{1}{1+x^2}\,dx + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}c\tan^{-1}(x) + \frac{1}{2}c\tanh^{-1}(x) + \frac{1}{2}d\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0147418, size = 42, normalized size = 1.75

$$\frac{1}{4}\left(-(c+d)\log(1-x)+c\log(x+1)+2c\tan^{-1}(x)+d\log\left(x^2+1\right)-d\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 - x^4),x]
```

```
[Out] (2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[1
 + x^2])/4
```

Maple [B] time = 0.004, size = 44, normalized size = 1.8

$$-\frac{\ln(-1+x)c}{4}-\frac{\ln(-1+x)d}{4}+\frac{\ln(1+x)c}{4}-\frac{\ln(1+x)d}{4}+\frac{d\ln\left(x^2+1\right)}{4}+\frac{c\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-x^4+1),x)
```

```
[Out] -1/4*ln(-1+x)*c-1/4*ln(-1+x)*d+1/4*ln(1+x)*c-1/4*ln(1+x)*d+1/4*d*ln(x^2+1)+
1/2*c*arctan(x)
```

Maxima [A] time = 1.4422, size = 47, normalized size = 1.96

$$\frac{1}{2}c\arctan(x)+\frac{1}{4}d\log\left(x^2+1\right)+\frac{1}{4}(c-d)\log(x+1)-\frac{1}{4}(c+d)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

Fricas [A] time = 1.29288, size = 119, normalized size = 4.96

$$\frac{1}{2}c\arctan(x)+\frac{1}{4}d\log\left(x^2+1\right)+\frac{1}{4}(c-d)\log(x+1)-\frac{1}{4}(c+d)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d)
*log(x - 1)
```

Sympy [C] time = 0.459214, size = 313, normalized size = 13.04

$$\frac{(c-d)\log\left(x+\frac{c^4(c-d)+5c^2d^3+c^2d(c-d)^2-2d^4(c-d)+2d^2(c-d)^3}{c^5+4cd^4}\right)}{4}-\frac{(c+d)\log\left(x+\frac{-c^4(c+d)+5c^2d^3+c^2d(c+d)^2+2d^4(c+d)-2d^2(c+d)^3}{c^5+4cd^4}\right)}{4}-\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x**4+1),x)
```

```
[Out] (c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(c
 - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*(c
 + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d)**
3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4) +
5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 128*d**
2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c**4*(I
*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c/4 - d/
4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

Giac [B] time = 1.05346, size = 50, normalized size = 2.08

$$\frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\log\left(x^2+1\right)+\frac{1}{4}\,(c-d)\log\left(|x+1|\right)-\frac{1}{4}\,(c+d)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(c
 + d)*log(abs(x - 1))
```

3.124 $\int \frac{c+dx}{1+x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\tan^{-1}\left(x^2\right)$$

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + S
qrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[
1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rubi [A] time = 0.0672962, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 203}

$$-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(1 + x^4),x]
```

```
[Out] (d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 + S
qrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Log[
1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{1+x^4}\,dx &= \int \left(\frac{c}{1+x^4}+\frac{dx}{1+x^4}\right) dx \\
&= c\int \frac{1}{1+x^4}\,dx + d\int \frac{x}{1+x^4}\,dx \\
&= \frac{1}{2}c\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}c\int \frac{1+x^2}{1+x^4}\,dx + \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right) + \frac{1}{4}c\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}c\int \frac{1}{1+\sqrt{2}x+x^2}\,dx - \frac{c\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} - \frac{c\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,d}{4\sqrt{2}} \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right) - \frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}} - \frac{c\,\text{S}}{} \\
&= \frac{1}{2}d\tan^{-1}\left(x^2\right) - \frac{c\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{c\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c\log\left(1+\sqrt{2}x+x^2\right.}{4\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.067398, size = 99, normalized size = 1.01

$$\frac{1}{4}\left(-\left(\sqrt[4]{-1}c+id\right)\log\left(\sqrt[4]{-1}-x\right)+\left(-(-1)^{3/4}c+id\right)\log\left((-1)^{3/4}-x\right)+\left(\sqrt[4]{-1}c-id\right)\log\left(x+\sqrt[4]{-1}\right)+\left((-1)^{3/4}c+id\right)\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(1 + x^4),x]
```

```
[Out] (-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log[
(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*c
+ I*d)*Log[(-1)^(3/4) + x])/4
```

Maple [A] time = 0.004, size = 68, normalized size = 0.7

$$\frac{c\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{c\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{c\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+\frac{d\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(x^4+1),x)
```

```
[Out] 1/4*c*arctan(1+x*2^(1/2))*2^(1/2)+1/4*c*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*c*
2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+1/2*d*arctan(x^2)
```

Maxima [A] time = 1.53606, size = 116, normalized size = 1.18

$$\frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)+\frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}c+2\,d\right)\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.411487, size = 83, normalized size = 0.85

$$\operatorname{RootSum}\left(256t^4+32t^2d^2-16tc^2d+c^4+d^4,\left(t\mapsto t\log\left(x+\frac{128t^3d^2+16t^2c^2d+4tc^4+8td^4-5c^2d^3}{c^5-4cd^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x**4+1),x)
```

```
[Out] RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, Lambda(_t,
_t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*c*
*2*d**3)/(c**5 - 4*c*d**4))))
```

Giac [A] time = 1.06388, size = 116, normalized size = 1.18

$$\frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)+\frac{1}{4}\left(\sqrt{2}c-2d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}c+2d\right)\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
+ 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

3.125 $\int \frac{c+dx+ex^2}{a-bx^4} dx$

Optimal. Leaf size=116

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0954763, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1876, 275, 208, 1167, 205}

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2}{a-bx^4}\,dx &= \int \left(\frac{dx}{a-bx^4}+\frac{c+ex^2}{a-bx^4}\right)dx\\ &= d\int \frac{x}{a-bx^4}\,dx+\int \frac{c+ex^2}{a-bx^4}\,dx\\ &= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{a}\sqrt{b}-\right.\\ &= \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.050743, size = 187, normalized size = 1.61

$$\frac{-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{a}e+\sqrt{b}c\right)+2\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)+\sqrt[4]{a}\sqrt[4]{b}d\log\left(\sqrt{a}+\sqrt{\right.}}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1/
4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4)
+ b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*e*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3
/4)*b^(3/4))
```

Maple [B] time = 0.002, size = 161, normalized size = 1.4

$$\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{d}{4}\ln\left(\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{e}{2b}\,\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)
^(1/4)/a*arctan(x/(1/b*a)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/
(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*e/b
/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 5.49401, size = 471, normalized size = 4.06

$$-\operatorname{RootSum}\left(256t^4a^3b^3+t^2\left(-64a^2b^2ce-32a^2b^2d^2\right)+t\left(-16a^2bde^2-16ab^2c^2d\right)-a^2e^4+2abc^2e^2-4abcd^2e+abd^4-b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2*b**2*d**2
) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e**4 + 2*a*b*c**2*e**2
 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*
a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 4
8*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2*b**3
*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b**2*c*
*3*e**2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3
*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5*e + 5*a*b**2*c
**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*c**2*e**4 - 8*a**2*b*c*d*
*2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*
a*b**2*c**2*d**4 - b**3*c**6))))
```

Giac [B] time = 1.08251, size = 396, normalized size = 3.41

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\text{a}}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3
) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (-a*b^3)^(1/4)*b^2*c - (-a*b^3)
^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*
b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt
```

```
(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*
c - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3
)
```

3.126 $\int \frac{c+dx+ex^2}{a+bx^4}\,dx$

Optimal. Leaf size=277

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.196624, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4))
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \int \left(\frac{dx}{a+bx^4}+\frac{c+ex^2}{a+bx^4}\right)dx$$
$$= d\int \frac{x}{a+bx^4}\,dx + \int \frac{c+ex^2}{a+bx^4}\,dx$$
$$= \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right) + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$
$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} - \frac{\left(\sqrt{bc}-\sqrt{ae}\right)\int \ldots}{4\sqrt{2}a^{3/4}b^{3}\ldots}$$
$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{bc}-\sqrt{ae}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{bc}-\sqrt{ae}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\ldots\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$
$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{bc}+\sqrt{ae}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{bc}+\sqrt{ae}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{bc}-\sqrt{a}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.100156, size = 229, normalized size = 0.83

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+\sqrt{2}\sqrt{b}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+\sqrt{2}\sqrt{b}c\right)-\sqrt{2}\left(\sqrt{b}c\ldots\right.}{8a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1
- (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d
 + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sq
rt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4)*b^(
3/4))
```

Maple [A] time = 0.003, size = 280, normalized size = 1.

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
)/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/
(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(
1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
```

```
/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 5.46942, size = 466, normalized size = 1.68

$$\text{RootSum}\left(256t^4a^3b^3 + t^2\left(64a^2b^2ce + 32a^2b^2d^2\right) + t\left(16a^2bde^2 - 16ab^2c^2d\right) + a^2e^4 + 2abc^2e^2 - 4abcd^2e + abd^4 + b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2)
+ _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2 -
4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**4
*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 48*_t
**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**3*c**
3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*c**3*e
**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b**3*c**
5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b**2*c**4*
d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b*c*d**2*e
**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*e - 4*a*b*
*2*c**2*d**4 + b**3*c**6))))
```

Giac [A] time = 1.08677, size = 371, normalized size = 1.34

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\right.}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1
/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8
*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/
4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.127 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan
[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.127515, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(3\sqrt{b}\,c-\sqrt{a}\,e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}\,e+3\sqrt{b}\,c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*ArcTan
[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Arc
Tanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^2}\,dx &= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}\\
&= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int \left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}\\
&= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a}+\frac{d\int \frac{x}{a-bx^4}\,dx}{2a}\\
&= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a}+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\int \frac{1}{\sqrt{a}\sqrt{b}}
\end{aligned}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.201098, size = 211, normalized size = 1.45

$$\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}-\frac{2\sqrt[4]{a}\left(\sqrt{a}e-3\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{4ax(c+x(d+ex))}{a-bx^4}+\frac{2\sqrt{a}d\log(\cdots}{\sqrt{\cdots}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c + Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*Sqrt[b]*c + 2*Sqrt[a
]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((3*a^(1/4)*Sq
rt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Maple [B] time = 0.003, size = 228, normalized size = 1.6

$$-\frac{cx}{4a\left(bx^4-a\right)}+\frac{3c}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{dx^2}{4a\left(bx^4-a\right)}-\frac{d}{8a}\ln\left(\left(-a+x^2\sqrt{ab}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*
a)^(1/4)))+3/8*c/a^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-1/4*d*x^2/a/(b*x
^4-a)-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/4
*e*x^3/a/(b*x^4-a)-1/8*e/a/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/16*e/a
/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 7.2781, size = 507, normalized size = 3.47

$$\mathrm{RootSum}\Big(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2e^2 - 48abcd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
2*d**4 - 729*b**3*c**6)))) - (c*x + d*x**2 + e*x**3)/(-4*a**2 + 4*a*b*x**4)
```

Giac [B] time = 1.08587, size = 440, normalized size = 3.01

$$-\frac{x^3e + dx^2 + cx}{4\left(bx^4 - a\right)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d + 3\left(-ab^3\right)^{\frac{1}{4}}b^2c + \left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d + 3\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 - a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-
a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt
(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(
2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(
1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b
^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.128 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan}{8\sqrt{2}a^{7/4}}$$

```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sq
rt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```

Rubi [A] time = 0.254637, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan}{8\sqrt{2}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```


```
[Out] (x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])
/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) - ((3*Sq
rt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4))
```


Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}\,dx}{4a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int \left(-\frac{2dx}{a+bx^4}+\frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a+bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a+bx^4}\,dx}{2a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{8ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{b}+}{a+bx^4}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,d}{16ab}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{\right.}}{}$$

$$= \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^{7/4}b^{3/4}}$$

Mathematica [A] time = 0.330297, size = 305, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} - \frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e+3\sqrt{2}\sqrt{b}\right.}{b^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

```
[Out] ((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c +
4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d + Sq
rt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2
]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2)
```

Maple [A] time = 0.004, size = 344, normalized size = 1.1

$$\frac{cx}{4a\left(bx^4+a\right)} + \frac{3c\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16*
c/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/16*c/a^2*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*d*x^2/a/(b*x^4+a)
+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/4*e*x^3/a/(b*x^4+a)+1/32*e/a
/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^
2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 7.28354, size = 505, normalized size = 1.64

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc^2e^2 - 48abcd^2e +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4*a*b*x**4)
```

Giac [A] time = 1.09527, size = 413, normalized size = 1.34

$$\frac{x^3e+dx^2+cx}{4\left(bx^4+a\right)a}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\left(ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*
b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(
x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.129 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=179

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(c+dx+\right.}{8a\left(a-b\right.}$$

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
16*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.167009, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{x\left(c+dx+\right.}{8a\left(a-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```


```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a - b*x^4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
16*a^(5/2)*Sqrt[b])
```


Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx &= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}-\frac{\int \frac{-7c-6dx-5ex^2}{\left(a-bx^4\right)^2}\,dx}{8a}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{16a^2}-\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}\cdots}}{64a^2}\\
&= \frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}}+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tanh^{-1}\left(\cdots\right.}{64a^{11/4}b^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.213492, size = 244, normalized size = 1.36

$$
\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{b}c-12\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{16a^2x(c+x(d+ex))}{\left(a-bx^4\right)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{b^{3/4}}\cdots}{128a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

```
[Out] ((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x)
))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)
/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a^(3
/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*Sqrt
[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*Sqrt[a
]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [B] time = 0.005, size = 286, normalized size = 1.6

$$\frac{cx}{8\,a\left(bx^4-a\right)^2}-\frac{7\,cx}{32\,a^2\left(bx^4-a\right)}+\frac{21\,c}{128\,a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{21\,c}{64\,a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{dx^2}{8\,a\left(bx^4-a\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(1/b*a)^(1/4)*ln(
(x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/
b*a)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*
b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+1/8*e*x^3/a/(b*x^4-a
)^2-5/32*e/a^2*x^3/(b*x^4-a)-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1
/4))+5/128*e/a^2/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 10.2057, size = 563, normalized size = 3.15

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^3+t^2\left(-6881280a^6b^2ce-4718592a^6b^2d^2\right)+t\left(-153600a^4bde^2-2709504a^3b^2c^2d\right)-625a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 47185
92*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d)
- 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4
 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2*e**
```

```
3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2
+ 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 1
820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a*
*5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a**4*b**2*
c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 +
 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b*d**5*e +
58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 27
5625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e*
*2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b*
*2*c**2*d**4 - 85766121*b**3*c**6)))) - (-11*a*c*x - 10*a*d*x**2 - 9*a*e*x*
*3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 - 64*a**3*b*x**4 + 32*a
**2*b**2*x**8)
```

Giac [B] time = 1.10124, size = 481, normalized size = 2.69

$$-\frac{5\,bx^7e+6\,bdx^6+7\,bcx^5-9\,ax^3e-10\,adx^2-11\,acx}{32\left(bx^4-a\right)^2a^2}-\frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{-ab}b^2d-21\,\left(-ab^3\right)^{\frac{1}{4}}b^2c-5\,\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\right.}{128\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*x^3*e - 10*a*d*x^2 - 11*a*c*
x)/((b*x^4 - a)^2*a^2) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-
a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*
(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b
)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-
a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) +
 sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3
)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3)
```

3.130 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx$

Optimal. Leaf size=341

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64\sqrt{2}a}$$

```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]
) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c
- 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))
```

Rubi [A] time = 0.310719, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)}{64\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```


```
[Out] (x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + (x*(7*c + 6*d*x + 5*e*x^2))/(32
*a^2*(a + b*x^4)) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]
) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*Sqrt[b]*c
- 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128
*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4))
```


Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{(a+bx^4)^2}\,dx}{8a}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{\int \frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{\int \left(\frac{12dx}{a+bx^4} + \frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{\int \frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{(3d)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{16a^2} + \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}} - 5e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}}{64a^2b}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}} - 5e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{128\sqrt{2}a^{9/4}b^{3/4}} - \frac{\left(21\right.}{}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}} - 5e\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \right.}{128\sqrt{2}a^{9/4}b^{3/4}}$$

$$= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} +$$

Mathematica [A] time = 0.306739, size = 337, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e-21\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}c-5a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{32a^2x(c+x(d+ex))}{(a+bx^4)^2} - \frac{2\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(24}{}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x)
))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d +
5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (2*
a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1/4)*S
qrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A] time = 0.006, size = 396, normalized size = 1.2

$$\frac{cx}{8a(bx^4+a)^2} + \frac{7cx}{32a^2(bx^4+a)} + \frac{21c\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21c\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(1/b*a)^(1/4)*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x+1)+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)
^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^
4+a)+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1
/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/128*e/a^2/b/(1/
b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+5/128*e/a^2/b/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 10.127, size = 558, normalized size = 1.64

$$\operatorname{RootSum}\left(268435456t^4a^{11}b^3 + t^2\left(6881280a^6b^2ce + 4718592a^6b^2d^2\right) + t\left(153600a^4bde^2 - 2709504a^3b^2c^2d\right) + 625a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 4718592
*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) + 6
25*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 +
194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**3 -
4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 30
9657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 18207
86688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b
```

```
*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b**2*c**2
*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112
500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**5*e + 5834
4300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625
*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 -
 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c
**2*d**4 + 85766121*b**3*c**6)))) + (11*a*c*x + 10*a*d*x**2 + 9*a*e*x**3 +
7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 + 64*a**3*b*x**4 + 32*a**2*b
**2*x**8)
```

Giac [A] time = 1.09774, size = 454, normalized size = 1.33

$$\frac{5bx^7e+6bdx^6+7bcx^5+9ax^3e+10adx^2+11acx}{32\left(bx^4+a\right)^2a^2}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{}\right.}{2\left(\frac{a}{b}\right)}\right.}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x^7*e + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*x^3*e + 10*a*d*x^2 + 11*a*c*x
)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b
^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

3.131 $\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}$$

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4
)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15
/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*
Sqrt[b])
```

Rubi [A] time = 0.211313, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a - b*x^4
)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15
/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*
Sqrt[b])
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a-bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a-bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-120dx-45ex^2}{a-bx^4}\,dx}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} - \frac{\int \left(-\frac{120dx}{a-bx^4} + \frac{-231c-45ex^2}{a-bx^4}\right) dx}{384a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} - \frac{\int \frac{-231c-45ex^2}{a-bx^4}\,dx}{384a^3} + \frac{(5d)\int \ldots}{16a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{(5d)\,\mathrm{Subst}\left(\int \frac{1}{a-bx^2}\,dx, x, x^2 \ldots\right.}{32a^3}$$

$$= \frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} + \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\ldots\right.}{256a^{15/4}b^{3/4}}$$

Mathematica [A] time = 0.258105, size = 276, normalized size = 1.31

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c+40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}} + \frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c-40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}} + \frac{128a^3x(c+x(d+ex))}{\left(a-bx^4\right)^3} + \frac{16a^2x(11c+x(10d+9ex))}{\left(a-bx^4\right)^2} + \ldots}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

```
[Out] ((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2 +
```

```
(6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/
4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^
(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^(1/4)
*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d*Log[S
qrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A] time = 0.013, size = 274, normalized size = 1.3

$$\frac{1}{\left(bx^4-a\right)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}-\frac{5\,b^2dx^{10}}{32\,a^3}-\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}-\frac{11\,dx^2}{32\,a}-\frac{51\,cx}{128\,a}\right)+\frac{77\,c}{512\,a^4}\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+21/64/a^2*
b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5-113/384/a*e*x^3-11/32*d/a*x^2-51
/128*c/a*x)/(b*x^4-a)^3+77/512/a^4*c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-
(1/b*a)^(1/4)))+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-5/64/a^3
*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-15/256/a^3*e/b
/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+15/512/a^3*e/b/(1/b*a)^(1/4)*ln((x+(
1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 16.4793, size = 612, normalized size = 2.9

$$\operatorname{RootSum}\left(68719476736t^4a^{15}b^3+t^2\left(-1211105280a^8b^2ce-838860800a^8b^2d^2\right)+t\left(18432000a^5bde^2+485703680a^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(-1211105280*a**8*b**2*c*e - 8
38860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 + 485703680*a**4*b**2
*c**2*d) - 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e +
 2560000*a*b*d**4 - 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832000
*_t**3*a**13*b**2*e**3 + 11936653639680*_t**3*a**12*b**3*c**2*e - 330712481
79200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 503
316480000*_t**2*a**9*b**2*d**3*e - 4787095470080*_t**2*a**8*b**3*c**3*d - 5
987520000*_t*a**6*b*c*e**4 - 8294400000*_t*a**6*b*d**2*e**3 - 210370406400*
_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 2018508800
00*_t*a**5*b**2*c*d**4 - 1385873488384*_t*a**4*b**3*c**5 + 91125000*a**3*d*
e**5 - 5544000000*a**2*b*c*d**3*e**2 + 3072000000*a**2*b*d**5*e + 105459123
000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 +
300155625*a**2*b*c**2*e**4 - 3326400000*a**2*b*c*d**2*e**3 + 2304000000*a**
2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c**3*d**2*
e - 60712960000*a*b**2*c**2*d**4 - 208422380089*b**3*c**6)))) - (153*a**2*c
*x + 132*a**2*d*x**2 + 113*a**2*e*x**3 - 198*a*b*c*x**5 - 160*a*b*d*x**6 -
126*a*b*e*x**7 + 77*b**2*c*x**9 + 60*b**2*d*x**10 + 45*b**2*e*x**11)/(-384*
a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Giac [B] time = 1.0982, size = 531, normalized size = 2.52

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c + 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c\right.}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c + 15*(
-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1
/4)*b^2*c + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 1
5*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3
) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4)*e)*log(x^2
- sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b^2*x^11*e + 6
0*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*x^7*e - 160*a*b*d*x^6 - 198*a*b*c*x^5
 + 113*a^2*x^3*e + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

3.132 $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^4}\,dx$

Optimal. Leaf size=372

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}e+7\right.}{2}$$

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4
)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a
]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rubi [A] time = 0.379463, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{a}e+7\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] (x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + (x*(11*c + 10*d*x + 9*e*x^2))/
(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x + 45*e*x^2))/(384*a^3*(a + b*x^4
)) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a
]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a
^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}dx = \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{(a+bx^4)^3}dx}{12a}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{\int \frac{77c+60dx+45ex^2}{(a+bx^4)^2}dx}{96a^2}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{\int \frac{-231c-120dx-45ex^2}{a+bx^4}dx}{384a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{\int \left(-\frac{120dx}{a+bx^4} + \frac{-231c-45ex^2}{a+bx^4}\right)}{384a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{\int \frac{-231c-45ex^2}{a+bx^4}dx}{384a^3} + \frac{(5d)\int}{1}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{a+bx^2}dx, x\right.}{32a^3}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} + \frac{\left(\frac{77\sqrt{b}}{\sqrt{a}}\right.}{}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{(77\sqrt{}}{}$$

$$= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{(77\sqrt{}}{}$$

Mathematica [A] time = 0.394003, size = 369, normalized size = 0.99

$$\frac{\frac{3\sqrt{2}(15a^{3/4}e-77\sqrt[4]{a}\sqrt{b}c)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{3\sqrt{2}(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{256a^3x(c+x(d+ex))}{(a+bx^4)^3} + \frac{32a^2x(11c+x(10}{(a+bx^4)^2}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

```
[Out] ((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d + 3
*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2 -
(6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*Sqrt
[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*c + 15
*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4)
 + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Maple [A] time = 0.013, size = 394, normalized size = 1.1

$$\frac{1}{\left(bx^4+a\right)^3}\left(\frac{15\,b^2ex^{11}}{128\,a^3}+\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}+\frac{113\,ex^3}{384\,a}+\frac{11\,dx^2}{32\,a}+\frac{51\,cx}{128\,a}\right)+\frac{77\,c\sqrt{2}}{1024\,a^4}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64/a^2*b
*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128*c/a*x)/(b*x^4+a)^3+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^
(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))
+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+77/51
2/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+5/32/a^3*d/
(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*l
n((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+
(1/b*a)^(1/2)))+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x+1)+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 16.3421, size = 610, normalized size = 1.64

$$\mathrm{RootSum}\left(68719476736t^4a^{15}b^3+t^2\left(1211105280a^8b^2ce+838860800a^8b^2d^2\right)+t\left(18432000a^5bde^2-485703680a^4b^2c^2d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(1211105280*a**8*b**2*c*e + 83
8860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 - 485703680*a**4*b**2*
c**2*d) + 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e +
2560000*a*b*d**4 + 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832000*
_t**3*a**13*b**2*e**3 - 11936653639680*_t**3*a**12*b**3*c**2*e + 3307124817
9200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2 - 5033
16480000*_t**2*a**9*b**2*d**3*e + 4787095470080*_t**2*a**8*b**3*c**3*d + 59
87520000*_t*a**6*b*c*e**4 + 8294400000*_t*a**6*b*d**2*e**3 - 210370406400*_
t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 20185088000
0*_t*a**5*b**2*c*d**4 + 1385873488384*_t*a**4*b**3*c**5 + 91125000*a**3*d*e
**5 + 5544000000*a**2*b*c*d**3*e**2 - 3072000000*a**2*b*d**5*e + 1054591230
00*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a**3*e**6 - 3
00155625*a**2*b*c**2*e**4 + 3326400000*a**2*b*c*d**2*e**3 - 2304000000*a**2
*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b**2*c**3*d**2*e
 - 60712960000*a*b**2*c**2*d**4 + 208422380089*b**3*c**6)))) + (153*a**2*c*
x + 132*a**2*d*x**2 + 113*a**2*e*x**3 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 1
26*a*b*e*x**7 + 77*b**2*c*x**9 + 60*b**2*d*x**10 + 45*b**2*e*x**11)/(384*a*
*6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*x**12)
```

Giac [A] time = 1.08912, size = 504, normalized size = 1.35

$$\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d+77\,\left(ab^3\right)^{\frac{1}{4}}b^2c+15\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^2d+77\,\left(ab^3\right)^{\frac{1}{4}}b^2c+15\right.}{512\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*x^11*e + 60*b^2*d*x^10 + 77*
b^2*c*x^9 + 126*a*b*x^7*e + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*x^3*e +
 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

3.133 $\int a\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=28

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rubi [A] time = 0.0113591, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 194}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[a*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int a\left(e+fx^4\right)^2\,dx &= a\int \left(e+fx^4\right)^2\,dx\\ &= a\int \left(e^2+2efx^4+f^2x^8\right)\,dx\\ &= ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9\end{aligned}$$

Mathematica [A] time = 0.0017667, size = 27, normalized size = 0.96

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*(e + f*x^4)^2,x]
```

```
[Out] a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Maple [A] time = 0.04, size = 24, normalized size = 0.9

$$a\left(\frac{f^2x^9}{9}+\frac{2\,ef\,x^5}{5}+e^2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(f*x^4+e)^2,x)
```

```
[Out] a*(1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)
```

Maxima [A] time = 0.955677, size = 34, normalized size = 1.21

$$\frac{1}{45}\left(5\,f^2x^9+18\,ef\,x^5+45\,e^2x\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

Fricas [A] time = 1.05641, size = 55, normalized size = 1.96

$$\frac{1}{9}x^9f^2a+\frac{2}{5}x^5fea+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*f^2*a + 2/5*x^5*f*e*a + x*e^2*a
```

Sympy [A] time = 0.059508, size = 27, normalized size = 0.96

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

Giac [A] time = 1.06919, size = 34, normalized size = 1.21

$$\frac{1}{45}\left(5\,f^2x^9+18\,fx^5e+45\,xe^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/45*(5*f^2*x^9 + 18*f*x^5*e + 45*x*e^2)*a
```

3.134 $\int bx\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rubi [A] time = 0.0143123, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 270}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[b*x*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int bx\left(e+fx^4\right)^2\,dx&=b\int x\left(e+fx^4\right)^2\,dx\\&=b\int\left(e^2x+2efx^5+f^2x^9\right)dx\\&=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.001375, size = 32, normalized size = 0.97

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x*(e + f*x^4)^2,x]
```

```
[Out] b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Maple [A] time = 0.039, size = 27, normalized size = 0.8

$$b\left(\frac{f^2x^{10}}{10}+\frac{efx^6}{3}+\frac{e^2x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x*(f*x^4+e)^2,x)
```

```
[Out] b*(1/10*f^2*x^10+1/3*e*f*x^6+1/2*e^2*x^2)
```

Maxima [A] time = 1.05494, size = 36, normalized size = 1.09

$$\frac{1}{30}\left(3\,f^2x^{10}+10\,efx^6+15\,e^2x^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

Fricas [A] time = 1.06542, size = 66, normalized size = 2.

$$\frac{1}{10}x^{10}f^2b+\frac{1}{3}x^6feb+\frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^2*b + 1/3*x^6*f*e*b + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.061032, size = 29, normalized size = 0.88

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

Giac [A] time = 1.04414, size = 36, normalized size = 1.09

$$\frac{1}{30}\left(3\,f^2x^{10}+10\,fx^6e+15\,x^2e^2\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/30*(3*f^2*x^10 + 10*f*x^6*e + 15*x^2*e^2)*b
```

3.135 $\int (a+bx) \left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=60

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Rubi [A] time = 0.0624218, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1850}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(e+fx^4\right)^2\,dx &= \int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)dx\\ &= ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.0019781, size = 60, normalized size = 1.

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10
```

Maple [A] time = 0.039, size = 51, normalized size = 0.9

$$ae^2x+\frac{be^2x^2}{2}+\frac{2\,aefx^5}{5}+\frac{befx^6}{3}+\frac{af^2x^9}{9}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*
x^10
```

Maxima [A] time = 0.981534, size = 68, normalized size = 1.13

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^2
*x^2 + a*e^2*x
```

Fricas [A] time = 1.00017, size = 123, normalized size = 2.05

$$\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 1/3*x^6*f*e*b + 2/5*x^5*f*e*a + 1/2*x^2*e
^2*b + x*e^2*a
```

Sympy [A] time = 0.066282, size = 58, normalized size = 0.97

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10
```

Giac [A] time = 1.05481, size = 68, normalized size = 1.13

$$\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*f*x^6*e + 2/5*a*f*x^5*e + 1/2*b*x^2
*e^2 + a*x*e^2
```

3.136 $\int cx^2 \left(e+fx^4\right)^2 dx$

Optimal. Leaf size=33

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.0132649, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 270}

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[c*x^2*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int cx^2\left(e+fx^4\right)^2 dx &= c\int x^2\left(e+fx^4\right)^2 dx\\
&= c\int \left(e^2x^2+2efx^6+f^2x^{10}\right) dx\\
&= \frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}
\end{aligned}$$

Mathematica [A] time = 0.0011796, size = 33, normalized size = 1.

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[c*x^2*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Maple [A] time = 0.041, size = 27, normalized size = 0.8

$$c\left(\frac{f^2x^{11}}{11}+\frac{2\,efx^7}{7}+\frac{e^2x^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2*(f*x^4+e)^2,x)
```

```
[Out] c*(1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)
```

Maxima [A] time = 0.986278, size = 36, normalized size = 1.09

$$\frac{1}{231}\left(21\,f^2x^{11}+66\,efx^7+77\,e^2x^3\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

Fricas [A] time = 1.05338, size = 66, normalized size = 2.

$$\frac{1}{11}x^{11}f^2c+\frac{2}{7}x^7fec+\frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 2/7*x^7*f*e*c + 1/3*x^3*e^2*c
```

Sympy [A] time = 0.059401, size = 31, normalized size = 0.94

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Giac [A] time = 1.08192, size = 36, normalized size = 1.09

$$\frac{1}{231}\left(21\,f^2x^{11}+66\,fx^7e+77\,x^3e^2\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/231*(21*f^2*x^11 + 66*f*x^7*e + 77*x^3*e^2)*c
```

3.137 $\int (a+cx^2)(e+fx^4)^2\,dx$

Optimal. Leaf size=60

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.0266964, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (a+cx^2)(e+fx^4)^2\,dx &= \int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)dx\\ &= ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A] time = 0.0022818, size = 60, normalized size = 1.

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11
```

Maple [A] time = 0.041, size = 51, normalized size = 0.9

$$ae^2x+\frac{ce^2x^3}{3}+\frac{2\,aefx^5}{5}+\frac{2\,cefx^7}{7}+\frac{af^2x^9}{9}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2*
x^11
```

Maxima [A] time = 0.974872, size = 68, normalized size = 1.13

$$\frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{2}{7}cefx^7 + \frac{2}{5}aefx^5 + \frac{1}{3}ce^2x^3 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^2
*x^3 + a*e^2*x
```

Fricas [A] time = 1.0562, size = 123, normalized size = 2.05

$$\frac{1}{11}x^{11}f^2c + \frac{1}{9}x^9f^2a + \frac{2}{7}x^7fec + \frac{2}{5}x^5fea + \frac{1}{3}x^3e^2c + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 2/5*x^5*f*e*a + 1/3*x^3*e
^2*c + x*e^2*a
```

Sympy [A] time = 0.069098, size = 60, normalized size = 1.

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11
```

Giac [A] time = 1.04692, size = 68, normalized size = 1.13

$$\frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{2}{7}cfx^7e + \frac{2}{5}afx^5e + \frac{1}{3}cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 2/5*a*f*x^5*e + 1/3*c*x^3
*e^2 + a*x*e^2
```

3.138 $\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.0975165, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1593, 1620}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx &= \int x(b+cx)\left(e+fx^4\right)^2\,dx\\ &= \int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right)dx\\ &= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}\end{aligned}$$

Mathematica [A] time = 0.0025941, size = 65, normalized size = 1.

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11
```

Maple [A] time = 0.04, size = 54, normalized size = 0.8

$$\frac{b e^2 x^2}{2}+\frac{c e^2 x^3}{3}+\frac{b e f x^6}{3}+\frac{2 c e f x^7}{7}+\frac{b f^2 x^{10}}{10}+\frac{c f^2 x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/1
1*c*f^2*x^11
```

Maxima [A] time = 1.01028, size = 72, normalized size = 1.11

$$\frac{1}{11}\,c f^2 x^{11}+\frac{1}{10}\,b f^2 x^{10}+\frac{2}{7}\,c e f x^7+\frac{1}{3}\,b e f x^6+\frac{1}{3}\,c e^2 x^3+\frac{1}{2}\,b e^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*e
^2*x^3 + 1/2*b*e^2*x^2
```

Fricas [A] time = 1.08661, size = 134, normalized size = 2.06

$$\frac{1}{11}x^{11}f^2c+\frac{1}{10}x^{10}f^2b+\frac{2}{7}x^7fec+\frac{1}{3}x^6feb+\frac{1}{3}x^3e^2c+\frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 2/7*x^7*f*e*c + 1/3*x^6*f*e*b + 1/3*x^3
*e^2*c + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.068121, size = 61, normalized size = 0.94

$$\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11
```

Giac [A] time = 1.0566, size = 72, normalized size = 1.11

$$\frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{2}{7}cfx^7e + \frac{1}{3}bfx^6e + \frac{1}{3}cx^3e^2 + \frac{1}{2}bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*f*x^7*e + 1/3*b*f*x^6*e + 1/3*c*x
^3*e^2 + 1/2*b*x^2*e^2
```

3.139 $\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=92

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rubi [A] time = 0.0531848, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1657}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx &= \int \left(ae^2+be^2x+ce^2x^2+2aefx^4+2befx^5+2cefx^6+af^2x^8+bf^2x^9+cf^2x^{10}\right)dx\\ &= ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2\end{aligned}$$

Mathematica [A] time = 0.0033933, size = 92, normalized size = 1.

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Maple [A] time = 0.041, size = 77, normalized size = 0.8

$$ae^2x+\frac{be^2x^2}{2}+\frac{ce^2x^3}{3}+\frac{2\,aefx^5}{5}+\frac{befx^6}{3}+\frac{2\,cefx^7}{7}+\frac{af^2x^9}{9}+\frac{bf^2x^{10}}{10}+\frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*x
^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Maxima [A] time = 0.981105, size = 103, normalized size = 1.12

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cefx^7 + \frac{1}{3}\,befx^6 + \frac{2}{5}\,aefx^5 + \frac{1}{3}\,ce^2x^3 + \frac{1}{2}\,be^2x^2 + ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

Fricas [A] time = 1.12252, size = 190, normalized size = 2.07

$$\frac{1}{11}x^{11}f^2c + \frac{1}{10}x^{10}f^2b + \frac{1}{9}x^9f^2a + \frac{2}{7}x^7fec + \frac{1}{3}x^6feb + \frac{2}{5}x^5fea + \frac{1}{3}x^3e^2c + \frac{1}{2}x^2e^2b + xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 2/7*x^7*f*e*c + 1/3*x^6
*f*e*b + 2/5*x^5*f*e*a + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b + x*e^2*a
```

Sympy [A] time = 0.069704, size = 90, normalized size = 0.98

$$ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Giac [A] time = 1.05879, size = 103, normalized size = 1.12

$$\frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{1}{9}\,af^2x^9 + \frac{2}{7}\,cfx^7e + \frac{1}{3}\,bfx^6e + \frac{2}{5}\,afx^5e + \frac{1}{3}\,cx^3e^2 + \frac{1}{2}\,bx^2e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*f*x^7*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

3.140 $\int dx^3 \left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=17

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.0050034, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 261}

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[d*x^3*(e + f*x^4)^2,x]

[Out] (d*(e + f*x^4)^3)/(12*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int dx^3 \left(e+fx^4\right)^2 \, dx &= d \int x^3 \left(e+fx^4\right)^2 \, dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.0008476, size = 33, normalized size = 1.94

$$\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x^3*(e + f*x^4)^2,x]

[Out] (d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

Maple [A] time = 0.039, size = 27, normalized size = 1.6

$$d\left(\frac{f^2x^{12}}{12}+\frac{efx^8}{4}+\frac{e^2x^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3*(f*x^4+e)^2,x)
```

```
[Out] d*(1/12*f^2*x^12+1/4*e*f*x^8+1/4*e^2*x^4)
```

Maxima [A] time = 1.01359, size = 20, normalized size = 1.18

$$\frac{\left(fx^4+e\right)^3d}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(f*x^4 + e)^3*d/f
```

Fricas [A] time = 1.05472, size = 66, normalized size = 3.88

$$\frac{1}{12}x^{12}f^2d+\frac{1}{4}x^8fed+\frac{1}{4}x^4e^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/4*x^8*f*e*d + 1/4*x^4*e^2*d
```

Sympy [B] time = 0.059406, size = 29, normalized size = 1.71

$$\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3*(f*x**4+e)**2,x)
```

```
[Out] d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A] time = 1.06087, size = 22, normalized size = 1.29

$$\frac{\left(fx^4+e\right)^3d}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*(f*x^4 + e)^3*d/f
```

3.141 $\int \left(a+dx^3\right) \left(e+fx^4\right)^2 \, dx$

Optimal. Leaf size=45

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.0190194, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1582, 12, 194}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (2*a*e*f*x^5)/5 + (a*f^2*x^9)/9 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+dx^3\right)\left(e+fx^4\right)^2 \, dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int a\left(e+fx^4\right)^2 \, dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}+a\int \left(e+fx^4\right)^2 \, dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}+a\int \left(e^2+2efx^4+f^2x^8\right) dx\\ &= ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.0022259, size = 60, normalized size = 1.33

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 +
 (d*f^2*x^12)/12
```

Maple [A] time = 0.038, size = 51, normalized size = 1.1

$$\frac{df^2x^{12}}{12}+\frac{af^2x^9}{9}+\frac{defx^8}{4}+\frac{2\,aefx^5}{5}+\frac{de^2x^4}{4}+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*e^2*x^4+a*e
^2*x
```

Maxima [A] time = 0.992621, size = 68, normalized size = 1.51

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^2
*x^4 + a*e^2*x
```

Fricas [A] time = 1.04397, size = 123, normalized size = 2.73

$$\frac{1}{12}x^{12}f^2d+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 2/5*x^5*f*e*a + 1/4*x^4*e
^2*d + x*e^2*a
```

Sympy [A] time = 0.064955, size = 58, normalized size = 1.29

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 +
d*f**2*x**12/12
```

Giac [A] time = 1.07443, size = 68, normalized size = 1.51

$$\frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}dfx^8e + \frac{2}{5}afx^5e + \frac{1}{4}dx^4e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/5*a*f*x^5*e + 1/4*d*x^4
*e^2 + a*x*e^2
```

3.142 $\int \left(bx+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.0198986, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1582, 12, 270}

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (b*e*f*x^6)/3 + (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \left(bx+dx^3\right)\left(e+fx^4\right)^2 dx = \frac{d\left(e+fx^4\right)^3}{12f} + \int bx\left(e+fx^4\right)^2 dx$$
$$= \frac{d\left(e+fx^4\right)^3}{12f} + b\int x\left(e+fx^4\right)^2 dx$$
$$= \frac{d\left(e+fx^4\right)^3}{12f} + b\int \left(e^2x+2efx^5+f^2x^9\right) dx$$
$$= \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d\left(e+fx^4\right)^3}{12f}$$

Mathematica [A] time = 0.002989, size = 65, normalized size = 1.3

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{4}de^2x^4 + \frac{1}{4}defx^8 + \frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^10
)/10 + (d*f^2*x^12)/12
```

Maple [A] time = 0.042, size = 54, normalized size = 1.1

$$\frac{df^2x^{12}}{12} + \frac{bf^2x^{10}}{10} + \frac{defx^8}{4} + \frac{befx^6}{3} + \frac{de^2x^4}{4} + \frac{be^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1
/2*b*e^2*x^2
```

Maxima [A] time = 1.00185, size = 72, normalized size = 1.44

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{10}\,bf^2x^{10} + \frac{1}{4}\,defx^8 + \frac{1}{3}\,befx^6 + \frac{1}{4}\,de^2x^4 + \frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*e
^2*x^4 + 1/2*b*e^2*x^2
```

Fricas [A] time = 1.02542, size = 134, normalized size = 2.68

$$\frac{1}{12}x^{12}f^2d + \frac{1}{10}x^{10}f^2b + \frac{1}{4}x^8fed + \frac{1}{3}x^6feb + \frac{1}{4}x^4e^2d + \frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/4*x^8*f*e*d + 1/3*x^6*f*e*b + 1/4*x^4
*e^2*d + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.065855, size = 60, normalized size = 1.2

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8
/4 + d*f**2*x**12/12
```

Giac [A] time = 1.05355, size = 72, normalized size = 1.44

$$\frac{1}{12}df^2x^{12} + \frac{1}{10}bf^2x^{10} + \frac{1}{4}dfx^8e + \frac{1}{3}bfx^6e + \frac{1}{4}dx^4e^2 + \frac{1}{2}bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 1/3*b*f*x^6*e + 1/4*d*x
^4*e^2 + 1/2*b*x^2*e^2
```

3.143 $\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.0511904, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1582, 1850}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9 +
 (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx&=\frac{d\left(e+fx^4\right)^3}{12f}+\int (a+bx)\left(e+fx^4\right)^2\,dx\\&=\frac{d\left(e+fx^4\right)^3}{12f}+\int \left(ae^2+be^2x+2aefx^4+2befx^5+af^2x^8+bf^2x^9\right)dx\\&=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.0033786, size = 92, normalized size = 1.19

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 +
 (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

Maple [A] time = 0.04, size = 77, normalized size = 1.

$$\frac{df^2x^{12}}{12}+\frac{bf^2x^{10}}{10}+\frac{af^2x^9}{9}+\frac{defx^8}{4}+\frac{befx^6}{3}+\frac{2\,aefx^5}{5}+\frac{de^2x^4}{4}+\frac{be^2x^2}{2}+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+2
/5*a*e*f*x^5+1/4*d*e^2*x^4+1/2*b*e^2*x^2+a*e^2*x
```

Maxima [A] time = 1.00375, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*e
*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Fricas [A] time = 1.09603, size = 190, normalized size = 2.47

$$\frac{1}{12}x^{12}f^2d+\frac{1}{10}x^{10}f^2b+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{1}{3}x^6feb+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+\frac{1}{2}x^2e^2b+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/10*x^10*f^2*b + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 1/3*x^6
*f*e*b + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + 1/2*x^2*e^2*b + x*e^2*a
```

Sympy [A] time = 0.068377, size = 88, normalized size = 1.14

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A] time = 1.05752, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,dfx^8e+\frac{1}{3}\,bfx^6e+\frac{2}{5}\,afx^5e+\frac{1}{4}\,dx^4e^2+\frac{1}{2}\,bx^2e^2+axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 1/3*b*f
*x^6*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/2*b*x^2*e^2 + a*x*e^2
```

3.144 $\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f
)
```

Rubi [A] time = 0.0234746, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1582, 12, 270}

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f
)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{d\left(e+fx^4\right)^3}{12f}+\int cx^2\left(e+fx^4\right)^2\,dx$$
$$=\frac{d\left(e+fx^4\right)^3}{12f}+c\int x^2\left(e+fx^4\right)^2\,dx$$
$$=\frac{d\left(e+fx^4\right)^3}{12f}+c\int \left(e^2x^2+2efx^6+f^2x^{10}\right)\,dx$$
$$=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Mathematica [A] time = 0.0028124, size = 65, normalized size = 1.3

$$\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x^
11)/11 + (d*f^2*x^12)/12
```

Maple [A] time = 0.039, size = 54, normalized size = 1.1

$$\frac{df^2x^{12}}{12}+\frac{cf^2x^{11}}{11}+\frac{defx^8}{4}+\frac{2\,cefx^7}{7}+\frac{de^2x^4}{4}+\frac{ce^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/4*d*e*f*x^8+2/7*c*e*f*x^7+1/4*d*e^2*x^4+1
/3*c*e^2*x^3
```

Maxima [A] time = 1.0436, size = 72, normalized size = 1.44

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*e
^2*x^4 + 1/3*c*e^2*x^3
```

Fricas [A] time = 1.08882, size = 134, normalized size = 2.68

$$\frac{1}{12}x^{12}f^2d+\frac{1}{11}x^{11}f^2c+\frac{1}{4}x^8fed+\frac{2}{7}x^7fec+\frac{1}{4}x^4e^2d+\frac{1}{3}x^3e^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/4*x^8*f*e*d + 2/7*x^7*f*e*c + 1/4*x^4
*e^2*d + 1/3*x^3*e^2*c
```

Sympy [A] time = 0.066073, size = 61, normalized size = 1.22

$$\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

```
[Out] c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x*
*8/4 + d*f**2*x**12/12
```

Giac [A] time = 1.05582, size = 72, normalized size = 1.44

$$\frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{4}dfx^8e+\frac{2}{7}cfx^7e+\frac{1}{4}dx^4e^2+\frac{1}{3}cx^3e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*f*x^8*e + 2/7*c*f*x^7*e + 1/4*d*x
^4*e^2 + 1/3*c*x^3*e^2
```

3.145 $\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

Optimal. Leaf size=77

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.04152, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1582, 1154}

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9
 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx &= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx\\ &= \frac{d\left(e+fx^4\right)^3}{12f}+\int \left(ae^2+ce^2x^2+2aefx^4+2cefx^6+af^2x^8+cf^2x^{10}\right)dx\\ &= ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}\end{aligned}$$

Mathematica [A] time = 0.0038174, size = 92, normalized size = 1.19

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7
 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A] time = 0.039, size = 77, normalized size = 1.

$$\frac{df^2x^{12}}{12}+\frac{cf^2x^{11}}{11}+\frac{af^2x^9}{9}+\frac{defx^8}{4}+\frac{2\,cefx^7}{7}+\frac{2\,aefx^5}{5}+\frac{de^2x^4}{4}+\frac{ce^2x^3}{3}+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+2
/5*a*e*f*x^5+1/4*d*e^2*x^4+1/3*c*e^2*x^3+a*e^2*x
```

Maxima [A] time = 0.942446, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*e
*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Fricas [A] time = 0.98629, size = 190, normalized size = 2.47

$$\frac{1}{12}x^{12}f^2d+\frac{1}{11}x^{11}f^2c+\frac{1}{9}x^9f^2a+\frac{1}{4}x^8fed+\frac{2}{7}x^7fec+\frac{2}{5}x^5fea+\frac{1}{4}x^4e^2d+\frac{1}{3}x^3e^2c+xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/9*x^9*f^2*a + 1/4*x^8*f*e*d + 2/7*x^7
*f*e*c + 2/5*x^5*f*e*a + 1/4*x^4*e^2*d + 1/3*x^3*e^2*c + x*e^2*a
```

Sympy [A] time = 0.068376, size = 90, normalized size = 1.17

$$ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

```
[Out] a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
+ c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A] time = 1.05124, size = 103, normalized size = 1.34

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{11}\,cf^2x^{11} + \frac{1}{9}\,af^2x^9 + \frac{1}{4}\,dfx^8e + \frac{2}{7}\,cfx^7e + \frac{2}{5}\,afx^5e + \frac{1}{4}\,dx^4e^2 + \frac{1}{3}\,cx^3e^2 + axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*f*x^8*e + 2/7*c*f
*x^7*e + 2/5*a*f*x^5*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + a*x*e^2
```

3.146 $\int \left(bx + cx^2 + dx^3\right) \left(e + fx^4\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{d\left(e + fx^4\right)^3}{12f}$$

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rubi [A] time = 0.0835611, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1582, 1593, 1620}

$$\frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{d\left(e + fx^4\right)^3}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x^
10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2 dx = \frac{d\left(e+fx^4\right)^3}{12f} + \int \left(bx+cx^2\right)\left(e+fx^4\right)^2 dx$$
$$= \frac{d\left(e+fx^4\right)^3}{12f} + \int x(b+cx)\left(e+fx^4\right)^2 dx$$
$$= \frac{d\left(e+fx^4\right)^3}{12f} + \int \left(be^2x+ce^2x^2+2befx^5+2cefx^6+bf^2x^9+cf^2x^{10}\right) dx$$
$$= \frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

Mathematica [A] time = 0.0041537, size = 97, normalized size = 1.18

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

```
[Out] (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x^
7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Maple [A] time = 0.04, size = 80, normalized size = 1.

$$\frac{df^2x^{12}}{12}+\frac{cf^2x^{11}}{11}+\frac{bf^2x^{10}}{10}+\frac{defx^8}{4}+\frac{2\,cefx^7}{7}+\frac{befx^6}{3}+\frac{de^2x^4}{4}+\frac{ce^2x^3}{3}+\frac{be^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x)
```

```
[Out] 1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/10*b*f^2*x^10+1/4*d*e*f*x^8+2/7*c*e*f*x^7
+1/3*b*e*f*x^6+1/4*d*e^2*x^4+1/3*c*e^2*x^3+1/2*b*e^2*x^2
```

Maxima [A] time = 0.942487, size = 107, normalized size = 1.3

$$\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*c
*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Fricas [A] time = 1.0713, size = 201, normalized size = 2.45

$$\frac{1}{12}x^{12}f^2d+\frac{1}{11}x^{11}f^2c+\frac{1}{10}x^{10}f^2b+\frac{1}{4}x^8fed+\frac{2}{7}x^7fec+\frac{1}{3}x^6feb+\frac{1}{4}x^4e^2d+\frac{1}{3}x^3e^2c+\frac{1}{2}x^2e^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^2*d + 1/11*x^11*f^2*c + 1/10*x^10*f^2*b + 1/4*x^8*f*e*d + 2/7*x
^7*f*e*c + 1/3*x^6*f*e*b + 1/4*x^4*e^2*d + 1/3*x^3*e^2*c + 1/2*x^2*e^2*b
```

Sympy [A] time = 0.070572, size = 92, normalized size = 1.12

$$\frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

```
[Out] b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x*
*7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Giac [A] time = 1.06003, size = 107, normalized size = 1.3

$$\frac{1}{12}\,df^2x^{12} + \frac{1}{11}\,cf^2x^{11} + \frac{1}{10}\,bf^2x^{10} + \frac{1}{4}\,dfx^8e + \frac{2}{7}\,cfx^7e + \frac{1}{3}\,bfx^6e + \frac{1}{4}\,dx^4e^2 + \frac{1}{3}\,cx^3e^2 + \frac{1}{2}\,bx^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

```
[Out] 1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*f*x^8*e + 2/7*c
*f*x^7*e + 1/3*b*f*x^6*e + 1/4*d*x^4*e^2 + 1/3*c*x^3*e^2 + 1/2*b*x^2*e^2
```

3.147 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rubi [A] time = 0.0804812, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1657}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx &= \frac{f\left(a+bx^4\right)^3}{12b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^2\,dx\\ &= \frac{f\left(a+bx^4\right)^3}{12b}+\int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abex^6+b^2cx^8+b^2\right.\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\end{aligned}$$

Mathematica [A] time = 0.0063405, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A] time = 0., size = 103, normalized size = 0.9

$$\frac{b^2fx^{12}}{12}+\frac{b^2ex^{11}}{11}+\frac{b^2dx^{10}}{10}+\frac{b^2cx^9}{9}+\frac{fabx^8}{4}+\frac{2\,abex^7}{7}+\frac{abdx^6}{3}+\frac{2\,abcx^5}{5}+\frac{fa^2x^4}{4}+\frac{a^2ex^3}{3}+\frac{a^2dx^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*f*a*b*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

Maxima [A] time = 0.932908, size = 138, normalized size = 1.27

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A] time = 1.15396, size = 258, normalized size = 2.37

$$\frac{1}{12}x^{12}fb^2+\frac{1}{11}x^{11}eb^2+\frac{1}{10}x^{10}db^2+\frac{1}{9}x^9cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba+\frac{1}{3}x^6dba+\frac{2}{5}x^5cba+\frac{1}{4}x^4fa^2+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x
^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*f*a^2 +
1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.073329, size = 121, normalized size = 1.11

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{a^2fx^4}{4} + \frac{2abcx^5}{5} + \frac{abdx^6}{3} + \frac{2abex^7}{7} + \frac{abfx^8}{4} + \frac{b^2cx^9}{9} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11} + \frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Giac [A] time = 1.05891, size = 142, normalized size = 1.3

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2x^{11}e + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abx^7e + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

3.148 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=151

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rubi [A] time = 0.105837, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1657}

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^3\,dx\\ &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2bex^6+3ab\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}a\end{aligned}$$

Mathematica [A] time = 0.0042748, size = 180, normalized size = 1.19

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] time = 0.001, size = 151, normalized size = 1.

$$\frac{b^3fx^{16}}{16}+\frac{b^3ex^{15}}{15}+\frac{b^3dx^{14}}{14}+\frac{b^3cx^{13}}{13}+\frac{ab^2fx^{12}}{4}+\frac{3\,ab^2ex^{11}}{11}+\frac{3\,ab^2dx^{10}}{10}+\frac{ab^2cx^9}{3}+\frac{3\,fba^2x^8}{8}+\frac{3\,a^2bex^7}{7}+\frac{a^2bdx^6}{2}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1/4*a*b^2*f
*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*f*b*a^2*x^8+3
/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*f*a^3*x^4+1/3*a^3*e*x^3+
1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 0.950192, size = 203, normalized size = 1.34

$$\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A] time = 0.924338, size = 375, normalized size = 2.48

$$\frac{1}{16}x^{16}fb^3+\frac{1}{15}x^{15}eb^3+\frac{1}{14}x^{14}db^3+\frac{1}{13}x^{13}cb^3+\frac{1}{4}x^{12}fb^2a+\frac{3}{11}x^{11}eb^2a+\frac{3}{10}x^{10}db^2a+\frac{1}{3}x^9cb^2a+\frac{3}{8}x^8fba^2+\frac{3}{7}x^7eba^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c*b^3 + 1/4
*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3
```

```
/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*
x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.084647, size = 180, normalized size = 1.19

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2f}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Giac [A] time = 1.05389, size = 208, normalized size = 1.38

$$\frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3x^{15}e + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2x^{11}e + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

3.149 $\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx$

Optimal. Leaf size=155

$$\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}$$

```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt
[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.118335, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1854, 1876, 275, 208, 1167, 205}

$$\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```


```
[Out] (a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + ((3*Sqrt[b]*c - Sqrt[a]
*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt
[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(3/4)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```


Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx = \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a-bx^4}\,dx}{4a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\frac{2dx}{a-bx^4}+\frac{-3c-ex^2}{a-bx^4}\right)dx}{4a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a-bx^4}\,dx}{4a} + \frac{d\int \frac{x}{a-bx^4}\,dx}{2a}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a} - \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx}{8a} + \frac{(3\sqrt{b}$$

$$= \frac{af+bx\left(c+dx+ex^2\right)}{4ab\left(a-bx^4\right)} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d}{}$$

Mathematica [A] time = 0.139363, size = 220, normalized size = 1.42

$$\frac{-\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d\right)+\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}e+3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}d\right)+\frac{4a(af+bx(c+x(}{a-bx^4}}{16a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*Sq
rt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*Sqrt[
b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*
(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/
4)*x] + 2*Sqrt[a]*Sqrt[b]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

Maple [B] time = 0.003, size = 248, normalized size = 1.6

$$-\frac{cx}{4\,a\left(bx^4-a\right)}+\frac{3\,c}{16\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{dx^2}{4\,a\left(bx^4-a\right)}-\frac{d}{8\,a}\ln\left(\left(-a+x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] -1/4*c*x/a/(b*x^4-a)+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*
a)^(1/4)))+3/8*c/a^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-1/4*d*x^2/a/(b*x
^4-a)-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/4
*e*x^3/a/(b*x^4-a)-1/8*e/a/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/16*e/a
/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/4*f*x^4/a/(b*x^4
-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 15.1272, size = 518, normalized size = 3.34

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2e^2 - 48abcd^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*a*
b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _t*l
og(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 98304*_t
**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2
*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_t*a**4
*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e
+ 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 - 120*a
**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c*
*3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3 + 64*a**2*
b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**
```

```
2*d**4 - 729*b**3*c**6)))) - (a*f + b*c*x + b*d*x**2 + b*e*x**3)/(-4*a**2*b
 + 4*a*b**2*x**4)
```

Giac [B] time = 1.0842, size = 452, normalized size = 2.92

$$-\frac{bx^3e+bdx^2+bcx+af}{4\left(bx^4-a\right)ab}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d+3\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b) + 1/16*sqrt(2)*(2*
sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*e)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/16*s
qrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3
) + 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)
*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(
a^2*b^3)
```

3.150 $\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx$

Optimal. Leaf size=188

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{af+bx\left(c\right.}{8ab\left(a\right.}$$

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*
x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.154601, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{\left(5\sqrt{a}e+21\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}+\frac{af+bx\left(c\right.}{8ab\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*
x^2))/(8*a*b*(a - b*x^4)^2) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(3/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/S
qrt[a]])/(16*a^(5/2)*Sqrt[b])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx = \frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}-\frac{\int \frac{-7c-6dx-5ex^2}{\left(a-bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{21c+12dx+5ex^2}{a-bx^4}\,dx}{32a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{21c+5ex^2}{a-bx^4}\right)dx}{32a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{21c+5ex^2}{a-bx^4}\,dx}{32a^2}+\frac{(3d)\int \frac{x}{a-bx^4}\,dx}{8a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{(3d)\,\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{16a^2}-\frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}}-\right.}{}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a-bx^4\right)}+\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}+\frac{\left(\frac{21\sqrt{bc}}{\sqrt{a}}-5e\right)\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{3/4}}+\frac{\left(21\sqrt{bc}+5\right.}{64}$$

Mathematica [A] time = 0.218945, size = 253, normalized size = 1.35

$$\frac{-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{bx}\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{bc}+12\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}}+\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{bx}\right)\left(5a^{3/4}e+21\sqrt[4]{a}\sqrt{bc}-12\sqrt{a}\sqrt[4]{bd}\right)}{b^{3/4}}+\frac{16a^2(af+bx(c+x(d+ex)))}{b\left(a-bx^4\right)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{bc}-5\sqrt{ae}\right)\tan}{b^{3/4}}}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(d
 + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1
/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[
b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4
) + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Maple [B] time = 0.006, size = 326, normalized size = 1.7

$$\frac{cx}{8a\left(bx^4-a\right)^2}-\frac{7cx}{32a^2\left(bx^4-a\right)}+\frac{21c}{128a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{21c}{64a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{dx^2}{8a\left(bx^4-a\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4-a)^2-7/32*c/a^2*x/(b*x^4-a)+21/128*c/a^3*(1/b*a)^(1/4)*ln(
(x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/
b*a)^(1/4))+1/8*d*x^2/a/(b*x^4-a)^2-3/16*d/a^2*x^2/(b*x^4-a)-3/32*d/a^2/(a*
b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+1/8*e*x^3/a/(b*x^4-a
)^2-5/32*e/a^2*x^3/(b*x^4-a)-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1
/4))+5/128*e/a^2/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/
8*f*x^4/a/(b*x^4-a)^2-1/8*f/a^2*x^4/(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 66.1839, size = 583, normalized size = 3.1

$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^3+t^2\left(-6881280a^6b^2ce-4718592a^6b^2d^2\right)+t\left(-153600a^4bde^2-2709504a^3b^2c^2d\right)-625\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] -RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 47185
92*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d)
- 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4
 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2*e**
3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2
+ 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e - 1
820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a*
*5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a**4*b**2*
c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 +
 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b*d**5*e +
58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 + 27
5625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a**2*b*d**4*e*
*2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b*
*2*c**2*d**4 - 85766121*b**3*c**6)))) - (-4*a**2*f - 11*a*b*c*x - 10*a*b*d*
x**2 - 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x**7)/(32*a*
*4*b - 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

Giac [B] time = 1.08412, size = 505, normalized size = 2.69

$$-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b^2c-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c - 5*(
-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^3*b^3) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1
/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1
/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 5*(
-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) -
 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 5*(-a*b^3)^(3/4)*e)*log(x^2 - sqr
t(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/32*(5*b^2*x^7*e + 6*b^2*d*x
^6 + 7*b^2*c*x^5 - 9*a*b*x^3*e - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x
^4 - a)^2*a^2*b)
```

3.151 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=220

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+$$

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a
 - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(3
2*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.188727, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{\left(15\sqrt{a}e+77\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}}+\frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2}+\frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a - b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a - b*x^4)) + (a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a
 - b*x^4)^3) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(256*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(256*a^(15/4)*b^(3/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(3
2*a^(7/2)*Sqrt[b])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^4}\,dx = \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a-bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a-bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-231c-120d}{a-bx}}{384a}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int\left(-\frac{120dx}{a-bx^4}+\right.}{3}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-231c-45ex^2}{a-bx^4}}{384a^3}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} + \frac{(5d)\,\text{Subst}}{}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a-bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a-bx^4\right)} + \frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3} + \frac{\left(77\sqrt{b}c-1\right.}{25}$$

Mathematica [A] time = 0.307724, size = 286, normalized size = 1.3

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c+40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(15a^{3/4}e+77\sqrt[4]{a}\sqrt{b}c-40\sqrt{a}\sqrt[4]{b}d\right)}{b^{3/4}}-\frac{128a^3(af+bx(c+x(d+ex)))}{b\left(bx^4-a\right)^3}+\frac{16a^2x(11c+x(10d+9ex}{\left(a-bx^4\right)^2}}{1536a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(-a
 + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^
(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*
Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*
Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Maple [A] time = 0.012, size = 280, normalized size = 1.3

$$\frac{1}{\left(bx^4-a\right)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}-\frac{5\,b^2dx^{10}}{32\,a^3}-\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}-\frac{11\,dx^2}{32\,a}-\frac{51\,cx}{128\,a}-\frac{f}{12\,b}\right)+\frac{77\,c}{512\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32*d/a^3*b^2*x^10-77/384*c/a^3*b^2*x^9+21/64/a^2*
b*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5-113/384/a*e*x^3-11/32*d/a*x^2-51
/128*c/a*x-1/12*f/b)/(b*x^4-a)^3+77/512/a^4*c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(
1/4))/(x-(1/b*a)^(1/4)))+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))
-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-15/25
6/a^3*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+15/512/a^3*e/b/(1/b*a)^(1/4
)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08736, size = 555, normalized size = 2.52

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d+77\left(-ab^3\right)^{\frac{1}{4}}b^2c+15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d+77\left(-ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c + 15*(
-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1
/4)*b^2*c + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(
1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 1
5*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3
) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 15*(-a*b^3)^(3/4)*e)*log(x^2
- sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b^3*x^11*e + 6
0*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2
*c*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*
x^4 - a)^3*a^3*b)
```

3.152 $\int \frac{a}{2+3x^4}\,dx$

Optimal. Leaf size=101

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}$$

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rubi [A] time = 0.0960809, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {12, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[a/(2 + 3*x^4),x]
```

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a}{2+3x^4}\,dx &= a\int \frac{1}{2+3x^4}\,dx \\
&= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} \\
&= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} \\
&= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\,\mathrm{Subst}}{} \\
&= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}
\end{aligned}
$$

Mathematica [A] time = 0.0290976, size = 78, normalized size = 0.77

$$
\frac{a\left(-\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{8\sqrt[4]{6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a/(2 + 3*x^4),x]
```

```
[Out] (a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))
```

Maple [A] time = 0.043, size = 114, normalized size = 1.1

$$
\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right) + \frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{a\sqrt{3}\sqrt[4]{6}}{24}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

Maxima [A] time = 1.477, size = 166, normalized size = 1.64

$$\frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

Fricas [B] time = 1.66575, size = 882, normalized size = 8.73

$$-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\arctan\left(-\frac{4a^3+2\cdot 24^{\frac{1}{4}}\sqrt{2}\left(a^4\right)^{\frac{3}{4}}x-24^{\frac{1}{4}}\sqrt{2}\sqrt{\frac{1}{3}}\left(a^4\right)^{\frac{3}{4}}\sqrt{\frac{12a^2x^2+24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}ax+4\sqrt{6}\sqrt{a^4}}{a^2}}}{4a^3}\right)-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\left(a^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*24^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(-1/4*(4*a^3 + 2*24^(1/4)*sqrt(2)*
(a^4)^(3/4)*x - 24^(1/4)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4)*sqrt((12*a^2*x^2 + 2
4^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))/a^2))/a^3) - 1/48*24
^(3/4)*sqrt(2)*(a^4)^(1/4)*arctan(1/4*(4*a^3 - 2*24^(1/4)*sqrt(2)*(a^4)^(3/
4)*x + 24^(1/4)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4)*sqrt((12*a^2*x^2 - 24^(3/4)*s
qrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))/a^2))/a^3) + 1/192*24^(3/4)*s
qrt(2)*(a^4)^(1/4)*log(12*a^2*x^2 + 24^(3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sq
rt(6)*sqrt(a^4)) - 1/192*24^(3/4)*sqrt(2)*(a^4)^(1/4)*log(12*a^2*x^2 - 24^(
3/4)*sqrt(2)*(a^4)^(1/4)*a*x + 4*sqrt(6)*sqrt(a^4))
```

Sympy [A] time = 0.359027, size = 88, normalized size = 0.87

$$a\left(-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x**4+2),x)
```

```
[Out] a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 +
6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4)*
atan(6**(1/4)*x + 1)/24)
```

Giac [A] time = 1.11121, size = 131, normalized size = 1.3

$$\frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+6^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}\left(\frac{2}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

3.153 $\int \frac{bx}{2+3x^4} dx$

Optimal. Leaf size=22

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rubi [A] time = 0.0130876, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {12, 275, 203}

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{bx}{2+3x^4}\,dx &= b\int \frac{x}{2+3x^4}\,dx\\
&= \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}
\end{aligned}$$

Mathematica [A] time = 0.0090014, size = 22, normalized size = 1.

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Maple [A] time = 0.04, size = 16, normalized size = 0.7

$$\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x/(3*x^4+2),x)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.42433, size = 20, normalized size = 0.91

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Fricas [A] time = 1.49792, size = 54, normalized size = 2.45

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Sympy [A] time = 0.089657, size = 19, normalized size = 0.86

$$\frac{\sqrt{6}b\operatorname{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x**4+2),x)
```

```
[Out] sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

Giac [A] time = 1.08175, size = 20, normalized size = 0.91

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

3.154 $\int \frac{a+bx}{2+3x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rubi [A] time = 0.102932, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 203}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx}{2+3x^4}\,dx &= \int \left(\frac{a}{2+3x^4}+\frac{bx}{2+3x^4}\right)dx\\
&= a\int \frac{1}{2+3x^4}\,dx + b\int \frac{x}{2+3x^4}\,dx\\
&= \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{\cdots}\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\cdots\right)}{4\sqrt[4]{6}}\\
&= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3\cdots}\right)}{8\sqrt[4]{6}}
\end{aligned}
$$

Mathematica [A] time = 0.0492228, size = 107, normalized size = 0.87

$$
\frac{-2\left(\sqrt[4]{6}a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt[4]{6}a-2b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+\sqrt[4]{6}a\left(\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+\cdots\right)\right)}{8\sqrt{6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1
+ 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2*6
^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

Maple [A] time = 0.003, size = 129, normalized size = 1.1

$$\frac{a\sqrt{3}\,\sqrt[4]{6}\,\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\,\sqrt[4]{6}\,\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\,\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\,\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\,\sqrt[4]{6}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*b*arctan(1/2*x^2*6^(1/
2))*6^(1/2)
```

Maxima [A] time = 1.48593, size = 198, normalized size = 1.61

$$\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/4
8*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*s
qrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt
(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*
arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.454476, size = 88, normalized size = 0.72

$$\operatorname{RootSum}\left(18432t^4+384t^2b^2-96ta^2b+3a^4+2b^4,\left(t\mapsto t\log\left(x+\frac{3072t^3b^2+192t^2a^2b+24ta^4+32tb^4-10a^2b^3}{3a^5-8ab^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lamb
da(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_t*
b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

Giac [A] time = 1.09838, size = 155, normalized size = 1.26

$$\frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)*
a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt(6
)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^
(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^
(1/4)))
```

3.155 $\int \frac{cx^2}{2+3x^4} dx$

Optimal. Leaf size=101

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}$$

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rubi [A] time = 0.0765918, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {12, 297, 1162, 617, 204, 1165, 628}

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{cx^2}{2+3x^4}\,dx = c\int \frac{x^2}{2+3x^4}\,dx$$
$$= -\frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$
$$= \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}}$$
$$= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} - \frac{c\,\text{Subst}\left(\int \frac{}{}\right.}{}$$
$$= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

Mathematica [A] time = 0.0147119, size = 78, normalized size = 0.77

$$\frac{c\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\right)}{4\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))
```

Maple [A] time = 0.041, size = 114, normalized size = 1.1

$$\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right) + \frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right) + \frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^2/(3*x^4+2),x)
```

```
[Out] 1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/14
4*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/
2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)
*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

Maxima [A] time = 1.45521, size = 166, normalized size = 1.64

$$\frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(
2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

Fricas [B] time = 1.44386, size = 869, normalized size = 8.6

$$-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}\arctan\left(-\frac{54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}x-54^{\frac{3}{4}}\sqrt{2}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{1}{4}}\sqrt{\frac{3c^3x^2+54^{\frac{1}{4}}\sqrt{2}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}{c^3}}+18c}{18c}\right)-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\left(c^4\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/108*54^(3/4)*sqrt(2)*(c^4)^(1/4)*arctan(-1/18*(54^(3/4)*sqrt(2)*(c^4)^(1
/4)*x - 54^(3/4)*sqrt(2)*sqrt(1/3)*(c^4)^(1/4)*sqrt((3*c^3*x^2 + 54^(1/4)*s
qrt(2)*(c^4)^(3/4)*x + sqrt(6)*sqrt(c^4)*c)/c^3) + 18*c)/c) - 1/108*54^(3/4
)*sqrt(2)*(c^4)^(1/4)*arctan(-1/18*(54^(3/4)*sqrt(2)*(c^4)^(1/4)*x - 54^(3/
4)*sqrt(2)*sqrt(1/3)*(c^4)^(1/4)*sqrt((3*c^3*x^2 - 54^(1/4)*sqrt(2)*(c^4)^(
3/4)*x + sqrt(6)*sqrt(c^4)*c)/c^3) - 18*c)/c) - 1/432*54^(3/4)*sqrt(2)*(c^4
)^(1/4)*log(9*c^3*x^2 + 3*54^(1/4)*sqrt(2)*(c^4)^(3/4)*x + 3*sqrt(6)*sqrt(c
^4)*c) + 1/432*54^(3/4)*sqrt(2)*(c^4)^(1/4)*log(9*c^3*x^2 - 3*54^(1/4)*sqrt
(2)*(c^4)^(3/4)*x + 3*sqrt(6)*sqrt(c^4)*c)
```

Sympy [A] time = 0.361851, size = 88, normalized size = 0.87

$$c\left(\frac{\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2/(3*x**4+2),x)
```

```
[Out] c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 + 6
**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*a
tan(6**(1/4)*x + 1)/12)
```

Giac [A] time = 1.0839, size = 131, normalized size = 1.3

$$\frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-6^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^2/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) -
6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 - sq
rt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

3.156 $\int \frac{a+cx^2}{2+3x^4} dx$

Optimal. Leaf size=141

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rubi [A] time = 0.0977998, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4))
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}+\frac{1}{24}\left(\sqrt{6}a+2c\right)\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx$$

$$= -\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\right.}{4\ 6^{3/4}}$$

$$= -\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+$$

Mathematica [A] time = 0.0505038, size = 113, normalized size = 0.8

$$\frac{-\left(\sqrt{6}a-2c\right)\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)\right)-2\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a+2c\right)\tan^{-1}}{8\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)*ArcTan[1
+ 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Log[
2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [B] time = 0.002, size = 226, normalized size = 1.6

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
```

```
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*c*3^(1/2)*6^(3/4)*2^(1
/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)
*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4
)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)
*3^(1/2)*6^(3/4)*x+1)
```

Maxima [A] time = 1.45568, size = 225, normalized size = 1.6

$$\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*
sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c
)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/48*3^(1/4
)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqr
t(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 - 3^(1
/4)*2^(3/4)*x + sqrt(2))
```

Fricas [B] time = 1.6198, size = 5063, normalized size = 35.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/144*(2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a
^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*s
qrt(1/3)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2
*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2
*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt
((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^
4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*s
qrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)
/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2
+ 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24
*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2
*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2
*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24
*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c
^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))
/(81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)) + 2*sqrt(6)*sqrt(2)*(54*a^4
+ 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 +
 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12
*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*sqrt(1/3)*(54*a^4 + 72*a^2*c^2 + 2
4*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^
2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*
c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^
4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*
```

```
x^2 - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 +
 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) +
sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4
*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4
+ 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqr
t(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2
 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 +
4*c^4)) - 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2
+ 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(81*a^8 + 108*a^6*c^2 - 48*a^2*c
^6 - 16*c^8)) - 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12*
a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 + 1
2*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a
^2*c^2 + 4*c^4))*log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 +
 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*
a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2
*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*(3*a^2 + 2*c^2)) + 3*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/
4)*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*
sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c
)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 - sq
rt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c
^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(
54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*
a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)
```

Sympy [A] time = 0.364615, size = 68, normalized size = 0.48

$$\operatorname{RootSum}\left(55296t^4 + 2304t^2ac + 9a^4 + 12a^2c^2 + 4c^4, \left(t \mapsto t\log\left(x + \frac{-4608t^3c + 72ta^3 - 144tac^2}{9a^4 - 4c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lamb
da(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 - 4*
c**4))))
```

Giac [A] time = 1.10751, size = 177, normalized size = 1.26

$$\frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2
+ sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x
^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.157 $\int \frac{bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=123

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Rubi [A] time = 0.117672, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1593, 1831, 275, 203, 297, 1162, 617, 204, 1165, 628}

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)/(2 + 3*x^4),x]
```


```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```


Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \int \frac{x(b+cx)}{2+3x^4}\,dx$$
$$= \int \left(\frac{bx}{2+3x^4}+\frac{cx^2}{2+3x^4}\right)dx$$
$$= b\int \frac{x}{2+3x^4}\,dx + c\int \frac{x^2}{2+3x^4}\,dx$$
$$= \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\,6^{3/4}} +$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\,6^{3/4}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\right.}{2\,6^{3/4}}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\,6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\,6^{3/4}} - \frac{c\log\left(\sqrt{6}\right.}{4}$$

Mathematica [A] time = 0.0436847, size = 99, normalized size = 0.8

$$\frac{-2\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\left(c-\sqrt[4]{6}b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*ArcTan[1 +
 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*Log[2 + 2*6^(1/4)*x
+ Sqrt[6]*x^2])/(4*6^(3/4))
```

Maple [A] time = 0.022, size = 129, normalized size = 1.1

$$\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arcta
n(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-
1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/
2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*
6^(3/4)*x+1)
```

Maxima [A] time = 1.45964, size = 198, normalized size = 1.61

$$\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c-2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c+2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c + 2*sqrt(
3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/24*3^
(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3^(1/
4)*2^(1/4)*c*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.458956, size = 85, normalized size = 0.69

$$\operatorname{RootSum}\left(27648t^4+576t^2b^2+96tbc^2+3b^4+2c^4,\left(t\mapsto t\log\left(x+\frac{-1152t^3c^2+288t^2b^3-36tb^2c^2+3b^5-3bc^4}{6b^4c-c^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lamb
da(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 + 3*
b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

Giac [A] time = 1.11023, size = 154, normalized size = 1.25

$$-\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)+\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4)
*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(sqr
t(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/
4)))
```

3.158 $\int \frac{a+bx+cx^2}{2+3x^4}\,dx$

Optimal. Leaf size=163

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rubi [A] time = 0.12368, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1876, 275, 203, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx=\int \left(\frac{bx}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right)dx$$

$$=b\int \frac{x}{2+3x^4}\,dx+\int \frac{a+cx^2}{2+3x^4}\,dx$$

$$=\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx$$

$$=\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}+\frac{1}{24}\left(\sqrt{6}a+2c\right)$$

$$=\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}\right.}{}$$

$$=\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\right.}{8\ 6}$$

Mathematica [A] time = 0.0713316, size = 129, normalized size = 0.79

$$\frac{-2\tan^{-1}\left(1-\sqrt[4]{6}x\right)\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)+2\tan^{-1}\left(\sqrt[4]{6}x+1\right)\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)\right.}{8\ 6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

```
[Out] (-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a -
2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(
1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Maple [B] time = 0.003, size = 241, normalized size = 1.5

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\sqrt[4]{6}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*b*arctan(1/2*x^2*6^(1/
2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/
4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/
2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(
1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

Maxima [A] time = 1.44739, size = 252, normalized size = 1.55

$$\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3
/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)
*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3)*s
qrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3
^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3^(1/4
)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 3.07534, size = 292, normalized size = 1.79

$$\operatorname{RootSum}\left(55296t^4 + t^2\left(2304ac + 1152b^2\right) + t\left(-288a^2b + 192bc^2\right) + 9a^4 + 12a^2c^2 - 24ab^2c + 6b^4 + 4c^4, \left(t \mapsto t\log\left(x \cdot\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192*
b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_t,
 _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3 + 1
728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**5 -
576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4 + 28
8*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**5*c +
24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4 - 12*a*
*2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6))))
```

Giac [A] time = 1.11269, size = 193, normalized size = 1.18

$$\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^
(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.159 $\int \frac{dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=13

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.0037983, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 260}

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (d*Log[2 + 3*x^4])/12
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{dx^3}{2+3x^4} \, dx &= d\int \frac{x^3}{2+3x^4} \, dx\\ &= \frac{1}{12}d\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0028887, size = 13, normalized size = 1.

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (d*Log[2 + 3*x^4])/12
```

Maple [A] time = 0.039, size = 12, normalized size = 0.9

$$\frac{d\ln\left(3\,x^{4}+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3/(3*x^4+2),x)
```

```
[Out] 1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 0.943257, size = 15, normalized size = 1.15

$$\frac{1}{12}\,d\log\left(3\,x^{4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/12*d*log(3*x^4 + 2)
```

Fricas [A] time = 1.39175, size = 31, normalized size = 2.38

$$\frac{1}{12}\,d\log\left(3\,x^{4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/12*d*log(3*x^4 + 2)
```

Sympy [A] time = 0.076402, size = 10, normalized size = 0.77

$$\frac{d\log\left(3x^{4}+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3/(3*x**4+2),x)
```

```
[Out] d*log(3*x**4 + 2)/12
```

Giac [A] time = 1.07502, size = 15, normalized size = 1.15

$$\frac{1}{12}\,d\log\left(3\,x^{4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*d*log(3*x^4 + 2)
```

3.160 $\int \frac{a+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.0991563, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 260}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -(a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/
4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+dx^3}{2+3x^4}\,dx &= \int \left(\frac{a}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx \\
&= a\int \frac{1}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}}+\frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{2}} \\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}+\frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}-\frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-}}{8\sqrt[4]{6}} \\
&= -\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+3x^4\right)+\frac{a\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\right.}{4\sqrt[4]{6}} \\
&= -\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+
\end{aligned}
$$

Mathematica [A] time = 0.031317, size = 108, normalized size = 0.95

$$\frac{1}{48}\left(-6^{3/4}a\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+6^{3/4}a\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\,6^{3/4}a\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\,6^{3/4}a\tan^{-1}\left(\sqrt[4]{6}x+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] - 6
^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A] time = 0.043, size = 125, normalized size = 1.1

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.45189, size = 201, normalized size = 1.76

$$\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^
(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(s
qrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4
)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [B] time = 1.61018, size = 1041, normalized size = 9.13

$$-\frac{4\cdot 6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^4\arctan\left(-\frac{6^{\frac{3}{4}}\sqrt{3}\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^4x-6^{\frac{3}{4}}\sqrt{3}\sqrt{2}\sqrt{\frac{1}{3}}\left(a^4\right)^{\frac{3}{4}}a^4\sqrt{\frac{3a^2x^2+6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}ax+\sqrt{6}\sqrt{a^4}}{a^2}}+6a^7}{6a^7}\right)+4\cdot 6^{\frac{1}{4}}\sqrt{3}\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^4\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*(4*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4*arctan(-1/6*(6^(3/4)*sqrt(
3)*sqrt(2)*(a^4)^(3/4)*a^4*x - 6^(3/4)*sqrt(3)*sqrt(2)*sqrt(1/3)*(a^4)^(3/4
)*a^4*sqrt((3*a^2*x^2 + 6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x + sqrt(6)*s
qrt(a^4))/a^2) + 6*a^7)/a^7) + 4*6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^4*ar
ctan(-1/6*(6^(3/4)*sqrt(3)*sqrt(2)*(a^4)^(3/4)*a^4*x - 6^(3/4)*sqrt(3)*sqrt
(2)*sqrt(1/3)*(a^4)^(3/4)*a^4*sqrt((3*a^2*x^2 - 6^(1/4)*sqrt(3)*sqrt(2)*(a^
4)^(1/4)*a*x + sqrt(6)*sqrt(a^4))/a^2) - 6*a^7)/a^7) - (6^(1/4)*sqrt(3)*sqr
t(2)*(a^4)^(1/4)*a^4 + 4*a^4*d)*log(3*a^2*x^2 + 6^(1/4)*sqrt(3)*sqrt(2)*(a^
4)^(1/4)*a*x + sqrt(6)*sqrt(a^4)) + (6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a^
4 - 4*a^4*d)*log(3*a^2*x^2 - 6^(1/4)*sqrt(3)*sqrt(2)*(a^4)^(1/4)*a*x + sqrt
```

```
(6)*sqrt(a^4)))/a^4
```

Sympy [A] time = 0.300339, size = 51, normalized size = 0.45

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + 6912t^2d^2 - 384td^3 + 27a^4 + 8d^4, \left(t \mapsto t\log\left(x + \frac{24t - 2d}{3a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*a
**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))
```

Giac [A] time = 1.13125, size = 147, normalized size = 1.29

$$\frac{1}{24} \cdot 6^{\frac{3}{4}} a \arctan\left(\frac{3}{4} \sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24} \cdot 6^{\frac{3}{4}} a \arctan\left(\frac{3}{4} \sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}} a + 4d\right) \log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1
/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.161 $\int \frac{bx+dx^3}{2+3x^4} \, dx$

Optimal. Leaf size=36

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12} d \log\left(3x^4+2\right)$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.0314551, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1593, 1248, 635, 203, 260}

$$\frac{b \tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12} d \log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) + (d*Log[2 + 3*x^4])/12
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{bx+dx^3}{2+3x^4}\,dx &= \int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx \\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right) \\ &= \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{x}{2+3x^2}\,dx,x,x^2\right) \\ &= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [C] time = 0.0304642, size = 65, normalized size = 1.81

$$\frac{1}{24}\left(2d+i\sqrt{6}b\right)\log\left(\sqrt{6}-3ix^2\right)+\frac{1}{24}\left(2d-i\sqrt{6}b\right)\log\left(\sqrt{6}+3ix^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] ((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d)
*Log[Sqrt[6] + (3*I)*x^2])/24
```

Maple [A] time = 0.042, size = 28, normalized size = 0.8

$$\frac{d\ln\left(3x^4+2\right)}{12}+\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Maxima [B] time = 1.46961, size = 153, normalized size = 4.25

$$-\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}d\log\left(\sqrt{3}x^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x -
 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2))
+ 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [A] time = 1.47641, size = 86, normalized size = 2.39

$$\frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)+\frac{1}{12}d\log\left(3x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2) + 1/12*d*log(3*x^4 + 2)
```

Sympy [C] time = 0.305611, size = 53, normalized size = 1.47

$$\left(-\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}i}{3}\right)+\left(\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)/(3*x**4+2),x)
```

```
[Out] (-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)*
log(x**2 + sqrt(6)*I/3)
```

Giac [B] time = 1.08778, size = 126, normalized size = 3.5

$$-\frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}d\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)
)) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.162 $\int \frac{a+bx+dx^3}{2+3x^4} dx$

Optimal. Leaf size=136

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*
Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.117721, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 1248, 635, 203, 260}

$$-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}+\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/4
)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*
Log[2 + 3*x^4])/12
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \int \left(\frac{a}{2+3x^4}+\frac{x(b+dx^2)}{2+3x^4}\right)dx$$
$$= a\int \frac{1}{2+3x^4}\,dx + \int \frac{x(b+dx^2)}{2+3x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx, x, x^2\right) + \frac{a\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}dx}{2\sqrt{2}} + \frac{a\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}dx}{2\sqrt{2}}$$
$$= \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}dx}{4\sqrt{6}} + \frac{a\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}dx}{4\sqrt{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}dx}{8\sqrt[4]{6}} - \frac{a\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}dx}{8\sqrt[4]{6}} + \frac{1}{2}b\,\text{Subs}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12}d\log\left(2+3x^4\right) + $$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log(}{}$$

Mathematica [A] time = 0.0584346, size = 128, normalized size = 0.94

$$\frac{1}{48}\left(-2\sqrt{6}\left(\sqrt[4]{6}a+2b\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt{6}\left(\sqrt[4]{6}a-2b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)-6^{3/4}a\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+6^{3/4}a\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a
- 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
 + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A] time = 0.041, size = 140, normalized size = 1.

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\sqrt[4]{6}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*b*arctan(1/2*x^2*6^(1/
2))*6^(1/2)+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.45488, size = 231, normalized size = 1.7

$$\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4)
*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)*l
og(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^(3/
4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/
4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^
(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.18933, size = 199, normalized size = 1.46

$$\mathrm{RootSum}\left(165888t^4 - 55296t^3d + t^2\left(3456b^2 + 6912d^2\right) + t\left(-864a^2b - 576b^2d - 384d^3\right) + 27a^4 + 72a^2bd + 18b^4 + 24l\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*(
-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 + 24
*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**2*a
**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4 + 5
76*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*d - 1
6*b**2*d**3)/(27*a**5 - 72*a*b**4))))
```

Giac [A] time = 1.10922, size = 169, normalized size = 1.24

$$\frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/
3)^(1/4)*x + sqrt(2/3))
```

3.163 $\int \frac{cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=114

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.120906, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1593, 1831, 297, 1162, 617, 204, 1165, 628, 260}

$$\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -(c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/
4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6
^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{cx^2+dx^3}{2+3x^4}\,dx &= \int \frac{x^2(c+dx)}{2+3x^4}\,dx \\
&= \int \left(\frac{cx^2}{2+3x^4}+\frac{dx^3}{2+3x^4}\right)dx \\
&= c\int \frac{x^2}{2+3x^4}\,dx + d\int \frac{x^3}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} \\
&= \frac{1}{12}d\log\left(2+3x^4\right) + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\ 6^{3/4}} + \\
&= \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{c\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\right.}{2\ 6^{3/4}} \\
&= -\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}
\end{aligned}
$$

Mathematica [A] time = 0.0276467, size = 108, normalized size = 0.95

$$
\frac{1}{24}\left(\sqrt[4]{6}c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\sqrt[4]{6}c\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\sqrt[4]{6}c\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}c\tan^{-1}\left(\sqrt[4]{6}x+1\right)+2d\,\text{lo}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1/4)*x] + 6
^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x
 + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [A] time = 0.001, size = 125, normalized size = 1.1

$$\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)/(3*x^4+2),x)
```

```
[Out] 1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/14
4*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/
2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)
*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.45013, size = 205, normalized size = 1.8

$$\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1
/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3
)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*3^(1/4)*2^(1/4)*
c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^(1/4
)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [B] time = 1.64733, size = 743, normalized size = 6.52

$$-\frac{4\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\arctan\left(-\frac{c^5+6^{\frac{1}{4}}\left(c^4\right)^{\frac{5}{4}}x-6^{\frac{1}{4}}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{5}{4}}\sqrt{\frac{3c^3x^2+6^{\frac{3}{4}}\left(c^4\right)^{\frac{3}{4}}x+\sqrt{6}\sqrt{c^4}c}{c^3}}}{c^5}\right)+4\cdot 6^{\frac{1}{4}}\left(c^4\right)^{\frac{1}{4}}c^4\arctan\left(\frac{c^5-6^{\frac{1}{4}}\left(c^4\right)^{\frac{5}{4}}x+6^{\frac{1}{4}}\sqrt{\frac{1}{3}}\left(c^4\right)^{\frac{5}{4}}\sqrt{3c}}{c^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*6^(1/4)*(c^4)^(1/4)*c^4*arctan(-(c^5 + 6^(1/4)*(c^4)^(5/4)*x - 6^(
1/4)*sqrt(1/3)*(c^4)^(5/4)*sqrt((3*c^3*x^2 + 6^(3/4)*(c^4)^(3/4)*x + sqrt(6
)*sqrt(c^4)*c)/c^3))/c^5) + 4*6^(1/4)*(c^4)^(1/4)*c^4*arctan((c^5 - 6^(1/4)
*(c^4)^(5/4)*x + 6^(1/4)*sqrt(1/3)*(c^4)^(5/4)*sqrt((3*c^3*x^2 - 6^(3/4)*(c
```

```
^4)^(3/4)*x + sqrt(6)*sqrt(c^4)*c)/c^3))/c^5) - (2*c^4*d - 6^(1/4)*(c^4)^(1
/4)*c^4)*log(3*c^3*x^2 + 6^(3/4)*(c^4)^(3/4)*x + sqrt(6)*sqrt(c^4)*c) - (2*
c^4*d + 6^(1/4)*(c^4)^(1/4)*c^4)*log(3*c^3*x^2 - 6^(3/4)*(c^4)^(3/4)*x + sq
rt(6)*sqrt(c^4)*c))/c^4
```

Sympy [A] time = 0.273372, size = 70, normalized size = 0.61

$$\operatorname{RootSum}\left(41472t^4 - 13824t^3d + 1728t^2d^2 - 96td^3 + 3c^4 + 2d^4, \left(t \mapsto t\log\left(x + \frac{3456t^3 - 864t^2d + 72td^2 - 2d^3}{3c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

```
[Out] RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**4
 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 - 2
*d**3)/(3*c**3))))
```

Giac [A] time = 1.1086, size = 147, normalized size = 1.29

$$\frac{1}{12} \cdot 6^{\frac{1}{4}} c \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12} \cdot 6^{\frac{1}{4}} c \arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2d\right)\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
+ 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))
) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1
/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.164 $\int \frac{a+cx^2+dx^3}{2+3x^4} dx$

Optimal. Leaf size=154

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.117019, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1876, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] -((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)
*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3
/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x +
3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx &= \int \left(\frac{dx^3}{2+3x^4}+\frac{a+cx^2}{2+3x^4}\right)dx \\
&= d\int \frac{x^3}{2+3x^4}\,dx+\int \frac{a+cx^2}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+3x^2}{2+3x^4}\,dx \\
&= \frac{1}{12}d\log\left(2+3x^4\right)-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}+\frac{1}{24}\left(\sqrt{6}a+2\right. \\
&= -\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)+ \\
&= -\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right.}{8\ 6^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.092163, size = 148, normalized size = 0.96

$$\frac{1}{48}\left(-\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)+\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}x^2+2\sqrt[4]{6}x+2\right)-2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(Sqrt[6]*a
+ 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4)*
x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[6]*
```

```
x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [B] time = 0.041, size = 237, normalized size = 1.5

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\sqrt[4]{6}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*c*3^(1/2)*6^(3/4)*2^(1
/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)
*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4
)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)
*3^(1/2)*6^(3/4)*x+1)+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.45753, size = 263, normalized size = 1.71

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arct
an(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*(3*3
^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(
3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [B] time = 1.95114, size = 5162, normalized size = 33.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/144*(2*sqrt(6)*sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a
^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*s
qrt(1/3)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2
*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 -
12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2
```

```
*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt
((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^
4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*s
qrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)
/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2
+ 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24
*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2
*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2
*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24
*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c
^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2 + 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))
/(81*a^8 + 108*a^6*c^2 - 48*a^2*c^6 - 16*c^8)) + 2*sqrt(6)*sqrt(2)*(54*a^4
+ 72*a^2*c^2 + 24*c^4)^(3/4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*sqrt((9*a^4 +
 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12
*a^2*c^2 + 4*c^4))*arctan(-1/12*(sqrt(2)*sqrt(1/3)*(54*a^4 + 72*a^2*c^2 + 2
4*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^
2*c^2 + 4*c^4)*a - 2*sqrt(6)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*
c^3))*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^
4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4))*sqrt((3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*
x^2 - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^
2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 +
 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) +
sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2))/(9*a^4 + 12*a^2*c^2 + 4
*c^4)) - sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(3/4)*(sqrt(6)*sqrt(54*a^4
+ 72*a^2*c^2 + 24*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4)*a*x - 2*sqrt(6)*sqr
t(9*a^4 - 12*a^2*c^2 + 4*c^4)*(3*a^2*c + 2*c^3)*x)*sqrt((9*a^4 + 12*a^2*c^2
 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 +
4*c^4)) - 2*sqrt(6)*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(9*a^4 + 12*a^2*c^2
+ 4*c^4)*sqrt(9*a^4 - 12*a^2*c^2 + 4*c^4))/(81*a^8 + 108*a^6*c^2 - 48*a^2*c
^6 - 16*c^8)) - 3*(sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12
*a^2*c^2 + 4*c^4 - 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 +
12*a^2*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*
a^2*c^2 + 4*c^4)) - 4*(9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*log(3*(9*a^4 + 12*a^2
*c^2 + 4*c^4)*x^2 + sqrt(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a
^4 + 72*a^2*c^2 + 24*c^4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2
*c^2 + 4*c^4 + 2*sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^
2 + 4*c^4)) + sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)) + 3*(sqrt
(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(9*a^4 + 12*a^2*c^2 + 4*c^4 - 2*sq
rt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*
sqrt(54*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + 4*(
9*a^4 + 12*a^2*c^2 + 4*c^4)*d)*log(3*(9*a^4 + 12*a^2*c^2 + 4*c^4)*x^2 - sqr
t(2)*(54*a^4 + 72*a^2*c^2 + 24*c^4)^(1/4)*(sqrt(54*a^4 + 72*a^2*c^2 + 24*c^
4)*c*x - 3*(3*a^3 + 2*a*c^2)*x)*sqrt((9*a^4 + 12*a^2*c^2 + 4*c^4 + 2*sqrt(5
4*a^4 + 72*a^2*c^2 + 24*c^4)*a*c)/(9*a^4 - 12*a^2*c^2 + 4*c^4)) + sqrt(54*a
^4 + 72*a^2*c^2 + 24*c^4)*(3*a^2 + 2*c^2)))/(9*a^4 + 12*a^2*c^2 + 4*c^4)
```

Sympy [A] time = 1.07802, size = 148, normalized size = 0.96

$$\operatorname{RootSum}\left(165888t^4 - 55296t^3d + t^2\left(6912ac + 6912d^2\right) + t\left(-1152acd - 384d^3\right) + 27a^4 + 36a^2c^2 + 48acd^2 + 12c^4 + 8d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(-
1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 + 8
```

```
*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a**3
 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/(27*
a**4 - 12*c**4))))
```

Giac [A] time = 1.10467, size = 185, normalized size = 1.2

$$\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(
3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)*lo
g(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c
- 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.165 $\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=136

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{1}{12}d$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*
Log[2 + 3*x^4])/12
```

Rubi [A] time = 0.139748, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1594, 1831, 297, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\tan^{-1}\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{1}{12}d$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```


```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/4
)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*
Log[2 + 3*x^4])/12
```


Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \int \frac{x\left(b+cx+dx^2\right)}{2+3x^4}\,dx$$
$$= \int \left(\frac{cx^2}{2+3x^4}+\frac{x\left(b+dx^2\right)}{2+3x^4}\right)dx$$
$$= c\int \frac{x^2}{2+3x^4}\,dx + \int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx, x, x^2\right) - \frac{c\int \frac{\sqrt{2}-\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{c\int \frac{\sqrt{2}+\sqrt{3}x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$
$$= \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right) + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{1}{12}c\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx + \frac{c\int}{}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right) + \frac{c}{}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{\right.}}{}$$

Mathematica [A] time = 0.0625569, size = 125, normalized size = 0.92

$$\frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}b+c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}\left(c-\sqrt[4]{6}b\right)\tan^{-1}\left(\sqrt[4]{6}x+1\right)+\sqrt[4]{6}c\log\left(\sqrt{6}x^2-2\sqrt[4]{6}x+2\right)-\sqrt[4]{6}c\log\left(\sqrt{6}x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(6^(1/4)*b)
 + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2]
- 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Maple [A] time = 0.046, size = 140, normalized size = 1.

$$\frac{b\sqrt{6}}{12}\arctan\left(\frac{x^2\sqrt{6}}{2}\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{72}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{c\sqrt{3}6^{\frac{3}{4}}\sqrt{2}}{144}\ln\left(\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)/(3*x^4+2),x)
```

```
[Out] 1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arcta
n(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-
1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/
2)+1/3*6^(1/2)))+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*
6^(3/4)*x+1)+1/12*d*ln(3*x^4+2)
```

Maxima [A] time = 1.45402, size = 235, normalized size = 1.73

$$\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c-6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c+6b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(
2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c +
 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/72*3^
(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(
3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*c)*lo
g(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.1706, size = 189, normalized size = 1.39

$$\operatorname{RootSum}\left(82944t^4-27648t^3d+t^2\left(1728b^2+3456d^2\right)+t\left(-288b^2d+288bc^2-192d^3\right)+9b^4+12b^2d^2-24bc^2d+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

```
[Out] RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(-
288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d +
 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b**3
+ 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2 + 9
*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4*c -
3*c**5))))
```

Giac [A] time = 1.10254, size = 167, normalized size = 1.23

$$-\frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\left(\sqrt{6}b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)
*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(
2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^
(1/4)*x + sqrt(2/3))
```

3.166 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x
^4])/12
```

Rubi [A] time = 0.143739, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```


```
[Out] (b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/
4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4))
- ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt
[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x
^4])/12
```


Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \int \left(\frac{a+cx^2}{2+3x^4}+\frac{x\left(b+dx^2\right)}{2+3x^4}\right)dx$$
$$= \int \frac{a+cx^2}{2+3x^4}\,dx + \int \frac{x\left(b+dx^2\right)}{2+3x^4}\,dx$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{b+dx}{2+3x^2}\,dx,x,x^2\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{2+3x^4}\,dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{\sqrt{6}+\ldots}{2+\ldots}$$
$$= \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{2+3x^2}\,dx,x,x^2\right)-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2\ldots}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}\ldots}{\sqrt[4]{3}}\ldots}}{8\ 6^{3/4}}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x\ldots\right)}{8\ 6^{3/4}}$$
$$= \frac{b\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\tan^{-1}\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-\ldots\right.}{}$$

Mathematica [A] time = 0.104701, size = 164, normalized size = 0.93

$$\frac{1}{48}\left(-2\sqrt[4]{6}\tan^{-1}\left(1-\sqrt[4]{6}x\right)\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)+2\sqrt[4]{6}\tan^{-1}\left(\sqrt[4]{6}x+1\right)\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)-\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(\sqrt{\ldots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

```
[Out] (-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/
4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]
*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Lo
g[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Maple [A] time = 0.003, size = 252, normalized size = 1.4

$$\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{24}\arctan\left(\frac{\sqrt{2}\sqrt{3}6^{\frac{3}{4}}x}{6}-1\right)+\frac{a\sqrt{3}\sqrt[4]{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)\left(x^2-\frac{\sqrt{3}\sqrt[4]{6}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}\right)^{-1}\right)+\frac{a\sqrt{3}\sqrt[4]{6}\ldots}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x)
```

```
[Out] 1/24*a*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/48
*a*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2
))/(x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/24*a*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*b*arctan(1/2*x^2*6^(1/
2))*6^(1/2)+1/72*c*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/
4)*x-1)+1/144*c*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/
2)+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))+1/72*c*3^(
1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/12*d*ln(3*x^
4+2)
```

Maxima [A] time = 1.48142, size = 279, normalized size = 1.59

$$-\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log(
sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(3)
*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x
 + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c - 6*s
qrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/7
2*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*arctan(
1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 6.53729, size = 580, normalized size = 3.3

$$\text{RootSum}\left(165888t^4-55296t^3d+t^2\left(6912ac+3456b^2+6912d^2\right)+t\left(-864a^2b-1152acd-576b^2d+576bc^2-384d^3\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

```
[Out] RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*d
**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3) +
 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b**4
 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x + (-4
1472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2*a**3
*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c**2 - 6
912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d - 1728
*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a*b**4 +
1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t*b**3*c*d
 + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c - 270*a**
3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d + 24*a**2*c*d
**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c**2*d**2 - 7
2*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*b*c**5 - 16*c
**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**2*b**4 - 36*a*
*2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6))))
```

Giac [A] time = 1.1216, size = 201, normalized size = 1.14

$$\frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)
*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)*c
)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/
4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/
48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2
/3))
```

3.167 $\int \frac{1+x+x^2+x^3}{1-x^4} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0076716, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2+x^3}{1-x^4} dx &= \int \frac{1}{1-x} dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] time = 0.0009808, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x)
```

```
[Out] -ln(-1+x)
```

Maxima [A] time = 0.953607, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 1.48741, size = 18, normalized size = 2.25

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.05629, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A] time = 1.0498, size = 9, normalized size = 1.12

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

3.168 $\int \frac{1+x+x^2+x^3}{1+x^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[
2] + Log[1 + x^4]/4
```

Rubi [A] time = 0.0418564, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1876, 1162, 617, 204, 1248, 635, 203, 260}

$$\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt[
2] + Log[1 + x^4]/4
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2+x^3}{1+x^4}\,dx &= \int \left(\frac{1+x^2}{1+x^4}+\frac{x\left(1+x^2\right)}{1+x^4}\right)dx\\ &= \int \frac{1+x^2}{1+x^4}\,dx+\int \frac{x\left(1+x^2\right)}{1+x^4}\,dx\\ &= \frac{1}{2}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,x^2\right)+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\right.}{\sqrt{2}}\\ &= \frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{\sqrt{2}}+\frac{1}{4}\log\left(1+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0351092, size = 50, normalized size = 0.94

$$\frac{1}{4}\left(\log\left(x^4+1\right)-2\left(1+\sqrt{2}\right)\tan^{-1}\left(1-\sqrt{2}x\right)+2\left(\sqrt{2}-1\right)\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

```
[Out] (-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt[
2]*x] + Log[1 + x^4])/4
```

Maple [B] time = 0.003, size = 102, normalized size = 1.9

$$\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(x^4+1),x)
```

```
[Out] 1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*
2^(1/2)))+1/2*arctan(1+x*2^(1/2))*2^(1/2)+1/2*arctan(x^2)+1/8*2^(1/2)*ln((1
+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))+1/4*ln(x^4+1)
```

Maxima [A] time = 1.43715, size = 103, normalized size = 1.94

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2
)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)
*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Fricas [B] time = 1.56056, size = 440, normalized size = 8.3

$$-\sqrt{-2\sqrt{2}+3}\arctan\left(\sqrt{x^2+\sqrt{2}x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+3}-\left(\sqrt{2}(x+1)+2x+1\right)\sqrt{-2\sqrt{2}+3}\right)+\sqrt{2\sqrt{2}+3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*sqrt(2) + 3)*arctan(sqrt(x^2 + sqrt(2)*x + 1)*(sqrt(2) + 2)*sqrt(-
2*sqrt(2) + 3) - (sqrt(2)*(x + 1) + 2*x + 1)*sqrt(-2*sqrt(2) + 3)) + sqrt(2
*sqrt(2) + 3)*arctan(-(sqrt(2)*(x + 1) - sqrt(x^2 - sqrt(2)*x + 1)*(sqrt(2)
 - 2) - 2*x - 1)*sqrt(2*sqrt(2) + 3)) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*
log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.324015, size = 73, normalized size = 1.38

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4}+2\left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right)+2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(x**4+1),x)
```

```
[Out] log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(2
)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

Giac [A] time = 1.06195, size = 95, normalized size = 1.79

$$\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log
(x^2 - sqrt(2)*x + 1)
```

3.169 $\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((
Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Rubi [A] time = 0.0870212, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1876, 1167, 205, 208, 1248, 635, 260}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}-\frac{\log\left(a-bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] -((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((
Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx &= \int \left(\frac{1+x^2}{a-bx^4}+\frac{x\left(1+x^2\right)}{a-bx^4}\right)dx\\
&= \int \frac{1+x^2}{a-bx^4}\,dx+\int \frac{x\left(1+x^2\right)}{a-bx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a}\sqrt{b}-b\ldots}\\
&= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\text{S}\ldots\\
&= -\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0499505, size = 203, normalized size = 1.64

$$-\frac{\left(a^{3/4}+\sqrt{a}\sqrt[4]{b}+\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}}-\frac{\left(-a^{3/4}+\sqrt{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}}+\frac{\left(\sqrt[4]{a}\sqrt{b}-a^{3/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

```
[Out] ((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)) -
((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1/4) - b^(1/4)*x])/(4
*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) - a^(1/4)*Sqrt[b])*Log[a^(1/4) +
 b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sqrt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt[b])
 - Log[a - b*x^4]/(4*b)
```

Maple [B] time = 0.004, size = 171, normalized size = 1.4

$$\frac{1}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{1}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{1}{4}\ln\left(\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{1}{2b}\text{arct}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-b*x^4+a),x)
```

```
[Out] 1/4*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*(1/b*a)^(1/
4)/a*arctan(x/(1/b*a)^(1/4))-1/4/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^
2*(a*b)^(1/2)))-1/2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4/b/(1/b*a)^(
1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/4/b*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 1.02863, size = 187, normalized size = 1.51

$$-\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\left(96a^3b^2 - 96a^2b^3\right) + t\left(-16a^3b + 32a^2b^2 - 16ab^3\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**2*
b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t**2*
a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*a*b*
*3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

Giac [B] time = 1.09042, size = 392, normalized size = 3.16

$$-\frac{\log\left(\left|bx^4 - a\right|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{-ab^3}b + \left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{-ab^3}b + \right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4)
)/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(-a
*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*lo
g(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3
)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b^3)
```

3.170 $\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$

Optimal. Leaf size=277

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt
[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) +
Log[a + b*x^4]/(4*b)
```

Rubi [A] time = 0.195501, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt
[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) +
Log[a + b*x^4]/(4*b)
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \int \left(\frac{1+x^2}{a+bx^4}+\frac{x\left(1+x^2\right)}{a+bx^4}\right)dx$$
$$= \int \frac{1+x^2}{a+bx^4}\,dx + \int \frac{x\left(1+x^2\right)}{a+bx^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{a+bx^2}\,dx,x,x^2\right) - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b} + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right) + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} +$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\text{lo}}{}$$

Mathematica [A] time = 0.188188, size = 283, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt[4]{b}\left(a^{3/4}-\sqrt[4]{a}\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}-a^{3/4}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+2a\log\left(a\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2*a
^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b])*b^(
1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log[a + b
*x^4])/(8*a*b)
```

Maple [A] time = 0.003, size = 286, normalized size = 1.

$$\frac{\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(b*x^4+a),x)
```

```
[Out] 1/8*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/
(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*(1/b*a)^(1/4)/a*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x-1)+1/2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8/b/(1/b*a)^(1
/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/
```

```
4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x+1)+1/4/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-
1)+1/4*ln(b*x^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.997206, size = 187, normalized size = 0.68

$$\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\left(96a^3b^2 + 96a^2b^3\right) + t\left(-16a^3b - 32a^2b^2 - 16ab^3\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 + 9
6*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**3) + a**3 + 3*a**2*b
 + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**3 - 48*_t**2*a*
*3*b**2 + 16*_t**2*a**2*b**3 + 12*_t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*b**3
 - a**3 - 2*a**2*b - a*b**2)/(a**2*b + 2*a*b**2 + b**3))))
```

Giac [A] time = 1.06264, size = 365, normalized size = 1.32

$$\frac{\log\left(\left|bx^4+a\right|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{ab^3}b + \left(ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{ab^3}b + \left(ab^3\right)^{\frac{3}{4}}\right.}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(a
*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)*b
+ (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a
*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.171 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.203053, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1885, 1248, 635, 208, 260, 1887, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] -((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/
(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[
a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx &= \int \left(\frac{x\left(d+fx^2\right)}{a-bx^4}+\frac{c+ex^2+gx^4}{a-bx^4}\right)dx\\ &= \int \frac{x\left(d+fx^2\right)}{a-bx^4}\,dx+\int \frac{c+ex^2+gx^4}{a-bx^4}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\left(a-bx^4\right)}\right)dx\\ &= -\frac{gx}{b}+\frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,dx}{b}+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\operatorname{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x\right.\\ &= -\frac{gx}{b}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\Big(\\ &= -\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{d\tanh}{2\sqrt{}}\end{aligned}$$

Mathematica [A] time = 0.0812388, size = 249, normalized size = 1.68

$$\frac{-a^{3/4}\sqrt[4]{b}f\log\left(a-bx^4\right)-4a^{3/4}\sqrt[4]{b}gx-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e+ag+bc\right)+\sqrt[4]{a}b^{3/4}d\log\left(\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}b^{3}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)
*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e + a*g)*Log[a^(1
/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(3/4)*d*Log[a^(
1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Log[a^
(1/4) + b^(1/4)*x] + a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3/4)
*b^(1/4)*f*Log[a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

Maple [B] time = 0.005, size = 244, normalized size = 1.7

$$-\frac{gx}{b}+\frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -g*x/b+1/2/b*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+1/2*c*(1/b*a)^(1/4)/a*
arctan(x/(1/b*a)^(1/4))+1/4/b*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)
^(1/4)))*g+1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/
4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(1/b*
a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))
/(x-(1/b*a)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 156.777, size = 2394, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**5 - 256*_t**3*a**3*b**4*f + _t**2*(-64*a**3*b**3
*e*g + 96*a**3*b**3*f**2 - 64*a**2*b**4*c*e - 32*a**2*b**4*d**2) + _t*(-16*
a**3*b**2*d*g**2 + 32*a**3*b**2*e*f*g - 16*a**3*b**2*f**3 - 32*a**2*b**3*c*
d*g + 32*a**2*b**3*c*e*f + 16*a**2*b**3*d**2*f - 16*a**2*b**3*d*e**2 - 16*a
*b**4*c**2*d) - a**4*g**4 - 4*a**3*b*c*g**3 + 4*a**3*b*d*f*g**2 + 2*a**3*b*
```

```
e**2*g**2 - 4*a**3*b*e*f**2*g + a**3*b*f**4 - 6*a**2*b**2*c**2*g**2 + 8*a**
2*b**2*c*d*f*g + 4*a**2*b**2*c*e**2*g - 4*a**2*b**2*c*e*f**2 - 4*a**2*b**2*
d**2*e*g - 2*a**2*b**2*d**2*f**2 + 4*a**2*b**2*d*e**2*f - a**2*b**2*e**4 -
4*a*b**3*c**3*g + 4*a*b**3*c**2*d*f + 2*a*b**3*c**2*e**2 - 4*a*b**3*c*d**2*
e + a*b**3*d**4 - b**4*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**5*b**4*e*g*
*2 + 128*_t**3*a**4*b**5*c*e*g - 128*_t**3*a**4*b**5*d**2*g + 64*_t**3*a**4
*b**5*e**3 + 64*_t**3*a**3*b**6*c**2*e - 128*_t**3*a**3*b**6*c*d**2 + 16*_t
**2*a**5*b**3*d*g**3 - 48*_t**2*a**5*b**3*e*f*g**2 + 48*_t**2*a**4*b**4*c*d
*g**2 - 96*_t**2*a**4*b**4*c*e*f*g + 96*_t**2*a**4*b**4*d**2*f*g - 48*_t**2
*a**4*b**4*d*e**2*g - 48*_t**2*a**4*b**4*e**3*f + 48*_t**2*a**3*b**5*c**2*d
*g - 48*_t**2*a**3*b**5*c**2*e*f + 96*_t**2*a**3*b**5*c*d**2*f - 48*_t**2*a
**3*b**5*c*d*e**2 + 32*_t**2*a**3*b**5*d**3*e + 16*_t**2*a**2*b**6*c**3*d -
 4*_t*a**6*b*g**5 - 20*_t*a**5*b**2*c*g**4 - 8*_t*a**5*b**2*d*f*g**3 - 16*_
t*a**5*b**2*e**2*g**3 + 12*_t*a**5*b**2*e*f**2*g**2 - 40*_t*a**4*b**3*c**2*
g**3 - 24*_t*a**4*b**3*c*d*f*g**2 - 48*_t*a**4*b**3*c*e**2*g**2 + 24*_t*a**
4*b**3*c*e*f**2*g + 36*_t*a**4*b**3*d**2*e*g**2 - 24*_t*a**4*b**3*d**2*f**2
*g + 24*_t*a**4*b**3*d*e**2*f*g - 12*_t*a**4*b**3*e**4*g + 12*_t*a**4*b**3*
e**3*f**2 - 40*_t*a**3*b**4*c**3*g**2 - 24*_t*a**3*b**4*c**2*d*f*g - 48*_t*
a**3*b**4*c**2*e**2*g + 12*_t*a**3*b**4*c**2*e*f**2 + 72*_t*a**3*b**4*c*d**
2*e*g - 24*_t*a**3*b**4*c*d**2*f**2 + 24*_t*a**3*b**4*c*d*e**2*f - 12*_t*a*
*3*b**4*c*e**4 + 8*_t*a**3*b**4*d**4*g - 16*_t*a**3*b**4*d**3*e*f - 12*_t*a
**3*b**4*d**2*e**3 - 20*_t*a**2*b**5*c**4*g - 8*_t*a**2*b**5*c**3*d*f - 16*
_t*a**2*b**5*c**3*e**2 + 36*_t*a**2*b**5*c**2*d**2*e + 8*_t*a**2*b**5*c*d**
4 - 4*_t*a*b**6*c**5 + a**6*f*g**5 + 5*a**5*b*c*f*g**4 - 5*a**5*b*d*e*g**4
+ a**5*b*d*f**2*g**3 + 4*a**5*b*e**2*f*g**3 - a**5*b*e*f**3*g**2 + 10*a**4*
b**2*c**2*f*g**3 - 20*a**4*b**2*c*d*e*g**3 + 3*a**4*b**2*c*d*f**2*g**2 + 12
*a**4*b**2*c*e**2*f*g**2 - 2*a**4*b**2*c*e*f**3*g + 5*a**4*b**2*d**3*g**3 -
 9*a**4*b**2*d**2*e*f*g**2 + 2*a**4*b**2*d**2*f**3*g - 3*a**4*b**2*d*e**2*f
**2*g + 3*a**4*b**2*e**4*f*g - a**4*b**2*e**3*f**3 + 10*a**3*b**3*c**3*f*g*
*2 - 30*a**3*b**3*c**2*d*e*g**2 + 3*a**3*b**3*c**2*d*f**2*g + 12*a**3*b**3*
c**2*e**2*f*g - a**3*b**3*c**2*e*f**3 + 15*a**3*b**3*c*d**3*g**2 - 18*a**3*
b**3*c*d**2*e*f*g + 2*a**3*b**3*c*d**2*f**3 - 3*a**3*b**3*c*d*e**2*f**2 + 3
*a**3*b**3*c*e**4*f - 2*a**3*b**3*d**4*f*g + 5*a**3*b**3*d**3*e**2*g + 2*a*
*3*b**3*d**3*e*f**2 + 3*a**3*b**3*d**2*e**3*f - 3*a**3*b**3*d*e**5 + 5*a**2
*b**4*c**4*f*g - 20*a**2*b**4*c**3*d*e*g + a**2*b**4*c**3*d*f**2 + 4*a**2*b
**4*c**3*e**2*f + 15*a**2*b**4*c**2*d**3*g - 9*a**2*b**4*c**2*d**2*e*f - 2*
a**2*b**4*c*d**4*f + 5*a**2*b**4*c*d**3*e**2 - 2*a**2*b**4*d**5*e + a*b**5*
c**5*f - 5*a*b**5*c**4*d*e + 5*a*b**5*c**3*d**3)/(a**6*g**6 + 6*a**5*b*c*g*
*5 + a**5*b*e**2*g**4 + 15*a**4*b**2*c**2*g**4 + 4*a**4*b**2*c*e**2*g**3 -
8*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 + 20*a**3*b**3*c**3*g**3 + 6*
a**3*b**3*c**2*e**2*g**2 - 24*a**3*b**3*c*d**2*e*g**2 - 2*a**3*b**3*c*e**4*
g + 4*a**3*b**3*d**4*g**2 + 8*a**3*b**3*d**2*e**3*g - a**3*b**3*e**6 + 15*a
**2*b**4*c**4*g**2 + 4*a**2*b**4*c**3*e**2*g - 24*a**2*b**4*c**2*d**2*e*g -
 a**2*b**4*c**2*e**4 + 8*a**2*b**4*c*d**4*g + 8*a**2*b**4*c*d**2*e**3 - 4*a
**2*b**4*d**4*e**2 + 6*a*b**5*c**5*g + a*b**5*c**4*e**2 - 8*a*b**5*c**3*d**
2*e + 4*a*b**5*c**2*d**4 + b**6*c**6)))) - g*x/b
```

Giac [B] time = 1.08823, size = 539, normalized size = 3.64

$$-\frac{gx}{b}-\frac{f\log\left(\left|bx^4-a\right|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}ab^4d+\left(-ab^3\right)^{\frac{1}{4}}ab^4c+\left(-ab^3\right)^{\frac{1}{4}}a^2b^3g+\left(-ab^3\right)^{\frac{3}{4}}ab^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -g*x/b - 1/4*f*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*a*b^
4*d + (-a*b^3)^(1/4)*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g + (-a*b^3)^(3/4)*a*
b^2*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b
^5) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*a*b^4*d + (-a*b^3)^(1/4)*a*b^4*c + (-
a*b^3)^(1/4)*a^2*b^3*g + (-a*b^3)^(3/4)*a*b^2*e)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^5) + 1/8*sqrt(2)*((-a*b^3)^(1/4)
*a*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g - (-a*b^3)^(3/4)*a*b^2*e)*log(x^2 + sqr
t(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^5) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*a
*b^4*c + (-a*b^3)^(1/4)*a^2*b^3*g - (-a*b^3)^(3/4)*a*b^2*e)*log(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^5)
```

3.172 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$

Optimal. Leaf size=172

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+\right.}{4ab\left(a-bx^4\right)}$$

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c -
 Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4))
+ ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.164898, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}+\frac{x\left(ag+bc+bdx+bex^2+\right.}{4ab\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + ((3*b*c -
 Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b^(5/4))
+ ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/
4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\int \frac{3bc-ag+2bdx+bex^2}{a-bx^4}\,dx}{4ab}$$
$$=\frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\int\left(\frac{2bdx}{a-bx^4}+\frac{3bc-ag+bex^2}{a-bx^4}\right)dx}{4ab}$$
$$=\frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\int \frac{3bc-ag+bex^2}{a-bx^4}\,dx}{4ab}+\frac{d\int\frac{x}{a-bx^4}\,dx}{2a}$$
$$=\frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{d\,\mathrm{Subst}\left(\int\frac{1}{a-bx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(3bc-\sqrt{a}\sqrt{b}e-a\right.}{8a^{3/}}$$
$$=\frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}+\frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}+\frac{\left(3bc+\sqrt{a}\right.}{}$$

Mathematica [A] time = 0.314004, size = 221, normalized size = 1.28

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4}-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-ag+3bc\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}\right.}{16a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*
(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c + 2
*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] + (3
*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(1/4)
*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5/4))
```

Maple [B] time = 0.008, size = 289, normalized size = 1.7

$$\frac{1}{bx^4-a}\left(-\frac{ex^3}{4a}-\frac{dx^2}{4a}-\frac{(ag+bc)x}{4ab}-\frac{f}{4b}\right)-\frac{g}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{g}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4/a*e*x^3-1/4*d/a*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/(b*x^4-a)-1/8/b/a*(1
/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+3/8*c/a^2*(1/b*a)^(1/4)*arctan(x/(1/b
*a)^(1/4))-1/16/b/a*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g
+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/8*d/a/(
a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/8*e/a/b/(1/b*a)^
(1/4)*arctan(x/(1/b*a)^(1/4))+1/16*e/a/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))
/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 169.346, size = 1406, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**5 + _t**2*(1024*a**5*b**3*e*g - 3072*a**4*b**4*
c*e - 2048*a**4*b**4*d**2) + _t*(128*a**4*b**2*d*g**2 - 768*a**3*b**3*c*d*g
 + 128*a**3*b**3*d*e**2 + 1152*a**2*b**4*c**2*d) - a**4*g**4 + 12*a**3*b*c*
g**3 + 2*a**3*b*e**2*g**2 - 54*a**2*b**2*c**2*g**2 - 12*a**2*b**2*c*e**2*g
+ 16*a**2*b**2*d**2*e*g - a**2*b**2*e**4 + 108*a*b**3*c**3*g + 18*a*b**3*c*
*2*e**2 - 48*a*b**3*c*d**2*e + 16*a*b**3*d**4 - 81*b**4*c**4, Lambda(_t, _t
*log(x + (-4096*_t**3*a**8*b**4*e*g**2 + 24576*_t**3*a**7*b**5*c*e*g - 3276
8*_t**3*a**7*b**5*d**2*g - 4096*_t**3*a**7*b**5*e**3 - 36864*_t**3*a**6*b**
6*c**2*e + 98304*_t**3*a**6*b**6*c*d**2 - 512*_t**2*a**7*b**3*d*g**3 + 4608
*_t**2*a**6*b**4*c*d*g**2 + 1536*_t**2*a**6*b**4*d*e**2*g - 13824*_t**2*a**
5*b**5*c**2*d*g - 4608*_t**2*a**5*b**5*c*d*e**2 + 4096*_t**2*a**5*b**5*d**3
```

```
*e + 13824*_t**2*a**4*b**6*c**3*d - 16*_t*a**7*b*g**5 + 240*_t*a**6*b**2*c*
g**4 - 64*_t*a**6*b**2*e**2*g**3 - 1440*_t*a**5*b**3*c**2*g**3 + 576*_t*a**
5*b**3*c*e**2*g**2 - 576*_t*a**5*b**3*d**2*e*g**2 - 48*_t*a**5*b**3*e**4*g
+ 4320*_t*a**4*b**4*c**3*g**2 - 1728*_t*a**4*b**4*c**2*e**2*g + 3456*_t*a**
4*b**4*c*d**2*e*g + 144*_t*a**4*b**4*c*e**4 + 512*_t*a**4*b**4*d**4*g + 192
*_t*a**4*b**4*d**2*e**3 - 6480*_t*a**3*b**5*c**4*g + 1728*_t*a**3*b**5*c**3
*e**2 - 5184*_t*a**3*b**5*c**2*d**2*e - 1536*_t*a**3*b**5*c*d**4 + 3888*_t*
a**2*b**6*c**5 - 10*a**5*b*d*e*g**4 + 120*a**4*b**2*c*d*e*g**3 - 40*a**4*b*
*2*d**3*g**3 - 540*a**3*b**3*c**2*d*e*g**2 + 360*a**3*b**3*c*d**3*g**2 - 40
*a**3*b**3*d**3*e**2*g - 6*a**3*b**3*d*e**5 + 1080*a**2*b**4*c**3*d*e*g - 1
080*a**2*b**4*c**2*d**3*g + 120*a**2*b**4*c*d**3*e**2 - 64*a**2*b**4*d**5*e
 - 810*a*b**5*c**4*d*e + 1080*a*b**5*c**3*d**3)/(a**6*g**6 - 18*a**5*b*c*g*
*5 + a**5*b*e**2*g**4 + 135*a**4*b**2*c**2*g**4 - 12*a**4*b**2*c*e**2*g**3
+ 32*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 - 540*a**3*b**3*c**3*g**3
+ 54*a**3*b**3*c**2*e**2*g**2 - 288*a**3*b**3*c*d**2*e*g**2 + 6*a**3*b**3*c
*e**4*g + 64*a**3*b**3*d**4*g**2 - 32*a**3*b**3*d**2*e**3*g - a**3*b**3*e**
6 + 1215*a**2*b**4*c**4*g**2 - 108*a**2*b**4*c**3*e**2*g + 864*a**2*b**4*c*
*2*d**2*e*g - 9*a**2*b**4*c**2*e**4 - 384*a**2*b**4*c*d**4*g + 96*a**2*b**4
*c*d**2*e**3 - 64*a**2*b**4*d**4*e**2 - 1458*a*b**5*c**5*g + 81*a*b**5*c**4
*e**2 - 864*a*b**5*c**3*d**2*e + 576*a*b**5*c**2*d**4 + 729*b**6*c**6))) -
 (a*f + b*d*x**2 + b*e*x**3 + x*(a*g + b*c))/(-4*a**2*b + 4*a*b**2*x**4)
```

Giac [B] time = 1.08497, size = 528, normalized size = 3.07

$$-\frac{bx^3e + bdx^2 + bcx + agx + af}{4\left(bx^4 - a\right)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d + 3\left(-ab^3\right)^{\frac{1}{4}}b^2c - \left(-ab^3\right)^{\frac{1}{4}}abg + \left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(-\right.}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b) + 1/16*sqr
t(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*
a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d + 3*(-a*b
^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(
-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sq
rt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/
4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.173 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3} \, dx$

Optimal. Leaf size=221

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{x\left(-ag+7bc+6bdx+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}$$

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f
+ x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c
- 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^
(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2
)*Sqrt[b])
```

Rubi [A] time = 0.263298, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{x\left(-ag+7bc+6bdx+5bex^2\right)+4af}{32a^2b\left(a-bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + (4*a*f
+ x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a - b*x^4)) + ((21*b*c
- 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^
(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4
)])/(64*a^(11/4)*b^(5/4)) + (3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2
)*Sqrt[b])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{\int \frac{7bc-ag+6bdx+5bex^2+4bfx^3}{\left(a-bx^4\right)^2}\,dx}{8ab}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}-\frac{\int \frac{-3(7bc-a}{}}{}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}-\frac{\int \left(-\frac{12bdx}{a-bx^4}\right.}{}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}-\frac{\int \frac{-3(7bc-a}{a-b}}{32}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}+\frac{(3d)\,\mathrm{Subs}}{}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}+\frac{4af+x\left(7bc-ag+6bdx+5bex^2\right)}{32a^2b\left(a-bx^4\right)}+\frac{\left(21bc-5\right.}{}$$

Mathematica [A] time = 0.491858, size = 263, normalized size = 1.19

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\left(a^2(4f+3gx)+abx\left(11c+x\left(10d+9ex+gx^3\right)\right)-b^2x^5(7c+x(6d+5ex))\right)}{\left(a-bx^4\right)^2}-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(12\sqrt[4]{a}b^{3/4}d+5\sqrt{a}\sqrt{b}e-3ag+21bc\right)+\log}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x)) +
a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2 + 2*(21*b*c - 5*Sqr
t[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/4)*
b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21*b*c
 - 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b^(1/4
)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^(5/
4))
```

Maple [A] time = 0.011, size = 328, normalized size = 1.5

$$-\frac{1}{\left(bx^4-a\right)^2}\left(\frac{5\,bex^7}{32\,a^2}+\frac{3\,bdx^6}{16\,a^2}-\frac{\left(ag-7\,bc\right)x^5}{32\,a^2}-\frac{9\,ex^3}{32\,a}-\frac{5\,dx^2}{16\,a}-\frac{\left(3\,ag+11\,bc\right)x}{32\,ab}-\frac{f}{8\,b}\right)-\frac{3\,g}{64\,ba^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(5/32/a^2*b*e*x^7+3/16/a^2*d*b*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32/a*e*x^3-5
/16*d/a*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8*f/b)/(b*x^4-a)^2-3/64/b/a^2*(1/b*
a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*
a)^(1/4))-3/128/b/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
*g+21/128*c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-3/32*
d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-5/64*e/a^2/
b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+5/128*e/a^2/b/(1/b*a)^(1/4)*ln((x+(
1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08181, size = 595, normalized size = 2.69

$$-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-21\left(-ab^3\right)^{\frac{1}{4}}b^2c+3\left(-ab^3\right)^{\frac{1}{4}}abg-5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}-\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c + 3*(
-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) - 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*
b)*b^2*d - 21*(-a*b^3)^(1/4)*b^2*c + 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b
^3) + 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(
-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) -
 1/256*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^
3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/32
*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*
b*d*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.174 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=266

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8}{96a^2b\left(a-bx^4\right)^2}$$

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*
(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f +
 x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b
*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/
4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32
*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.320272, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8}{96a^2b\left(a-bx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + (x*(7*
(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a - b*x^4)) + (8*a*f +
 x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a - b*x^4)^2) + ((77*b
*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/
4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(256*a^(15/4)*b^(5/4)) + (5*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32
*a^(7/2)*Sqrt[b])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{\int \frac{11bc-ag+10bdx+9bex^2+8bfx^3}{\left(a-bx^4\right)^3}\,dx}{12ab}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{8af+x\left(11bc-ag+10bdx+9bex^2\right)}{96a^2b\left(a-bx^4\right)^2} - \frac{\int \frac{-7(11bc-a}{(}}{}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x\left(11b\right.}{96}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x\left(11b\right.}{96}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x\left(11b\right.}{96}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x\left(11b\right.}{96}$$

$$= \frac{x\left(bc+ag+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{384a^3b\left(a-bx^4\right)} + \frac{8af+x\left(11b\right.}{96}$$

Mathematica [A] time = 0.323845, size = 313, normalized size = 1.18

$$\frac{\frac{128a^{11/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{\left(a-bx^4\right)^3} + \frac{16a^{7/4}\sqrt[4]{b}x(-ag+11bc+bx(10d+9ex))}{\left(a-bx^4\right)^2} + \frac{4a^{3/4}\sqrt[4]{b}x(-7ag+77bc+15bx(4d+3ex))}{a-bx^4} - 3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(40\sqrt[4]{a}b^{3/}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4)
+ (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^2
+ (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4)^
3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] -
 3*(77*b*c + 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[a^(1/
4) - b^(1/4)*x] + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e -
 7*a*g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt
[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

Maple [A] time = 0.012, size = 368, normalized size = 1.4

$$\frac{1}{\left(bx^4-a\right)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3} - \frac{5\,b^2dx^{10}}{32\,a^3} + \frac{\left(7\,ag-77\,bc\right)bx^9}{384\,a^3} + \frac{21\,bex^7}{64\,a^2} + \frac{5\,bdx^6}{12\,a^2} - \frac{\left(3\,ag-33\,bc\right)x^5}{64\,a^2} - \frac{113\,ex^3}{384\,a} - \frac{11\,dx^2}{32\,a} - \frac{(7}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11-5/32*d/a^3*b^2*x^10+7/384*(a*g-11*b*c)/a^3*b*x^9+21
/64/a^2*b*e*x^7+5/12/a^2*d*b*x^6-3/64/a^2*(a*g-11*b*c)*x^5-113/384/a*e*x^3-
11/32*d/a*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x^4-a)^3-7/512/b/a^3*
(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+77/512/a^4*c*(1/b*a
)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-7/256/b/a^3*(1/b*a)^(1/4)*a
rctan(x/(1/b*a)^(1/4))*g+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))
-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-15/25
6/a^3*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+15/512/a^3*e/b/(1/b*a)^(1/4
)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09935, size = 662, normalized size = 2.49

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d + 77\left(-ab^3\right)^{\frac{1}{4}}b^2c - 7\left(-ab^3\right)^{\frac{1}{4}}abg + 15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c - 7*(-
a*b^3)^(1/4)*a*b*g + 15*(-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*
b)*b^2*d + 77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g + 15*(-a*b^3)^(
3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*
b^3) + 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 1
5*(-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3
) - 1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(
-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) -
 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*
a*b^2*x^7*e - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*
b*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4
- a)^3*a^3*b)
```

3.175 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$

Optimal. Leaf size=319

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\tan^{-1}$$

```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + S
qrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
- a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*
x^4])/(4*b)
```

Rubi [A] time = 0.350505, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1885, 1248, 635, 205, 260, 1887, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c + S
qrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b]*e
- a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log[a + b*
x^4])/(4*b)
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \int \left(\frac{x\left(d+fx^2\right)}{a+bx^4}+\frac{c+ex^2+gx^4}{a+bx^4}\right)dx$$
$$=\int \frac{x\left(d+fx^2\right)}{a+bx^4}\,dx+\int \frac{c+ex^2+gx^4}{a+bx^4}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\left(a+bx^4\right)}\right)dx$$
$$=\frac{gx}{b}+\frac{\int \frac{bc-ag+bex^2}{a+bx^4}\,dx}{b}+\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{1}{2}f\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx\right.$$
$$=\frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2\sqrt{a}b^{3/2}}+\frac{(bc+}{}$$
$$=\frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}}-$$
$$=\frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-}{}$$
$$=\frac{gx}{b}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right.}{2\sqrt{2}a}$$

Mathematica [A] time = 0.318302, size = 311, normalized size = 0.97

$$2a^{3/4}\sqrt[4]{b}f\log\left(a+bx^4\right)+8a^{3/4}\sqrt[4]{b}gx-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag+\sqrt{2}bc\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqr
t[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*a*g
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2
]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/4))
```

Maple [A] time = 0.005, size = 429, normalized size = 1.3

$$\frac{gx}{b}-\frac{\sqrt{2}g}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}g}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] g*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+1/4*c
*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4/b*(1/b*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/
2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1
/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(
1/2)))*g+1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b
*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/2*d/(a*b)^(1/2)*a
rctan(x^2*(b/a)^(1/2))+1/8*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e
/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 158.354, size = 2392, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 - 256*_t**3*a**3*b**4*f + _t**2*(-64*a**3*b**3*
e*g + 96*a**3*b**3*f**2 + 64*a**2*b**4*c*e + 32*a**2*b**4*d**2) + _t*(-16*a
**3*b**2*d*g**2 + 32*a**3*b**2*e*f*g - 16*a**3*b**2*f**3 + 32*a**2*b**3*c*d
*g - 32*a**2*b**3*c*e*f - 16*a**2*b**3*d**2*f + 16*a**2*b**3*d*e**2 - 16*a*
b**4*c**2*d) + a**4*g**4 - 4*a**3*b*c*g**3 + 4*a**3*b*d*f*g**2 + 2*a**3*b*e
**2*g**2 - 4*a**3*b*e*f**2*g + a**3*b*f**4 + 6*a**2*b**2*c**2*g**2 - 8*a**2
*b**2*c*d*f*g - 4*a**2*b**2*c*e**2*g + 4*a**2*b**2*c*e*f**2 + 4*a**2*b**2*d
**2*e*g + 2*a**2*b**2*d**2*f**2 - 4*a**2*b**2*d*e**2*f + a**2*b**2*e**4 - 4
*a*b**3*c**3*g + 4*a*b**3*c**2*d*f + 2*a*b**3*c**2*e**2 - 4*a*b**3*c*d**2*e
 + a*b**3*d**4 + b**4*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**5*b**4*e*g*
*2 + 128*_t**3*a**4*b**5*c*e*g - 128*_t**3*a**4*b**5*d**2*g + 64*_t**3*a**4
*b**5*e**3 - 64*_t**3*a**3*b**6*c**2*e + 128*_t**3*a**3*b**6*c*d**2 - 16*_t
```

```
**2*a**5*b**3*d*g**3 + 48*_t**2*a**5*b**3*e*f*g**2 + 48*_t**2*a**4*b**4*c*d
*g**2 - 96*_t**2*a**4*b**4*c*e*f*g + 96*_t**2*a**4*b**4*d**2*f*g - 48*_t**2
*a**4*b**4*d*e**2*g - 48*_t**2*a**4*b**4*e**3*f - 48*_t**2*a**3*b**5*c**2*d
*g + 48*_t**2*a**3*b**5*c**2*e*f - 96*_t**2*a**3*b**5*c*d**2*f + 48*_t**2*a
**3*b**5*c*d*e**2 - 32*_t**2*a**3*b**5*d**3*e + 16*_t**2*a**2*b**6*c**3*d -
 4*_t*a**6*b*g**5 + 20*_t*a**5*b**2*c*g**4 + 8*_t*a**5*b**2*d*f*g**3 + 16*_
t*a**5*b**2*e**2*g**3 - 12*_t*a**5*b**2*e*f**2*g**2 - 40*_t*a**4*b**3*c**2*
g**3 - 24*_t*a**4*b**3*c*d*f*g**2 - 48*_t*a**4*b**3*c*e**2*g**2 + 24*_t*a**
4*b**3*c*e*f**2*g + 36*_t*a**4*b**3*d**2*e*g**2 - 24*_t*a**4*b**3*d**2*f**2
*g + 24*_t*a**4*b**3*d*e**2*f*g - 12*_t*a**4*b**3*e**4*g + 12*_t*a**4*b**3*
e**3*f**2 + 40*_t*a**3*b**4*c**3*g**2 + 24*_t*a**3*b**4*c**2*d*f*g + 48*_t*
a**3*b**4*c**2*e**2*g - 12*_t*a**3*b**4*c**2*e*f**2 - 72*_t*a**3*b**4*c*d**
2*e*g + 24*_t*a**3*b**4*c*d**2*f**2 - 24*_t*a**3*b**4*c*d*e**2*f + 12*_t*a*
*3*b**4*c*e**4 - 8*_t*a**3*b**4*d**4*g + 16*_t*a**3*b**4*d**3*e*f + 12*_t*a
**3*b**4*d**2*e**3 - 20*_t*a**2*b**5*c**4*g - 8*_t*a**2*b**5*c**3*d*f - 16*
_t*a**2*b**5*c**3*e**2 + 36*_t*a**2*b**5*c**2*d**2*e + 8*_t*a**2*b**5*c*d**
4 + 4*_t*a*b**6*c**5 + a**6*f*g**5 - 5*a**5*b*c*f*g**4 + 5*a**5*b*d*e*g**4
 - a**5*b*d*f**2*g**3 - 4*a**5*b*e**2*f*g**3 + a**5*b*e*f**3*g**2 + 10*a**4*
b**2*c**2*f*g**3 - 20*a**4*b**2*c*d*e*g**3 + 3*a**4*b**2*c*d*f**2*g**2 + 12
*a**4*b**2*c*e**2*f*g**2 - 2*a**4*b**2*c*e*f**3*g + 5*a**4*b**2*d**3*g**3 -
 9*a**4*b**2*d**2*e*f*g**2 + 2*a**4*b**2*d**2*f**3*g - 3*a**4*b**2*d*e**2*f
**2*g + 3*a**4*b**2*e**4*f*g - a**4*b**2*e**3*f**3 - 10*a**3*b**3*c**3*f*g*
*2 + 30*a**3*b**3*c**2*d*e*g**2 - 3*a**3*b**3*c**2*d*f**2*g - 12*a**3*b**3*
c**2*e**2*f*g + a**3*b**3*c**2*e*f**3 - 15*a**3*b**3*c*d**3*g**2 + 18*a**3*
b**3*c*d**2*e*f*g - 2*a**3*b**3*c*d**2*f**3 + 3*a**3*b**3*c*d*e**2*f**2 - 3
*a**3*b**3*c*e**4*f + 2*a**3*b**3*d**4*f*g - 5*a**3*b**3*d**3*e**2*g - 2*a*
*3*b**3*d**3*e*f**2 - 3*a**3*b**3*d**2*e**3*f + 3*a**3*b**3*d*e**5 + 5*a**2
*b**4*c**4*f*g - 20*a**2*b**4*c**3*d*e*g + a**2*b**4*c**3*d*f**2 + 4*a**2*b
**4*c**3*e**2*f + 15*a**2*b**4*c**2*d**3*g - 9*a**2*b**4*c**2*d**2*e*f - 2*
a**2*b**4*c*d**4*f + 5*a**2*b**4*c*d**3*e**2 - 2*a**2*b**4*d**5*e - a*b**5*
c**5*f + 5*a*b**5*c**4*d*e - 5*a*b**5*c**3*d**3)/(a**6*g**6 - 6*a**5*b*c*g*
*5 - a**5*b*e**2*g**4 + 15*a**4*b**2*c**2*g**4 + 4*a**4*b**2*c*e**2*g**3 -
8*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 - 20*a**3*b**3*c**3*g**3 - 6*
a**3*b**3*c**2*e**2*g**2 + 24*a**3*b**3*c*d**2*e*g**2 + 2*a**3*b**3*c*e**4*
g - 4*a**3*b**3*d**4*g**2 - 8*a**3*b**3*d**2*e**3*g + a**3*b**3*e**6 + 15*a
**2*b**4*c**4*g**2 + 4*a**2*b**4*c**3*e**2*g - 24*a**2*b**4*c**2*d**2*e*g -
 a**2*b**4*c**2*e**4 + 8*a**2*b**4*c*d**4*g + 8*a**2*b**4*c*d**2*e**3 - 4*a
**2*b**4*d**4*e**2 - 6*a*b**5*c**5*g - a*b**5*c**4*e**2 + 8*a*b**5*c**3*d**
2*e - 4*a*b**5*c**2*d**4 + b**6*c**6)))) + g*x/b
```

Giac [A] time = 1.10431, size = 459, normalized size = 1.44

$$\frac{gx}{b} + \frac{f\log\left(\left|bx^4+a\right|\right)}{4b} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d + \left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] g*x/b + 1/4*f*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
+ (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt
(2)*sqrt(a*b)*b^2*d + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(
3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e
```

```
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.176 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=341

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\text{ta}$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e
+ a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.305006, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```


```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (d*ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*b*c + Sqrt[a]*Sqrt[b]*e
+ a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/
4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} - \frac{\int \frac{-3bc-ag-2bdx-bex^2}{a+bx^4}dx}{4ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} - \frac{\int \left(-\frac{2bdx}{a+bx^4}+\frac{-3bc-ag-bex^2}{a+bx^4}\right)dx}{4ab}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} - \frac{\int \frac{-3bc-ag-bex^2}{a+bx^4}dx}{4ab} + \frac{d\int \frac{x}{a+bx^4}dx}{2a}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}dx, x, x^2\right)}{4a} + \frac{(3bc-\sqrt{a}\sqrt{b}e+a}{8a^{3/2}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{(3bc-\sqrt{a}\sqrt{b}e+ag)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{2}}}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{(3bc-\sqrt{a}\sqrt{b}e+ag)\log(\sqrt{a}}{16\sqrt{2}a^{7/4}b^{5}}$$

$$= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{(3bc+\sqrt{a}\sqrt{b}e+ag)\tan^{-1}(1}{8\sqrt{2}a^{7/4}b^{5/4}}$$

Mathematica [A] time = 0.196124, size = 319, normalized size = 0.94

$$-\frac{\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4} - 2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e+\sqrt{2}ag+3\sqrt{2}bc\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2
*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]
*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(1/4
)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[a]*Sqr
t[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a
^(7/4)*b^(5/4))
```

Maple [A] time = 0.009, size = 482, normalized size = 1.4

$$\frac{1}{bx^4+a}\left(\frac{ex^3}{4a}+\frac{dx^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}\right)+\frac{\sqrt{2}g}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (1/4/a*e*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+3/16*c/a^2*(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16/b/a*(1/b*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+3/16*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/32/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b
*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/
2)))*g+3/32*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/
b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*d/a/(a*b)^(1/2
)*arctan(x^2*(b/a)^(1/2))+1/32*e/a/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^
(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))
+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16*e/
a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 170.756, size = 1406, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**5 + _t**2*(1024*a**5*b**3*e*g + 3072*a**4*b**4*
c*e + 2048*a**4*b**4*d**2) + _t*(-128*a**4*b**2*d*g**2 - 768*a**3*b**3*c*d*
g + 128*a**3*b**3*d*e**2 - 1152*a**2*b**4*c**2*d) + a**4*g**4 + 12*a**3*b*c
*g**3 + 2*a**3*b*e**2*g**2 + 54*a**2*b**2*c**2*g**2 + 12*a**2*b**2*c*e**2*g
 - 16*a**2*b**2*d**2*e*g + a**2*b**2*e**4 + 108*a*b**3*c**3*g + 18*a*b**3*c
**2*e**2 - 48*a*b**3*c*d**2*e + 16*a*b**3*d**4 + 81*b**4*c**4, Lambda(_t, _
t*log(x + (-4096*_t**3*a**8*b**4*e*g**2 - 24576*_t**3*a**7*b**5*c*e*g + 327
68*_t**3*a**7*b**5*d**2*g + 4096*_t**3*a**7*b**5*e**3 - 36864*_t**3*a**6*b*
*6*c**2*e + 98304*_t**3*a**6*b**6*c*d**2 + 512*_t**2*a**7*b**3*d*g**3 + 460
8*_t**2*a**6*b**4*c*d*g**2 + 1536*_t**2*a**6*b**4*d*e**2*g + 13824*_t**2*a*
*5*b**5*c**2*d*g + 4608*_t**2*a**5*b**5*c*d*e**2 - 4096*_t**2*a**5*b**5*d**
```

```
3*e + 13824*_t**2*a**4*b**6*c**3*d + 16*_t*a**7*b*g**5 + 240*_t*a**6*b**2*c
*g**4 - 64*_t*a**6*b**2*e**2*g**3 + 1440*_t*a**5*b**3*c**2*g**3 - 576*_t*a*
*5*b**3*c*e**2*g**2 + 576*_t*a**5*b**3*d**2*e*g**2 + 48*_t*a**5*b**3*e**4*g
 + 4320*_t*a**4*b**4*c**3*g**2 - 1728*_t*a**4*b**4*c**2*e**2*g + 3456*_t*a*
*4*b**4*c*d**2*e*g + 144*_t*a**4*b**4*c*e**4 + 512*_t*a**4*b**4*d**4*g + 19
2*_t*a**4*b**4*d**2*e**3 + 6480*_t*a**3*b**5*c**4*g - 1728*_t*a**3*b**5*c**
3*e**2 + 5184*_t*a**3*b**5*c**2*d**2*e + 1536*_t*a**3*b**5*c*d**4 + 3888*_t
*a**2*b**6*c**5 + 10*a**5*b*d*e*g**4 + 120*a**4*b**2*c*d*e*g**3 - 40*a**4*b
**2*d**3*g**3 + 540*a**3*b**3*c**2*d*e*g**2 - 360*a**3*b**3*c*d**3*g**2 + 4
0*a**3*b**3*d**3*e**2*g + 6*a**3*b**3*d*e**5 + 1080*a**2*b**4*c**3*d*e*g -
1080*a**2*b**4*c**2*d**3*g + 120*a**2*b**4*c*d**3*e**2 - 64*a**2*b**4*d**5*
e + 810*a*b**5*c**4*d*e - 1080*a*b**5*c**3*d**3)/(a**6*g**6 + 18*a**5*b*c*g
**5 - a**5*b*e**2*g**4 + 135*a**4*b**2*c**2*g**4 - 12*a**4*b**2*c*e**2*g**3
 + 32*a**4*b**2*d**2*e*g**3 - a**4*b**2*e**4*g**2 + 540*a**3*b**3*c**3*g**3
 - 54*a**3*b**3*c**2*e**2*g**2 + 288*a**3*b**3*c*d**2*e*g**2 - 6*a**3*b**3*
c*e**4*g - 64*a**3*b**3*d**4*g**2 + 32*a**3*b**3*d**2*e**3*g + a**3*b**3*e*
*6 + 1215*a**2*b**4*c**4*g**2 - 108*a**2*b**4*c**3*e**2*g + 864*a**2*b**4*c
**2*d**2*e*g - 9*a**2*b**4*c**2*e**4 - 384*a**2*b**4*c*d**4*g + 96*a**2*b**
4*c*d**2*e**3 - 64*a**2*b**4*d**4*e**2 + 1458*a*b**5*c**5*g - 81*a*b**5*c**
4*e**2 + 864*a*b**5*c**3*d**2*e - 576*a*b**5*c**2*d**4 + 729*b**6*c**6))))
+ (-a*f + b*d*x**2 + b*e*x**3 + x*(-a*g + b*c))/(4*a**2*b + 4*a*b**2*x**4)
```

Giac [A] time = 1.07941, size = 493, normalized size = 1.45

$$\frac{bx^3e + bdx^2 + bcx - agx - af}{4\left(bx^4 + a\right)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)
*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*
b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^
(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a^2*b^3)
```

3.177 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f
- x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt
[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4
)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqr
t[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^
(5/4))
```

Rubi [A] time = 0.438579, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```


```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) - (4*a*f
- x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(32*a^2*b*(a + b*x^4)) + (3*d*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[b]) - ((21*b*c + 5*Sqrt[a]*Sqrt
[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4
)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) - ((21*b*c - 5*Sqrt[a]*Sqr
t[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(12
8*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^
(5/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^3}\,dx = \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-7bc-ag-6bdx-5bex^2-4bfx^3}{\left(a+bx^4\right)^2}\,dx}{8ab}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{\int \frac{-3(-7bc-ag)+\cdots}{a+\cdots}}{32\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{\int \left(\frac{12bdx}{a+bx^4}+\cdots\right)}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{\int \frac{-3(-7bc-ag)+\cdots}{a+bx^4}}{32a^2b\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{(3d)\,\mathrm{Subst}\left(\cdots\right)}{\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}\right)}{16a^{5/2}\sqrt{b}\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}\right)}{16a^{5/2}\sqrt{b}\cdots}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+6bdx+5bex^2\right)}{32a^2b\left(a+bx^4\right)} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}\right)}{16a^{5/2}\sqrt{b}\cdots}$$

Mathematica [A] time = 0.277805, size = 366, normalized size = 0.93

$$\frac{-\frac{32a^{7/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}\sqrt[4]{b}x(ag+7bc+bx(6d+5ex))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\sqrt{2}ag+21\sqrt{\cdots}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32*
a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^2 - 2*(2
1*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt
[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c - 24*a
^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a
```

```
*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(21*b*
c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

Maple [A] time = 0.01, size = 519, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (5/32/a^2*b*e*x^7+3/16/a^2*d*b*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*e*x^3+5/
16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+3/128/b/a^2*(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+21/128*c/a^3*(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b/a^2*(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+3/256/b/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x
^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b
*a)^(1/2)))*g+21/256*c/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16*d/a^
2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*l
n((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+
(1/b*a)^(1/2)))+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^
(1/4)*x+1)+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)
*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09804, size = 562, normalized size = 1.43

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d + 21\left(ab^3\right)^{\frac{1}{4}}b^2c + 3\left(ab^3\right)^{\frac{1}{4}}abg + 5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d + 21\left(ab^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b
^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d
 + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*s
qrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21
*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d
*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x -
3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.178 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4} dx$

Optimal. Leaf size=437

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+7\right.}{512\sqrt{2}a^{15/4}b^{5/4}}$$

```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*
(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f -
 x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*A
rcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^
(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqr
t[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a
*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(15/4)*b^(5/4))
```

Rubi [A] time = 0.531377, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+7\right.}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```


```
[Out] (x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (x*(7*
(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(384*a^3*b*(a + b*x^4)) - (8*a*f -
 x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(96*a^2*b*(a + b*x^4)^2) + (5*d*A
rcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - ((77*b*c + 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^
(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqr
t[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a
*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^
(15/4)*b^(5/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-11bc-ag-10bdx-9bex^2-8bfx^3}{\left(a+bx^4\right)^3}\,dx}{12ab}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x\left(11bc+ag+10bdx+9bex^2\right)}{96a^2b\left(a+bx^4\right)^2} + \frac{\int \frac{-7(-11}{}}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

$$= \frac{x\left(bc-ag+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+60bdx+45bex^2\right)}{384a^3b\left(a+bx^4\right)} - \frac{8af-x\left(1\right.}{}$$

Mathematica [A] time = 0.366398, size = 411, normalized size = 0.94

$$-\frac{256a^{11/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{\left(a+bx^4\right)^3} + \frac{32a^{7/4}\sqrt[4]{b}x(ag+11bc+bx(10d+9ex))}{\left(a+bx^4\right)^2} + \frac{8a^{3/4}\sqrt[4]{b}x(7ag+77bc+15bx(4d+3ex))}{a+bx^4} - 6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(80\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4)
+ (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^2
- (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^
3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e
 + 7*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b
*c - 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*a*g)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqr
t[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*
Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))
```

Maple [A] time = 0.014, size = 560, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21/
64/a^2*b*e*x^7+5/12/a^2*d*b*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3+1
1/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+7/512/b/a^3*(
1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+77/512/a^4*c*(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/b/a^3*(1/b*a)^(1/4)*2^(1/2)*
ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)
+(1/b*a)^(1/2)))*g+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4
)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/3
2/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)*2
^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*
2^(1/2)+(1/b*a)^(1/2)))+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/b*a)^(1/4)*x+1)+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09294, size = 629, normalized size = 1.44

$$\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^{2}d+77\,\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c+7\,\left(ab^{3}\right)^{\frac{1}{4}}abg+15\,\left(ab^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^{4}b^{3}}+\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{ab}b^{2}d+77\,\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b
^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*
d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/102
4*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4
)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2
)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e
+ 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^
2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*
x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.179 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{4}(1-x)^4$$

```
[Out] -(1 - x)^4/4
```

Rubi [A] time = 0.0125012, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$-\frac{1}{4}(1-x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -(1 - x)^4/4
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx &= \int (1-x)^3\,dx\\ &= -\frac{1}{4}(1-x)^4\end{aligned}$$

Mathematica [A] time = 0.0013102, size = 9, normalized size = 0.82

$$-\frac{1}{4}(x-1)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

```
[Out] -(-1 + x)^4/4
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$-\frac{(-1+x)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^3/(x^3+x^2+x+1)^3,x)
```

```
[Out] -1/4*(-1+x)^4
```

Maxima [B] time = 0.964346, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Fricas [B] time = 1.72758, size = 41, normalized size = 3.73

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

Sympy [B] time = 0.075679, size = 15, normalized size = 1.36

$$-\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
```

```
[Out] -x**4/4 + x**3 - 3*x**2/2 + x
```

Giac [B] time = 1.10327, size = 20, normalized size = 1.82

$$-\frac{1}{4}x^4 + x^3 - \frac{3}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")
```

```
[Out] -1/4*x^4 + x^3 - 3/2*x^2 + x
```

3.180 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx$

Optimal. Leaf size=11

$$-\frac{1}{3}(1-x)^3$$

```
[Out] -(1 - x)^3/3
```

Rubi [A] time = 0.0122637, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$-\frac{1}{3}(1-x)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]

[Out] -(1 - x)^3/3
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx &= \int (1-x)^2 dx \\ &= -\frac{1}{3}(1-x)^3\end{aligned}$$

Mathematica [A] time = 0.0005153, size = 14, normalized size = 1.27

$$\frac{x^3}{3} - x^2 + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]

[Out] x - x^2 + x^3/3
```

Maple [A] time = 0.001, size = 8, normalized size = 0.7

$$\frac{(-1+x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^2/(x^3+x^2+x+1)^2,x)
```

```
[Out] 1/3*(-1+x)^3
```

Maxima [A] time = 0.939669, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x^2 + x
```

Fricas [A] time = 1.68165, size = 26, normalized size = 2.36

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - x^2 + x
```

Sympy [A] time = 0.066356, size = 8, normalized size = 0.73

$$\frac{x^3}{3} - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)
```

```
[Out] x**3/3 - x**2 + x
```

Giac [A] time = 1.06477, size = 16, normalized size = 1.45

$$\frac{1}{3}x^3 - x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x^2 + x
```

3.181 $\int \frac{1-x^4}{1+x+x^2+x^3} \, dx$

Optimal. Leaf size=9

$$x-\frac{x^2}{2}$$

```
[Out] x - x^2/2
```

Rubi [A] time = 0.0086773, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1586}

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

```
[Out] x - x^2/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^4}{1+x+x^2+x^3} \, dx &= \int (1-x) \, dx \\ &= x-\frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0004395, size = 9, normalized size = 1.

$$x-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

```
[Out] x - x^2/2
```

Maple [A] time = 0.001, size = 8, normalized size = 0.9

$$x-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^3+x^2+x+1),x)
```

```
[Out] x-1/2*x^2
```

Maxima [A] time = 0.93638, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x
```

Fricas [A] time = 1.67103, size = 19, normalized size = 2.11

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x
```

Sympy [A] time = 0.057147, size = 5, normalized size = 0.56

$$-\frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**3+x**2+x+1),x)
```

```
[Out] -x**2/2 + x
```

Giac [A] time = 1.0496, size = 9, normalized size = 1.

$$-\frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x
```

3.182 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0074175, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2+x^3}{1-x^4} \, dx &= \int \frac{1}{1-x} \, dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] time = 0.0008638, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]

[Out] -Log[1 - x]
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)/(-x^4+1),x)
```

```
[Out] -ln(-1+x)
```

Maxima [A] time = 0.943333, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 1.70181, size = 18, normalized size = 2.25

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.056644, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A] time = 1.05278, size = 9, normalized size = 1.12

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

3.183 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx$

Optimal. Leaf size=7

$$\frac{1}{1-x}$$

```
[Out] (1 - x)^(-1)
```

Rubi [A] time = 0.015811, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$\frac{1}{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

```
[Out] (1 - x)^(-1)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx &= \int \frac{1}{(1-x)^2} dx \\ &= \frac{1}{1-x} \end{aligned}$$

Mathematica [A] time = 0.0008222, size = 7, normalized size = 1.

$$-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

```
[Out] -(-1 + x)^(-1)
```

Maple [A] time = 0.001, size = 8, normalized size = 1.1

$$-(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^2/(-x^4+1)^2,x)
```

```
[Out] -1/(-1+x)
```

Maxima [A] time = 0.960074, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x - 1)
```

Fricas [A] time = 1.66394, size = 16, normalized size = 2.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")
```

```
[Out] -1/(x - 1)
```

Sympy [A] time = 0.080331, size = 5, normalized size = 0.71

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
```

```
[Out] -1/(x - 1)
```

Giac [A] time = 1.05921, size = 9, normalized size = 1.29

$$-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")
```

```
[Out] -1/(x - 1)
```

3.184 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} \, dx$

Optimal. Leaf size=11

$$\frac{1}{2(1-x)^2}$$

```
[Out] 1/(2*(1 - x)^2)
```

Rubi [A] time = 0.0181356, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$\frac{1}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(1 - x)^2)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3} \, dx &= \int \frac{1}{(1-x)^3} \, dx \\ &= \frac{1}{2(1-x)^2}\end{aligned}$$

Mathematica [A] time = 0.0013108, size = 9, normalized size = 0.82

$$\frac{1}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

```
[Out] 1/(2*(-1 + x)^2)
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$\frac{1}{2\left(-1+x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^3/(-x^4+1)^3,x)
```

```
[Out] 1/2/(-1+x)^2
```

Maxima [A] time = 0.936703, size = 16, normalized size = 1.45

$$\frac{1}{2\left(x^{2}-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 - 2*x + 1)
```

Fricas [A] time = 1.66084, size = 28, normalized size = 2.55

$$\frac{1}{2\left(x^{2}-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.096645, size = 10, normalized size = 0.91

$$\frac{1}{2x^{2}-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)
```

```
[Out] 1/(2*x**2 - 4*x + 2)
```

Giac [A] time = 1.06814, size = 9, normalized size = 0.82

$$\frac{1}{2\left(x-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")
```

```
[Out] 1/2/(x - 1)^2
```

3.185 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3(1-x)^3}$$

```
[Out] 1/(3*(1 - x)^3)
```

Rubi [A] time = 0.0193226, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 32}

$$\frac{1}{3(1-x)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]

[Out] 1/(3*(1 - x)^3)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x+x^2+x^3\right)^4}{\left(1-x^4\right)^4}\,dx &= \int \frac{1}{(1-x)^4}\,dx\\ &= \frac{1}{3(1-x)^3}\end{aligned}$$

Mathematica [A] time = 0.0010176, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]

[Out] -1/(3*(-1 + x)^3)
```

Maple [A] time = 0., size = 8, normalized size = 0.7

$$-\frac{1}{3\left(-1+x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+1)^4/(-x^4+1)^4,x)
```

```
[Out] -1/3/(-1+x)^3
```

Maxima [B] time = 0.941818, size = 23, normalized size = 2.09

$$-\frac{1}{3\left(x^{3}-3\,x^{2}+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Fricas [B] time = 1.69488, size = 41, normalized size = 3.73

$$-\frac{1}{3\left(x^{3}-3\,x^{2}+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [B] time = 0.102664, size = 17, normalized size = 1.55

$$-\frac{1}{3x^{3}-9x^{2}+9x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
```

```
[Out] -1/(3*x**3 - 9*x**2 + 9*x - 3)
```

Giac [A] time = 1.05314, size = 9, normalized size = 0.82

$$-\frac{1}{3\left(x-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")
```

```
[Out] -1/3/(x - 1)^3
```

3.186 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$

Optimal. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-$$

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcT
anh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.260628, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1885, 1887, 1167, 205, 208, 1819, 1810, 635, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```


```
[Out] -((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcT
anh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx &= \int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\right)dx\\
&= \int \frac{c+ex^2+gx^4}{a-bx^4}\,dx+\int \frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}+\frac{bc+ag+bex^2}{b\left(a-bx^4\right)}\right)dx\\
&= -\frac{gx}{b}+\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\left(a-bx^2\right)}\right)dx,x,x^2\right)+\frac{\int \frac{bc+ag+bex^2}{a-bx^4}\,dx}{b}\\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\operatorname{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx,x,x^2\right)}{2b}+\frac{1}{2}\left(e-\frac{bc+ag}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}
\\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\tanh^{-1}\left(
\right)}{2a^{3/4}b^{5/4}}
\\
&= -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\tanh^{-1}\left(
\right)}{2a^{3/4}b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.257312, size = 256, normalized size = 1.55

$$\frac{-\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}h+\sqrt{a}b^{3/4}e+\sqrt[4]{a}bd+a\sqrt[4]{b}g+b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{5/4}(-h)+\sqrt{a}b^{3/4}e-\sqrt[4]{a}bd+a\sqrt[4]{b}g+b^{5/4}\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

```
[Out] (-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*c - Sqrt[a
]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/4)*c + a^(1/4)*b*d +
 Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (b
^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*Log[a
^(1/4) + b^(1/4)*x] + a^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] - a^(3
/4)*Sqrt[b]*f*Log[a - b*x^4])/(4*a^(3/4)*b^(3/2))
```

Maple [B] time = 0.045, size = 296, normalized size = 1.8

$$-\frac{hx^2}{2b}-\frac{gx}{b}+\frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/2*h*x^2/b-g*x/b+1/2/b*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+1/2*c*(1/b
*a)^(1/4)/a*arctan(x/(1/b*a)^(1/4))+1/4/b*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4)
)/(x-(1/b*a)^(1/4)))*g+1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a
)^(1/4)))-1/4/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*a
*h-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2*e/b/
(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(
1/4))/(x-(1/b*a)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08378, size = 539, normalized size = 3.27

$$-\frac{f\log\left(\left|bx^4-a\right|\right)}{4b}-\frac{bhx^2+2\,bgx}{2\,b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\sqrt{2}\sqrt{-ab}abh+\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg+\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\right.}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*f*log(abs(b*x^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*sqrt(2)*(s
qrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c +
 (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*
d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g
+ (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^
(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g -
 (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) -
 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*
e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3)
```

3.187 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} dx$

Optimal. Leaf size=188

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx\right.}{4b}$$

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/
Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (
Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4
)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.326583, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.22, Rules used = {1885, 1819, 1810, 635, 208, 260, 1887, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```


```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))/
Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e + (
Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4
)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2
)) - (f*Log[a - b*x^4])/(4*b)
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+187x^6}{a-bx^4}\,dx = \int \left(\frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}+\frac{c+ex^2+gx^4+187x^6}{a-bx^4}\right)dx$$

$$= \int \frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\,dx + \int \frac{c+ex^2+gx^4+187x^6}{a-bx^4}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+fx+hx^2}{a-bx^2}\,dx, x, x^2\right) + \int \left(-\frac{g}{b}-\frac{187x^2}{b}+\frac{bc+ag+(18}{b\left(a-b\right.}\right.$$

$$= -\frac{gx}{b}-\frac{187x^3}{3b}+\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{h}{b}+\frac{bd+ah+bfx}{b\left(a-bx^2\right)}\right)dx, x, x^2\right)+\frac{\int \frac{bc+a}{}}{}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{187x^3}{3b}+\frac{\text{Subst}\left(\int \frac{bd+ah+bfx}{a-bx^2}\,dx, x, x^2\right)}{2b}+\frac{\left(187a+be-\sqrt{b}\right.}{}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{187x^3}{3b}-\frac{\left(187a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\left(187a+b\right.}{}$$

$$= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{187x^3}{3b}-\frac{\left(187a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\left(187a+b\right.}{}$$

Mathematica [A] time = 0.375655, size = 301, normalized size = 1.6

$$\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}\sqrt[4]{b}h+a^{3/2}i+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{5/4}\sqrt[4]{b}h+a^{3/2}i-\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(a^{3/2}(-i\right.}{a^{3}}}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 + (6*(b^(3/2)*c - Sqrt
[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(3/4) - (
3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b^(1
/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c - a^(1
/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/Sqrt[a] - 3*b^(3/4)*f*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [B] time = 0.045, size = 367, normalized size = 2.

$$-\frac{ix^3}{3b}-\frac{hx^2}{2b}-\frac{gx}{b}+\frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{4a}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/3*i*x^3/b-1/2*h*x^2/b-g*x/b+1/2/b*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*
g+1/2*c*(1/b*a)^(1/4)/a*arctan(x/(1/b*a)^(1/4))+1/4/b*(1/b*a)^(1/4)*ln((x+(
1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4
))/(x-(1/b*a)^(1/4)))-1/4/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*
b)^(1/2)))*a*h-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2
)))-1/2/b^2/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*a*i-1/2*e/b/(1/b*a)^(1/4)
*arctan(x/(1/b*a)^(1/4))+1/4/b^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b
*a)^(1/4)))*a*i+1/4*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)
))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08977, size = 807, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="gi
ac")
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1
/4))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/b^4) + 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*
log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4) - 1/4*f*log(abs(b*x^4 -
 a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h +
 (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(
sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c
+ (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c +
 (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*
a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a
*b^3) - 1/6*(2*b^2*i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/b^3
```

3.188 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$

Optimal. Leaf size=205

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log}{4b^2}$$

```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*e - (Sqrt[
b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7
/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
2*Sqrt[a]*b^(3/2)) - ((b*f + a*j)*Log[a - b*x^4])/(4*b^2)
```

Rubi [A] time = 0.313444, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.196, Rules used = {1885, 1887, 1167, 205, 208, 1819, 1810, 635, 260}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{(ah+bd)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),x
]
```


```
[Out] -((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*e - (Sqrt[
b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7
/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(
2*Sqrt[a]*b^(3/2)) - ((b*f + a*j)*Log[a - b*x^4])/(4*b^2)
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+188x^6+jx^7}{a-bx^4}\,dx &= \int \left(\frac{c+ex^2+gx^4+188x^6}{a-bx^4}+\frac{x\left(d+fx^2+hx^4+jx^6\right)}{a-bx^4}\right)dx \\
&= \int \frac{c+ex^2+gx^4+188x^6}{a-bx^4}\,dx+\int \frac{x\left(d+fx^2+hx^4+jx^6\right)}{a-bx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a-bx^2}\,dx,x,x^2\right)+\int \left(-\frac{g}{b}-\frac{188x^2}{b}+bc \right. \\
&= -\frac{gx}{b}-\frac{188x^3}{3b}+\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{h}{b}-\frac{jx}{b}+\frac{bd+ah+(bf+aj)x}{b\left(a-bx^2\right)}\right) d \right. \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{188x^3}{3b}-\frac{jx^4}{4b}+\frac{\operatorname{Subst}\left(\int \frac{bd+ah+(bf+aj)x}{a-bx^2}\,dx,x,x^2\right)}{2b}+ \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{188x^3}{3b}-\frac{jx^4}{4b}-\frac{\left(188a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{188x^3}{3b}-\frac{jx^4}{4b}-\frac{\left(188a+be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}}
\end{aligned}
$$

Mathematica [A] time = 0.351059, size = 318, normalized size = 1.55

$$
\frac{-\frac{3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{5/4}\sqrt[4]{b}h+a^{3/2}i+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{5/4}\sqrt[4]{b}h+a^{3/2}i-\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(a^{3/2}(-i \right.}{a^{3}}}{12b^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4),x]
```

```
[Out] (-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*j*x^4 + (6
*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(
1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b
]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3
*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/
4)*h + a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h
)*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4])/b^(1
/4))/(12*b^(7/4))
```

Maple [B] time = 0.045, size = 393, normalized size = 1.9

$$-\frac{jx^4}{4b} - \frac{ix^3}{3b} - \frac{hx^2}{2b} - \frac{gx}{b} + \frac{g}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right) + \frac{g}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right) + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/4*j*x^4/b-1/3*i*x^3/b-1/2*h*x^2/b-g*x/b+1/2/b*(1/b*a)^(1/4)*arctan(x/(1/
b*a)^(1/4))*g+1/2*c*(1/b*a)^(1/4)/a*arctan(x/(1/b*a)^(1/4))+1/4/b*(1/b*a)^(
1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+1/4*c*(1/b*a)^(1/4)/a*ln((x+
(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/4/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))
/(-a-x^2*(a*b)^(1/2)))*a*h-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^
2*(a*b)^(1/2)))-1/2/b^2/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*a*i-1/2*e/b/(
1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4/b^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1
/4))/(x-(1/b*a)^(1/4)))*a*i+1/4*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(
1/b*a)^(1/4)))-1/4/b^2*ln(b*x^4-a)*a*j-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10172, size = 828, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorit
hm="giac")
```

```
[Out] 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1
/4))/(-a/b)^(1/4))/b^4 - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/b^4) + 1/8*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + sqrt(2)*(-a*b^3)^(3/4)*
log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4) - 1/4*(b*f + a*j)*log(a
bs(b*x^4 - a))/b^2 + 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-
a*b)*a*b*h + (-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1
/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - sqrt(2)*sqrt(-a*b)*a*b*h + (-a*b^3)^
(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g + (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(
1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a
*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/(a*b^3) - 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*
g*x)/b^4
```

3.189 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$

Optimal. Leaf size=337

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\tan^{-}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a
*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) -
((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.399356, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {1885, 1887, 1168, 1162, 617, 204, 1165, 628, 1819, 1810, 635, 205, 260}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\tan^{-}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a
*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) -
((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4
)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}\right)dx$$

$$= \int \frac{c+ex^2+gx^4}{a+bx^4}\,dx + \int \frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{a+bx^2}\,dx, x, x^2\right) + \int \left(\frac{g}{b}+\frac{bc-ag+bex^2}{b\left(a+bx^4\right)}\right)dx$$

$$= \frac{gx}{b} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{bd-ah+bfx}{b\left(a+bx^2\right)}\right)dx, x, x^2\right) + \frac{\int \frac{bc-ag+bex^2}{a+bx^4}\,dx}{b}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx, x, x^2\right)}{2b} + \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,d}{2\sqrt{a}b^{3/2}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{1}{2}f\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right) - \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{4\sqrt{2}a^{3/4}b^{5/4}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

Mathematica [A] time = 0.324556, size = 342, normalized size = 1.01

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-2a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}e+2\sqrt[4]{a}bd-\sqrt{2}a\sqrt[4]{b}g+\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}h+\sqrt{2}\sqrt{a}b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2
]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(S
qrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a*b^
(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*(S
qrt[2]*(-(b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x*(2*g +
 h*x) + f*Log[a + b*x^4]))))/(8*a^(3/4)*b^(3/2))
```

Maple [A] time = 0.003, size = 462, normalized size = 1.4

$$\frac{hx^2}{2b} + \frac{gx}{b} - \frac{\sqrt{2}g}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) - \frac{\sqrt{2}g}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) + \frac{c}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/2*h*x^2/b+g*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*
x+1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4/
b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+1/4*c*(1/b*a)^(
1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/b*(1/b*a)^(1/4)*2^(1/2
)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/
2)+(1/b*a)^(1/2)))*g+1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*
2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/2/b/(
a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(
1/2))+1/8*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)
^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e/b/(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08285, size = 506, normalized size = 1.5

$$\frac{f\log\left(\left|bx^4+a\right|\right)}{4\,b}+\frac{bhx^2+2\,bgx}{2\,b^2}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh+\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{1}{4}}abg+\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x\right.}{2}\right.}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 + 1/4*sqrt(2)*(sq
rt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*
b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)
^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(
2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3
/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3)
+ 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.190 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$

Optimal. Leaf size=384

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-a\right.}{4\sqrt{2}a^{3/4}b^{7/4}}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i
))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + (
(Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*
e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4))
+ (f*Log[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.565177, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {1885, 1819, 1810, 635, 205, 260, 1887, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-a\right.}{4\sqrt{2}a^{3/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i
))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) + (
(Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*
e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4))
+ (f*Log[a + b*x^4])/(4*b)
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 190x^6}{a + bx^4}\,dx = \int \left(\frac{x\left(d + fx^2 + hx^4\right)}{a + bx^4} + \frac{c + ex^2 + gx^4 + 190x^6}{a + bx^4}\right) dx$$

$$= \int \frac{x\left(d + fx^2 + hx^4\right)}{a + bx^4}\,dx + \int \frac{c + ex^2 + gx^4 + 190x^6}{a + bx^4}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{d + fx + hx^2}{a + bx^2}\,dx, x, x^2\right) + \int \left(\frac{g}{b} + \frac{190x^2}{b} + \frac{bc - ag - (190 \ldots}{b\left(a + bx \ldots\right.}\right.$$

$$= \frac{gx}{b} + \frac{190x^3}{3b} + \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{h}{b} + \frac{bd - ah + bfx}{b\left(a + bx^2\right)}\right) dx, x, x^2\right) + \int \frac{bc-ag+(\ldots}{a \ldots}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{\text{Subst}\left(\int \frac{bd-ah+bfx}{a+bx^2}\,dx, x, x^2\right)}{2b} - \frac{\left(190a - be - \frac{\sqrt{b}(bc \ldots}{\sqrt{\ldots}}\right.}{2 \ldots}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{1}{2}f\,\text{Subst}\left(\int \frac{x}{a + bx^2}\,dx, x, x^2\right) - \frac{\left(190a - be - \frac{\sqrt{\ldots}}{\ldots}\right.}{\ldots}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{(bd - ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{\left(190a - be + \frac{\sqrt{b}(bc-ag)}{\sqrt{a}}\right) \ldots}{4\sqrt{\ldots}}$$

$$= \frac{gx}{b} + \frac{hx^2}{2b} + \frac{190x^3}{3b} + \frac{(bd - ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} + \frac{\left(190a - be - \frac{\sqrt{b}(bc-ag)}{\sqrt{a}}\right) \ldots}{2\sqrt{2}\sqrt[4]{a}b \ldots}$$

Mathematica [A] time = 0.306906, size = 427, normalized size = 1.11

$$\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g-\sqrt{2}b^{3/2}c\right)}{a^{3/4}} + \frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}a\sqrt{b}g+\sqrt{2}b \ldots\right.}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt[2]*b^(3/2
)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 2*
a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a
]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(3/2)*c - Sqrt
[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g
 + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/
4) + 6*b^(3/4)*f*Log[a + b*x^4])/(24*b^(7/4))
```

Maple [B] time = 0.004, size = 603, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/3*i*x^3/b+1/2*h*x^2/b+g*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x+1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x+1)-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+1/4
*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/b*(1/b*a)^
(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(
1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b
*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/
2)))-1/2/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+1/2*d/(a*b)^(1/2)*arctan
(x^2*(b/a)^(1/2))-1/8/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(
1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*a*i+1/8*e/
b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2
+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/4/b^2/(1/b*a)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(1/b*a)^(1/4)*x+1)*a*i+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/b*a)^(1/4)*x+1)-1/4/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x-1)*a*i+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*
x-1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09652, size = 759, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="gia
c")
```

```
[Out] -1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/b^4 - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/b^4) - 1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + sqrt(2)*(a*b^3)^(3/4)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4) + 1/4*f*log(abs(b*x^4 + a))/b + 1/
4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4
)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*
b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g
 + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*
b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sq
rt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/6*(2*b^2*i*x^3 + 3*b^2*h*
x^2 + 6*b^2*g*x)/b^3
```

3.191 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4} dx$

Optimal. Leaf size=402

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be\right.}{4\sqrt{2}a^{3/4}b^{7/4}}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sq
rt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*
g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e -
 a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)
```

Rubi [A] time = 0.566919, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {1885, 1887, 1168, 1162, 617, 204, 1165, 628, 1819, 1810, 635, 205, 260}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be\right.}{4\sqrt{2}a^{3/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),x
]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sq
rt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*
g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e -
 a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+191x^6+jx^7}{a+bx^4}\,dx = \int \left(\frac{c+ex^2+gx^4+191x^6}{a+bx^4}+\frac{x\left(d+fx^2+hx^4+jx^6\right)}{a+bx^4}\right)dx$$

$$= \int \frac{c+ex^2+gx^4+191x^6}{a+bx^4}\,dx+\int \frac{x\left(d+fx^2+hx^4+jx^6\right)}{a+bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2+jx^3}{a+bx^2}\,dx,x,x^2\right)+\int \left(\frac{g}{b}+\frac{191x^2}{b}+\right.$$

$$= \frac{gx}{b}+\frac{191x^3}{3b}+\frac{1}{2}\text{Subst}\left(\int \left(\frac{h}{b}+\frac{jx}{b}+\frac{bd-ah+(bf-aj)x}{b\left(a+bx^2\right)}\right)d\right.$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{191x^3}{3b}+\frac{jx^4}{4b}+\frac{\text{Subst}\left(\int \frac{bd-ah+(bf-aj)x}{a+bx^2}\,dx,x,x^2\right)}{2b}-$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{191x^3}{3b}+\frac{jx^4}{4b}-\frac{\left(191a-be-\frac{\sqrt{b}(bc-ag)}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}}}{4b^2}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{191x^3}{3b}+\frac{jx^4}{4b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(191a-b}{}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{191x^3}{3b}+\frac{jx^4}{4b}+\frac{(bd-ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}+\frac{(191a-b}{}$$

Mathematica [A] time = 0.308731, size = 445, normalized size = 1.11

$$\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g-\sqrt{2}b^{3/2}c\right)}{a^{3/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(2a^{5/4}\sqrt[4]{b}h-\sqrt{2}a^{3/2}i-2\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be-\sqrt{2}a\sqrt{b}g+\right.}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4),x]
```

```
[Out] (24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*j*x^4 + (6
*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2
]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*
d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[
2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]
*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]
*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
```

```
qrt[b]*x^2])/a^(3/4) + (6*(b*f - a*j)*Log[a + b*x^4])/b^(1/4))/(24*b^(7/4))
```

Maple [B] time = 0.006, size = 627, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/4*j*x^4/b+1/3*i*x^3/b+1/2*h*x^2/b+g*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(1/b*a)^(1/4)*x+1)-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4
)*x-1)*g+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/
8/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x
^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*
ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)
+(1/b*a)^(1/2)))-1/2/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*a*h+1/2*d/(a*b)^
(1/2)*arctan(x^2*(b/a)^(1/2))-1/4/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x+1)*a*i+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)
^(1/4)*x+1)-1/4/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
*a*i+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/b^
2/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2
+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*a*i+1/8*e/b/(1/b*a)^(1/4)*2^(1/2)*
ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)
+(1/b*a)^(1/2)))-1/4/b^2*ln(b*x^4+a)*a*j+1/4*f*ln(b*x^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0966, size = 780, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorith
m="giac")
```

```
[Out] -1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/b^4 - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/b^4) - 1/8*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + sqrt(2)*(a*b^3)^(3/4)*log(x^2 -
 sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4) + 1/4*(b*f - a*j)*log(abs(b*x^4 +
a))/b^2 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h +
(a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*(sqrt(2
)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h + (a*b^3)^(1/4)*b^2*c - (a*b^3)
^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)
*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b
^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4
)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/12*(3*b^3*j*x
^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

3.192 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^2} \, dx$

Optimal. Leaf size=184

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+a\right.}{4ab\left(\right.}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (
(3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b
^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/
2)*b^(3/2))
```

Rubi [A] time = 0.203857, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{8a^{7/4}b^{5/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd)+a\right.}{4ab\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (
(3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(8*a^(7/4)*b
^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/
(8*a^(7/4)*b^(5/4)) + ((b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/
2)*b^(3/2))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-2b(bd-ah)x-b^2ex^2}{a-bx^4}\,dx}{4ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4}+\frac{-b(3bc-ag)-b^2ex^2}{a-bx^4}\right) dx}{4ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-b^2ex^2}{a-bx^4}\,dx}{4ab^2} + \frac{(bd-ah)\int}{2ab}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,d}{8a^{3/2}\sqrt{b}}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}}$$

Mathematica [A] time = 0.163229, size = 257, normalized size = 1.4

$$\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}h-\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd+a\sqrt[4]{b}g-3b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}h+\sqrt{a}b^{3/4}e-2\sqrt[4]{a}bd-a\sqrt[4]{b}g+\right.}{16a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*x
^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/
4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + 2*a
^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[a]
*b^(3/4)*e - a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4
)*(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(3/2))
```

Maple [B] time = 0.008, size = 340, normalized size = 1.9

$$\frac{1}{bx^4-a}\left(-\frac{ex^3}{4a}-\frac{(ah+bd)\,x^2}{4ab}-\frac{(ag+bc)\,x}{4ab}-\frac{f}{4b}\right)-\frac{g}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3c}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{g}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4/a*e*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/(b*x^4-a)-
1/8/b/a*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+3/8*c/a^2*(1/b*a)^(1/4)*arc
tan(x/(1/b*a)^(1/4))-1/16/b/a*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)
^(1/4)))*g+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
+1/8/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-1/8*d/a/
(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/8*e/a/b/(1/b*a)
^(1/4)*arctan(x/(1/b*a)^(1/4))+1/16*e/a/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4)
)/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.07615, size = 574, normalized size = 3.12

$$-\frac{bx^3e+bdx^2+ahx^2+bcx+agx+af}{4\left(bx^4-a\right)ab}-\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}b^2d-2\sqrt{2}\sqrt{-ab}abh-3\left(-ab^3\right)^{\frac{1}{4}}b^2c+\left(-ab^3\right)^{\frac{1}{4}}abg-\left(-ab^3\right)\right.}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*a*b)
- 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3
*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) - 1/16*sqrt(2
)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/
4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^3)
^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x
*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*
c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(a^2*b^3)
```

3.193 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^2} dx$

Optimal. Leaf size=203

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd\right.}{}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x
^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3
*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.27428, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1858, 1876, 275, 208, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{x\left(x(ah+bd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*x
^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/
a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3
*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a*h)*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+193x^6}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)-2b(bd\cdots}{a-\cdots}}{4\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \left(-\frac{2b(bd-ah)x}{a-bx^4}+\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b(3bc-ag)+b(579\cdots}{a-bx^4}}{4ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\left(579a-be-\frac{\sqrt{b}(\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(193a+be)x^2+bfx^3\right)}{4ab\left(a-bx^4\right)} + \frac{\left(579a-be+\frac{\sqrt{b}(\cdots}{\cdots}\right.}{8a\cdots}$$

Mathematica [A] time = 0.221807, size = 302, normalized size = 1.49

$$\frac{\frac{4a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{a-bx^4}+\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}be+a\sqrt{b}g-3b^{3/2}c\right)+\log\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x
]
```

```
[Out] ((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))/
(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Arc
Tan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]*b*
e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*
x] + (3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(
5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^(1/4)*
(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))
```

Maple [B] time = 0.009, size = 409, normalized size = 2.

$$\frac{1}{bx^4-a}\left(-\frac{(ai+be)\,x^3}{4\,ab}-\frac{(ah+bd)\,x^2}{4\,ab}-\frac{\left(ag+bc\right)x}{4\,ab}-\frac{f}{4\,b}\right)-\frac{g}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{g}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*f/b)/
(b*x^4-a)-1/8/b/a*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+3/8*c/a^2*(1/b*a)
^(1/4)*arctan(x/(1/b*a)^(1/4))-1/16/b/a*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/
(x-(1/b*a)^(1/4)))*g+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*
a)^(1/4)))+1/8/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*
h-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+3/8/b^2
/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*i-1/8*e/a/b/(1/b*a)^(1/4)*arctan(x/(
1/b*a)^(1/4))-3/16/b^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4))
)*i+1/16*e/a/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09331, size = 848, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="
giac")
```

```
[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^
(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*
(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 1/
4*(a*i*x^3 + b*x^3*e + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a
)*a*b) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*
b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) - 1/16
*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b
^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(
-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sq
rt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/
4)*b^2*c - (-a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.194 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^2}\,dx$

Optimal. Leaf size=225

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a-bx^4\right)}{4b^2}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*
(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(
1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqr
t[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a
*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4]
)/(4*b^2)
```

Rubi [A] time = 0.310455, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1858, 1876, 1167, 205, 208, 1248, 635, 260}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{8a^{5/4}b^{7/4}}+\frac{(bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{j\log\left(a-bx^4\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^2
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b*
(a - b*x^4)) - ((b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(
1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*e + (Sqrt[b]*(3*b*c - a*g))/Sqr
t[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(8*a^(5/4)*b^(7/4)) + ((b*d - a
*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) + (j*Log[a - b*x^4]
)/(4*b^2)
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+194x^6+jx^7}{\left(a-bx^4\right)^2}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)} - \int \frac{-b}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int\Big(}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int \frac{-b}{}}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)} - \frac{\text{Sub}}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)} + \frac{\big(582}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(194a+be)x^2+(bf+aj)x^3\right)}{4ab\left(a-bx^4\right)} + \frac{\big(582}{}$$

Mathematica [A] time = 0.239555, size = 338, normalized size = 1.5

$$\frac{\frac{4\left(a^2j+ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{a\left(a-bx^4\right)}+\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-2\sqrt[4]{a}b^{5/4}d-\sqrt{a}be+a\sqrt{b}g-3b^{3/2}c\right)}{a^{7/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(2a^{5/4}\sqrt[4]{b}h-3a^{3/2}i-\right.}{a^{7/4}}}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^2,x]
```

```
[Out] ((4*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/
2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(7/4) + (b^(1/4)*(-3*b^(3/2)*c - 2*a^(
1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2
)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(1/4)*
b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*
Log[a^(1/4) + b^(1/4)*x])/a^(7/4) + (2*Sqrt[b]*(b*d - a*h)*Log[Sqrt[a] + Sq
rt[b]*x^2])/a^(3/2) + 4*j*Log[a - b*x^4])/(16*b^2)
```

Maple [B] time = 0.011, size = 431, normalized size = 1.9

$$\frac{1}{bx^4-a}\left(-\frac{(ai+be)\,x^3}{4\,ab}-\frac{(ah+bd)\,x^2}{4\,ab}-\frac{\left(ag+bc\right)x}{4\,ab}-\frac{aj+bf}{4\,b^2}\right)-\frac{g}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3\,c}{8\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i+b*e)/a/b*x^3-1/4*(a*h+b*d)/a/b*x^2-1/4*(a*g+b*c)/a/b*x-1/4*(a*j+
b*f)/b^2)/(b*x^4-a)-1/8/b/a*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+3/8*c/a
^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-1/16/b/a*(1/b*a)^(1/4)*ln((x+(1/b*
a)^(1/4))/(x-(1/b*a)^(1/4)))*g+3/16*c/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4)
)/(x-(1/b*a)^(1/4)))+1/8/b/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b
)^(1/2)))*h-1/8*d/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)
))+3/8/b^2/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*i-1/8*e/a/b/(1/b*a)^(1/4)*
arctan(x/(1/b*a)^(1/4))-3/16/b^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b
*a)^(1/4)))*i+1/16*e/a/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4
)))+1/4/b^2*j*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11957, size = 884, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algor
ithm="giac")
```

```
[Out] -3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^
(1/4))/(-a/b)^(1/4))/(a*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) - 3/32*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^4) + sqrt(2)*
(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^4)) + 1/
4*j*log(abs(b*x^4 - a))/b^2 - 1/4*((a*i + b*e)*x^3 + (b*d + a*h)*x^2 + (b*c
 + a*g)*x + (a*b*f + a^2*j)/b)/((b*x^4 - a)*a*b) - 1/16*sqrt(2)*(2*sqrt(2)*
sqrt(-a*b)*b^2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-
a*b^3)^(1/4)*a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^3) - 1/16*sqrt(2)*(2*sqrt(2)*sqrt(-a*b)*b^
2*d - 2*sqrt(2)*sqrt(-a*b)*a*b*h - 3*(-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(1/4)*
a*b*g - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1
/4)*a*b*g - (-a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a^2*b^3) - 1/32*sqrt(2)*(3*(-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(1/4)*a*b*g -
 (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^3)
```

3.195 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}dx$

Optimal. Leaf size=353

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}}{}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (
(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((3*b*c +
Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.340409, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3bc\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```



```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + (
(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((3*b*c +
Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(5/4)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c - Sqrt[a]*Sqrt[
b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^2}dx = \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+ah)x-b^2ex^2}{a+bx^4}dx}{4ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \left(-\frac{2b(bd+ah)x}{a+bx^4}+\frac{-b(3bc+ag)-b^2ex^2}{a+bx^4}\right)dx}{4ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-b^2ex^2}{a+bx^4}dx}{4ab^2} + \frac{(bd+ah)\int \frac{x}{a+b\ldots}}{2ab}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}dx}{8a^{3/2}b^{3/2}} + \ldots$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\left(3bc-\sqrt{a}\ldots\right.}{\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\left(3bc-\sqrt{a}\ldots\right.}{\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\left(3bc+\sqrt{a}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.241713, size = 359, normalized size = 1.02

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}e+4\sqrt[4]{a}bd+\sqrt{2}a\sqrt[4]{b}g+3\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4a^{5/4}h+\sqrt{2}\sqrt{a}b^{3/4}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a +
 b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*
e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e +
Sqrt[2]*a*b^(1/4)*g - 4*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
+ Sqrt[2]*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*b*c - Sqrt[a]*Sqrt[b]
*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/
4)*b^(3/2))
```

Maple [A] time = 0.009, size = 515, normalized size = 1.5

$$\frac{1}{bx^4+a}\left(\frac{ex^3}{4\,a}-\frac{(ah-bd)\,x^2}{4\,ab}-\frac{\left(ag-bc\right)x}{4\,ab}-\frac{f}{4\,b}\right)+\frac{\sqrt{2}g}{16\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\,c\sqrt{2}}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1
/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+3/16*c/a^
2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16/b/a*(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+3/16*c/a^2*(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/32/b/a*(1/b*a)^(1/4)*2^(1/2)*ln
((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(
1/b*a)^(1/2)))*g+3/32*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2
^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/b/(a
*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1
/2))+1/32*e/a/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*
a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16*e/a/b/(1/b*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16*e/a/b/(1/b*a)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10694, size = 537, normalized size = 1.52

$$\frac{bx^3e + bdx^2 - ahx^2 + bcx - agx - af}{4\left(bx^4 + a\right)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d + 2\sqrt{2}\sqrt{ab}abh + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan}{16\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) +
 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a
*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqr
t(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c +
(a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (
a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b
*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3
)
```

3.196 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=395

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{\ }\right.}{16\sqrt{2}a^{7/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x
^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((
Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a
]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (
(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rubi [A] time = 0.492387, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1858, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{\ }\right.}{16\sqrt{2}a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*x
^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/2)) - ((
Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a
]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (
(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
```

```
0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+196x^6}{\left(a+bx^4\right)^2}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-2b(bd+\cdots}{a+\cdots}}{4\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int\left(-\frac{2b(bd+ah)x}{a+bx^4}+\cdots\right.}{\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+ag)-b(588a\cdots}{a+bx^4}}{4ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} - \frac{\left(588a+be-\frac{\sqrt{b}(3\cdots}{\cdots}\right.}{8\cdots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\cdots\right.}{4a^{3/2}b^{3/2}}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\cdots\right.}{4a^{3/2}b^{3/2}}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(196a-be)x^2+bfx^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah)\tan^{-1}\left(\cdots\right.}{4a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.344446, size = 415, normalized size = 1.05

$$\frac{-\frac{8a^{3/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x
]
```

```
[Out] ((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x))
)))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sq
rt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*a^(1
/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*b^(1/
4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[
2]*(-3*b^(3/2)*c + Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b^(3/2)*c - Sqrt[a]*b*
e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(32*a^(7/4)*b^(7/4))
```

Maple [B] time = 0.01, size = 654, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/
(b*x^4+a)+1/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*
g+3/16*c/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16/b
/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+3/16*c/a^2*(1/
b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/32/b/a*(1/b*a)^(1/4)
*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2)))*g+3/32*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a
)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
))+1/4/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+1/4*d/a/(a*b)^(1/2)*arctan(x
^2*(b/a)^(1/2))+3/32/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1
/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*i+1/32*e/a/
b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2
+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x+1)*i+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x+1)+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b
*a)^(1/4)*x-1)*i+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1
/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09558, size = 795, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="g
iac")
```

```
[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^4)) + 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3
/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)) - 1/4*(a*i*x^3 -
b*x^3*e - b*d*x^2 + a*h*x^2 - b*c*x + a*g*x + a*f)/((b*x^4 + a)*a*b) + 1/16
*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)
^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*
sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^
3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3
)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g -
(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.197 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=417

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3\right.}{16\sqrt{2}a^{7/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*
(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/
2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g)
+ Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7
/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[
a + b*x^4])/(4*b^2)
```

Rubi [A] time = 0.53602, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1858, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3ai+be)\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(ag+3bc)-\sqrt{a}(3\right.}{16\sqrt{2}a^{7/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^2
,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b*
(a + b*x^4)) + ((b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(4*a^(3/2)*b^(3/
2)) - ((Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(7/4)) + ((Sqrt[b]*(3*b*c + a*g)
+ Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*b^(7/4)) - ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7
/4)) + ((Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(7/4)) + (j*Log[
a + b*x^4])/(4*b^2)
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+197x^6+jx^7}{\left(a+bx^4\right)^2}dx = \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+}{}}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int\left(\frac{-b(3bc}{}\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-b(3bc+}{}}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} + \frac{\Big(591a+}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah}{4}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(197a-be)x^2+(bf-aj)x^3\right)}{4ab\left(a+bx^4\right)} + \frac{(bd+ah}{4}$$

Mathematica [A] time = 0.328068, size = 460, normalized size = 1.1

$$\frac{8\left(a^2j-ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{a\left(a+bx^4\right)} - \frac{2\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+3\sqrt{2}b^{3/2}c\right)}{a^{7/4}} + \frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^2,x]
```

```
[Out] ((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a*(
a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[
2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3
/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3*Sqr
t[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt
[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*b^(1/4)*(-3*b^(3/2)*c + Sqrt[a]*b*e - a
*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
```

```
*x^2])/a^(7/4) + (Sqrt[2]*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g
- 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7
/4) + 8*j*Log[a + b*x^4])/(32*b^2)
```

Maple [B] time = 0.011, size = 675, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x+1/4*(a*j-
b*f)/b^2)/(b*x^4+a)+1/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x-1)*g+3/16*c/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
-1)+1/32/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^
(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+3/32*c/a^2*(1/b*a)^(1
/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/
4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x+1)*g+3/16*c/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*x+1)+1/4/b/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+1/4*d/a/(a*b)^(1/2
)*arctan(x^2*(b/a)^(1/2))+3/32/b^2/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1
/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*i
+1/32*e/a/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(
1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16/b^2/(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*i+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+3/16/b^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x+1)*i+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x+1)+1/4*j*ln(b*x^4+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.11231, size = 833, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algori
thm="giac")
```

```
[Out] 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(a*b^4)) + 3/32*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + sqrt(2)*(a*b^3)^(3
/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)) + 1/4*j*log(abs(b
*x^4 + a))/b^2 - 1/4*((a*i - b*e)*x^3 - (b*d - a*h)*x^2 - (b*c - a*g)*x + (
a*b*f - a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2
*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*
g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(
a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/
32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a
*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2
)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.198 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=241

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(2x(}{}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) +
 (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b
*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.338917, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{64a^{11/4}b^{5/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(2x(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) +
 (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a - b
*x^4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)
])/(64*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(64*a^(11/4)*b^(5/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc-ag)-2b(3bd-ah)x-5b^2ex^2-4b^2fx^3}{\left(a-bx^4\right)^2}\,dx}{8ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-ah)x+5\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-ah)x+5\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-ah)x+5\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-ah)x+5\right.}{32a^2b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+2(3bd-ah)x+5\right.}{32a^2b\left(a-bx^4\right)}$$

Mathematica [A] time = 0.28654, size = 309, normalized size = 1.28

$$\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}h-5\sqrt{a}b^{3/4}e-12\sqrt[4]{a}bd+3a\sqrt[4]{b}g-21b^{5/4}c\right)+\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(4a^{5/4}h+5\sqrt{a}b^{3/4}e-12\sqrt[4]{a}bd-3a\sqrt[4]{\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) - a*(g + 2*h*x)))/(a - b*x
^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a
 - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1
/4)*x)/a^(1/4)] + (-21*b^(5/4)*c - 12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e + 3
*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c - 12*a
^(1/4)*b*d + 5*Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4)
 + b^(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a
^(11/4)*b^(3/2))
```

Maple [A] time = 0.011, size = 389, normalized size = 1.6

$$-\frac{1}{\left(bx^4-a\right)^2}\left(\frac{5\,bex^7}{32\,a^2}-\frac{(ah-3\,bd)\,x^6}{16\,a^2}-\frac{\left(ag-7\,bc\right)x^5}{32\,a^2}-\frac{9\,ex^3}{32\,a}-\frac{(ah+5\,bd)\,x^2}{16\,ab}-\frac{\left(3\,ag+11\,bc\right)x}{32\,ab}-\frac{f}{8\,b}\right)-\frac{3\,g}{64\,ba^2}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(5/32/a^2*b*e*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2*x^5-9/32/a
*e*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/a/b*x-1/8*f/b)/(b*x^4-a
)^2-3/64/b/a^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+21/64*c/a^3*(1/b*a)^
(1/4)*arctan(x/(1/b*a)^(1/4))-3/128/b/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4)
)/(x-(1/b*a)^(1/4)))*g+21/128*c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(
1/b*a)^(1/4)))+1/32/b/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(
1/2)))*h-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)
))-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+5/128*e/a^2/b/(1/b*a)
^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12877, size = 656, normalized size = 2.72

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{-ab}b^2d-4\sqrt{2}\sqrt{-ab}abh+21\left(-ab^3\right)^{\frac{1}{4}}b^2c-3\left(-ab^3\right)^{\frac{1}{4}}abg+5\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}+\frac{\sqrt{2}(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h + 2
1*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/128*
sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h + 21*(-a*
b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2
)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*l
og(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21
*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e)*log(x^
2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/32*(5*b^2*x^7*e + 6*
b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*x^3*e - 10*a*b*d*
x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.199 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$

Optimal. Leaf size=268

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(2\cdots}{\cdots}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x
^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2)
)/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*
a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4
)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(
3/2))
```

Rubi [A] time = 0.434678, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\frac{(3bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x(2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*x
^4)^2) + (4*a*f + x*(7*b*c - a*g + 2*(3*b*d - a*h)*x + (5*b*e - 3*a*i)*x^2)
)/(32*a^2*b*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*
a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((5*b*e + (3*Sqrt[
b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4
)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(
3/2))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+199x^6}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc-ag)-2b(3bd-ah\ldots}{(a\ldots}}{8\ldots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+\ldots\right.}{3\ldots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+\ldots\right.}{3\ldots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+\ldots\right.}{3\ldots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+\ldots\right.}{3\ldots}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(199a+be)x^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4af+x\left(7bc-ag+\ldots\right.}{3\ldots}$$

Mathematica [A] time = 0.352227, size = 359, normalized size = 1.34

$$\frac{16a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{\left(a-bx^4\right)^2}-\frac{4a^{3/4}b^{3/4}x(a(g+x(2h+3ix))-b(7c+x(6d+5ex)))}{a-bx^4}+\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h+3a^{3/2}i-12\sqrt[4]{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x
]
```

```
[Out] ((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(2*h + 3*i*
x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(
g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*a*S
qrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c - 12*a
^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*
a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d
+ 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^
(1/4) + b^(1/4)*x] - 4*a^(1/4)*b^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]
*x^2])/(128*a^(11/4)*b^(7/4))
```

Maple [B] time = 0.011, size = 472, normalized size = 1.8

$$-\frac{1}{\left(bx^4-a\right)^2}\left(-\frac{(3\,ai-5\,be)\,x^7}{32\,a^2}-\frac{(ah-3\,bd)\,x^6}{16\,a^2}-\frac{\left(ag-7\,bc\right)x^5}{32\,a^2}-\frac{(ai+9\,be)\,x^3}{32\,ab}-\frac{(ah+5\,bd)\,x^2}{16\,ab}-\frac{\left(3\,ag+11\,bc\right)x}{32\,ab}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2
*x^5-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*a*g+11*b*c)/
a/b*x-1/8*f/b)/(b*x^4-a)^2-3/64/b/a^2*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))
*g+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-3/128/b/a^2*(1/b*a)^(1
/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+21/128*c/a^3*(1/b*a)^(1/4)*ln
((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/32/b/a/(a*b)^(1/2)*ln((-a+x^2*(a*b)
^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-3/32*d/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/
2))/(-a-x^2*(a*b)^(1/2)))+3/64/b^2/a/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*
i-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-3/128/b^2/a/(1/b*a)^(1
/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*i+5/128*e/a^2/b/(1/b*a)^(1/4)*l
n((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12048, size = 942, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="
giac")
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) - 3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) +
sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*
b^4)) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a
*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3)
+ 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h +
 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/25
6*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3
/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqr
t(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e
)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) + 1/32*(3*a*b*i*
x^7 - 5*b^2*x^7*e - 6*b^2*d*x^6 + 2*a*b*h*x^6 - 7*b^2*c*x^5 + a*b*g*x^5 + a
^2*i*x^3 + 9*a*b*x^3*e + 10*a*b*d*x^2 + 2*a^2*h*x^2 + 11*a*b*c*x + 3*a^2*g*
x + 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.200 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^3} dx$

Optimal. Leaf size=285

$$\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{32a^2b^2\left(a-bx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\tanh^{-1}(\cdots$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*
(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*
x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]
*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b
^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(
1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.391448, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1858, 1854, 1876, 275, 208, 1167, 205}

$$\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{32a^2b^2\left(a-bx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{64a^{9/4}b^{7/4}}+\tanh^{-1}(\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^3
,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b*
(a - b*x^4)^2) + (4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*
x + b*(5*b*e - 3*a*i)*x^2))/(32*a^2*b^2*(a - b*x^4)) - ((5*b*e - (3*Sqrt[b]
*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(64*a^(9/4)*b
^(7/4)) + ((5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(
1/4)*x)/a^(1/4)])/(64*a^(9/4)*b^(7/4)) + ((3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
```

```
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+200x^6+jx^7}{\left(a-bx^4\right)^3}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} - \frac{\int \frac{-b(7bc}{}}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4a(bf-}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4a(bf-}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4a(bf-}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4a(bf-}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(200a+be)x^2+(bf+aj)x^3\right)}{8ab\left(a-bx^4\right)^2} + \frac{4a(bf-}{}$$

Mathematica [A] time = 0.277334, size = 380, normalized size = 1.33

$$\frac{\frac{16a^{7/4}\left(a^2j+ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{\left(a-bx^4\right)^2}-\frac{4a^{3/4}\left(8a^2j+abx(g+x(2h+3ix))-b^2x(7c+x(6d+5ex))\right)}{a-bx^4}+\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(4a^{5/4}\sqrt[4]{b}h+3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^3,x]
```

```
[Out] ((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b
*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b^(3/2)*c - 5*Sq
rt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + b^(1
/4)*(-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g +
 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4)*(21*
b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4
)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*Sqrt[b]*(-3
*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^2)
```

Maple [B] time = 0.012, size = 488, normalized size = 1.7

$$-\frac{1}{\left(bx^4-a\right)^2}\left(-\frac{(3\,ai-5\,be)\,x^7}{32\,a^2}-\frac{(ah-3\,bd)\,x^6}{16\,a^2}-\frac{\left(ag-7\,bc\right)x^5}{32\,a^2}-\frac{jx^4}{4\,b}-\frac{(ai+9\,be)\,x^3}{32\,ab}-\frac{(ah+5\,bd)\,x^2}{16\,ab}-\frac{\left(3\,ag+11\,bc\right.}{32\,ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

```
[Out] -(-1/32*(3*a*i-5*b*e)/a^2*x^7-1/16*(a*h-3*b*d)/a^2*x^6-1/32*(a*g-7*b*c)/a^2
*x^5-1/4*j*x^4/b-1/32*(a*i+9*b*e)/a/b*x^3-1/16*(a*h+5*b*d)/a/b*x^2-1/32*(3*
a*g+11*b*c)/a/b*x+1/8*(a*j-b*f)/b^2)/(b*x^4-a)^2-3/64/b/a^2*(1/b*a)^(1/4)*a
rctan(x/(1/b*a)^(1/4))*g+21/64*c/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-
3/128/b/a^2*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+21/128*
c/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/32/b/a/(a*b)^
(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-3/32*d/a^2/(a*b)^(1/2
)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+3/64/b^2/a/(1/b*a)^(1/4)*ar
ctan(x/(1/b*a)^(1/4))*i-5/64*e/a^2/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-
3/128/b^2/a/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*i+5/128*e
/a^2/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12438, size = 986, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algor
ithm="giac")
```

```
[Out] -3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)
^(1/4))/(-a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*b^4)) - 3/256*i*(2*sqrt(2)*(-a*b^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^4) +
sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^2*
b^4)) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a
*b*h + 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3)
+ 1/128*sqrt(2)*(12*sqrt(2)*sqrt(-a*b)*b^2*d - 4*sqrt(2)*sqrt(-a*b)*a*b*h +
 21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g + 5*(-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^3) + 1/25
6*sqrt(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3
/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) - 1/256*sqr
t(2)*(21*(-a*b^3)^(1/4)*b^2*c - 3*(-a*b^3)^(1/4)*a*b*g - 5*(-a*b^3)^(3/4)*e
)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^3) + 1/32*(3*a*b^2*
i*x^7 - 5*b^3*x^7*e - 6*b^3*d*x^6 + 2*a*b^2*h*x^6 - 7*b^3*c*x^5 + a*b^2*g*x
^5 + 8*a^2*b*j*x^4 + a^2*b*i*x^3 + 9*a*b^2*x^3*e + 10*a*b^2*d*x^2 + 2*a^2*b
*h*x^2 + 11*a*b^2*c*x + 3*a^2*b*g*x + 4*a^2*b*f - 4*a^3*j)/((b*x^4 - a)^2*a
^2*b^2)
```

3.201 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^3} dx$

Optimal. Leaf size=413

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) -
 (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b
*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
 - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3
*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)
) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt
[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.48642, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{128\sqrt{2}a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) -
 (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + 5*b*e*x^2))/(32*a^2*b*(a + b
*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2))
 - ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3
*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(5/4)
) - ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + ((21*b*c - 5*Sqrt
[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(128*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^3}dx = \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah)x-5b^2ex^2-4b^2fx^3}{\left(a+bx^4\right)^2}}{8ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+2(3bd+ah)x\right.}{32a^2b\left(a+bx^4\right)}$$

Mathematica [A] time = 0.364447, size = 411, normalized size = 1.

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}h+5\sqrt{2}\sqrt{a}b^{3/4}e+24\sqrt[4]{a}bd+3\sqrt{2}a\sqrt[4]{b}g+21\sqrt{2}b^{5/4}c\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8a^{5/4}h+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x)))/(a + b*x
^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))
/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(5/4)*c + 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqrt[
a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g + 8*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b^(5/4)*c - 24*a^(1/4)*b*d + 5*Sqrt[2]*Sqr
t[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g - 8*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b
```

```
^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(3/2))
```

Maple [A] time = 0.012, size = 561, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (5/32/a^2*b*e*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*
e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)
^2+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+21
/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b/
a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+21/128*c/a^3*
(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+3/256/b/a^2*(1/b*a)
^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^
(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+21/256*c/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^
2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*
a)^(1/2)))+1/16/b/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+3/16*d/a^2/(a*b)^
(1/2)*arctan(x^2*(b/a)^(1/2))+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(
1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^
(1/2)))+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+
1)+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10831, size = 620, normalized size = 1.5

$$\frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+4\,\sqrt{2}\sqrt{ab}abh+21\,\left(ab^3\right)^{\frac{1}{4}}b^2c+3\,\left(ab^3\right)^{\frac{1}{4}}abg+5\,\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}+\frac{\sqrt{2}\left(12\,\sqrt{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*
(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)
*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^
(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a*b*h*x^6
+ 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*x^3*e + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a
*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.202 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3} dx$

Optimal. Leaf size=463

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{}\right.}{128\sqrt{2}a^{11/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x
^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2)
)/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3
*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) -
 Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]
*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi [A] time = 0.685778, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{a}(3ai+5be)\right)}{128\sqrt{2}a^{11/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\sqrt{}\right.}{128\sqrt{2}a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```



```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*x
^4)^2) - (4*a*f - x*(7*b*c + a*g + 2*(3*b*d + a*h)*x + (5*b*e + 3*a*i)*x^2)
)/(32*a^2*b*(a + b*x^4)) + ((3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(1
6*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3
*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) - ((3*Sqrt[b]*(7*b*c + a*g) -
 Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]
*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
128*Sqrt[2]*a^(11/4)*b^(7/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+202x^6}{\left(a+bx^4\right)^3}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-b(7bc+ag)-2b(3bd+ah}{(a+}}{8}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(202a-be)x^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2} - \frac{4af-x\left(7bc+ag+\right.}{32}$$

Mathematica [A] time = 0.530798, size = 473, normalized size = 1.02

$$-\frac{32a^{7/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{\left(a+bx^4\right)^2}+\frac{8a^{3/4}b^{3/4}x(ag+ax(2h+3ix)+7bc+bx(6d+5ex))}{a+bx^4}-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x
]
```

```
[Out] ((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h + 3*i*x))
)/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g
 + x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*b^(
5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*
```

```
h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sq
rt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*
a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b
]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] +
 Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(7/4))
```

Maple [A] time = 0.011, size = 716, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/
b*x-1/8*f/b)/(b*x^4+a)^2+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x+1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x+1)+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4
)*x-1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-
1)+3/256/b/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a
)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+21/256*c/a^3*(1/b*a
)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)
^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16/b/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2
))*h+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/256/b^2/a/(1/b*a)^(1/
4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4
)*x*2^(1/2)+(1/b*a)^(1/2)))*i+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(
1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^
(1/2)))+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)
*i+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/
128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*i+5/128*e
/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12375, size = 892, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="g
iac")
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a
*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128
*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^
3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*
sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3
*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b*i*x^7 + 5*b^2*x^7*e + 6*b^2*d*x^6 + 2*a
*b*h*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 - a^2*i*x^3 + 9*a*b*x^3*e + 10*a*b*d*x^2
 - 2*a^2*h*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.203 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=480

$$-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{32a^2b^2\left(a+bx^4\right)}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\right.}{128\sqrt{2}a^{11/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*
(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*
x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTa
n[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g)
+ Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i
))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) -
 ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqr
t[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi [A] time = 0.665761, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {1858, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{32a^2b^2\left(a+bx^4\right)}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(3\sqrt{b}(ag+7bc)-\right.}{128\sqrt{2}a^{11/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^3
,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b*
(a + b*x^4)^2) - (4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*
x + b*(5*b*e + 3*a*i)*x^2))/(32*a^2*b^2*(a + b*x^4)) + ((3*b*d + a*h)*ArcTa
n[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/2)*b^(3/2)) - ((3*Sqrt[b]*(7*b*c + a*g)
+ Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*b^(7/4)) + ((3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i
))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(7/4)) -
 ((3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4)) + ((3*Sqr
t[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(7/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + 203x^6 + jx^7}{\left(a + bx^4\right)^3} dx = \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{\int \frac{-b(}{}}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

$$= \frac{x\left(bc - ag + (bd - ah)x - (203a - be)x^2 + (bf - aj)x^3\right)}{8ab\left(a + bx^4\right)^2} - \frac{4a(bj}{}$$

Mathematica [A] time = 0.429171, size = 500, normalized size = 1.04

$$\frac{\frac{32a^{7/4}\left(a^2j - ab(f + x(g + x(h + ix))) + b^2x(c + x(d + ex))\right)}{\left(a + bx^4\right)^2} + \frac{8a^{3/4}\left(-8a^2j + abx(g + x(2h + 3ix)) + b^2x(7c + x(6d + 5ex))\right)}{a + bx^4} - 2\sqrt[4]{b}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8a^{5/4}\sqrt[4]{b}h\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^3,x]
```

```
[Out] ((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h +
 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b
*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*
c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g +
```

```
8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)] + 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2
]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(
3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b^(3
/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(21*b^(3/2)*c - 5*Sqrt[a
]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(256*a^(11/4)*b^2)
```

Maple [A] time = 0.013, size = 731, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x
^5-1/4*j*x^4/b-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*
g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^2+3/128/b/a^2*(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/128/b/a^2*(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)+3/256/b/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^
(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))
*g+21/256*c/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*
a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16/b/a/(a*b)^(1/2)
*arctan(x^2*(b/a)^(1/2))*h+3/16*d/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3
/256/b^2/a/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1
/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*i+5/256*e/a^2/b/(1/b*a)^(
1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1
/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/b*a)^(1/4)*x+1)*i+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/b*a)^(1/4)*x+1)+3/128/b^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*x-1)*i+5/128*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.11724, size = 936, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algori
thm="giac")
```

```
[Out] 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^2*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 3/256*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + sqrt(2)*(a
*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)) + 1/128
*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^
3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*
sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(1/4)*b^2*
c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)
*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3
*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a^3*b^3) + 1/32*(3*a*b^2*i*x^7 + 5*b^3*x^7*e + 6*b^3*d*x^6 + 2
*a*b^2*h*x^6 + 7*b^3*c*x^5 + a*b^2*g*x^5 - 8*a^2*b*j*x^4 - a^2*b*i*x^3 + 9*
a*b^2*x^3*e + 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2 + 11*a*b^2*c*x - 3*a^2*b*g*x -
 4*a^2*b*f - 4*a^3*j)/((b*x^4 + a)^2*a^2*b^2)
```

3.204 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4} dx$

Optimal. Leaf size=293

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x(2x(5}{}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3)
+ (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a -
b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e -
7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h
)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.430552, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\frac{x(2x(5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3)
+ (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a -
b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a - b*x^4)^2) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/
4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e -
7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(15/4)*b^(5/4)) + ((5*b*d - a*h
)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5bd-ah)x-9b^2ex^2-8b^2fx^3}{\left(a-bx^4\right)^3}\,dx}{12ab^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{8af+x\left(11bc-ag+2(5bd-ah)x+\right.}{96a^2b\left(a-bx^4\right)^2}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah)x+45\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah)x+45\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah)x+45\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah)x+45\right.}{384a^3b\left(a-bx^4\right)}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)+12(5bd-ah)x+45\right.}{384a^3b\left(a-bx^4\right)}$$

Mathematica [A] time = 0.39516, size = 360, normalized size = 1.23

$$-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(-8a^{5/4}h+15\sqrt{a}b^{3/4}e+40\sqrt[4]{a}bd-7a\sqrt[4]{b}g+77b^{5/4}c\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(8a^{5/4}h+15\sqrt{a}b^{3/4}e-40\sqrt[4]{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

```
[Out] ((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*b*e*x^2))/
(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) - a*(g + 2
*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f
 + x*(g + h*x))))/(a - b*x^4)^3 + 6*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e
- 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*d + 1
5*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x]
 + 3*(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g
+ 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d + a*h)*Log[Sqr
t[a] + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(3/2))
```

Maple [A] time = 0.013, size = 434, normalized size = 1.5

$$\frac{1}{\left(bx^4-a\right)^3}\left(-\frac{15\,b^2ex^{11}}{128\,a^3}+\frac{(ah-5\,bd)\,bx^{10}}{32\,a^3}+\frac{\left(7\,ag-77\,bc\right)bx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}-\frac{(ah-5\,bd)\,x^6}{12\,a^2}-\frac{\left(3\,ag-33\,bc\right)x^5}{64\,a^2}-\frac{113\,ex^3}{384\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (-15/128*e/a^3*b^2*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c)/a^3*
b*x^9+21/64/a^2*b*e*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a^2*(a*g-11*b*c)*x^5-
113/384/a*e*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*(7*a*g+51*b*c)/a/b*x-1/12*f
/b)/(b*x^4-a)^3-7/256/b/a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+77/256/
a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-7/512/b/a^3*(1/b*a)^(1/4)*ln((x
+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+77/512/a^4*c*(1/b*a)^(1/4)*ln((x+(1/b*
a)^(1/4))/(x-(1/b*a)^(1/4)))+1/64/b/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))
/(-a-x^2*(a*b)^(1/2)))*h-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a
-x^2*(a*b)^(1/2)))-15/256/a^3*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+15/
512/a^3*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.08415, size = 738, normalized size = 2.52

$$-\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}b^2d-8\sqrt{2}\sqrt{-ab}abh-77\left(-ab^3\right)^{\frac{1}{4}}b^2c+7\left(-ab^3\right)^{\frac{1}{4}}abg-15\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h -
77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) - 1/51
2*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*h - 77*(-
a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) + 1/1024*sq
rt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1024*sqrt(
2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/384*(45*b^3*x
^11*e + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 12
6*a*b^2*x^7*e - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^5 + 18*a^2
*b*g*x^5 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 153*a^2*b*c*x
 + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

3.205 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a-bx^4\right)^4}\,dx$

Optimal. Leaf size=331

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\ x(\ldots$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*
x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2)
)/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x +
3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g
))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^
(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^
(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.567183, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}+\ x(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b*
x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a*i)*x^2)
)/(384*a^3*b*(a - b*x^4)) + (8*a*f + x*(11*b*c - a*g + 2*(5*b*d - a*h)*x +
3*(3*b*e - a*i)*x^2))/(96*a^2*b*(a - b*x^4)^2) + (((7*Sqrt[b]*(11*b*c - a*g
))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^
(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^
(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((5*b*d - a*h)*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
```

```
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+205x^6}{\left(a-bx^4\right)^4}\,dx = \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11bc-ag)-2b(5}{}}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{8af+x\left(11bc-\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(205a+be)x^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11bc-ag)\right.}{}$$

Mathematica [A] time = 0.368419, size = 422, normalized size = 1.27

$$\frac{\frac{128a^3b^{3/4}(a(f+x(g+x(h+ix)))+bx(c+x(d+ex)))}{\left(a-bx^4\right)^3} - \frac{16a^2b^{3/4}x(a(g+x(2h+3ix))-b(11c+x(10d+9ex)))}{\left(a-bx^4\right)^2} + 3\sqrt[4]{a}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h+5a^{3/2}i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x
]
```

```
[Out] ((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i
*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(10*d + 9*e*x))) + a*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*b^(3/4)*(b*x*(c + x*(d + e*
x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/2)*c
 - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)
] + 3*a^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*
Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] - 3
*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[
b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqr
t[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^4*b^(7/4))
```

Maple [A] time = 0.015, size = 522, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c
)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a^2*(a*g
-11*b*c)*x^5-1/384*(5*a*i+113*b*e)/a/b*x^3-1/32*(a*h+11*b*d)/a/b*x^2-1/128*
(7*a*g+51*b*c)/a/b*x-1/12*f/b)/(b*x^4-a)^3-7/256/b/a^3*(1/b*a)^(1/4)*arctan
(x/(1/b*a)^(1/4))*g+77/256/a^4*c*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))-7/51
2/b/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*g+77/512/a^4*
c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/64/b/a^2/(a*b)^(1
/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h-5/64/a^3*d/(a*b)^(1/2)*
ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+5/256/b^2/a^2/(1/b*a)^(1/4)*a
rctan(x/(1/b*a)^(1/4))*i-15/256/a^3*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4
))-5/512/b^2/a^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))*i+15
/512/a^3*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10286, size = 1044, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="
giac")
```

```
[Out] -5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4)) - 5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4)
+ sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^
3*b^4)) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)
*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/
4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^
3) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*
h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) +
1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1
024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)
^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) + 1/384*
(15*a*b^2*i*x^11 - 45*b^3*x^11*e - 60*b^3*d*x^10 + 12*a*b^2*h*x^10 - 77*b^3
*c*x^9 + 7*a*b^2*g*x^9 - 42*a^2*b*i*x^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6
 - 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 - 18*a^2*b*g*x^5 - 5*a^3*i*x^3 - 113*a^
2*b*x^3*e - 132*a^2*b*d*x^2 - 12*a^3*h*x^2 - 153*a^2*b*c*x - 21*a^3*g*x - 3
2*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$

Optimal. Leaf size=349

$$\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{96a^2b^2\left(a-bx^4\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b
*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a
*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g
) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2
) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g)
)/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + (
(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```

Rubi [A] time = 0.523832, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1858, 1854, 1855, 1876, 275, 208, 1167, 205}

$$\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{96a^2b^2\left(a-bx^4\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)}{256a^{13/4}b^{7/4}}+\frac{\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^4
,x]
```


```
[Out] (x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*b
*(a - b*x^4)^3) + (x*(7*(11*b*c - a*g) + 12*(5*b*d - a*h)*x + 15*(3*b*e - a
*i)*x^2))/(384*a^3*b*(a - b*x^4)) + (4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g
) + 2*b*(5*b*d - a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(96*a^2*b^2*(a - b*x^4)^2
) + (((7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*(3*b*e - a*i))*ArcTan[(b^(1/4)
*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + ((15*b*e + (7*Sqrt[b]*(11*b*c - a*g)
)/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(256*a^(13/4)*b^(7/4)) + (
(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
```

```
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+206x^6+jx^7}{\left(a-bx^4\right)^4}dx = \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} - \frac{\int \frac{-b(11b}{}}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{4a(2bf}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc+ag+(bd+ah)x+(206a+be)x^2+(bf+aj)x^3\right)}{12ab\left(a-bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

Mathematica [A] time = 0.36775, size = 439, normalized size = 1.26

$$\frac{\frac{128a^3\left(a^2j+ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{\left(a-bx^4\right)^3} - \frac{16a^2\left(12a^2j+abx(g+x(2h+3ix))-b^2x(11c+x(10d+9ex))\right)}{\left(a-bx^4\right)^2} + 3\sqrt[4]{a}\sqrt[4]{b}\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(8a^{5/4}\sqrt[4]{b}h+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*
x^4)^4,x]
```

```
[Out] ((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x)))/
(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) + a*b*x*(
g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*(a^2*j + b^2*x*(c + x*(d +
e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*b^(1/4)*(
77*b^(3/2)*c - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4
)*x)/a^(1/4)] + 3*a^(1/4)*b^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 1
5*Sqrt[a]*b*e + 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1
/4) - b^(1/4)*x] + 3*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d +
 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h - 5*a^(3/2)*i)*Log[a^
(1/4) + b^(1/4)*x] - 24*Sqrt[a]*Sqrt[b]*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b
]*x^2])/(1536*a^4*b^2)
```

Maple [A] time = 0.015, size = 538, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i-3*b*e)/a^3*b*x^11+1/32*(a*h-5*b*d)/a^3*b*x^10+7/384*(a*g-11*b*c
)/a^3*b*x^9-7/64/a^2*(a*i-3*b*e)*x^7-1/12/a^2*(a*h-5*b*d)*x^6-3/64/a^2*(a*g
-11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i+113*b*e)/a/b*x^3-1/32*(a*h+11*b*d)/a/
b*x^2-1/128*(7*a*g+51*b*c)/a/b*x+1/24*(a*j-2*b*f)/b^2)/(b*x^4-a)^3-7/256/b/
a^3*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*g+77/256/a^4*c*(1/b*a)^(1/4)*arct
an(x/(1/b*a)^(1/4))-7/512/b/a^3*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*
a)^(1/4)))*g+77/512/a^4*c*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/
4)))+1/64/b/a^2/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))*h
-5/64/a^3*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))+5/256
/b^2/a^2/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))*i-15/256/a^3*e/b/(1/b*a)^(1/
4)*arctan(x/(1/b*a)^(1/4))-5/512/b^2/a^2/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))
/(x-(1/b*a)^(1/4)))*i+15/512/a^3*e/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-
(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11209, size = 1087, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algor
ithm="giac")
```

```
[Out] -5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(-a*b^3)^(3/4)*log(x^2 + sqrt(2)
*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^3*b^4)) - 5/1024*i*(2*sqrt(2)*(-a*b^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^3*b^4)
+ sqrt(2)*(-a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^
3*b^4)) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)
*a*b*h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/
4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^
3) - 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b^2*d - 8*sqrt(2)*sqrt(-a*b)*a*b*
h - 77*(-a*b^3)^(1/4)*b^2*c + 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)^(3/4)*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^4*b^3) +
1/1024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) - 1/1
024*sqrt(2)*(77*(-a*b^3)^(1/4)*b^2*c - 7*(-a*b^3)^(1/4)*a*b*g - 15*(-a*b^3)
^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a^4*b^3) + 1/384*
(15*a*b^3*i*x^11 - 45*b^4*x^11*e - 60*b^4*d*x^10 + 12*a*b^3*h*x^10 - 77*b^4
*c*x^9 + 7*a*b^3*g*x^9 - 42*a^2*b^2*i*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x
^6 - 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 - 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4
 - 5*a^3*b*i*x^3 - 113*a^2*b^2*x^3*e - 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 -
 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f + 16*a^4*j)/((b*x^4 - a)^3*a^3
*b^2)
```

3.207 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^4\right)^4} dx$

Optimal. Leaf size=462

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+7\right.}{512\sqrt{2}a^{15/4}b^{5/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3)
+ (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a +
b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a
]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) +
((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```

Rubi [A] time = 0.618692, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{512\sqrt{2}a^{15/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+7\right.}{512\sqrt{2}a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3)
+ (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 45*b*e*x^2))/(384*a^3*b*(a +
b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x + 9*b*e*x^2))/(96*a^
2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7
/2)*b^(3/2)) - ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(5/4)) + ((77*b*c + 15*Sqrt[a
]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*
a^(15/4)*b^(5/4)) - ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4)) +
((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(5/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx = \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+ah)x-9b^2ex^2-8b^2fx^3}{(a+bx^4)^3}}{12ab^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x\left(11bc+ag+2(5bd+ah)x\right.}{96a^2b\left(a+bx^4\right)^2}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

$$= \frac{x\left(bc-ag+(bd-ah)x+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12(5bd+ah)x+\right.}{384a^3b\left(a+bx^4\right)}$$

Mathematica [A] time = 0.461843, size = 461, normalized size = 1.

$$-6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16a^{5/4}h+15\sqrt{2}\sqrt{a}b^{3/4}e+80\sqrt[4]{a}bd+7\sqrt{2}a\sqrt[4]{b}g+77\sqrt{2}b^{5/4}c\right)+6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16a^{5/4}h\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*b*e*x^2))/
(a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) + a*(g + 2
*h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a
*(f + x*(g + h*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/4)*b
*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)*h)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80*a^(1/
4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a^(5/4)*
h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/4)*(77*b*c - 15
*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(3/2))
```

Maple [A] time = 0.014, size = 607, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*b
*x^9+21/64/a^2*b*e*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5+1
13/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/
b)/(b*x^4+a)^3+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x+1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
+1)+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+7
7/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/
b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/
(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+77/1024/a^4*c*(1/b*a)^(1/4)*
2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x
*2^(1/2)+(1/b*a)^(1/2)))+1/32/b/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+5
/32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)
*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2)))+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/b*a)^(1/4)*x+1)+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09323, size = 703, normalized size = 1.52

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+8\sqrt{2}\sqrt{ab}abh+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*
(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2
)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4
)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)
*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 1
2*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*x^7*e + 160*a*b^2
*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*x^3*
e + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)
/((b*x^4 + a)^3*a^3*b)
```

3.208 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\left(a+bx^4\right)^4} dx$

Optimal. Leaf size=516

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-\right.}{512\sqrt{2}a^{15/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*
x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2)
)/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x +
3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*S
qrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]
*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - (
(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```

Rubi [A] time = 0.850028, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-5\sqrt{a}(ai+3be)\right)}{512\sqrt{2}a^{15/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-\right.}{512\sqrt{2}a^{15/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b*
x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a*i)*x^2)
)/(384*a^3*b*(a + b*x^4)) - (8*a*f - x*(11*b*c + a*g + 2*(5*b*d + a*h)*x +
3*(3*b*e + a*i)*x^2))/(96*a^2*b*(a + b*x^4)^2) + ((5*b*d + a*h)*ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((7*Sqrt[b]*(11*b*c + a*g) + 5*S
qrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]
*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) - (
(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt
[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(7/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
```

```
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+208x^6}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-b(11bc+ag)-2b(5bd+a\ldots}{(a\ldots}}{1\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{8af-x\left(11bc+ag+\ldots\right.}{9\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(208a-be)x^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11bc+ag)+12\ldots\right.}{38\ldots}$$

Mathematica [A] time = 0.702842, size = 530, normalized size = 1.03

$$\frac{-\frac{256a^{11/4}b^{3/4}(a(f+x(g+x(h+ix)))-bx(c+x(d+ex)))}{(a+bx^4)^3}+\frac{32a^{7/4}b^{3/4}x(ag+ax(2h+3ix)+11bc+bx(10d+9ex))}{(a+bx^4)^2}+\frac{8a^{3/4}b^{3/4}x(7ag+3ax(4h+5ix)+77bc+15bx(4d+3e}{a+bx^4}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x
]
```

```
[Out] ((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(2*h + 3*i*
x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3
*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (256*a^(11/4)*b^(3/4)*(-(b*x*(c
 + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*(77*Sqrt
[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a
*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)*d + 15
*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h + 5*Sqr
t[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*(-77*b^
(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*(77*b^(3/2)*c - 15*Sqrt[a]*
b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(3072*a^(15/4)*b^(7/4))
```

Maple [A] time = 0.014, size = 767, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g
+11*b*c)*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*
(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)*g+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(1/b*a)^(1/4)*x-1)+7/1024/b/a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/
4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*g+
77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)
^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/32/b/a^2/(a*b)^(1/2)
*arctan(x^2*(b/a)^(1/2))*h+5/32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+5
/512/b^2/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*i+15/5
12/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+5/512/b^
2/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*i+15/512/a^3*
e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+5/1024/b^2/a^2/
(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(
1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*i+15/1024/a^3*e/b/(1/b*a)^(1/4)*2^(1
/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(
1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10734, size = 992, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="g
iac")
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*
(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 1/5
12*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*
b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(
40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b
^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
```

```
- sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^
(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^
2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^2*i*x^11 + 45*b^3*x^11*e + 60
*b^3*d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 42*a^2*b*i*x
^7 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 +
 18*a^2*b*g*x^5 - 5*a^3*i*x^3 + 113*a^2*b*x^3*e + 132*a^2*b*d*x^2 - 12*a^3*
h*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.209 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=534

$$-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{96a^2b^2\left(a+bx^4\right)^2}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-\right.}{512\sqrt{2}a^{15/4}b^{7/4}}$$

```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b
*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a
*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g
) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2
) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((
7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*
g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e
+ a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2
]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(7/4))
```

Rubi [A] time = 0.824189, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1858, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{96a^2b^2\left(a+bx^4\right)^2}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(7\sqrt{b}(ag+11bc)-\right.}{512\sqrt{2}a^{15/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^4
,x]
```


```
[Out] (x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*b
*(a + b*x^4)^3) + (x*(7*(11*b*c + a*g) + 12*(5*b*d + a*h)*x + 15*(3*b*e + a
*i)*x^2))/(384*a^3*b*(a + b*x^4)) - (4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g
) + 2*b*(5*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(96*a^2*b^2*(a + b*x^4)^2
) + ((5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*b^(3/2)) - ((
7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*
g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(7/4)) - ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e
+ a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2
]*a^(15/4)*b^(7/4)) + ((7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15
/4)*b^(7/4))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+209x^6+jx^7}{\left(a+bx^4\right)^4}\,dx = \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-b(11bc}{}}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} - \frac{4a(2bf}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

$$= \frac{x\left(bc-ag+(bd-ah)x-(209a-be)x^2+(bf-aj)x^3\right)}{12ab\left(a+bx^4\right)^3} + \frac{x\left(7(11b\right.}{}$$

Mathematica [A] time = 0.553262, size = 555, normalized size = 1.04

$$\frac{\frac{256a^{11/4}\left(a^2j-ab(f+x(g+x(h+ix)))+b^2x(c+x(d+ex))\right)}{\left(a+bx^4\right)^3}-\frac{32a^{7/4}\left(12a^2j-abx(g+x(2h+3ix))-b^2x(11c+x(10d+9ex))\right)}{\left(a+bx^4\right)^2}-6\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16a^{5/4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*
x^4)^4,x]
```

```
[Out] ((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x
)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) -
 a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4)^2 + (256*a^(11/4)*(a^2*j + b^2*x
*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*b^(1
/4)*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e +
 7*Sqrt[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c - 80*a^
(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/
4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^
(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]
*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^2)
```

Maple [A] time = 0.014, size = 783, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c
)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g
+11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/
b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+7/512/b/
a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*g+77/512/a^4*c*
(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*(1/b*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*g+77/512/a^4*c*(1/b*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+7/1024/b/a^3*(1/b*a)^(1/4)*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))*g+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)
^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))
)+1/32/b/a^2/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))*h+5/32/a^3*d/(a*b)^(1/2)*a
rctan(x^2*(b/a)^(1/2))+5/512/b^2/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x+1)*i+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x+1)+5/512/b^2/a^2/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^
(1/4)*x-1)*i+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1
/4)*x-1)+5/1024/b^2/a^2/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/
2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*i+15/1024/a^
3*e/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/
(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x
)
```

```
[Out] Timed out
```

Giac [A] time = 1.10909, size = 1035, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algori
thm="giac")
```

```
[Out] 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a^3*b^4) - sqrt(2)*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 5/1024*i*(2*sqrt(2)*(a*b^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + sqrt(2)*
(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)) + 1/5
12*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*
b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(
40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^(1/4)*b
```

```
^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^
(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^
2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(15*a*b^3*i*x^11 + 45*b^4*x^11*e + 60
*b^4*d*x^10 + 12*a*b^3*h*x^10 + 77*b^4*c*x^9 + 7*a*b^3*g*x^9 + 42*a^2*b^2*i
*x^7 + 126*a*b^3*x^7*e + 160*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x
^5 + 18*a^2*b^2*g*x^5 - 48*a^3*b*j*x^4 - 5*a^3*b*i*x^3 + 113*a^2*b^2*x^3*e
+ 132*a^2*b^2*d*x^2 - 12*a^3*b*h*x^2 + 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*
a^3*b*f - 16*a^4*j)/((b*x^4 + a)^3*a^3*b^2)
```

3.210 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.0678343, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1885, 220, 275, 217, 206}

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{a+bx^4}}\,dx+d\int \frac{x}{\sqrt{a+bx^4}}\,dx\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{a+bx^4}}\right)\\
&= \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0469559, size = 79, normalized size = 0.65

$$\frac{cx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] time = 0.073, size = 96, normalized size = 0.8

$$\frac{d}{2}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+c\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^4 + a), x)
```

Sympy [C] time = 1.84028, size = 61, normalized size = 0.5

$$\frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

3.211 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 -
 (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b
*x^4])
```

Rubi [A] time = 0.0621967, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1885, 224, 221, 275, 217, 203}

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1 -
 (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b
*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{a-bx^4}}+\frac{dx}{\sqrt{a-bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{a-bx^4}}\,dx+d\int \frac{x}{\sqrt{a-bx^4}}\,dx\\
&= \frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,x^2\right)+\frac{\left(c\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{a-bx^4}}\\
&= \frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{1}{2}d\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x^2}{\sqrt{a-bx^4}}\right)\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0472358, size = 81, normalized size = 0.93

$$
\frac{cx\sqrt{1-\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{bx^4}{a}\right)}{\sqrt{a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x^
4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[a - b*x^4]
```

Maple [A] time = 0.015, size = 90, normalized size = 1.

$$
\frac{d}{2}\arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{b}}+c\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)+c/(1/a^(1/2)*b^(1/2))^(1
/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^
```

```
(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-bx^4+a}(dx+c)}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)*(d*x + c)/(b*x^4 - a), x)
```

Sympy [A] time = 1.96213, size = 97, normalized size = 1.11

$$d\left(\begin{cases} -\frac{i\,\mathrm{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \frac{|bx^4|}{|a|} > 1 \\ \frac{\mathrm{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
```

```
[Out] d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4)/Abs(a)
 > 1), (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x*gamma(1/4)*hyp
er((1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

3.212 $\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1
 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-a
+ b*x^4])
```

Rubi [A] time = 0.0599722, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1885, 224, 221, 275, 217, 206}

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1
 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-a
+ b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{-a+bx^4}}+\frac{dx}{\sqrt{-a+bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{-a+bx^4}}\,dx+d\int \frac{x}{\sqrt{-a+bx^4}}\,dx\\
&= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+bx^2}}\,dx,x,x^2\right)+\frac{\left(c\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{-a+bx^4}}\\
&= \frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}+\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{-a+bx^4}}\right)\\
&= \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0420012, size = 83, normalized size = 0.93

$$\frac{cx\sqrt{1-\frac{bx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{bx^4}{a}\right)}{\sqrt{bx^4-a}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[-a + b*x^4]
```

Maple [A] time = 0.019, size = 95, normalized size = 1.1

$$\frac{d}{2}\ln\left(x^2\sqrt{b}+\sqrt{bx^4-a}\right)\frac{1}{\sqrt{b}}+c\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{-\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^4-a)^(1/2),x)
```

```
[Out] 1/2*d*ln(x^2*b^(1/2)+(b*x^4-a)^(1/2))/b^(1/2)+c/(-1/a^(1/2)*b^(1/2))^(1/2)*
(1+x^2*b^(1/2)/a^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)/(b*x^4-a)^(1/2)
```

```
*EllipticF(x*(-1/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx+c}{\sqrt{bx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/sqrt(b*x^4 - a), x)
```

Sympy [A] time = 1.96492, size = 92, normalized size = 1.03

$$d\left(\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for}\ \frac{|bx^4|}{|a|}>1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) - \frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

```
[Out] d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4)/Abs(a) >
1), (-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c*x*gamma(1/4)*h
yper((1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

3.213 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$

Optimal. Leaf size=127

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])
```

Rubi [A] time = 0.0721643, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1885, 220, 275, 217, 203}

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx &= \int \left(\frac{c}{\sqrt{-a-bx^4}}+\frac{dx}{\sqrt{-a-bx^4}}\right)dx\\
&= c\int \frac{1}{\sqrt{-a-bx^4}}\,dx+d\int \frac{x}{\sqrt{-a-bx^4}}\,dx\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{\sqrt{-a-bx^2}}\,dx,x,x^2\right)\\
&= \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x^2}{\sqrt{-a-bx^4}}\right)\\
&= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.0451096, size = 85, normalized size = 0.67

$$\frac{cx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{-a-bx^4}}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/Sqrt[-a - b*x^4],x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[-a - b*x^4]
```

Maple [C] time = 0.018, size = 101, normalized size = 0.8

$$\frac{d}{2}\arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4-a}}\right)\frac{1}{\sqrt{b}}+c\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(-b*x^4-a)^(1/2),x)
```

```
[Out] 1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)+c/(-I/a^(1/2)*b^(1/2))^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(-b*x^
4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-bx^4-a}(dx+c)}{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 - a)*(d*x + c)/(b*x^4 + a), x)
```

Sympy [C] time = 1.96542, size = 66, normalized size = 0.52

$$-\frac{id\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}-\frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

```
[Out] -I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*hyper((1/4,
 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

3.214 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=257

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] +
 e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.12442, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\,\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] +
 e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{dx}{\sqrt{a+bx^4}}+\frac{c+ex^2}{\sqrt{a+bx^4}}\right)dx$$

$$= d\int \frac{x}{\sqrt{a+bx^4}}\,dx+\int \frac{c+ex^2}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\left(c+\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{\right.}}{}$$

$$= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+$$

Mathematica [C] time = 0.103442, size = 131, normalized size = 0.51

$$\frac{cx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

```
[Out] (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (e
```

```
*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3
*Sqrt[a + b*x^4])
```

Maple [C] time = 0.005, size = 193, normalized size = 0.8

$$ie\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}+\frac{d}{2}\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^2*b^
(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+dx+c}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Sympy [C] time = 2.15659, size = 102, normalized size = 0.4

$$\frac{d\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(3
/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/
4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.215 $\int \frac{ag-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=14

$$\frac{gx}{\sqrt{a+bx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

Rubi [A] time = 0.0055367, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {383}

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{ag-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{gx}{\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.0086365, size = 14, normalized size = 1.

$$\frac{gx}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^4]
```

Maple [A] time = 0.043, size = 13, normalized size = 0.9

$$gx\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x)
```

```
[Out] g*x/(b*x^4+a)^(1/2)
```

Maxima [A] time = 1.05881, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Fricas [A] time = 1.3868, size = 28, normalized size = 2.

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

Sympy [C] time = 7.55679, size = 80, normalized size = 5.71

$$\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Giac [A] time = 1.07768, size = 16, normalized size = 1.14

$$\frac{gx}{\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] g*x/sqrt(b*x^4 + a)
```

3.216 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

```
[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.0230818, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1856}

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Rule 1856

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+ex-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.0368414, size = 27, normalized size = 0.93

$$\frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])
```

Maple [A] time = 0.042, size = 24, normalized size = 0.8

$$\frac{x\left(2\,ag+ex\right)}{2\,a}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*x*(2*a*g+e*x)/(b*x^4+a)^(1/2)/a
```

Maxima [A] time = 1.07298, size = 34, normalized size = 1.17

$$\frac{2\,agx + ex^2}{2\,\sqrt{bx^4 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)
```

Fricas [A] time = 1.34993, size = 74, normalized size = 2.55

$$\frac{\sqrt{bx^4 + a}\left(2\,agx + ex^2\right)}{2\,\left(abx^4 + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

Sympy [C] time = 11.0955, size = 104, normalized size = 3.59

$$\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/
a))
```

Giac [A] time = 1.0737, size = 43, normalized size = 1.48

$$-\frac{x\left(\frac{2g}{a^2b^4} + \frac{xe}{a^3b^4}\right)}{64\,\sqrt{bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*x*(2*g/(a^2*b^4) + x*e/(a^3*b^4))/sqrt(b*x^4 + a)
```

3.217 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

```
[Out] -(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.0282155, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1856}

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] -(f - 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

Rule 1856

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x, 0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]] /; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.0366762, size = 27, normalized size = 1.08

$$\frac{2bgx-f}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

Maple [A] time = 0.043, size = 24, normalized size = 1.

$$\frac{2\,bgx-f}{2\,b}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

Maxima [A] time = 1.08228, size = 31, normalized size = 1.24

$$\frac{2\,bgx-f}{2\,\sqrt{bx^4+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

Fricas [A] time = 1.50489, size = 69, normalized size = 2.76

$$\frac{\sqrt{bx^4+a}\left(2\,bgx-f\right)}{2\left(b^2x^4+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

Sympy [A] time = 10.4613, size = 109, normalized size = 4.36

$$f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Giac [A] time = 1.08497, size = 30, normalized size = 1.2

$$\frac{2\,gx-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

3.218 $\int \frac{ag+ex+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

[Out] -(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])

Rubi [A] time = 0.0298362, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1856}

$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] -(a*f - 2*a*b*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])

Rule 1856

```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, -Simp[(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Rubi steps

$$\int \frac{ag+ex+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=-\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.0415902, size = 38, normalized size = 1.

$$\frac{2abgx-af+bex^2}{2ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]

[Out] (-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])

Maple [A] time = 0.042, size = 35, normalized size = 0.9

$$\frac{2\,abgx+bex^2-af}{2\,ab}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)
```

Maxima [A] time = 1.07834, size = 59, normalized size = 1.55

$$\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\,\left(ab^2x^4+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

Fricas [A] time = 1.30205, size = 93, normalized size = 2.45

$$\frac{\sqrt{bx^4+a}\left(2\,abgx+bex^2-af\right)}{2\,\left(ab^2x^4+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

Sympy [A] time = 13.7758, size = 133, normalized size = 3.5

$$f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a))
```

Giac [A] time = 1.09686, size = 42, normalized size = 1.11

$$\frac{\left(2\,g+\frac{xe}{a}\right)x-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*g + x*e/a)*x - f/b)/sqrt(b*x^4 + a)
```

3.219 $\int \frac{-1+x^4}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{x^4+1}}$$

[Out] -(x/Sqrt[1 + x^4])

Rubi [A] time = 0.0031383, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {383}

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4)/(1 + x^4)^(3/2),x]

[Out] -(x/Sqrt[1 + x^4])

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\left(1+x^4\right)^{3/2}}\,dx = -\frac{x}{\sqrt{1+x^4}}$$

Mathematica [A] time = 0.0050631, size = 12, normalized size = 1.

$$-\frac{x}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]

[Out] -(x/Sqrt[1 + x^4])

Maple [A] time = 0.004, size = 11, normalized size = 0.9

$$-x\frac{1}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)^(3/2),x)
```

```
[Out] -x/(x^4+1)^(1/2)
```

Maxima [A] time = 1.54444, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Fricas [A] time = 1.30916, size = 24, normalized size = 2.

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -x/sqrt(x^4 + 1)
```

Sympy [C] time = 2.61457, size = 58, normalized size = 4.83

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+1)**(3/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma
(5/4))
```

Giac [A] time = 1.06654, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -x/sqrt(x^4 + 1)
```

3.220 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+$$

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a + b
*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a +
b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*
e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.423748, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1885, 1819, 1815, 641, 217, 206, 1888, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{(2bd-ah)\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{x\sqrt{a+b}}{5b^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```


```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a + b
*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a +
b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(15*b*
e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```


Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+220x^6}{\sqrt{a+bx^4}}dx = \int \left(\frac{x(d+fx^2+hx^4)}{\sqrt{a+bx^4}}+\frac{c+ex^2+gx^4+220x^6}{\sqrt{a+bx^4}}\right)dx$$
$$= \int \frac{x(d+fx^2+hx^4)}{\sqrt{a+bx^4}}dx + \int \frac{c+ex^2+gx^4+220x^6}{\sqrt{a+bx^4}}dx$$
$$= \frac{44x^3\sqrt{a+bx^4}}{b} + \frac{1}{2}\text{Subst}\left(\int \frac{d+fx+hx^2}{\sqrt{a+bx^2}}dx, x, x^2\right) + \frac{\int \frac{5bc-5(132a-}{\sqrt{a}}}{5}$$
$$= \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a+bx^4}}{b} + \frac{\int \frac{5b(3bc-ag)-15b(132a-}{\sqrt{a+bx^4}}}{15b^2}$$
$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a+bx^4}}{b} + \frac{(\sqrt{a}(1}{}$$
$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a+bx^4}}{b} - \frac{(132a}{b^{3/2}}$$
$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{gx\sqrt{a+bx^4}}{3b} + \frac{hx^2\sqrt{a+bx^4}}{4b} + \frac{44x^3\sqrt{a+bx^4}}{b} - \frac{(132a}{b^{3/2}}$$

Mathematica [C] time = 0.223516, size = 281, normalized size = 0.73

$$\frac{-20\sqrt{b}x\sqrt{\frac{bx^4}{a}+1}(ag-3bc)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+30bd\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+4\sqrt{b}x^3\sqrt{\frac{bx^4}{a}+1}(5be-3ai)\,{}_2F_1\left(\frac{1}{2},\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4]
,x]
```

```
[Out] (30*a*Sqrt[b]*f + 20*a*Sqrt[b]*g*x + 15*a*Sqrt[b]*h*x^2 + 12*a*Sqrt[b]*i*x^
3 + 30*b^(3/2)*f*x^4 + 20*b^(3/2)*g*x^5 + 15*b^(3/2)*h*x^6 + 12*b^(3/2)*i*x
^7 + 30*b*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 15*a*h
*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*Sqrt[b]*(-3*b*
c + a*g)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a
)] + 4*Sqrt[b]*(5*b*e - 3*a*i)*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/
2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.017, size = 516, normalized size = 1.3

$$\frac{ix^3}{5b}\sqrt{bx^4+a} - \frac{3i}{5}ia^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{3i}{5}ia^{\frac{3}{2}}\sqrt{1-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/5*i*x^3*(b*x^4+a)^(1/2)/b-3/5*I*i/b^(3/2)*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*i/b^(3/2)*a^(3/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*h
*x^2*(b*x^4+a)^(1/2)/b-1/4*h*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*
g*x*(b*x^4+a)^(1/2)/b-1/3*g/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*(b*x^4+a)^(1/2)/b+I*e*a^(1/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
/b^(1/2)+c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a),
 x)
```

Sympy [A] time = 5.73435, size = 260, normalized size = 0.68

$$\frac{\sqrt{a}hx^2\sqrt{1 + \frac{bx^4}{a}}}{4b} - \frac{ah\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b = 0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*h
yper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
 e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(7/4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/2, 7/4)
, (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ix^6 + hx^5 + gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a)
, x)
```

3.221 $\int \frac{1+x}{1+x^5} dx$

Optimal. Leaf size=109

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left(x+(-1)^{2/5}\right)-\frac{1}{5}$$

```
[Out] -((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1/5) - x])/5 + ((-1)^(4/5)*(1 - (-1
)^(4/5))*Log[-(-1)^(4/5) - x])/5 + ((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2
/5) + x])/5 - ((-1)^(3/5)*(1 + (-1)^(3/5))*Log[-(-1)^(3/5) + x])/5
```

Rubi [A] time = 0.0641871, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1586, 2068}

$$-\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-x-(-1)^{4/5}\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left(x+(-1)^{2/5}\right)-\frac{1}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(1 + x^5),x]
```

```
[Out] -((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1/5) - x])/5 + ((-1)^(4/5)*(1 - (-1
)^(4/5))*Log[-(-1)^(4/5) - x])/5 + ((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2
/5) + x])/5 - ((-1)^(3/5)*(1 + (-1)^(3/5))*Log[-(-1)^(3/5) + x])/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2068

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{1+x^5} dx &= \int \frac{1}{1-x+x^2-x^3+x^4} dx \\
&= \int \left(\frac{-1+(-1)^{4/5}}{5\left(-1+\sqrt[5]{-1}x\right)}+\frac{-1-(-1)^{3/5}}{5\left(-1-(-1)^{2/5}x\right)}+\frac{-1+(-1)^{2/5}}{5\left(-1+(-1)^{3/5}x\right)}+\frac{-1-\sqrt[5]{-1}}{5\left(-1-(-1)^{4/5}x\right)}\right) dx \\
&= -\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(\sqrt[5]{-1}-x\right)+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\log\left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log(
\end{aligned}$$

Mathematica [C] time = 0.0098981, size = 51, normalized size = 0.47

$$\text{RootSum}\left[\#1^4-\#1^3+\#1^2-\#1+1\&,\frac{\log(x-\#1)}{4\#1^3-3\#1^2+2\#1-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(1 + x^5),x]
```

```
[Out] RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 + 4
*#1^3) & ]
```

Maple [B] time = 0.019, size = 173, normalized size = 1.6

$$-\frac{\sqrt{5}\ln\left(-x\sqrt{5}+2x^2-x+2\right)}{10}+\frac{1}{\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)+\frac{\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^5+1),x)
```

```
[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2-x+2)+1/(10-2*5^(1/2))^(1/2)*arctan((-5^(1
/2)+4*x-1)/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/2)*arctan((-5^(1/2)+
4*x-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(1/2)*ln(x*5^(1/2)+2*x^2-x+2)+1
/(10+2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))-1/5/(10+
2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(x^5 + 1), x)
```

Fricas [B] time = 8.73671, size = 2955, normalized size = 27.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))*log(3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + 1/8*(3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 3/8*((sqrt(5) + 5*sqrt(-2/25*sqr
t(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 11*x + 1) -
1/10*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*log(-3/8*(sqrt(5) + 5*sqrt(-2/
25*sqrt(5) - 1/5))^3 + (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 11*x - 9
/2*sqrt(5) - 45/2*sqrt(-2/25*sqrt(5) - 1/5) - 14) + 1/10*(sqrt(5) + 5*sqrt(
-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(
5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5) + 15*sqrt(-2/25*s
```

```
qrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - (sqrt(5) + 5
*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5)
)^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 5/4*sqrt(-3/100*(sqrt(5
) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) -
 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt(5) - 5*sqrt(-2
/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqr
t(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqrt(-2/25*sqrt(5) - 1
/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5) - 1/5) + 2) + 1/10
*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*
(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1
/5)) - 3/100*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5
) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5
))^2 - (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 5/4*
sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*s
qrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(
sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(
5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqr
t(-2/25*sqrt(5) - 1/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5)
 - 1/5) + 2)
```

Sympy [A] time = 0.372474, size = 36, normalized size = 0.33

$$\operatorname{RootSum}\left(125t^4 - 5t + 1, \left(t \mapsto t\log\left(\frac{375t^3}{11} + \frac{100t^2}{11} + \frac{45t}{11} + x - \frac{14}{11}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**5+1),x)
```

```
[Out] RootSum(125*_t**4 - 5*_t + 1, Lambda(_t, _t*log(375*_t**3/11 + 100*_t**2/11
 + 45*_t/11 + x - 14/11)))
```

Giac [A] time = 1.10738, size = 136, normalized size = 1.25

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{1}{10}\sqrt{5}\log\left(x^2 - \frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) + \frac{1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))
- 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2 +
1/2*x*(sqrt(5) - 1) + 1)
```

3.222 $\int \frac{1-x}{1-x^5} \, dx$

Optimal. Leaf size=109

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(x+\sqrt[5]{-1}\right)$$

```
[Out] -((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2/5) - x])/5 + ((-1)^(3/5)*(1 + (-1
)^(3/5))*Log[-(-1)^(3/5) - x])/5 + ((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1
/5) + x])/5 - ((-1)^(4/5)*(1 - (-1)^(4/5))*Log[-(-1)^(4/5) + x])/5
```

Rubi [A] time = 0.0407916, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1586, 2068}

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-x-(-1)^{3/5}\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)\log\left(x+\sqrt[5]{-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 - x^5),x]
```

```
[Out] -((-1)^(2/5)*(1 - (-1)^(2/5))*Log[(-1)^(2/5) - x])/5 + ((-1)^(3/5)*(1 + (-1
)^(3/5))*Log[-(-1)^(3/5) - x])/5 + ((-1)^(1/5)*(1 + (-1)^(1/5))*Log[(-1)^(1
/5) + x])/5 - ((-1)^(4/5)*(1 - (-1)^(4/5))*Log[-(-1)^(4/5) + x])/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2068

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[1/a^
(3*p), Int[ExpandIntegrand[1/((a - b*x)^p/(a^5 - b^5*x^5)^p), x], x], x] /;
 NeQ[a, 0] && EqQ[c, b^2/a] && EqQ[d, b^3/a^2] && EqQ[e, b^4/a^3]] /; FreeQ
[p, x] && PolyQ[P4, x, 4] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{1-x^5} \, dx &= \int \frac{1}{1+x+x^2+x^3+x^4} \, dx\\
&= \int \left(\frac{1-(-1)^{4/5}}{5\left(1+\sqrt[5]{-1}x\right)}+\frac{1+(-1)^{3/5}}{5\left(1-(-1)^{2/5}x\right)}+\frac{1-(-1)^{2/5}}{5\left(1+(-1)^{3/5}x\right)}+\frac{1+\sqrt[5]{-1}}{5\left(1-(-1)^{4/5}x\right)}\right) dx\\
&= -\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\log\left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\log\left(-(-1)^{3/5}-x\right)+\frac{1}{5}\sqrt[5]{-1}\left(1+\sqrt[5]{-1}\right)
\end{aligned}$$

Mathematica [C] time = 0.0087526, size = 47, normalized size = 0.43

$$\text{RootSum}\left[\#1^4+\#1^3+\#1^2+\#1+1\&,\frac{\log(x-\#1)}{4\#1^3+3\#1^2+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 - x^5),x]
```

```
[Out] RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4*
#1^3) & ]
```

Maple [B] time = 0.017, size = 169, normalized size = 1.6

$$-\frac{\sqrt{5}\ln\left(-x\sqrt{5}+2x^2+x+2\right)}{10}+\frac{1}{\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1+4x-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)-\frac{\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1+4x-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)+\frac{\sqrt{5}\ln\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(-x^5+1),x)
```

```
[Out] -1/10*5^(1/2)*ln(-x*5^(1/2)+2*x^2+x+2)+1/(10+2*5^(1/2))^(1/2)*arctan((1+4*x
-5^(1/2))/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1/2))^(1/2)*arctan((1+4*x-5^(1
/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/10*5^(1/2)*ln(x*5^(1/2)+2*x^2+x+2)+1/(
10-2*5^(1/2))^(1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))+1/5/(10-2*
5^(1/2))^(1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-1}{x^5-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(x^5 - 1), x)
```

Fricas [B] time = 8.55768, size = 2493, normalized size = 22.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(5) - sqrt(2*sqrt(5) - 5))*log(3/8*(sqrt(5) + sqrt(2*sqrt(5) - 5
))^3 + 1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5
) - 5))^2 + 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*
sqrt(5) - 5)) + 11*x - 1) - 1/10*(sqrt(5) + sqrt(2*sqrt(5) - 5))*log(-3/8*(
sqrt(5) + sqrt(2*sqrt(5) - 5))^3 - (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 + 11*x
 - 9/2*sqrt(5) - 9/2*sqrt(2*sqrt(5) - 5) + 14) + 1/10*(sqrt(5) + 5*sqrt(-3/
100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5)
)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2
))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(
5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqr
```

```
t(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) + 5/4*sqrt(-3/100*(sqrt(
5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5)
 - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt
(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5
) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) -
 5) - 2) + 1/10*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2
- 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/
100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt
(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5)
 - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*s
qrt(5) - 5)) - 5/4*sqrt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sq
rt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(
5) - sqrt(2*sqrt(5) - 5))^2)*((3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt
(5) - sqrt(2*sqrt(5) - 5)) - 8*sqrt(5) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x
 + 9/2*sqrt(5) + 9/2*sqrt(2*sqrt(5) - 5) - 2)
```

Sympy [A] time = 0.372795, size = 36, normalized size = 0.33

$$\text{RootSum}\left(125t^4+5t+1,\left(t\mapsto t\log\left(\frac{375t^3}{11}-\frac{100t^2}{11}+\frac{45t}{11}+x+\frac{14}{11}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x**5+1),x)
```

```
[Out] RootSum(125*_t**4 + 5*_t + 1, Lambda(_t, _t*log(375*_t**3/11 - 100*_t**2/11
 + 45*_t/11 + x + 14/11)))
```

Giac [A] time = 1.09347, size = 136, normalized size = 1.25

$$\frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(-x^5+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10)) +
 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10))
+ 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2 -
1/2*x*(sqrt(5) - 1) + 1)
```

3.223 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=208

$$\frac{x^9\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{9b^4}-\frac{ax^6\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5}+\frac{a^2x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6}-\frac{a^3\log\left(a+b\right.}{}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9
)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(1
5*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a +
 b*x^3])/(3*b^7)
```

Rubi [A] time = 0.315686, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^9\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{9b^4}-\frac{ax^6\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5}+\frac{a^2x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6}-\frac{a^3\log\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^6) - (a*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^6)/(6*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9
)/(9*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(12*b^3) + ((b*e - a*f)*x^15)/(1
5*b^2) + (f*x^18)/(18*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a +
 b*x^3])/(3*b^7)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a+bx}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^6}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{b^5}+\frac{(b^3}{}\right.\right.$$

$$=\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^6}-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{6b^5}+\frac{\left(b^3c-ab^2d+a^2be\right.}{9b^4}$$

Mathematica [A] time = 0.0902507, size = 187, normalized size = 0.9

$$\frac{bx^3\left(5a^2b^3\left(12c+6dx^3+4ex^6+3fx^9\right)-10a^3b^2\left(6d+3ex^3+2fx^6\right)+30a^4b\left(2e+fx^3\right)-60a^5f-ab^4x^3\left(30c+20dx\right.\right.}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*e*x^3 + 2*
f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^5*x^6*(20*c + 1
5*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 + 12
*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3])/(1
80*b^7)
```

Maple [A] time = 0.005, size = 266, normalized size = 1.3

$$\frac{fx^{18}}{18\,b}-\frac{x^{15}af}{15\,b^2}+\frac{x^{15}e}{15\,b}+\frac{x^{12}a^2f}{12\,b^3}-\frac{x^{12}ae}{12\,b^2}+\frac{x^{12}d}{12\,b}-\frac{x^9a^3f}{9\,b^4}+\frac{x^9a^2e}{9\,b^3}-\frac{x^9ad}{9\,b^2}+\frac{x^9c}{9\,b}+\frac{x^6a^4f}{6\,b^5}-\frac{a^3ex^6}{6\,b^4}+\frac{a^2dx^6}{6\,b^3}-\frac{acx^6}{6\,b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/18*f*x^18/b-1/15/b^2*x^15*a*f+1/15/b*x^15*e+1/12/b^3*x^12*a^2*f-1/12/b^2*
x^12*a*e+1/12/b*x^12*d-1/9/b^4*x^9*a^3*f+1/9/b^3*x^9*a^2*e-1/9/b^2*x^9*a*d+
1/9/b*x^9*c+1/6/b^5*x^6*a^4*f-1/6/b^4*x^6*a^3*e+1/6/b^3*x^6*a^2*d-1/6/b^2*x
^6*a*c-1/3/b^6*a^5*f*x^3+1/3/b^5*a^4*e*x^3-1/3/b^4*a^3*d*x^3+1/3/b^3*a^2*c*
x^3+1/3*a^6/b^7*ln(b*x^3+a)*f-1/3*a^5/b^6*ln(b*x^3+a)*e+1/3*a^4/b^5*ln(b*x^
3+a)*d-1/3*a^3/b^4*ln(b*x^3+a)*c
```

Maxima [A] time = 0.956447, size = 282, normalized size = 1.36

$$\frac{10\,b^5fx^{18}+12\left(b^5e-ab^4f\right)x^{15}+15\left(b^5d-ab^4e+a^2b^3f\right)x^{12}+20\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^9-30\left(ab^4c-a^2b^3d\right.}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(10*b^5*f*x^18 + 12*(b^5*e - a*b^4*f)*x^15 + 15*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^12 + 20*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^9 - 30*(a*b^
4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^6 + 60*(a^2*b^3*c - a^3*b^2*d + a^
4*b*e - a^5*f)*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log
(b*x^3 + a)/b^7
```

Fricas [A] time = 1.27343, size = 429, normalized size = 2.06

$$\frac{10\,b^6fx^{18}+12\left(b^6e-ab^5f\right)x^{15}+15\left(b^6d-ab^5e+a^2b^4f\right)x^{12}+20\left(b^6c-ab^5d+a^2b^4e-a^3b^3f\right)x^9-30\left(ab^5c-a^2b^4d\right.}{180\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(10*b^6*f*x^18 + 12*(b^6*e - a*b^5*f)*x^15 + 15*(b^6*d - a*b^5*e + a^
2*b^4*f)*x^12 + 20*(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*x^9 - 30*(a*b^
5*c - a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 + 60*(a^2*b^4*c - a^3*b^3*d +
a^4*b^2*e - a^5*b*f)*x^3 - 60*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log
(b*x^3 + a))/b^7
```

Sympy [A] time = 1.16717, size = 192, normalized size = 0.92

$$\frac{a^3\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^7}+\frac{fx^{18}}{18b}-\frac{x^{15}\left(af-be\right)}{15b^2}+\frac{x^{12}\left(a^2f-abe+b^2d\right)}{12b^3}-\frac{x^9\left(a^3f-a^2be+ab^2d-b^3c\right)}{9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**7) + f*x
**18/(18*b) - x**15*(a*f - b*e)/(15*b**2) + x**12*(a**2*f - a*b*e + b**2*d)
/(12*b**3) - x**9*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(9*b**4) + x**6*(
a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(6*b**5) - x**3*(a**5*f - a**4*
b*e + a**3*b**2*d - a**2*b**3*c)/(3*b**6)
```

Giac [A] time = 1.05679, size = 332, normalized size = 1.6

$$\frac{10\,b^5fx^{18}-12\,ab^4fx^{15}+12\,b^5x^{15}e+15\,b^5dx^{12}+15\,a^2b^3fx^{12}-15\,ab^4x^{12}e+20\,b^5cx^9-20\,ab^4dx^9-20\,a^3b^2fx^9+20\,a^2b}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(10*b^5*f*x^18 - 12*a*b^4*f*x^15 + 12*b^5*x^15*e + 15*b^5*d*x^12 + 15
*a^2*b^3*f*x^12 - 15*a*b^4*x^12*e + 20*b^5*c*x^9 - 20*a*b^4*d*x^9 - 20*a^3*
b^2*f*x^9 + 20*a^2*b^3*x^9*e - 30*a*b^4*c*x^6 + 30*a^2*b^3*d*x^6 + 30*a^4*b
*f*x^6 - 30*a^3*b^2*x^6*e + 60*a^2*b^3*c*x^3 - 60*a^3*b^2*d*x^3 - 60*a^5*f*
x^3 + 60*a^4*b*x^3*e)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*l
og(abs(b*x^3 + a))/b^7
```

3.224 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal. Leaf size=170

$$\frac{x^6\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4}-\frac{ax^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{a^2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6}$$

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b
*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi [A] time = 0.242332, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^6\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4}-\frac{ax^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{a^2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^6)/(6*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(9*b^3) + ((b
*e - a*f)*x^12)/(12*b^2) + (f*x^15)/(15*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{a+bx}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int\left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-\right.}{}\right.\right.$$

$$=-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{6b^4}+\frac{\left(b^2d-abe+a^2f\right.}{9b^3}$$

Mathematica [A] time = 0.0659395, size = 154, normalized size = 0.91

$$\frac{bx^3\left(10a^2b^2\left(6d+3ex^3+2fx^6\right)-30a^3b\left(2e+fx^3\right)+60a^4f-5ab^3\left(12c+6dx^3+4ex^6+3fx^9\right)+b^4x^3\left(30c+20dx^3+15\right.\right.}{180b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e*x^3 + 2*f
*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^4*x^3*(30*c + 20*d
*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*Log[a + b*x^3])/(180*b^6)
```

Maple [A] time = 0.004, size = 218, normalized size = 1.3

$$\frac{fx^{15}}{15b}-\frac{x^{12}af}{12b^2}+\frac{x^{12}e}{12b}+\frac{x^9a^2f}{9b^3}-\frac{x^9ae}{9b^2}+\frac{x^9d}{9b}-\frac{a^3fx^6}{6b^4}+\frac{a^2ex^6}{6b^3}-\frac{adx^6}{6b^2}+\frac{x^6c}{6b}+\frac{a^4fx^3}{3b^5}-\frac{a^3ex^3}{3b^4}+\frac{a^2dx^3}{3b^3}-\frac{acx^3}{3b^2}-\frac{a^5\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/15*f*x^15/b-1/12/b^2*x^12*a*f+1/12/b*x^12*e+1/9/b^3*x^9*a^2*f-1/9/b^2*x^9
*a*e+1/9/b*x^9*d-1/6/b^4*x^6*a^3*f+1/6/b^3*x^6*a^2*e-1/6/b^2*x^6*a*d+1/6/b*
x^6*c+1/3/b^5*a^4*f*x^3-1/3/b^4*a^3*e*x^3+1/3/b^3*a^2*d*x^3-1/3/b^2*a*c*x^3
-1/3*a^5/b^6*ln(b*x^3+a)*f+1/3*a^4/b^5*ln(b*x^3+a)*e-1/3*a^3/b^4*ln(b*x^3+a
)*d+1/3*a^2/b^3*ln(b*x^3+a)*c
```

Maxima [A] time = 0.95531, size = 228, normalized size = 1.34

$$\frac{12b^4fx^{15}+15\left(b^4e-ab^3f\right)x^{12}+20\left(b^4d-ab^3e+a^2b^2f\right)x^9+30\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^6-60\left(ab^3c-a^2b^2d+a^3b\right.}{180b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/180*(12*b^4*f*x^15 + 15*(b^4*e - a*b^3*f)*x^12 + 20*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^9 + 30*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^6 - 60*(a*b^3*c
 - a^2*b^2*d + a^3*b*e - a^4*f)*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a)/b^6
```

Fricas [A] time = 1.234, size = 351, normalized size = 2.06

$$\frac{12b^5fx^{15}+15\left(b^5e-ab^4f\right)x^{12}+20\left(b^5d-ab^4e+a^2b^3f\right)x^9+30\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^6-60\left(ab^4c-a^2b^3d+a^3\right.}{180b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*b^4*e + a^
2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^6 - 60*(a*b^4
*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 60*(a^2*b^3*c - a^3*b^2*d + a^4
*b*e - a^5*f)*log(b*x^3 + a))/b^6
```

Sympy [A] time = 1.13667, size = 155, normalized size = 0.91

$$-\frac{a^2\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^6}+\frac{fx^{15}}{15b}-\frac{x^{12}\left(af-be\right)}{12b^2}+\frac{x^9\left(a^2f-abe+b^2d\right)}{9b^3}-\frac{x^6\left(a^3f-a^2be+ab^2d-\right.}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] -a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**6) + f*
x**15/(15*b) - x**12*(a*f - b*e)/(12*b**2) + x**9*(a**2*f - a*b*e + b**2*d)
/(9*b**3) - x**6*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(6*b**4) + x**3*(a
**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*b**5)
```

Giac [A] time = 1.06393, size = 266, normalized size = 1.56

$$\frac{12\,b^4fx^{15}-15\,ab^3fx^{12}+15\,b^4x^{12}e+20\,b^4dx^9+20\,a^2b^2fx^9-20\,ab^3x^9e+30\,b^4cx^6-30\,ab^3dx^6-30\,a^3bfx^6+30\,a^2b^2}{180\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/180*(12*b^4*f*x^15 - 15*a*b^3*f*x^12 + 15*b^4*x^12*e + 20*b^4*d*x^9 + 20*
a^2*b^2*f*x^9 - 20*a*b^3*x^9*e + 30*b^4*c*x^6 - 30*a*b^3*d*x^6 - 30*a^3*b*f
*x^6 + 30*a^2*b^2*x^6*e - 60*a*b^3*c*x^3 + 60*a^2*b^2*d*x^3 + 60*a^4*f*x^3
- 60*a^3*b*x^3*e)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*log(a
bs(b*x^3 + a))/b^6
```

3.225 $\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

Optimal. Leaf size=132

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{a\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{x^6\left(a^2f-abe+b^2d\right)}{6b^3}+\frac{x^9(be-af)}{9b^2}+$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rubi [A] time = 0.183319, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{a\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{x^6\left(a^2f-abe+b^2d\right)}{6b^3}+\frac{x^9(be-af)}{9b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^6)/(6*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (f*x^12)/(12*b) - (a*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{a+bx}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^2}{b^2}+\frac{fx^3}{b}+\frac{a}{}\right.\right.$$
$$=\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe+a^2f\right)x^6}{6b^3}+\frac{(be-af)x^9}{9b^2}+\frac{fx^{12}}{12b}-\frac{a\left(b^3c-\right.}{}$$

Mathematica [A] time = 0.052875, size = 119, normalized size = 0.9

$$\frac{bx^3\left(6a^2b\left(2e+fx^3\right)-12a^3f-2ab^2\left(6d+3ex^3+2fx^6\right)+b^3\left(12c+6dx^3+4ex^6+3fx^9\right)\right)+12a\log\left(a+bx^3\right)\left(-a^2be+\right.}{36b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^3 + 2*f*x^
6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)
```

Maple [A] time = 0.003, size = 170, normalized size = 1.3

$$\frac{fx^{12}}{12b}-\frac{x^9af}{9b^2}+\frac{x^9e}{9b}+\frac{a^2fx^6}{6b^3}-\frac{aex^6}{6b^2}+\frac{dx^6}{6b}-\frac{a^3fx^3}{3b^4}+\frac{a^2ex^3}{3b^3}-\frac{adx^3}{3b^2}+\frac{cx^3}{3b}+\frac{a^4\ln\left(bx^3+a\right)f}{3b^5}-\frac{a^3\ln\left(bx^3+a\right)e}{3b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/12*f*x^12/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/6/b^3*x^6*a^2*f-1/6/b^2*x^6*a*e
+1/6/b*x^6*d-1/3/b^4*a^3*f*x^3+1/3/b^3*a^2*e*x^3-1/3/b^2*a*d*x^3+1/3/b*c*x^
3+1/3*a^4/b^5*ln(b*x^3+a)*f-1/3*a^3/b^4*ln(b*x^3+a)*e+1/3*a^2/b^3*ln(b*x^3+
a)*d-1/3*a/b^2*ln(b*x^3+a)*c
```

Maxima [A] time = 0.953117, size = 174, normalized size = 1.32

$$\frac{3b^3fx^{12}+4\left(b^3e-ab^2f\right)x^9+6\left(b^3d-ab^2e+a^2bf\right)x^6+12\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{36b^4}-\frac{\left(ab^3c-a^2b^2d+a^3be-a^4f\right.}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/36*(3*b^3*f*x^12 + 4*(b^3*e - a*b^2*f)*x^9 + 6*(b^3*d - a*b^2*e + a^2*b*f
)*x^6 + 12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4 - 1/3*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a)/b^5
```

Fricas [A] time = 1.27328, size = 267, normalized size = 2.02

$$\frac{3b^4fx^{12}+4\left(b^4e-ab^3f\right)x^9+6\left(b^4d-ab^3e+a^2b^2f\right)x^6+12\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^3-12\left(ab^3c-a^2b^2d+a^3be\right.}{36b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 12*(a*b^3*c - a^
2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^5
```

Sympy [A] time = 1.08249, size = 117, normalized size = 0.89

$$\frac{a\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)\log\left(a+bx^{3}\right)}{3b^{5}}+\frac{fx^{12}}{12b}-\frac{x^{9}\left(af-be\right)}{9b^{2}}+\frac{x^{6}\left(a^{2}f-abe+b^{2}d\right)}{6b^{3}}-\frac{x^{3}\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)}{3b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5) + f*x**1
2/(12*b) - x**9*(a*f - b*e)/(9*b**2) + x**6*(a**2*f - a*b*e + b**2*d)/(6*b*
*3) - x**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*b**4)
```

Giac [A] time = 1.06881, size = 200, normalized size = 1.52

$$\frac{3\,b^{3}fx^{12}-4\,ab^{2}fx^{9}+4\,b^{3}x^{9}e+6\,b^{3}dx^{6}+6\,a^{2}bfx^{6}-6\,ab^{2}x^{6}e+12\,b^{3}cx^{3}-12\,ab^{2}dx^{3}-12\,a^{3}fx^{3}+12\,a^{2}bx^{3}e}{36\,b^{4}}-\frac{\left(ab^{3}c-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/36*(3*b^3*f*x^12 - 4*a*b^2*f*x^9 + 4*b^3*x^9*e + 6*b^3*d*x^6 + 6*a^2*b*f*
x^6 - 6*a*b^2*x^6*e + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 - 12*a^3*f*x^3 + 12*a^2
*b*x^3*e)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*log(abs(b*x^3 +
 a))/b^5
```

3.226 $\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

Optimal. Leaf size=96

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)
/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^4)
```

Rubi [A] time = 0.139912, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1819, 1850}

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^6(be-af)}{6b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^3)/(3*b^3) + ((b*e - a*f)*x^6)/(6*b^2) + (f*x^9)
/(9*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*b^4)
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{a+bx}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x}{b^2}+\frac{fx^2}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)}\right)dx,\right.\\ &= \frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^6}{6b^2}+\frac{fx^9}{9b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.0430123, size = 88, normalized size = 0.92

$$\frac{6\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+bx^3\left(6a^2f-3ab\left(2e+fx^3\right)+b^2\left(6d+3ex^3+2fx^6\right)\right)}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*x^6)) + 6*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*b^4)
```

Maple [A] time = 0.003, size = 124, normalized size = 1.3

$$\frac{fx^9}{9b}-\frac{x^6af}{6b^2}+\frac{ex^6}{6b}+\frac{a^2fx^3}{3b^3}-\frac{aex^3}{3b^2}+\frac{dx^3}{3b}-\frac{\ln\left(bx^3+a\right)a^3f}{3b^4}+\frac{\ln\left(bx^3+a\right)a^2e}{3b^3}-\frac{\ln\left(bx^3+a\right)ad}{3b^2}+\frac{c\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/9*f*x^9/b-1/6/b^2*x^6*a*f+1/6/b*x^6*e+1/3/b^3*a^2*f*x^3-1/3/b^2*a*e*x^3+1
/3/b*d*x^3-1/3/b^4*ln(b*x^3+a)*a^3*f+1/3/b^3*ln(b*x^3+a)*a^2*e-1/3/b^2*ln(b
*x^3+a)*a*d+1/3*c*ln(b*x^3+a)/b
```

Maxima [A] time = 0.948623, size = 123, normalized size = 1.28

$$\frac{2\,b^2fx^9+3\left(b^2e-abf\right)x^6+6\left(b^2d-abe+a^2f\right)x^3}{18\,b^3}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b^2*f*x^9 + 3*(b^2*e - a*b*f)*x^6 + 6*(b^2*d - a*b*e + a^2*f)*x^3)/
b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/b^4
```

Fricas [A] time = 1.19505, size = 190, normalized size = 1.98

$$\frac{2\,b^3fx^9+3\left(b^3e-ab^2f\right)x^6+6\left(b^3d-ab^2e+a^2bf\right)x^3+6\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{18\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e + a^2*b*f)
*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/b^4
```

Sympy [A] time = 1.00665, size = 83, normalized size = 0.86

$$\frac{fx^9}{9b}-\frac{x^6\left(af-be\right)}{6b^2}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] f*x**9/(9*b) - x**6*(a*f - b*e)/(6*b**2) + x**3*(a**2*f - a*b*e + b**2*d)/(
3*b**3) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**4)
```

Giac [A] time = 1.06947, size = 136, normalized size = 1.42

$$\frac{2\,b^2fx^9 - 3\,abfx^6 + 3\,b^2x^6e + 6\,b^2dx^3 + 6\,a^2fx^3 - 6\,abx^3e}{18\,b^3} + \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/18*(2*b^2*f*x^9 - 3*a*b*f*x^6 + 3*b^2*x^6*e + 6*b^2*d*x^3 + 6*a^2*f*x^3 -
 6*a*b*x^3*e)/b^3 + 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 +
 a))/b^4
```

3.227 $\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=80

$$-\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rubi [A] time = 0.120113, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3}+\frac{x^3(be-af)}{3b^2}+\frac{c\log(x)}{a}+\frac{fx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] ((b*e - a*f)*x^3)/(3*b^2) + (f*x^6)/(6*b) + (c*Log[x])/a - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a*b^3)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{be-af}{b^2}+\frac{c}{ax}+\frac{fx}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)}\right)dx,x,x^3\right)\\&=\frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3ab^3}\end{aligned}$$

Mathematica [A] time = 0.0306707, size = 75, normalized size = 0.94

$$\frac{-2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+abx^3\left(-2af+2be+bfx^3\right)+6b^3c\log(x)}{6ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

```
[Out] (a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)
```

Maple [A] time = 0.005, size = 97, normalized size = 1.2

$$\frac{fx^6}{6b}-\frac{ax^3f}{3b^2}+\frac{ex^3}{3b}+\frac{a^2\ln\left(bx^3+a\right)f}{3b^3}-\frac{ae\ln\left(bx^3+a\right)}{3b^2}+\frac{d\ln\left(bx^3+a\right)}{3b}-\frac{c\ln\left(bx^3+a\right)}{3a}+\frac{c\ln\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x)
```

```
[Out] 1/6*f*x^6/b-1/3/b^2*x^3*a*f+1/3*e*x^3/b+1/3*a^2/b^3*ln(b*x^3+a)*f-1/3*a*e*l
n(b*x^3+a)/b^2+1/3*d*ln(b*x^3+a)/b-1/3*c*ln(b*x^3+a)/a+c*ln(x)/a
```

Maxima [A] time = 0.958248, size = 104, normalized size = 1.3

$$\frac{c\log\left(x^3\right)}{3a}+\frac{bfx^6+2\left(be-af\right)x^3}{6b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*c*log(x^3)/a + 1/6*(b*f*x^6 + 2*(b*e - a*f)*x^3)/b^2 - 1/3*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/(a*b^3)
```

Fricas [A] time = 1.58809, size = 171, normalized size = 2.14

$$\frac{ab^2fx^6+6b^3c\log\left(x\right)+2\left(ab^2e-a^2bf\right)x^3-2\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{6ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(a*b^2*f*x^6 + 6*b^3*c*log(x) + 2*(a*b^2*e - a^2*b*f)*x^3 - 2*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/(a*b^3)
```

Sympy [A] time = 4.91323, size = 68, normalized size = 0.85

$$\frac{fx^6}{6b}-\frac{x^3\left(af-be\right)}{3b^2}+\frac{c\log\left(x\right)}{a}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
```

```
[Out] f*x**6/(6*b) - x**3*(a*f - b*e)/(3*b**2) + c*log(x)/a + (a**3*f - a**2*b*e
+ a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a*b**3)
```

Giac [A] time = 1.05752, size = 107, normalized size = 1.34

$$\frac{c\log\left(|x|\right)}{a}+\frac{bfx^{6}-2\,afx^{3}+2\,bx^{3}e}{6\,b^{2}}-\frac{\left(b^{3}c-ab^{2}d-a^{3}f+a^{2}be\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a + 1/6*(b*f*x^6 - 2*a*f*x^3 + 2*b*x^3*e)/b^2 - 1/3*(b^3*c -
a*b^2*d - a^3*f + a^2*b*e)*log(abs(b*x^3 + a))/(a*b^3)
```

3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)} dx$

Optimal. Leaf size=81

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

```
[Out] -c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rubi [A] time = 0.116136, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{3ax^3}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] -c/(3*a*x^3) + (f*x^3)/(3*b) - ((b*c - a*d)*Log[x])/a^2 + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^2*b^2)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{f}{b}+\frac{c}{ax^2}+\frac{-bc+ad}{a^2x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0434588, size = 77, normalized size = 0.95

$$\frac{1}{3}\left(\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^2b^2}+\frac{3\log(x)(ad-bc)}{a^2}-\frac{c}{ax^3}+\frac{fx^3}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*Log[x])/a^2 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

Maple [A] time = 0.007, size = 94, normalized size = 1.2

$$\frac{fx^3}{3\,b}-\frac{a\ln\left(bx^3+a\right)f}{3\,b^2}+\frac{e\ln\left(bx^3+a\right)}{3\,b}-\frac{d\ln\left(bx^3+a\right)}{3\,a}+\frac{b\ln\left(bx^3+a\right)c}{3\,a^2}-\frac{c}{3\,ax^3}+\frac{d\ln(x)}{a}-\frac{\ln(x)\,bc}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3*f*x^3/b-1/3*a/b^2*ln(b*x^3+a)*f+1/3*e*ln(b*x^3+a)/b-1/3*d*ln(b*x^3+a)/a
+1/3/a^2*b*ln(b*x^3+a)*c-1/3*c/a/x^3+d*ln(x)/a-1/a^2*ln(x)*b*c
```

Maxima [A] time = 0.966186, size = 104, normalized size = 1.28

$$\frac{fx^3}{3\,b}-\frac{(bc-ad)\log\left(x^3\right)}{3\,a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^2b^2}-\frac{c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b - 1/3*(b*c - a*d)*log(x^3)/a^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(b*x^3 + a)/(a^2*b^2) - 1/3*c/(a*x^3)
```

Fricas [A] time = 1.50273, size = 180, normalized size = 2.22

$$\frac{a^2bfx^6+\left(b^3c-ab^2d+a^2be-a^3f\right)x^3\log\left(bx^3+a\right)-3\left(b^3c-ab^2d\right)x^3\log(x)-ab^2c}{3\,a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b*f*x^6 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(b*x^3 + a) -
 3*(b^3*c - a*b^2*d)*x^3*log(x) - a*b^2*c)/(a^2*b^2*x^3)
```

Sympy [A] time = 11.4391, size = 70, normalized size = 0.86

$$\frac{fx^3}{3b}-\frac{c}{3ax^3}+\frac{(ad-bc)\log(x)}{a^2}-\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)
```

```
[Out] f*x**3/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x)/a**2 - (a**3*f - a**2*b*e
+ a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**2*b**2)
```

Giac [A] time = 1.05868, size = 128, normalized size = 1.58

$$\frac{fx^3}{3\,b} - \frac{(bc - ad)\log(|x|)}{a^2} + \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^2b^2} + \frac{bcx^3 - adx^3 - ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b - (b*c - a*d)*log(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d - a^3*f +
a^2*b*e)*log(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a*d*x^3 - a*c)/(a^2
*x^3)
```

3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx$

Optimal. Leaf size=95

$$-\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}+\frac{bc-ad}{3a^2x^3}-\frac{c}{6ax^6}$$

```
[Out] -c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*Log[x])/a
^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^3*b)
```

Rubi [A] time = 0.128976, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b}+\frac{\log(x)\left(a^2e-abd+b^2c\right)}{a^3}+\frac{bc-ad}{3a^2x^3}-\frac{c}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] -c/(6*a*x^6) + (b*c - a*d)/(3*a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*Log[x])/a
^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^3*b)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^3}+\frac{-bc+ad}{a^2x^2}+\frac{b^2c-abd+a^2e}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)}\right)dx,x,x^3\right)\\ &= -\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{\left(b^2c-abd+a^2e\right)\log(x)}{a^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^3b}\end{aligned}$$

Mathematica [A] time = 0.0686564, size = 88, normalized size = 0.93

$$\frac{\log\left(a+bx^3\right)\left(\frac{2a^3f}{b}-2a^2e+2abd-2b^2c\right)+6\log(x)\left(a^2e-abd+b^2c\right)-\frac{a\left(ac+2adx^3-2bcx^3\right)}{x^6}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^2*e)*Log[x
] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a + b*x^3])/(6*a^3)
```

Maple [A] time = 0.007, size = 116, normalized size = 1.2

$$\frac{\ln\left(bx^3+a\right)f}{3b}-\frac{e\ln\left(bx^3+a\right)}{3a}+\frac{b\ln\left(bx^3+a\right)d}{3a^2}-\frac{b^2\ln\left(bx^3+a\right)c}{3a^3}-\frac{c}{6ax^6}-\frac{d}{3ax^3}+\frac{bc}{3x^3a^2}+\frac{e\ln(x)}{a}-\frac{\ln(x)bd}{a^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x)
```

```
[Out] 1/3/b*ln(b*x^3+a)*f-1/3*e*ln(b*x^3+a)/a+1/3/a^2*b*ln(b*x^3+a)*d-1/3/a^3*b^2
*ln(b*x^3+a)*c-1/6*c/a/x^6-1/3/a/x^3*d+1/3/a^2/x^3*b*c+e*ln(x)/a-1/a^2*ln(x
)*b*d+1/a^3*ln(x)*b^2*c
```

Maxima [A] time = 0.948345, size = 126, normalized size = 1.33

$$\frac{\left(b^2c-abd+a^2e\right)\log\left(x^3\right)}{3a^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3a^3b}+\frac{2(bc-ad)x^3-ac}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c - a*b*d + a^2*e)*log(x^3)/a^3 - 1/3*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*log(b*x^3 + a)/(a^3*b) + 1/6*(2*(b*c - a*d)*x^3 - a*c)/(a^2*x^6)
```

Fricas [A] time = 1.52655, size = 213, normalized size = 2.24

$$-\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6\log\left(bx^3+a\right)-6\left(b^3c-ab^2d+a^2be\right)x^6\log(x)+a^2bc-2\left(ab^2c-a^2bd\right)x^3}{6a^3bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6*log(b*x^3 + a) - 6*(b^3*c -
 a*b^2*d + a^2*b*e)*x^6*log(x) + a^2*b*c - 2*(a*b^2*c - a^2*b*d)*x^3)/(a^3*
b*x^6)
```

Sympy [A] time = 51.4964, size = 85, normalized size = 0.89

$$-\frac{ac+x^3(2ad-2bc)}{6a^2x^6}+\frac{\left(a^2e-abd+b^2c\right)\log(x)}{a^3}+\frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

```
[Out] -(a*c + x**3*(2*a*d - 2*b*c))/(6*a**2*x**6) + (a**2*e - a*b*d + b**2*c)*log
(x)/a**3 + (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**3*
b)
```

Giac [A] time = 1.05506, size = 170, normalized size = 1.79

$$\frac{\left(b^2c-abd+a^2e\right)\log\left(|x|\right)}{a^3}-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^3b}-\frac{3\,b^2cx^6-3\,abdx^6+3\,a^2x^6e-2\,abcx^3+2\,a^2dx^3}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d + a^2*e)*log(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d - a^3*f + a^
2*b*e)*log(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3*a*b*d*x^6 + 3*a^2
*x^6*e - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^3*x^6)
```

3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx$

Optimal. Leaf size=128

$$\frac{\log(a+bx^3)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^4}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{6a^2x^6}-\frac{c}{9ax}$$

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 0.161649, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{\log(a+bx^3)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4}-\frac{\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^4}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{6a^2x^6}-\frac{c}{9ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^3}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x}-\frac{b\left(-b^3c\right.}{}\right.\right.$$
$$=-\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log(x)}{a^4}+\frac{\left(b^3c-ab^2d+\right.}{}$$

Mathematica [A] time = 0.0643549, size = 128, normalized size = 1.

$$\frac{\log(a+bx^3)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4}+\frac{\log(x)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^4}+\frac{a^2(-e)+abd-b^2c}{3a^3x^3}+\frac{bc-ad}{6a^2x^6}-\frac{}{9a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(6*a^2*x^6) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^3*
x^3) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/a^4 + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

Maple [A] time = 0.007, size = 161, normalized size = 1.3

$$-\frac{\ln\left(bx^3+a\right)f}{3a}+\frac{\ln\left(bx^3+a\right)be}{3a^2}-\frac{\ln\left(bx^3+a\right)b^2d}{3a^3}+\frac{\ln\left(bx^3+a\right)b^3c}{3a^4}-\frac{c}{9ax^9}-\frac{d}{6ax^6}+\frac{bc}{6a^2x^6}-\frac{e}{3ax^3}+\frac{bd}{3x^3a^2}-\frac{b}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x)
```

```
[Out] -1/3/a*ln(b*x^3+a)*f+1/3/a^2*ln(b*x^3+a)*b*e-1/3/a^3*ln(b*x^3+a)*b^2*d+1/3/
a^4*ln(b*x^3+a)*b^3*c-1/9*c/a/x^9-1/6/a/x^6*d+1/6/a^2/x^6*b*c-1/3/a/x^3*e+1
/3/a^2/x^3*b*d-1/3/a^3/x^3*b^2*c+1/a*ln(x)*f-1/a^2*ln(x)*b*e+1/a^3*ln(x)*b^
2*d-1/a^4*ln(x)*b^3*c
```

Maxima [A] time = 0.95327, size = 169, normalized size = 1.32

$$\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3a^4}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(x^3\right)}{3a^4}-\frac{6\left(b^2c-abd+a^2e\right)x^6-3\left(abc-a^2d\right)x^3+}{18a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/a^4 - 1/3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*log(x^3)/a^4 - 1/18*(6*(b^2*c - a*b*d + a^2*e)*x^
6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9)
```

Fricas [A] time = 1.47109, size = 269, normalized size = 2.1

$$\frac{6\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log\left(bx^3+a\right)-18\left(b^3c-ab^2d+a^2be-a^3f\right)x^9\log\left(x\right)-6\left(ab^2c-a^2bd+a^3e\right)x^6-2a^3c+3}{18a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) - 18*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6
 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06691, size = 248, normalized size = 1.94

$$-\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\log\left(|x|\right)}{a^4} + \frac{\left(b^4c - ab^3d - a^3bf + a^2b^2e\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^4b} + \frac{11\,b^3cx^9 - 11\,ab^2dx^9 - 11\,a^3fx^9 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*log(abs(x))/a^4 + 1/3*(b^4*c - a*b^3*d
 - a^3*b*f + a^2*b^2*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/18*(11*b^3*c*x^9 -
11*a*b^2*d*x^9 - 11*a^3*f*x^9 + 11*a^2*b*x^9*e - 6*a*b^2*c*x^6 + 6*a^2*b*d*
x^6 - 6*a^3*x^6*e + 3*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)
```

3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=164

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5}-\frac{a^2\ldots}{}$$

```
[Out] -c/(12*a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3*x
^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Lo
g[a + b*x^3])/(3*a^5)
```

Rubi [A] time = 0.181439, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5}+\frac{b\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5}-\frac{a^2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```


```
[Out] -c/(12*a*x^12) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(6*a^3*x
^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[x])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Lo
g[a + b*x^3])/(3*a^5)
```


Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\frac{b\left(-b^3c+a\ldots\right.}{}\right.\right.$$
$$=-\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b\left(b^3c-ab^2d+a^2be-a\ldots\right.}{a^5}$$

Mathematica [A] time = 0.0624507, size = 164, normalized size = 1.

$$\frac{36bx^{12}\log(x)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)-12bx^{12}\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)-6a^2b^2x^6\left(c+2dx^3\right)}{36a^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

```
[Out] (12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c + 3*d*x^3
+ 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + 36*b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^12*Log[a + b*x^3])/(36*a^5*x^12)
```

Maple [A] time = 0.009, size = 210, normalized size = 1.3

$$\frac{b\ln\left(bx^3+a\right)f}{3a^2}-\frac{b^2\ln\left(bx^3+a\right)e}{3a^3}+\frac{b^3\ln\left(bx^3+a\right)d}{3a^4}-\frac{b^4\ln\left(bx^3+a\right)c}{3a^5}-\frac{c}{12ax^{12}}-\frac{d}{9ax^9}+\frac{bc}{9a^2x^9}-\frac{e}{6ax^6}+\frac{bd}{6a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x)
```

```
[Out] 1/3*b/a^2*ln(b*x^3+a)*f-1/3*b^2/a^3*ln(b*x^3+a)*e+1/3*b^3/a^4*ln(b*x^3+a)*d
-1/3*b^4/a^5*ln(b*x^3+a)*c-1/12*c/a/x^12-1/9/a/x^9*d+1/9/a^2/x^9*b*c-1/6/a/
x^6*e+1/6/a^2/x^6*b*d-1/6/a^3/x^6*b^2*c-1/3/a/x^3*f+1/3/a^2/x^3*b*e-1/3/a^3
/x^3*b^2*d+1/3/a^4/x^3*b^3*c-1/a^2*b*ln(x)*f+1/a^3*b^2*ln(x)*e-1/a^4*b^3*ln
(x)*d+1/a^5*b^4*ln(x)*c
```

Maxima [A] time = 0.955154, size = 224, normalized size = 1.37

$$-\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(bx^3+a\right)}{3a^5}+\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(x^3\right)}{3a^5}+\frac{12\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(b*x^3 + a)/a^5 + 1/3*(b^4*
c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x^3)/a^5 + 1/36*(12*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^9 - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 3*a^3*c + 4*(
a^2*b*c - a^3*d)*x^3)/(a^4*x^12)
```

Fricas [A] time = 1.5128, size = 355, normalized size = 2.16

$$-\frac{12\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{12}\log\left(bx^3+a\right)-36\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{12}\log(x)-12\left(ab^3c-a^2b^2d+a^3b\right.}{36a^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(b*x^3 + a) - 36*
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(x) - 12*(a*b^3*c - a^2*b^2
*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 3*a^4*c -
 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06825, size = 317, normalized size = 1.93

$$\frac{\left(b^4c - ab^3d - a^3bf + a^2b^2e\right)\log\left(|x|\right)}{a^5} - \frac{\left(b^5c - ab^4d - a^3b^2f + a^2b^3e\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^5b} - \frac{25\,b^4cx^{12} - 25\,ab^3dx^{12} - 25\,a^3bfx^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*log(abs(x))/a^5 - 1/3*(b^5*c - a*b^
4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/36*(25*b^4*c*x
^12 - 25*a*b^3*d*x^12 - 25*a^3*b*f*x^12 + 25*a^2*b^2*x^12*e - 12*a*b^3*c*x^
9 + 12*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 12*a^3*b*x^9*e + 6*a^2*b^2*c*x^6 - 6*
a^3*b*d*x^6 + 6*a^4*x^6*e - 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^5*x^1
2)
```

3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$

Optimal. Leaf size=205

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^4x^6}+\frac{b^2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^6}-\frac{b^2\,\mathrm{l}}{}$$

```
[Out] -c/(15*a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2*e)/(9*a^3
*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(
3*a^6)
```

Rubi [A] time = 0.208872, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^4x^6}+\frac{b^2\log\left(a+bx^3\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^6}-\frac{b^2\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -c/(15*a*x^15) + (b*c - a*d)/(12*a^2*x^12) - (b^2*c - a*b*d + a^2*e)/(9*a^3
*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*x^6) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(3*a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*Log[x])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(
3*a^6)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int\frac{c+dx^3+ex^6+fx^9}{x^{16}\left(a+bx^3\right)}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int\frac{c+dx+ex^2+fx^3}{x^6(a+bx)}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^5}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3}-\frac{b\left(-b^3c\right.}{}\right.\right.$$

$$=-\frac{c}{15ax^{15}}+\frac{bc-ad}{12a^2x^{12}}-\frac{b^2c-abd+a^2e}{9a^3x^9}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6}-\frac{b\left(b^3c-ab^2d+a^2be\right.}{3a^5x^3}$$

Mathematica [A] time = 0.143637, size = 194, normalized size = 0.95

$$-\frac{\frac{a(10a^2b^2x^6(2c+3dx^3+6ex^6)-5a^3bx^3(3c+4dx^3+6ex^6+12fx^9)+a^4(12c+15dx^3+20ex^6+30fx^9)-30ab^3x^9(c+2dx^3)+60b^4cx^{12})}{x^{15}}-60b^2\log\left(a+bx^3\right)\left(a^2be\right.}{180a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

```
[Out] -((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6*(2*c + 3*
d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^4*(
12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15 + 180*b^2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
+ b*x^3])/(180*a^6)
```

Maple [A] time = 0.01, size = 260, normalized size = 1.3

$$-\frac{b^2\ln\left(bx^3+a\right)f}{3\,a^3}+\frac{b^3\ln\left(bx^3+a\right)e}{3\,a^4}-\frac{b^4\ln\left(bx^3+a\right)d}{3\,a^5}+\frac{b^5\ln\left(bx^3+a\right)c}{3\,a^6}-\frac{c}{15\,ax^{15}}-\frac{d}{12\,ax^{12}}+\frac{bc}{12\,a^2x^{12}}-\frac{e}{9\,ax^9}+\frac{}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x)
```

```
[Out] -1/3*b^2/a^3*ln(b*x^3+a)*f+1/3*b^3/a^4*ln(b*x^3+a)*e-1/3*b^4/a^5*ln(b*x^3+a
)*d+1/3*b^5/a^6*ln(b*x^3+a)*c-1/15*c/a/x^15-1/12/a/x^12*d+1/12/a^2/x^12*b*c
-1/9/a/x^9*e+1/9/a^2/x^9*b*d-1/9/a^3/x^9*b^2*c-1/6/a/x^6*f+1/6/a^2/x^6*b*e-
1/6/a^3/x^6*b^2*d+1/6/a^4/x^6*b^3*c+1/a^3*b^2*ln(x)*f-1/a^4*b^3*ln(x)*e+1/a
^5*b^4*ln(x)*d-1/a^6*b^5*ln(x)*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3*e+1/3/a^4*
b^3/x^3*d-1/3/a^5*b^4/x^3*c
```

Maxima [A] time = 0.958911, size = 281, normalized size = 1.37

$$\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(bx^3+a\right)}{3\,a^6}-\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\log\left(x^3\right)}{3\,a^6}-\frac{60\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{12}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(b*x^3 + a)/a^6 - 1/3*(b^5
*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x^3)/a^6 - 1/180*(60*(b^4*c - a*b
^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 30*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*
f)*x^9 + 20*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c - a^
4*d)*x^3)/(a^5*x^15)
```

Fricas [A] time = 1.6669, size = 441, normalized size = 2.15

$$\frac{60\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{15}\log\left(bx^3+a\right)-180\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{15}\log\left(x\right)-60\left(ab^4c-a^2b^3d+a^3b^2\right.}{180\,a^6x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(b*x^3 + a) - 1
80*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(x) - 60*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 30*(a^2*b^3*c - a^3*b^2*d + a^4*b*e -
 a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4*b*c
 - a^5*d)*x^3)/(a^6*x^15)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06632, size = 387, normalized size = 1.89

$$-\frac{\left(b^5c - ab^4d - a^3b^2f + a^2b^3e\right)\log\left(|x|\right)}{a^6} + \frac{\left(b^6c - ab^5d - a^3b^3f + a^2b^4e\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^6b} + \frac{137\,b^5cx^{15} - 137\,ab^4dx^{15} - 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*log(abs(x))/a^6 + 1/3*(b^6*c - a
*b^5*d - a^3*b^3*f + a^2*b^4*e)*log(abs(b*x^3 + a))/(a^6*b) + 1/180*(137*b^
5*c*x^15 - 137*a*b^4*d*x^15 - 137*a^3*b^2*f*x^15 + 137*a^2*b^3*x^15*e - 60*
a*b^4*c*x^12 + 60*a^2*b^3*d*x^12 + 60*a^4*b*f*x^12 - 60*a^3*b^2*x^12*e + 30
*a^2*b^3*c*x^9 - 30*a^3*b^2*d*x^9 - 30*a^5*f*x^9 + 30*a^4*b*x^9*e - 20*a^3*
b^2*c*x^6 + 20*a^4*b*d*x^6 - 20*a^5*x^6*e + 15*a^4*b*c*x^3 - 15*a^5*d*x^3 -
 12*a^5*c)/(a^6*x^15)
```

3.233 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal. Leaf size=348

$$\frac{x^7\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{7b^4}-\frac{ax^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^5}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-\right.}{6b^{19/3}}$$

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2)
 + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (
a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*b^(19/3))
```

Rubi [A] time = 0.332909, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x^7\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{7b^4}-\frac{ax^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^5}+\frac{a^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-\right.}{6b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```


```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7)/(7*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)/(13*b^2)
 + (f*x^16)/(16*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) - (a^(7/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(19/3)) + (
a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*b^(19/3))
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9 \left(c + dx^3 + ex^6 + fx^9\right)}{a + bx^3} dx = \frac{fx^{16}}{16b} + \frac{\int \frac{x^9\left(16bc+16bdx^3+16(be-af)x^6\right)}{a+bx^3} dx}{16b}$$

$$= \frac{fx^{16}}{16b} + \frac{\int \left(\frac{16a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^5} - \frac{16a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{b^4} + \frac{16\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{b^3} + \frac{16(b^2 \cdots}{\cdots}\right.}{16b}$$

$$= \frac{a^2\left(b^3c - ab^2d + a^2be - a^3f\right)x}{b^6} - \frac{a\left(b^3c - ab^2d + a^2be - a^3f\right)x^4}{4b^5} + \frac{\left(b^3c - ab^2d + a^2be - \cdots\right.}{7b^4}$$

$$= \frac{a^2\left(b^3c - ab^2d + a^2be - a^3f\right)x}{b^6} - \frac{a\left(b^3c - ab^2d + a^2be - a^3f\right)x^4}{4b^5} + \frac{\left(b^3c - ab^2d + a^2be - \cdots\right.}{7b^4}$$

$$= \frac{a^2\left(b^3c - ab^2d + a^2be - a^3f\right)x}{b^6} - \frac{a\left(b^3c - ab^2d + a^2be - a^3f\right)x^4}{4b^5} + \frac{\left(b^3c - ab^2d + a^2be - \cdots\right.}{7b^4}$$

$$= \frac{a^2\left(b^3c - ab^2d + a^2be - a^3f\right)x}{b^6} - \frac{a\left(b^3c - ab^2d + a^2be - a^3f\right)x^4}{4b^5} + \frac{\left(b^3c - ab^2d + a^2be - \cdots\right.}{7b^4}$$

$$= \frac{a^2\left(b^3c - ab^2d + a^2be - a^3f\right)x}{b^6} - \frac{a\left(b^3c - ab^2d + a^2be - a^3f\right)x^4}{4b^5} + \frac{\left(b^3c - ab^2d + a^2be - \cdots\right.}{7b^4}$$

Mathematica [A] time = 0.0786584, size = 351, normalized size = 1.01

$$\frac{x^7\left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{7b^4} + \frac{ax^4\left(-a^2be + a^3f + ab^2d - b^3c\right)}{4b^5} - \frac{a^{7/3}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)\left(-a^2be + a^3f + ab \cdots\right.}{6b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^13)
/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/3)) +
 (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

Maple [A] time = 0.003, size = 592, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/b^4*a^3*d*x+1/b^3*a^2*c*x+1/10/b^3*x^10*a^2*f-1/10/b^2*x^10*a*e-1/7/b^4*
x^7*a^3*f-1/13/b^2*x^13*a*f-1/b^6*a^5*f*x+1/b^5*a^4*e*x-1/6*a^6/b^7/(1/b*a)
^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6*a^5/b^6/(1/b*a)^(2/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3*a^3/b^4/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*a^4/b^5/(1/b*a)^(2/3)*3^(1/2)*
```

```
arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/7/b*x^7*c+1/13/b*x^13*e+1/10/
b*x^10*d+1/3*a^6/b^7/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3*a^5/b^6/(1/b*a
)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/3*a^4/b^5/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))
*d-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/6*a^4/b^5/(1/b*a)^(2/3
)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6*a^3/b^4/(1/b*a)^(2/3)*ln(x^2-
(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*a^6/b^7/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/7/b^3*x^7*a^2*e-1/7/b^2*x^7*a*d+1/4/b
^5*x^4*a^4*f-1/4/b^4*x^4*a^3*e+1/4/b^3*x^4*a^2*d-1/4/b^2*x^4*a*c+1/16*f*x^1
6/b-1/3*a^5/b^6/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39803, size = 767, normalized size = 2.2

$$1365\, b^5 f x^{16} + 1680 \left(b^5 e - a b^4 f\right) x^{13} + 2184 \left(b^5 d - a b^4 e + a^2 b^3 f\right) x^{10} + 3120 \left(b^5 c - a b^4 d + a^2 b^3 e - a^3 b^2 f\right) x^7 - 5460 \left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(1365*b^5*f*x^16 + 1680*(b^5*e - a*b^4*f)*x^13 + 2184*(b^5*d - a*b^
4*e + a^2*b^3*f)*x^10 + 3120*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7
- 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 - 7280*sqrt(3)*(a^2*
b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*
(a/b)^(2/3) - sqrt(3)*a)/a) + 3640*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f
)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 7280*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 21840*(a^2*b^
3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

Sympy [A] time = 1.87413, size = 450, normalized size = 1.29

$$\operatorname{RootSum}\left(27t^3 b^{19} - a^{16} f^3 + 3a^{15} b e f^2 - 3a^{14} b^2 d f^2 - 3a^{14} b^2 e^2 f + 3a^{13} b^3 c f^2 + 6a^{13} b^3 d e f + a^{13} b^3 e^3 - 6a^{12} b^4 c e f - 3a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**19 - a**16*f**3 + 3*a**15*b*e*f**2 - 3*a**14*b**2*d*f**
2 - 3*a**14*b**2*e**2*f + 3*a**13*b**3*c*f**2 + 6*a**13*b**3*d*e*f + a**13*
```

```
b**3*e**3 - 6*a**12*b**4*c*e*f - 3*a**12*b**4*d**2*f - 3*a**12*b**4*d*e**2
+ 6*a**11*b**5*c*d*f + 3*a**11*b**5*c*e**2 + 3*a**11*b**5*d**2*e - 3*a**10*
b**6*c**2*f - 6*a**10*b**6*c*d*e - a**10*b**6*d**3 + 3*a**9*b**7*c**2*e + 3
*a**9*b**7*c*d**2 - 3*a**8*b**8*c**2*d + a**7*b**9*c**3, Lambda(_t, _t*log(
3*_t*b**6/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x))) + f*x**16/
(16*b) - x**13*(a*f - b*e)/(13*b**2) + x**10*(a**2*f - a*b*e + b**2*d)/(10*
b**3) - x**7*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(7*b**4) + x**4*(a**4*
f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(4*b**5) - x*(a**5*f - a**4*b*e + a*
*3*b**2*d - a**2*b**3*c)/b**6
```

Giac [A] time = 1.0624, size = 613, normalized size = 1.76

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2b^3c-\left(-ab^2\right)^{\frac{1}{3}}a^3b^2d-\left(-ab^2\right)^{\frac{1}{3}}a^5f+\left(-ab^2\right)^{\frac{1}{3}}a^4be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}a^2b^3c-\left(-ab^2\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d - (-a*b^2
)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/b^7 - 1/6*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3
*b^2*d - (-a*b^2)^(1/3)*a^5*f + (-a*b^2)^(1/3)*a^4*b*e)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/b^7 + 1/3*(a^3*b^13*c - a^4*b^12*d - a^6*b^10*f + a^5
*b^11*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^16) + 1/7280*(455*b^1
5*f*x^16 - 560*a*b^14*f*x^13 + 560*b^15*x^13*e + 728*b^15*d*x^10 + 728*a^2*
b^13*f*x^10 - 728*a*b^14*x^10*e + 1040*b^15*c*x^7 - 1040*a*b^14*d*x^7 - 104
0*a^3*b^12*f*x^7 + 1040*a^2*b^13*x^7*e - 1820*a*b^14*c*x^4 + 1820*a^2*b^13*
d*x^4 + 1820*a^4*b^11*f*x^4 - 1820*a^3*b^12*x^4*e + 7280*a^2*b^13*c*x - 728
0*a^3*b^12*d*x - 7280*a^5*b^10*f*x + 7280*a^4*b^11*x*e)/b^16
```

3.234 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=316

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}-\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^5}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f\right.}{6b^{17/3}}$$

```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b
*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) - (a^(5/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(17/3)) - (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(17/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Rubi [A] time = 0.306078, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}-\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^5}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f\right.}{6b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```


```
[Out] -(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + ((b
*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) - (a^(5/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(17/3)) - (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(17/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}dx = \frac{fx^{14}}{14b}+\frac{\int \frac{x^7\left(14bc+14bdx^3+14(be-af)x^6\right)}{a+bx^3}dx}{14b}$$

$$= \frac{fx^{14}}{14b}+\frac{\int\left(-\frac{14a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{14\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{b^3}+\frac{14\left(b^2d-abe+a^2f\right)x^7}{b^2}+\frac{14(be-\ldots}{b\ldots}\right.}{14b}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-abe+a^2f\ldots\right.}{8b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-abe+a^2f\ldots\right.}{8b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-abe+a^2f\ldots\right.}{8b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-abe+a^2f\ldots\right.}{8b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^5}{5b^4}+\frac{\left(b^2d-abe+a^2f\ldots\right.}{8b^3}$$

Mathematica [A] time = 0.0975826, size = 311, normalized size = 0.98

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}+\frac{ax^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2b^5}-\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+\ldots\right.}{6b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) + (
(b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) + (a^(5/3)*(-(b^3*c) + a*b^2*d
 - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b
^(17/3)) + (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(3*b^(17/3)) - (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

Maple [B] time = 0.003, size = 554, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/3*a^3/b^4/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/3*a^2/b^3/(1/b*a)^(1/3)*l
n(x+(1/b*a)^(1/3))*c-1/6*a^5/b^6/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))*f+1/6*a^4/b^5/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))
*e-1/6*a^3/b^4/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6*a^
2/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3*a^4/b^5/(1/
```

```
b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/11/b*x^11*e+1/8/b*x^8*d+1/5/b*x^5*c-1/3*
a^5/b^6*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1
/3*a^4/b^5*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
e-1/3*a^3/b^4*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))*d+1/3*a^2/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*c+1/3*a^5/b^6/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+1/2/b^5*x^2*a^4*f-1
/2/b^4*x^2*a^3*e-1/11/b^2*x^11*a*f+1/8/b^3*x^8*a^2*f-1/8/b^2*x^8*a*e-1/2/b^
2*x^2*a*c-1/5/b^4*x^5*a^3*f+1/5/b^3*x^5*a^2*e-1/5/b^2*x^5*a*d+1/2/b^3*x^2*a
^2*d+1/14*f*x^14/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44431, size = 726, normalized size = 2.3

$$\frac{660\,b^4fx^{14}+840\left(b^4e-ab^3f\right)x^{11}+1155\left(b^4d-ab^3e+a^2b^2f\right)x^8+1848\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^5-4620\left(ab^3c-a^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9240*(660*b^4*f*x^14 + 840*(b^4*e - a*b^3*f)*x^11 + 1155*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^8 + 1848*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 4620
*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2 + 3080*sqrt(3)*(a*b^3*c - a^2*
b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2
)^(1/3) - sqrt(3)*a)/a) + 1540*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2
/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 3080*(a*
b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^
(2/3)))/b^5
```

Sympy [A] time = 1.45072, size = 496, normalized size = 1.57

$$\operatorname{RootSum}\left(27t^3b^{17}-a^{14}f^3+3a^{13}bef^2-3a^{12}b^2df^2-3a^{12}b^2e^2f+3a^{11}b^3cf^2+6a^{11}b^3def+a^{11}b^3e^3-6a^{10}b^4cef-3a^{10}b\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*b**2*d*f**
2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11*b**3*d*e*f + a**11*
```

```
b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4*d**2*f - 3*a**10*b**4*d*e**2
+ 6*a**9*b**5*c*d*f + 3*a**9*b**5*c*e**2 + 3*a**9*b**5*d**2*e - 3*a**8*b**6
*c**2*f - 6*a**8*b**6*c*d*e - a**8*b**6*d**3 + 3*a**7*b**7*c**2*e + 3*a**7*
b**7*c*d**2 - 3*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t, _t*log(9*_t**
2*b**11/(a**9*f**2 - 2*a**8*b*e*f + 2*a**7*b**2*d*f + a**7*b**2*e**2 - 2*a*
*6*b**3*c*f - 2*a**6*b**3*d*e + 2*a**5*b**4*c*e + a**5*b**4*d**2 - 2*a**4*b
**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b) - x**11*(a*f - b*e)/(11*b
**2) + x**8*(a**2*f - a*b*e + b**2*d)/(8*b**3) - x**5*(a**3*f - a**2*b*e +
a*b**2*d - b**3*c)/(5*b**4) + x**2*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**
3*c)/(2*b**5)
```

Giac [A] time = 1.0825, size = 595, normalized size = 1.88

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}ab^3c-\left(-ab^2\right)^{\frac{2}{3}}a^2b^2d-\left(-ab^2\right)^{\frac{2}{3}}a^4f+\left(-ab^2\right)^{\frac{2}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}+\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}ab^3c-\left(-ab^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d - (-a*b^2)^
(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/b^7 + 1/6*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2
*d - (-a*b^2)^(2/3)*a^4*f + (-a*b^2)^(2/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/b^7 - 1/3*(a^2*b^12*c*(-a/b)^(1/3) - a^3*b^11*d*(-a/b)^(1
/3) - a^5*b^9*f*(-a/b)^(1/3) + a^4*b^10*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b^14) + 1/3080*(220*b^13*f*x^14 - 280*a*b^12*f*x^11
 + 280*b^13*x^11*e + 385*b^13*d*x^8 + 385*a^2*b^11*f*x^8 - 385*a*b^12*x^8*e
 + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 - 616*a^3*b^10*f*x^5 + 616*a^2*b^11*x^
5*e - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*x^2 + 1540*a^4*b^9*f*x^2 - 1540*a
^3*b^10*x^2*e)/b^14
```

3.235 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

Optimal. Leaf size=312

$$\frac{x^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^4}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{16/3}}-\frac{ax\left(a^2be+a^3(-f)-\right.}{b^5}$$

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1
6/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Rubi [A] time = 0.297768, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^4}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{16/3}}-\frac{ax\left(a^2be+a^3(-f)-\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```


```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1
6/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{13}}{13b} + \frac{\int \frac{x^6\left(13bc+13bdx^3+13(be-af)x^6\right)}{a+bx^3}\,dx}{13b}$$
$$= \frac{fx^{13}}{13b} + \frac{\int \left(-\frac{13a\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^4} + \frac{13\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{b^3} + \frac{13\left(b^2d-abe+a^2f\right)x^6}{b^2} + \frac{13(be-af)x^9}{b}\right.}{13b}$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4} + \frac{\left(b^2d-abe+a^2f\right)x^7}{7b^3}$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4} + \frac{\left(b^2d-abe+a^2f\right)x^7}{7b^3}$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4} + \frac{\left(b^2d-abe+a^2f\right)x^7}{7b^3}$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4} + \frac{\left(b^2d-abe+a^2f\right)x^7}{7b^3}$$
$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^4}{4b^4} + \frac{\left(b^2d-abe+a^2f\right)x^7}{7b^3}$$

Mathematica [A] time = 0.0934935, size = 306, normalized size = 0.98

$$\frac{x^4\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4b^4} + \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{6b^{16/3}} + \frac{ax\left(-a^2be+a^3f+ab^2\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e -
 a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(16/3
)) - (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(16/3)) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

Maple [B] time = 0.004, size = 544, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] -1/7/b^2*x^7*a*e-1/4/b^4*x^4*a^3*f-1/10/b^2*x^10*a*f+1/b^3*a^2*d*x-1/b^2*a*
c*x+1/4/b^3*x^4*a^2*e-1/4/b^2*x^4*a*d+1/b^5*a^4*f*x-1/b^4*a^3*e*x-1/6*a^2/b
^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/6*a^3/b^4/(1/b*a
)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/3*a^3/b^4/(1/b*a)^(2/3)*3
```

```
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*a^2/b^3/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*a^4/b^5/(1/b*a)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*a^5/b^6/(1/b*a
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3*a^4/b^5/(1/
b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/
3))*d+1/3*a^2/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c+1/6*a^5/b^6/(1/b*a)^(
2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/6*a^4/b^5/(1/b*a)^(2/3)*ln(x
^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/10/b*x^10*e+1/7/b*x^7*d+1/4/b*x^4*c-1
/3*a^5/b^6/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+1/7/b^3*x^7*a^2*f+1/13*f*x^1
3/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37597, size = 679, normalized size = 2.18

$$\frac{420\,b^4fx^{13} + 546\left(b^4e - ab^3f\right)x^{10} + 780\left(b^4d - ab^3e + a^2b^2f\right)x^7 + 1365\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)x^4 - 1820\,\sqrt{3}\left(ab^3c \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/5460*(420*b^4*f*x^13 + 546*(b^4*e - a*b^3*f)*x^10 + 780*(b^4*d - a*b^3*e
+ a^2*b^2*f)*x^7 + 1365*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820*
sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(a*b^3*c - a^2*b^2*d + a^3*b
*e - a^4*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) -
5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

Sympy [A] time = 1.82208, size = 411, normalized size = 1.32

$$\operatorname{RootSum}\left(27t^3b^{16} + a^{13}f^3 - 3a^{12}bef^2 + 3a^{11}b^2df^2 + 3a^{11}b^2e^2f - 3a^{10}b^3cf^2 - 6a^{10}b^3def - a^{10}b^3e^3 + 6a^9b^4cef + 3a^{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**16 + a**13*f**3 - 3*a**12*b*e*f**2 + 3*a**11*b**2*d*f**
2 + 3*a**11*b**2*e**2*f - 3*a**10*b**3*c*f**2 - 6*a**10*b**3*d*e*f - a**10*
b**3*e**3 + 6*a**9*b**4*c*e*f + 3*a**9*b**4*d**2*f + 3*a**9*b**4*d*e**2 - 6
```

```
*a**8*b**5*c*d*f - 3*a**8*b**5*c*e**2 - 3*a**8*b**5*d**2*e + 3*a**7*b**6*c*
*2*f + 6*a**7*b**6*c*d*e + a**7*b**6*d**3 - 3*a**6*b**7*c**2*e - 3*a**6*b**
7*c*d**2 + 3*a**5*b**8*c**2*d - a**4*b**9*c**3, Lambda(_t, _t*log(-3*_t*b**
5/(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c) + x))) + f*x**13/(13*b) - x*
*10*(a*f - b*e)/(10*b**2) + x**7*(a**2*f - a*b*e + b**2*d)/(7*b**3) - x**4*
(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(4*b**4) + x*(a**4*f - a**3*b*e + a
**2*b**2*d - a*b**3*c)/b**5
```

Giac [A] time = 1.07153, size = 541, normalized size = 1.73

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c-\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d-\left(-ab^2\right)^{\frac{1}{3}}a^4f+\left(-ab^2\right)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^3c-\left(-ab^2\right)^{\frac{1}{3}}a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*d - (-a*b^2)^(
1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/b^6 + 1/6*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*
d - (-a*b^2)^(1/3)*a^4*f + (-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/b^6 - 1/3*(a^2*b^11*c - a^3*b^10*d - a^5*b^8*f + a^4*b^9*e
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^13) + 1/1820*(140*b^12*f*x^1
3 - 182*a*b^11*f*x^10 + 182*b^12*x^10*e + 260*b^12*d*x^7 + 260*a^2*b^10*f*x
^7 - 260*a*b^11*x^7*e + 455*b^12*c*x^4 - 455*a*b^11*d*x^4 - 455*a^3*b^9*f*x
^4 + 455*a^2*b^10*x^4*e - 1820*a*b^11*c*x + 1820*a^2*b^10*d*x + 1820*a^4*b^
8*f*x - 1820*a^3*b^9*x*e)/b^13
```

3.236 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=279

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^4}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{14/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(11*b) + (a^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(1/3) + b^(1/3)*x]])/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```

Rubi [A] time = 0.273338, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2b^4}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{14/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```


```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^4) + ((b^2*d - a*b*e + a^2*f
)*x^5)/(5*b^3) + ((b*e - a*f)*x^8)/(8*b^2) + (f*x^11)/(11*b) + (a^(2/3)*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*b^(14/3)) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(1/3) + b^(1/3)*x]])/(3*b^(14/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3))
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{11}}{11b}+\frac{\int \frac{x^4\left(11bc+11bdx^3+11(be-af)x^6\right)}{a+bx^3}\,dx}{11b}$$

$$= \frac{fx^{11}}{11b}+\frac{\int \left(\frac{11\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^3}+\frac{11\left(b^2d-abe+a^2f\right)x^4}{b^2}+\frac{11(be-af)x^7}{b}+\frac{11\left(-ab^3c+a^2b^2d-a^3be+a^4f\right)\cdots}{b^3\left(a+bx^3\right)}\cdots\right.}{11b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}-\frac{\left(a\left(b^3c-\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}+\frac{\left(a^{2/3}\left(b^3c\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}+\frac{a^{2/3}\left(b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}+\frac{a^{2/3}\left(b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{2b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}+\frac{fx^{11}}{11b}+\frac{a^{2/3}\left(b^3c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.0915794, size = 266, normalized size = 0.95

$$660b^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+220a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)-440a^{2/3}\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3)*(b^2*d -
 a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120*b^(11/3)*f*x^11 - 4
40*Sqrt[3]*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^
3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*b^(14/3))
```

Maple [B] time = 0.004, size = 502, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/11*f*x^11/b-1/8/b^2*x^8*a*f+1/8/b*x^8*e+1/5/b^3*x^5*a^2*f-1/5/b^2*x^5*a*e
+1/5/b*x^5*d-1/2/b^4*x^2*a^3*f+1/2/b^3*x^2*a^2*e-1/2/b^2*x^2*a*d+1/2/b*x^2*
c-1/3*a^4/b^5/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+1/3*a^3/b^4/(1/b*a)^(1/3)
*ln(x+(1/b*a)^(1/3))*e-1/3*a^2/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d+1/3*
a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/6*a^4/b^5/(1/b*a)^(1/3)*ln(x^2-
(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/6*a^3/b^4/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(
1/3)*x+(1/b*a)^(2/3))*e+1/6*a^2/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))*d-1/6*a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3)
)*c+1/3*a^4/b^5*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))*f-1/3*a^3/b^4*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3
)*x-1))*e+1/3*a^2/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))*d-1/3*a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^
(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40658, size = 635, normalized size = 2.28

$$120\,b^3fx^{11}+165\left(b^3e-ab^2f\right)x^8+264\left(b^3d-ab^2e+a^2bf\right)x^5+660\left(b^3c-ab^2d+a^2be-a^3f\right)x^2-440\,\sqrt{3}\left(b^3c-ab^2d+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/1320*(120*b^3*f*x^11 + 165*(b^3*e - a*b^2*f)*x^8 + 264*(b^3*d - a*b^2*e +
 a^2*b*f)*x^5 + 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 - 440*sqrt(3)*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b
*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
- 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^
2/b^2)^(2/3)))/b^4
```

Sympy [A] time = 1.38275, size = 459, normalized size = 1.65

$$\operatorname{RootSum}\left(27t^3b^{14}+a^{11}f^3-3a^{10}bef^2+3a^9b^2df^2+3a^9b^2e^2f-3a^8b^3cf^2-6a^8b^3def-a^8b^3e^3+6a^7b^4cef+3a^7b^4d^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b**2*d*f**2
 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**3*d*e*f - a**8*b**3*
e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f + 3*a**7*b**4*d*e**2 - 6*a**6
*b**5*c*d*f - 3*a**6*b**5*c*e**2 - 3*a**6*b**5*d**2*e + 3*a**5*b**6*c**2*f
+ 6*a**5*b**6*c*d*e + a**5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4*b**7*c*d
**2 + 3*a**3*b**8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t**2*b**9/(
a**7*f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*a**4*b**3*c
*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*a**2*b**5*c*d +
 a*b**6*c**2) + x))) + f*x**11/(11*b) - x**8*(a*f - b*e)/(8*b**2) + x**5*(a
**2*f - a*b*e + b**2*d)/(5*b**3) - x**2*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)/(2*b**4)
```

Giac [A] time = 1.07492, size = 521, normalized size = 1.87

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\left(-ab^2\right)^{\frac{2}{3}}a^3f+\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6}-\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)
*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
```

```
a/b)^(1/3))/b^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*
b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/b^6 + 1/3*(a*b^10*c*(-a/b)^(1/3) - a^2*b^9*d*(-a/b)^(1/3) - a^4*b^
7*f*(-a/b)^(1/3) + a^3*b^8*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b^11) + 1/440*(40*b^10*f*x^11 - 55*a*b^9*f*x^8 + 55*b^10*x^8*e +
 88*b^10*d*x^5 + 88*a^2*b^8*f*x^5 - 88*a*b^9*x^5*e + 220*b^10*c*x^2 - 220*a
*b^9*d*x^2 - 220*a^3*b^7*f*x^2 + 220*a^2*b^8*x^2*e)/b^11
```

3.237 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{13/3}}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a$$

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)
/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```

Rubi [A] time = 0.26722, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1836, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^{13/3}}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```


```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^4)
/(4*b^3) + ((b*e - a*f)*x^7)/(7*b^2) + (f*x^10)/(10*b) + (a^(1/3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*b^(13/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(3*b^(13/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3))
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^{10}}{10b}+\frac{\int \frac{x^3\left(10bc+10bdx^3+10(be-af)x^6\right)}{a+bx^3}\,dx}{10b}$$

$$= \frac{fx^{10}}{10b}+\frac{\int \left(\frac{10\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^3}+\frac{10\left(b^2d-abe+a^2f\right)x^3}{b^2}+\frac{10(be-af)x^6}{b}+\frac{10\left(-ab^3c+a^2b^2d-a^3be+a^4 \cdots\right.}{b^3\left(a+bx^3\right)} \cdots}{10b}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\left(a\left(b^3c \cdots\right.\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\left(\sqrt[3]{a}\left(b \cdots\right.\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\sqrt[3]{a}\left(b^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}-\frac{\sqrt[3]{a}\left(b^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^4}{4b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^{10}}{10b}+\frac{\sqrt[3]{a}\left(b^3 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.0911862, size = 264, normalized size = 0.96

$$-70\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)+420\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+140\sqrt[3]{a}\log\left(\sqrt[3]{\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*(b^2*d - a
*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(10/3)*f*x^10 - 140*S
qrt[3]*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*b^(13/3))
```

Maple [B] time = 0.003, size = 492, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/10*f*x^10/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/4/b^3*x^4*a^2*f-1/4/b^2*x^4*a*e
+1/4/b*x^4*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+c*x/b+1/3*a^4/b^5/(1/b
*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3
))*e+1/3*a^2/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/3*a/b^2/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))*c-1/6*a^4/b^5/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))*f+1/6*a^3/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/
3))*e-1/6*a^2/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6
*a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*a^4/b^5/(1
/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3*a^3/b^4
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3*a^2/
b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3*a
/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29716, size = 566, normalized size = 2.07

$$42\,b^3fx^{10}+60\left(b^3e-ab^2f\right)x^7+105\left(b^3d-ab^2e+a^2bf\right)x^4-140\,\sqrt{3}\left(b^3c-ab^2d+a^2be-a^3f\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{}}{3\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(42*b^3*f*x^10 + 60*(b^3*e - a*b^2*f)*x^7 + 105*(b^3*d - a*b^2*e + a^
2*b*f)*x^4 - 140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*ar
ctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

Sympy [A] time = 1.71881, size = 371, normalized size = 1.35

$$\mathrm{RootSum}\left(27t^3b^{13} - a^{10}f^3 + 3a^9bef^2 - 3a^8b^2df^2 - 3a^8b^2e^2f + 3a^7b^3cf^2 + 6a^7b^3def + a^7b^3e^3 - 6a^6b^4cef - 3a^6b^4d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b**2*d*f**2
- 3*a**8*b**2*e**2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3*d*e*f + a**7*b**3*e
**3 - 6*a**6*b**4*c*e*f - 3*a**6*b**4*d**2*f - 3*a**6*b**4*d*e**2 + 6*a**5*
b**5*c*d*f + 3*a**5*b**5*c*e**2 + 3*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f -
 6*a**4*b**6*c*d*e - a**4*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d*
*2 - 3*a**2*b**8*c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f
- a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**10/(10*b) - x**7*(a*f - b*e)/
(7*b**2) + x**4*(a**2*f - a*b*e + b**2*d)/(4*b**3) - x*(a**3*f - a**2*b*e +
 a*b**2*d - b**3*c)/b**4
```

Giac [A] time = 1.08937, size = 467, normalized size = 1.7

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^5} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}al\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3
)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/b^5 + 1/3*(a*b^9*c - a^2*b^8*d - a^4*b^6*f + a^3*b^7*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 - 20*a*b^8*f
*x^7 + 20*b^9*x^7*e + 35*b^9*d*x^4 + 35*a^2*b^7*f*x^4 - 35*a*b^8*x^4*e + 14
0*b^9*c*x - 140*a*b^8*d*x - 140*a^3*b^6*f*x + 140*a^2*b^7*x*e)/b^10
```

3.238 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

Optimal. Leaf size=245

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6\sqrt[3]{a}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3\sqrt[3]{a}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)
/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/3)) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
a^(1/3)*b^(11/3))
```

Rubi [A] time = 0.215567, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6\sqrt[3]{a}b^{11/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3\sqrt[3]{a}b^{11/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```


```
[Out] ((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^8)
/(8*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(11/3)) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(11/3)) + ((b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
a^(1/3)*b^(11/3))
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx = \frac{fx^8}{8b}+\frac{\int \frac{x\left(8bc+8bdx^3+8(be-af)x^6\right)}{a+bx^3}\,dx}{8b}$$

$$= \frac{fx^8}{8b}+\frac{\int \left(\frac{8\left(b^2d-abe+a^2f\right)x}{b^2}+\frac{8(be-af)x^4}{b}+\frac{8\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^2\left(a+bx^3\right)}\right)dx}{8b}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{x}{a+bx^3}\,dx}{b^3}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b^{10/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3\sqrt[3]{a}b^{11/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3\sqrt[3]{a}b^{11/3}}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{\sqrt{3}\sqrt[3]{a}b^{11/3}}$$

Mathematica [A] time = 0.160058, size = 231, normalized size = 0.94

$$\frac{\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt[3]{a}}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\sqrt[3]{a}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\sqrt[3]{a}}+60b}{120b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*x^5 + 15*b
^(8/3)*f*x^8 + (40*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(12
0*b^(11/3))
```

Maple [B] time = 0.003, size = 450, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/8*f*x^8/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/2/b^3*x^2*a^2*f-1/2/b^2*x^2*a*e+1
/2*d*x^2/b+1/3/b^4/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a^3*f-1/3/b^3/(1/b*a)^
(1/3)*ln(x+(1/b*a)^(1/3))*a^2*e+1/3/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a
*d-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b^4/(1/b*a)^(1/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^3*f+1/6/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1
/3)*x+(1/b*a)^(2/3))*a^2*e-1/6/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))*a*d+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c
-1/3/b^4*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^
3*f+1/3/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*a^2*e-1/3/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-
1))*a*d+1/3/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51902, size = 1289, normalized size = 5.26

$$\left[\frac{15\,ab^4fx^8 + 24\left(ab^4e - a^2b^3f\right)x^5 + 60\left(ab^4d - a^2b^3e + a^3b^2f\right)x^2 - 60\sqrt{\frac{1}{3}}\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b
^3*e + a^3*b^2*f)*x^2 - 60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*
(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/
3)*x)/(b*x^3 + a)) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*l
og(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5), 1/120*(15*a*
b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e + a^3*b^
2*f)*x^2 - 120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((
a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3
)/a)/b) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b^2*x^2
- (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5)]
```

Sympy [A] time = 1.47288, size = 422, normalized size = 1.72

$$\mathrm{RootSum}\left(27t^3ab^{11} - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2
 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*
e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4
*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f
- 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d
**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a*b**7/(a**6*f
**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2
*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**
2) + x))) + f*x**8/(8*b) - x**5*(a*f - b*e)/(5*b**2) + x**2*(a**2*f - a*b*e
 + b**2*d)/(2*b**3)
```

Giac [A] time = 1.08358, size = 466, normalized size = 1.9

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}a^3f + \left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^5} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3
)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a*b^5) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d -
 (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a*b^5) - 1/3*(b^8*c*(-a/b)^(1/3) - a*b^7*d*(-a/b)^(1/3) - a^
3*b^5*f*(-a/b)^(1/3) + a^2*b^6*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a*b^8) + 1/40*(5*b^7*f*x^8 - 8*a*b^6*f*x^5 + 8*b^7*x^5*e + 20*
b^7*d*x^2 + 20*a^2*b^5*f*x^2 - 20*a*b^6*x^2*e)/b^8
```

3.239 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$

Optimal. Leaf size=240

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{2/3}b^{10/3}}-\frac{\tan^{-1}}{}$$

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b)
 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*b^(10/3))
```

Rubi [A] time = 0.152689, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1887, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{2/3}b^{10/3}}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b)
 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*b^(10/3))
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^3}{b^2}+\frac{fx^6}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\left(a+bx^3\right)}\right)dx$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^3}+\frac{\left(b^3c\right.}{}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(b}{}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(b}{}$$

$$= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{(}{}$$

Mathematica [A] time = 0.147161, size = 229, normalized size = 0.95

$$\frac{\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{2/3}}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{2/3}}+84\sqrt[3]{}}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^4 + 12*b^(
7/3)*f*x^7 + (28*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b*e
```

```
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b
^(10/3))
```

Maple [B] time = 0.004, size = 442, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x)
```

```
[Out] 1/7*f*x^7/b-1/4/b^2*x^4*a*f+1/4/b*x^4*e+1/b^3*a^2*f*x-1/b^2*a*e*x+d*x/b-1/3
/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^3*f+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/
b*a)^(1/3))*a^2*e-1/3/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a*d+1/3*c/b/(1/
b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/6/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))*a^3*f-1/6/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))*a^2*e+1/6/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a*
d-1/6*c/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3/b^4/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^3*f+1/3/b^3/(1
/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*e-1/3/b^2
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*d+1/3*c/
b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36121, size = 1350, normalized size = 5.62

$$\left[\frac{12\,a^2b^3fx^7 + 21\left(a^2b^3e - a^3b^2f\right)x^4 - 42\sqrt{\frac{1}{3}}\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2 \cdots\right.}{bx^3+\cdots}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 - 42*sqrt(1/3)*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*
```

```
x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*
x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-
a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log
(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a^4*b*f)*x)/(a^2*b^4
), 1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 + 84*sqrt(1/3)*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(s
qrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^
2) - 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-
a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a
^4*b*f)*x)/(a^2*b^4)]
```

Sympy [A] time = 1.89, size = 340, normalized size = 1.42

$$\operatorname{RootSum}\left(27t^3a^2b^{10} + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*
*3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*
f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b) - x**4*(a*f - b*e)/
(4*b**2) + x*(a**2*f - a*b*e + b**2*d)/b**3
```

Giac [A] time = 1.07513, size = 414, normalized size = 1.72

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)
*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(a*b^4) + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d -
(-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/(a*b^4) - 1/3*(b^7*c - a*b^6*d - a^3*b^4*f + a^2*b^5*e)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*b^6*f*x^7 - 7*a*b^5*f*x
^4 + 7*b^6*x^4*e + 28*b^6*d*x + 28*a^2*b^4*f*x - 28*a*b^5*x*e)/b^7
```

3.240 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}}+\tan^{-1}$$

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

Rubi [A] time = 0.192612, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{4/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{4/3}b^{8/3}}+\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```



```
[Out] -(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)}dx = \int \left(\frac{c}{ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{ab^2\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{x}{a+bx^3}dx}{ab^2}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{3a^{4/3}b^{7/3}}-\frac{\left(b^3c-ab^2d+a^2be-\right.}{3}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}}-\frac{\left(b^3c-ab^2d+a^2b\right.}{}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}}-\frac{\left(b^3c-ab^2d+a^2b\right.}{}$$

$$= -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{8/3}}+\frac{\left(b^3c-ab^2d+a^2\right.}{}$$

Mathematica [A] time = 0.128406, size = 224, normalized size = 0.99

$$\frac{-5x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+10x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+10\sqrt{}}{30a^{4/3}b^{8/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-30*a^(1/3)*b^(8/3)*c + 15*a^(4/3)*b^(2/3)*(b*e - a*f)*x^3 + 6*a^(4/3)*b^(
5/3)*f*x^6 + 10*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*L
og[a^(1/3) + b^(1/3)*x] - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*Log[a^(2/
```

```
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(4/3)*b^(8/3)*x)
```

Maple [B] time = 0.004, size = 419, normalized size = 1.9

$$\frac{fx^5}{5b} - \frac{ax^2f}{2b^2} + \frac{ex^2}{2b} - \frac{a^2f}{3b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{ae}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{d}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{c}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x)
```

```
[Out] 1/5*f*x^5/b-1/2/b^2*x^2*a*f+1/2*e*x^2/b-1/3/b^3*a^2/(1/b*a)^(1/3)*ln(x+(1/b
*a)^(1/3))*f+1/3/b^2*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e-1/3*d/b/(1/b*a)^
(1/3)*ln(x+(1/b*a)^(1/3))+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/6/b^3
*a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/6/b^2*a/(1/b*a
)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/6*d/b/(1/b*a)^(1/3)*ln(x^
2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))*c+1/3/b^3*a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))*f-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/
(1/b*a)^(1/3)*x-1))*e+1/3*d*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))-1/3/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a
)^(1/3)*x-1))*c-c/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44421, size = 1260, normalized size = 5.55

$$\left[\frac{6\,a^2b^3fx^6 - 30\,ab^4c + 15\left(a^2b^3e - a^3b^2f\right)x^3 - 15\sqrt{\frac{1}{3}}\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)x\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\sqrt{\frac{1}{3}}\left(abx + 2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 15*s
qrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt((-a*b^2)^(1/3)/
```

```
a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)
^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b
^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x), 1/30*(6*a^2*b^3*f*x^6 -
 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 30*sqrt(1/3)*(a*b^4*c - a^2*
b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b*x -
(-a*b^2)^(1/3)))/(a^2*b^4*x)]
```

Sympy [A] time = 2.51496, size = 406, normalized size = 1.79

$$\operatorname{RootSum}\left(27t^3a^4b^8 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**4*b**8 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f*
*2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**
3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a*
*4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*
f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c
*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**3*b**5/(a
**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*
f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**
6*c**2) + x))) + f*x**5/(5*b) - x**2*(a*f - b*e)/(2*b**2) - c/(a*x)
```

Giac [A] time = 1.09501, size = 428, normalized size = 1.89

$$-\frac{c}{ax} + \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -c/(a*x) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1
/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^
2) + 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^
(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^2*b^4) + 1/10*(2*b^4*f*x^5 - 5*a*b^3*f*x^2 + 5*b^4*x^2*
e)/b^5 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3
)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a^2*b^4)
```

3.241 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=224

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] -c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

Rubi [A] time = 0.169822, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```


```
[Out] -c/(2*a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx = \int \left(\frac{be-af}{b^2}+\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^3}\,dx}{ab^2}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}b^2}-\frac{\left(b^3c-ab^2d+a^2be\right.}{}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}}+\frac{\left(b^3c-ab^2d+a^2\right.}{}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}}+\frac{\left(b^3c-ab^2d+a^2\right.}{}$$

$$= -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{7/3}}-\frac{\left(b^3c-ab^2d+a\right.}{}$$

Mathematica [A] time = 0.117203, size = 218, normalized size = 0.97

$$\frac{1}{12}\left(\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{5/3}b^{7/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{5/3}b^{7/3}}+\frac{4\sqrt{3}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*Sqrt[3]*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(
```

```
a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) +
b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3)))/12
```

Maple [B] time = 0.005, size = 414, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x)
```

```
[Out] 1/4*f*x^4/b-1/b^2*a*f*x+e*x/b+1/3/b^3*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))
*f-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/3/b/(1/b*a)^(2/3)*ln(x+(
1/b*a)^(1/3))*d-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b^3*a^2/(1/b*
a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6/b^2*a/(1/b*a)^(2/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1
/3)*x+(1/b*a)^(2/3))*d+1/6/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*c+1/3/b^3*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))*f-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1
/3)*x-1))*e+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)
*x-1))*d-1/3/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-
1))*c-1/2*c/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53733, size = 1280, normalized size = 5.71

$$\left[\frac{3\,a^{3}b^{2}fx^{6}-6\,a^{2}b^{3}c-6\,\sqrt{\frac{1}{3}}\left(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x^{2}\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\left(a^{2}b\right)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}\cdots}{bx^{3}+a}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a
^3*b^2*e - a^4*b*f)*x^2*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(
```

```
1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(
2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2), 1/12*(3*a^3*b^2*f*x^6
- 6*a^2*b^3*c - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
2*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2
*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*b)^(2/3)) +
12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2)]
```

Sympy [A] time = 3.05793, size = 326, normalized size = 1.46

$$\text{RootSum}\left(27t^3a^5b^7 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f*
*2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**
3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a*
*4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*
f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c
*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**2*b**2/(a**3
*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**4/(4*b) - x*(a*f - b*e)/b*
*2 - c/(2*a*x**2)
```

Giac [A] time = 1.07595, size = 378, normalized size = 1.69

$$\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} - \frac{c}{2\,ax^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}\right.}{3\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/(a^2*b^2) - 1/2*c/(a*x^2) - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^
2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/4*(b^3*f*x^4 - 4
*a*b^2*f*x + 4*b^3*x*e)/b^4 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*
b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)} dx$

Optimal. Leaf size=227

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{7/3}b^{5/3}}-\frac{\tan^{-1}\left(\cdots\right.}{}$$

```
[Out] -c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rubi [A] time = 0.185703, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{7/3}b^{5/3}}-\frac{\tan^{-1}\left(\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] -c/(4*a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^5}+\frac{-bc+ad}{a^2x^2}+\frac{fx}{b}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a^2b\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{x}{a+bx^3}\,dx}{a^2b}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}b^{4/3}}+\frac{\left(b^3c-ab^2d+a^2be-a\right.}{3a^7}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}+\frac{\left(b^3c-ab^2d+a^2be\right.}{}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}}+\frac{\left(b^3c-ab^2d+a^2be\right.}{}$$

$$= -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}b^{5/3}}-\frac{\left(b^3c-ab^2d+a^2be\right.}{3}$$

Mathematica [A] time = 0.111304, size = 220, normalized size = 0.97

$$\frac{1}{12}\left(\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{7/3}b^{5/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^{7/3}b^{5/3}}+\frac{4\sqrt{3}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*Sqrt[3]*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
```

```
]])/(a^(7/3)*b^(5/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/(a^(7/3)*b^(5/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(5/3)))/12
```

Maple [B] time = 0.007, size = 412, normalized size = 1.8

$$\frac{fx^2}{2b}+\frac{af}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{d}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{bc}{3a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{af}{6b^2}\ln\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x)
```

```
[Out] 1/2*f*x^2/b+1/3/b^2*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-1/3/b/(1/b*a)^(1/
3)*ln(x+(1/b*a)^(1/3))*e+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/3*b/a^
2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*f+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))*e-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6
*b/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/b^2*a*3^(1
/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3/b*3^(1/2)
/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3/a*3^(1/2)/(1
/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*b/a^2*3^(1/2)/(
1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/4*c/a/x^4-d/a/x+
1/a^2/x*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38581, size = 1251, normalized size = 5.51

$$\left[\frac{6a^3b^2fx^6-6\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^4\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab-3\sqrt{\frac{1}{3}}\left(abx+2\left(ab^2\right)^{\frac{2}{3}}x^2-\left(ab^2\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}-3\left(ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^3*b^2*f*x^6 - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^4*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x
+ 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3
)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*
c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4), 1/12*(6*a^3*b^2*f*x^6 - 12
*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt((a*b^2)^(1/
3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) +
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x^2 - (a*b^
2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^
2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4*c - a^2*b^3
*d)*x^3)/(a^3*b^3*x^4)]
```

Sympy [A] time = 8.04656, size = 411, normalized size = 1.81

$$\operatorname{RootSum}\left(27t^3a^7b^5 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f*
*2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**
3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a*
*4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*
f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c
*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**5*b**3/(a
**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*
f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**
6*c**2) + x))) + f*x**2/(2*b) - (a*c + x**3*(4*a*d - 4*b*c))/(4*a**2*x**4)
```

Giac [A] time = 1.08082, size = 417, normalized size = 1.84

$$\frac{fx^2}{2b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)
^(1/3) + a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3
*b) - 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)
^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^3*b^3) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*
b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/4*(4*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^2*x^4
)
```

3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$

Optimal. Leaf size=225

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{\tan^{-1}}{}$$

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Rubi [A] time = 0.169774, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```


```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx &= \int \left(\frac{f}{b}+\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\left(a+bx^3\right)}\right)dx \\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^3}\,dx}{a^2b} \\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}b}+\frac{\left(b^3c-ab^2d+a^2be-a^3\right.}{3a} \\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}}-\frac{\left(b^3c-ab^2d+a^2be-\right.}{6} \\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}}-\frac{\left(b^3c-ab^2d+a^2be-\right.}{} \\
&= -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}b^{4/3}}+\frac{\left(b^3c-ab^2d+a^2be\right.}{3a}
\end{aligned}$$

Mathematica [A] time = 0.08175, size = 220, normalized size = 0.98

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b + ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(8/
3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x]
```

```
)/(3*a^(8/3)*b^(4/3)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

Maple [B] time = 0.005, size = 410, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x)
```

```
[Out] f*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+1/3/b/(1/b*a)^(2/3)*ln(
x+(1/b*a)^(1/3))*e-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+1/3*b/a^2/(1/b
*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3
)*x+(1/b*a)^(2/3))*f-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/
3))*e+1/6/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/6*b/a^2
/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/b^2*a/(1/b*a)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3/b/(1/b*a)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3/a/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*b/a^2/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/5*c/a/x^5-1/2*d/a/x^2+
1/2/a^2/x^2*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30809, size = 1312, normalized size = 5.83

$$\left[\frac{30\,a^4bfx^6 - 15\sqrt{\frac{1}{3}}\left(ab^4c - a^2b^3d + a^3b^2e - a^4bf\right)x^5\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\left(-\cdots\right.}}{bx^3+a}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/30*(30*a^4*b*f*x^6 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^5*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^
2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^
```

```
2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b
)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(2/3))
 - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5), 1/30*(30*a^
4*b*f*x^6 + 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^5*sq
rt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)
*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-
a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^(
2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5)]
```

Sympy [A] time = 9.84072, size = 328, normalized size = 1.46

$$\operatorname{RootSum}\left(27t^3a^8b^4 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f*
*2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**
3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a*
*4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*
f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c
*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(a**3*f
 - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b - (2*a*c + x**3*(5*a*d - 5*
b*c))/(10*a**2*x**5)
```

Giac [A] time = 1.07951, size = 370, normalized size = 1.64

$$\frac{fx}{b} - \frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2b\right.}{3\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] f*x/b - 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^3*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a
*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/6*((-a*b^2)^(1/3)*b^3*c -
(-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2) + 1/10*(5*b*c*x^3 - 5*a*d*x
^3 - 2*a*c)/(a^2*x^5)
```

3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$

Optimal. Leaf size=242

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{10/3}b^{2/3}}+\frac{\tan^{-1}}{}$$

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x) +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3)*
b^(2/3))
```

Rubi [A] time = 0.187964, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{10/3}b^{2/3}}+\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x) +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)*b^(2/3)) - ((b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3)*
b^(2/3))
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^5}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{a^3\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{x}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}\sqrt[3]{b}}-\frac{\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}}-\frac{\left(b^3c-a\right.}{}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}}-\frac{\left(b^3c-a\right.}{}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}b^{2/3}}+\frac{\left(b^3c-\right.}{}$$

Mathematica [A] time = 0.111245, size = 231, normalized size = 0.95

$$\frac{\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b^{2/3}}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{2/3}}+\frac{28\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{2/3}}-\frac{84}{}}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)*(b^2*c -
a*b*d + a^2*e))/x + (28*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[
```

```
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(
84*a^(10/3))
```

Maple [B] time = 0.007, size = 440, normalized size = 1.8

$$-\frac{f}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{e}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{bd}{3a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{b^2c}{3a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{f}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x)
```

```
[Out] -1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)
^(1/3))*e-1/3/a^2*b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d+1/3/a^3*b^2/(1/b*a)
^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))*f-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1
/6/a^2*b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/6/a^3*b^2/
(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*3^(1/2)/b/(1/b*a)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3/a*3^(1/2)/(1/b*a)^(1
/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/a^2*3^(1/2)*b/(1/b*a)^(
1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3/a^3*3^(1/2)*b^2/(1/b*a
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/7*c/a/x^7-1/4/a/x^4*d
+1/4/a^2/x^4*b*c-e/a/x+1/a^2/x*b*d-1/a^3/x*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43548, size = 1359, normalized size = 5.62

$$\left[-\frac{42\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^7\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+14\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt((
-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/
3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*
x^3 + a)) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b
^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*c - a^
2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)*x^3)/(
a^4*b^2*x^7), -1/84*(84*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*
f)*x^7*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sq
rt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^
(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 8
4*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^
3*b^2*d)*x^3)/(a^4*b^2*x^7)]
```

Sympy [A] time = 30.4187, size = 432, normalized size = 1.79

$$\operatorname{RootSum}\left(27t^3a^{10}b^2 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4cef + 3a^5b^4d^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b*
*3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a
**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2
*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*
c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*b/(a**
6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f
- 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*
c**2) + x))) - (4*a**2*c + x**6*(28*a**2*e - 28*a*b*d + 28*b**2*c) + x**3*(
7*a**2*d - 7*a*b*c))/(28*a**3*x**7)
```

Giac [A] time = 1.07466, size = 444, normalized size = 1.83

$$\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(-ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3) + a^2*b
*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/3*sqrt(3)*
((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + (-a
*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a^4*b^2) - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(
2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^4*b^2) - 1/28*(28*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*x^6*e - 7*a*b*c*x
^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^3*x^7)
```

3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)} dx$

Optimal. Leaf size=244

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x^2
) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11
/3)*b^(1/3))
```

Rubi [A] time = 0.174033, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```


```
[Out] -c/(8*a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*x^2
) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11
/3)*b^(1/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^9}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^3}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{11/3}}-\frac{\left(b^3c-ab^2d\right.}{}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}}-\frac{\left(b^3c-a\right.}{}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}}+\frac{\left(b^3c-a\right.}{}$$

$$= -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{\left(b^3c-\right.}{}$$

Mathematica [A] time = 0.111757, size = 231, normalized size = 0.95

$$\frac{\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt[3]{b}}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt[3]{b}}-60a}{120a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

```
[Out] ((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)*(b^2*c -
a*b*d + a^2*e))/x^2 + (40*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTa
```

```
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-(b^3*c) + a*b^2*d -
 a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))
/(120*a^(11/3))
```

Maple [B] time = 0.008, size = 441, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^
(1/3))*e+1/3/a^2*b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/3/a^3*b^2/(1/b*a)^
(2/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))*f+1/6/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/
6/a^2*b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6/a^3*b^2/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3/a/(1/b*a)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/a^2*b/(1/b*a)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3/a^3*b^2/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/8*c/a/x^8-1/5/a/x^5*d+
1/5/a^2/x^5*b*c-1/2/a/x^2*e+1/2/a^2/x^2*b*d-1/2/a^3/x^2*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37799, size = 1353, normalized size = 5.55

$$\left[-\frac{60\sqrt{\frac{1}{3}}\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^8\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-20\left(b^3\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*sqrt(
-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*
(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) + 60*(a^2*b^3*c - a^3
*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4*b*d)*x^3)/(a^5*b*x
^8), -1/120*(120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*
sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*
b)^(2/3)*x^8*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) +
60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4
*b*d)*x^3)/(a^5*b*x^8)]
```

Sympy [A] time = 52.7116, size = 348, normalized size = 1.43

$$\operatorname{RootSum}\left(27t^3a^{11}b - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4d^2f - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**11*b - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f**2
 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b**3*
e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6*a**4
*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c**2*f
- 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b**7*c*d
**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(3*_t*a**4/(a**3*f - a*
*2*b*e + a*b**2*d - b**3*c) + x))) - (5*a**2*c + x**6*(20*a**2*e - 20*a*b*d
 + 20*b**2*c) + x**3*(8*a**2*d - 8*a*b*c))/(40*a**3*x**8)
```

Giac [A] time = 1.07419, size = 401, normalized size = 1.64

$$\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c - \left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{1}{3}}a^3f + \left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\mathrm{arc}}{3\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/
3)))/a^4 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a
*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/40*(20*b^2*c*x^6 - 20*a*b*d*x^6 + 20*a^
2*x^6*e - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)
```

3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{13/3}}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2\right.}{3}$$

```
[Out] -c/(10*a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3*x
^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^(1/3)*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(13/3)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

Rubi [A] time = 0.222478, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{13/3}}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```


```
[Out] -c/(10*a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^3*x
^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^(1/3)*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(13/3)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{11}} + \frac{-bc+ad}{a^2x^8} + \frac{b^2c-abd+a^2e}{a^3x^5} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2} - \frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^4\left(a+bx^3\right.}\right.$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} + \frac{\left(b\left(b^3c-ab^2d+a^2be-\right.\right.}{a^4}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\left(b^{2/3}\left(b^3c-ab^2d+a^2be\right.\right.}{3a^{13/}}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt[3]{b}\left(b^3c-ab^2d+a^2be-\right.}{3a^{13}}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt[3]{b}\left(b^3c-ab^2d+a^2be-\right.}{3a^{13}}$$

$$= -\frac{c}{10ax^{10}} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{4a^3x^4} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt[3]{b}\left(b^3c-ab^2d+a^2be-\right.}{\sqrt{3}a}$$

Mathematica [A] time = 0.112328, size = 266, normalized size = 0.96

$$\frac{70\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)+\frac{420\sqrt[3]{a}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{x}+140\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\right.}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

```
[Out] ((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4/3)*(b^2*c
 - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/
x - 140*Sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*
```

```
b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))
```

Maple [B] time = 0.009, size = 491, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x)
```

```
[Out] 1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-1/3*b/a^2/(1/b*a)^(1/3)*ln(x+(1/b
*a)^(1/3))*e+1/3*b^2/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/3*b^3/a^4/(1
/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))*f+1/6*b/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*e-1/6*b^2/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1
/6*b^3/a^4/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/a*3^(1
/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3*b/a^2*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*b^2/a^3*
3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*b^3/a
^4*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/10*c
/a/x^10-1/7/a/x^7*d+1/7/a^2/x^7*b*c-1/4/a/x^4*e+1/4/a^2/x^4*b*d-1/4/a^3/x^4
*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3/x*b^2*d+1/a^4/x*b^3*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40998, size = 597, normalized size = 2.16

$$140\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + 70\left(b^3c - ab^2d + a^2be - a^3f\right)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\Bigg(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*arc
tan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) -
140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x + a*(b/a)^
(2/3)) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 105*(a*b^2*c - a^2*b
*d + a^3*e)*x^6 - 42*a^3*c + 60*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

Sympy [A] time = 142.622, size = 473, normalized size = 1.71

$$\mathrm{RootSum}\Big(27t^3a^{13} - a^9bf^3 + 3a^8b^2ef^2 - 3a^7b^3df^2 - 3a^7b^3e^2f + 3a^6b^4cf^2 + 6a^6b^4def + a^6b^4e^3 - 6a^5b^5cef - 3a^5b^5d^2f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**13 - a**9*b*f**3 + 3*a**8*b**2*e*f**2 - 3*a**7*b**3*d*f
**2 - 3*a**7*b**3*e**2*f + 3*a**6*b**4*c*f**2 + 6*a**6*b**4*d*e*f + a**6*b*
*4*e**3 - 6*a**5*b**5*c*e*f - 3*a**5*b**5*d**2*f - 3*a**5*b**5*d*e**2 + 6*a
**4*b**6*c*d*f + 3*a**4*b**6*c*e**2 + 3*a**4*b**6*d**2*e - 3*a**3*b**7*c**2
*f - 6*a**3*b**7*c*d*e - a**3*b**7*d**3 + 3*a**2*b**8*c**2*e + 3*a**2*b**8*
c*d**2 - 3*a*b**9*c**2*d + b**10*c**3, Lambda(_t, _t*log(9*_t**2*a**9/(a**6
*b*f**2 - 2*a**5*b**2*e*f + 2*a**4*b**3*d*f + a**4*b**3*e**2 - 2*a**3*b**4*
c*f - 2*a**3*b**4*d*e + 2*a**2*b**5*c*e + a**2*b**5*d**2 - 2*a*b**6*c*d + b
**7*c**2) + x))) - (14*a**3*c + x**9*(140*a**3*f - 140*a**2*b*e + 140*a*b**
2*d - 140*b**3*c) + x**6*(35*a**3*e - 35*a**2*b*d + 35*a*b**2*c) + x**3*(20
*a**3*d - 20*a**2*b*c))/(140*a**4*x**10)
```

Giac [A] time = 1.08536, size = 508, normalized size = 1.83

$$-\frac{\left(b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^5} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d - \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) - a^3*b*f*(-a/b)^(1/3) + a^
2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*sqr
t(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)*a^3*f
+ (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a^5*b) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2
)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^5*b) + 1/140*(140*b^3*c*x^9 - 140*a*b^2*d*x^9 - 140*a^3*f*x^9 + 14
0*a^2*b*x^9*e - 35*a*b^2*c*x^6 + 35*a^2*b*d*x^6 - 35*a^3*x^6*e + 20*a^2*b*c
*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^4*x^10)
```

3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$

Optimal. Leaf size=280

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{2a^4x^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a$$

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x
^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^(2/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

Rubi [A] time = 0.199354, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{2a^4x^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```


```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x
^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^(2/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^9} + \frac{b^2c-abd+a^2e}{a^3x^6} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^3} - \frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^4\left(a+bx^3\right)}\right.$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{\left(b\left(b^3c-ab^2d+a^2be-\right.\right.}{a^4}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{\left(b\left(b^3c-ab^2d+a^2be-\right.\right.}{3a^{14/3}}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}\left(b^3c-ab^2d+a^2be-\right.}{3a^{14}}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} + \frac{b^{2/3}\left(b^3c-ab^2d+a^2be-\right.}{3a^{14}}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{8a^2x^8} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} - \frac{b^{2/3}\left(b^3c-ab^2d+a^2be-\right.}{\sqrt{3}a}$$

Mathematica [A] time = 0.114654, size = 266, normalized size = 0.95

$$\frac{\frac{660a^{2/3}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{x^2} + 220b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right) + 440b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

```
[Out] ((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^(5/3)*(b^2
*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/x^2 - 440*Sqrt[3]*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 -
```

```
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*a^(14/3))
```

Maple [B] time = 0.008, size = 493, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x)
```

```
[Out] -1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+1/3*b/a^2/(1/b*a)^(2/3)*ln(x+(1/
b*a)^(1/3))*e-1/3*b^2/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+1/3*b^3/a^4/(
1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c+1/6/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*f-1/6*b/a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))*e+1/6*b^2/a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-
1/6*b^3/a^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/a/(1/
b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3*b/a^2/(1
/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*b^2/a^3
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*b^3/
a^4/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/11*
c/a/x^11-1/8/a/x^8*d+1/8/a^2/x^8*b*c-1/5/a/x^5*e+1/5/a^2/x^5*b*d-1/5/a^3/x^
5*b^2*c-1/2/a/x^2*f+1/2/a^2/x^2*b*e-1/2/a^3/x^2*b^2*d+1/2/a^4/x^2*b^3*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39613, size = 667, normalized size = 2.38

$$-\frac{440\sqrt{3}\left(b^3c - ab^2d + a^2be - a^3f\right)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3}b}{3b}\right) - 220\left(b^3c - ab^2d + a^2be - a^3f\right)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 220*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 440*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 660*(b^3*c - a*b
```

```
^2*d + a^2*b*e - a^3*f)*x^9 + 264*(a*b^2*c - a^2*b*d + a^3*e)*x^6 + 120*a^3
*c - 165*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07223, size = 456, normalized size = 1.63

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d-\left(-ab^2\right)^{\frac{1}{3}}a^3f+\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5}-\frac{\left(b^4c-ab^3d-a^3bf+a^2b^2e\right)\left(-\right.}{3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(1/3)
*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/a^5 + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)
*a*b^2*d - (-a*b^2)^(1/3)*a^3*f + (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9 - 220*
a^3*f*x^9 + 220*a^2*b*x^9*e - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a^3*x^6*
e + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

Optimal. Leaf size=313

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{4a^4x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{16/3}}-\frac{b\left(a^2be+a^3(-f)-ab^2\right.}{a^5x}$$

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3)) +
(b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(16/3)) - (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rubi [A] time = 0.238236, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{4a^4x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^{16/3}}-\frac{b\left(a^2be+a^3(-f)-ab^2\right.}{a^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3)) +
(b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(16/3)) - (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx = \int\left(\frac{c}{ax^{14}} + \frac{-bc+ad}{a^2x^{11}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^5} - \frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^5x^2}\right.$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{a^5x}$$

$$= -\frac{c}{13ax^{13}} + \frac{bc-ad}{10a^2x^{10}} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{a^5x}$$

Mathematica [A] time = 0.0985146, size = 308, normalized size = 0.98

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{4a^4x^4} + \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+a^3f+ab^2d-b^3c\right)}{6a^{16/3}} + \frac{b\left(-a^2be+a^3f+ab^2d-b^3\right.}{a^5x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

```
[Out] -c/(13*a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) + (b*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(16/3)) + (b^
(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1
6/3)) + (b^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

Maple [B] time = 0.009, size = 546, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x)
```

```
[Out] -1/10/a/x^10*d-1/7/a/x^7*e-1/4/a/x^4*f+1/4/a^2/x^4*b*e-1/4/a^3/x^4*b^2*d+1/
4/a^4/x^4*b^3*c+1/a^2*b/x*f-1/a^3*b^2/x*e+1/a^4*b^3/x*d-1/a^5*b^4/x*c+1/7/a
^2/x^7*b*d-1/7/a^3/x^7*b^2*c+1/10/a^2/x^10*b*c-1/13*c/a/x^13-1/6*b^2/a^3/(1
/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/6*b^4/a^5/(1/b*a)^(1/
3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/6*b^3/a^4/(1/b*a)^(1/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/3*b^3/a^4/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1
/3))*d+1/3*b^4/a^5/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/6*b/a^2/(1/b*a)^(1
/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/3*b/a^2/(1/b*a)^(1/3)*ln(x+(1
/b*a)^(1/3))*f+1/3*b^2/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/3*b/a^2*3^
(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3*b^2/a^3
*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3*b^3/
a^4*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3*b
^4/a^5*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34294, size = 711, normalized size = 2.27

$$-\frac{1820\,\sqrt{3}\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-910\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^{13}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/5460*(1820*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(
1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*(b^4*c - a*b^3*
```

```
d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a
*(-b/a)^(1/3)) + 1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(
1/3)*log(b*x + a*(-b/a)^(2/3)) + 5460*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*
f)*x^12 - 1365*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 780*(a^2*b^2*c
 - a^3*b*d + a^4*e)*x^6 + 420*a^4*c - 546*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07779, size = 566, normalized size = 1.81

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}} b^3 c - \left(-ab^2\right)^{\frac{2}{3}} ab^2 d - \left(-ab^2\right)^{\frac{2}{3}} a^3 f + \left(-ab^2\right)^{\frac{2}{3}} a^2 be\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6} + \frac{\left(b^5 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^4 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3 b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3)
*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/a^6 + 1/3*(b^5*c*(-a/b)^(1/3) - a*b^4*d*(-a/b)^(1/3) - a^3*b^2*
f*(-a/b)^(1/3) + a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/a^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(
2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/a^6 - 1/1820*(1820*b^4*c*x^12 - 1820*a*b^3*d*x^12 - 1820*a^3*b*f*x^12 +
1820*a^2*b^2*x^12*e - 455*a*b^3*c*x^9 + 455*a^2*b^2*d*x^9 + 455*a^4*f*x^9 -
 455*a^3*b*x^9*e + 260*a^2*b^2*c*x^6 - 260*a^3*b*d*x^6 + 260*a^4*x^6*e - 18
2*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^5*x^13)
```

3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)} dx$

Optimal. Leaf size=315

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5x^2}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-a\right.}{6a^{17/3}}$$

```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a^3
*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/3)
) - (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Rubi [A] time = 0.228995, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 200, 31, 634, 617, 204, 628}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5x^2}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-a\right.}{6a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```


```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a^3
*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(17/3)
) - (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{15}} + \frac{-bc+ad}{a^2x^{12}} + \frac{b^2c-abd+a^2e}{a^3x^9} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^5x^3}\right.$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{2a^5x^2}$$

$$= -\frac{c}{14ax^{14}} + \frac{bc-ad}{11a^2x^{11}} - \frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{2a^5x^2}$$

Mathematica [A] time = 0.0974872, size = 311, normalized size = 0.99

$$\frac{b\left(-a^2be+a^3f+ab^2d-b^3c\right)}{2a^5x^2} + \frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5} + \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b\right.}{6a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

```
[Out] -c/(14*a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*a^3
*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(17/3)) +
 (b^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

Maple [B] time = 0.009, size = 548, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x)
```

```
[Out] -1/6*b/a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6*b^2/a^
3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/6*b^3/a^4/(1/b*a)
^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6*b^4/a^5/(1/b*a)^(2/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*b/a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^
(1/3))*f-1/3*b^2/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/5/a/x^5*f-1/11/a
/x^11*d-1/8/a/x^8*e+1/11/a^2/x^11*b*c+1/8/a^2/x^8*b*d-1/8/a^3/x^8*b^2*c+1/5
/a^2/x^5*b*e-1/5/a^3/x^5*b^2*d+1/5/a^4/x^5*b^3*c+1/2/a^2*b/x^2*f-1/14*c/a/x
^14+1/3*b^3/a^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/3*b^4/a^5/(1/b*a)^(2/
3)*ln(x+(1/b*a)^(1/3))*c-1/3*b^4/a^5/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(1/b*a)^(1/3)*x-1))*c-1/2/a^3*b^2/x^2*e+1/2/a^4*b^3/x^2*d-1/2/a^5*b^4
/x^2*c+1/3*b/a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*f-1/3*b^2/a^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))*e+1/3*b^3/a^4/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^
(1/3)*x-1))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38373, size = 759, normalized size = 2.41

$$-\frac{3080 \sqrt{3}\left(b^4 c - a b^3 d + a^2 b^2 e - a^3 b f\right) x^{14} \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3} a x \left(\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3} b}{3 b}\right) - 1540 \left(b^4 c - a b^3 d + a^2 b^2 e - a^3 b f\right) x^{14} \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/9240*(3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 1540*(b
^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 3080*(b^4*c - a*b^3*d + a^2*b^
2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 4620*(b^
4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1848*(a*b^3*c - a^2*b^2*d + a^3
*b*e - a^4*f)*x^9 + 1155*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 660*a^4*c - 84
0*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07733, size = 531, normalized size = 1.69

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c - \left(-ab^2\right)^{\frac{1}{3}}ab^3d - \left(-ab^2\right)^{\frac{1}{3}}a^3bf + \left(-ab^2\right)^{\frac{1}{3}}a^2b^2e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6} + \frac{\left(b^5c - ab^4d - a^3b^2f + a^2b^3\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d - (-a*b^2)^(1/3
)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a*b^2
)^(1/3)*a*b^3*d - (-a*b^2)^(1/3)*a^3*b*f + (-a*b^2)^(1/3)*a^2*b^2*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 1540*a*b
^3*d*x^12 - 1540*a^3*b*f*x^12 + 1540*a^2*b^2*x^12*e - 616*a*b^3*c*x^9 + 616
*a^2*b^2*d*x^9 + 616*a^4*f*x^9 - 616*a^3*b*x^9*e + 385*a^2*b^2*c*x^6 - 385*
a^3*b*d*x^6 + 385*a^4*x^6*e - 280*a^3*b*c*x^3 + 280*a^4*d*x^3 + 220*a^4*c)/
(a^5*x^14)
```

3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=351

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5x^4}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{7a^4x^7}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-a\right.}{6a^{19/3}}$$

```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*a^
3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(7/
3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(19/3))
```

Rubi [A] time = 0.257811, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5x^4}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{7a^4x^7}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-a\right.}{6a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```


```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*a^
3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^(7/
3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(19/3))
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^{17}} + \frac{-bc+ad}{a^2x^{14}} + \frac{b^2c-abd+a^2e}{a^3x^{11}} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^8} - \frac{b\left(-b^3c+ab^2d-a^2b\right.}{a^5x^5}\right.$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{4a^5x^4}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{4a^5x^4}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{4a^5x^4}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{4a^5x^4}$$

$$= -\frac{c}{16ax^{16}} + \frac{bc-ad}{13a^2x^{13}} - \frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7} - \frac{b\left(b^3c-ab^2d+a^2be-a\right.}{4a^5x^4}$$

Mathematica [A] time = 0.110059, size = 346, normalized size = 0.99

$$\frac{b\left(-a^2be+a^3f+ab^2d-b^3c\right)}{4a^5x^4} + \frac{a^2be+a^3(-f)-ab^2d+b^3c}{7a^4x^7} + \frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+a^3(-f)-ab^2d+b\right.}{6a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

```
[Out] -c/(16*a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10*a^
3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f))/(a^6*x) + (b^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(19/3)) + (b^(7/3)*(-(b^3*c
) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3)) + (b^
(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(19/3))
```

Maple [A] time = 0.01, size = 600, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x)
```

```
[Out] -1/10/a/x^10*e-1/7/a/x^7*f-1/13/a/x^13*d-1/16*c/a/x^16+1/6*b^3/a^4/(1/b*a)^
(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/6*b^4/a^5/(1/b*a)^(1/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/6*b^5/a^6/(1/b*a)^(1/3)*ln(x^2-(1/b*
a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*b^2/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f
-1/3*b^3/a^4/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/3*b^4/a^5/(1/b*a)^(1/3)*
ln(x+(1/b*a)^(1/3))*d-1/3*b^5/a^6/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6*b
^2/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/3*b^2/a^3*3^
(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3*b^3/a^4
*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*b^4/
a^5*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/3*b
^5/a^6*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/
a^6*b^5/x*c+1/4/a^2*b/x^4*f-1/4/a^3*b^2/x^4*e+1/4/a^4*b^3/x^4*d-1/4/a^5*b^4
/x^4*c+1/13/a^2/x^13*b*c+1/10/a^2/x^10*b*d-1/10/a^3/x^10*b^2*c+1/7/a^2/x^7*
b*e-1/7/a^3/x^7*b^2*d+1/7/a^4/x^7*b^3*c-1/a^3*b^2/x*f+1/a^4*b^3/x*e-1/a^5*b
^4/x*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2725, size = 798, normalized size = 2.27

$$7280\sqrt{3}\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + 3640\left(b^5c - ab^4d + a^2b^3e - a^3b^2f\right)x^{16}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/21840*(7280*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^
(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 3640*(b^5*c - a*b^4
*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) +
a*(b/a)^(1/3)) - 7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^
(1/3)*log(b*x + a*(b/a)^(2/3)) + 21840*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b
^2*f)*x^15 - 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 3120*(
a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 2184*(a^3*b^2*c - a^4*b*d +
a^5*e)*x^6 - 1365*a^5*c + 1680*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10368, size = 640, normalized size = 1.82

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}} b^4 c - \left(-ab^2\right)^{\frac{2}{3}} ab^3 d - \left(-ab^2\right)^{\frac{2}{3}} a^3 b f + \left(-ab^2\right)^{\frac{2}{3}} a^2 b^2 e\right) \arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^7} - \frac{\left(b^6 c \left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^5 d \left(-\frac{a}{b}\right)^{\frac{1}{3}} - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b^2)^(2/3
)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/a^7 - 1/3*(b^6*c*(-a/b)^(1/3) - a*b^5*d*(-a/b)^(1/3) - a^3
*b^3*f*(-a/b)^(1/3) + a^2*b^4*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^7 + 1/6*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d - (-a*b
^2)^(2/3)*a^3*b*f + (-a*b^2)^(2/3)*a^2*b^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/a^7 + 1/7280*(7280*b^5*c*x^15 - 7280*a*b^4*d*x^15 - 7280*a^3*b^
2*f*x^15 + 7280*a^2*b^3*x^15*e - 1820*a*b^4*c*x^12 + 1820*a^2*b^3*d*x^12 +
1820*a^4*b*f*x^12 - 1820*a^3*b^2*x^12*e + 1040*a^2*b^3*c*x^9 - 1040*a^3*b^2
*d*x^9 - 1040*a^5*f*x^9 + 1040*a^4*b*x^9*e - 728*a^3*b^2*c*x^6 + 728*a^4*b*
d*x^6 - 728*a^5*x^6*e + 560*a^4*b*c*x^3 - 560*a^5*d*x^3 - 455*a^5*c)/(a^6*x
^16)
```

3.251 $\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=220

$$\frac{x^6\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{6b^5}-\frac{ax^3\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^2\log\left(a\right.}{}$$

```
[Out] -(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*
x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3) + (f*x^15)/(15*b^2) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a
*b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi [A] time = 0.341178, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^6\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{6b^5}-\frac{ax^3\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^2\log\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/(3*b^6) + ((b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(6*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*
x^9)/(9*b^4) + ((b*e - 2*a*f)*x^12)/(12*b^3) + (f*x^15)/(15*b^2) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a
*b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{(a+bx)^2}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a\left(-2b^3c+3ab^2d-4a^2be+5a^3f\right)}{b^6}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5}+\right.\right.$$
$$=-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x^3}{3b^6}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^6}{6b^5}+\frac{\left(b^2d-2ab\right.}{9}$$

Mathematica [A] time = 0.138523, size = 205, normalized size = 0.93

$$\frac{30b^2x^6\left(3a^2be-4a^3f-2ab^2d+b^3c\right)+60abx^3\left(-4a^2be+5a^3f+3ab^2d-2b^3c\right)-\frac{60a^3\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a+bx^3}+60a^2\log\left(a+\right.}{180b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2*(b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d - 2*a*b*e + 3*a^2*f)*x
^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^15 - (60*a^3*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f))/(a + b*x^3) + 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2*b*e
 - 6*a^3*f)*Log[a + b*x^3])/(180*b^7)
```

Maple [A] time = 0.012, size = 288, normalized size = 1.3

$$\frac{fx^{15}}{15\,b^2}-\frac{x^{12}af}{6\,b^3}+\frac{x^{12}e}{12\,b^2}+\frac{x^9a^2f}{3\,b^4}-\frac{2\,x^9ae}{9\,b^3}+\frac{x^9d}{9\,b^2}-\frac{2\,a^3fx^6}{3\,b^5}+\frac{a^2ex^6}{2\,b^4}-\frac{adx^6}{3\,b^3}+\frac{x^6c}{6\,b^2}+\frac{5\,a^4fx^3}{3\,b^6}-\frac{4\,a^3ex^3}{3\,b^5}+\frac{a^2dx^3}{b^4}-\frac{2\,acx^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/15*f*x^15/b^2-1/6/b^3*x^12*a*f+1/12/b^2*x^12*e+1/3/b^4*x^9*a^2*f-2/9/b^3*
x^9*a*e+1/9/b^2*x^9*d-2/3/b^5*x^6*a^3*f+1/2/b^4*x^6*a^2*e-1/3/b^3*x^6*a*d+1
/6/b^2*x^6*c+5/3/b^6*a^4*f*x^3-4/3/b^5*a^3*e*x^3+1/b^4*a^2*d*x^3-2/3/b^3*a*
c*x^3-2*a^5/b^7*ln(b*x^3+a)*f+5/3*a^4/b^6*ln(b*x^3+a)*e-4/3*a^3/b^5*ln(b*x^
3+a)*d+a^2/b^4*ln(b*x^3+a)*c-1/3*a^6/b^7/(b*x^3+a)*f+1/3*a^5/b^6/(b*x^3+a)*
e-1/3*a^4/b^5/(b*x^3+a)*d+1/3*a^3/b^4/(b*x^3+a)*c
```

Maxima [A] time = 0.958348, size = 300, normalized size = 1.36

$$\frac{a^3b^3c-a^4b^2d+a^5be-a^6f}{3\left(b^8x^3+ab^7\right)}+\frac{12\,b^4fx^{15}+15\left(b^4e-2\,ab^3f\right)x^{12}+20\left(b^4d-2\,ab^3e+3\,a^2b^2f\right)x^9+30\left(b^4c-2\,ab^3d+3\,a^2b\right.}{180\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)/(b^8*x^3 + a*b^7) + 1/180*(12
*b^4*f*x^15 + 15*(b^4*e - 2*a*b^3*f)*x^12 + 20*(b^4*d - 2*a*b^3*e + 3*a^2*b
^2*f)*x^9 + 30*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^6 - 60*(2*a*
b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x^3)/b^6 + 1/3*(3*a^2*b^3*c - 4*
a^3*b^2*d + 5*a^4*b*e - 6*a^5*f)*log(b*x^3 + a)/b^7
```

Fricas [A] time = 1.244, size = 656, normalized size = 2.98

$$\frac{12\,b^6fx^{18} + 3\left(5\,b^6e - 6\,ab^5f\right)x^{15} + 5\left(4\,b^6d - 5\,ab^5e + 6\,a^2b^4f\right)x^{12} + 10\left(3\,b^6c - 4\,ab^5d + 5\,a^2b^4e - 6\,a^3b^3f\right)x^9 + 60\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^6*f*x^18 + 3*(5*b^6*e - 6*a*b^5*f)*x^15 + 5*(4*b^6*d - 5*a*b^5*
e + 6*a^2*b^4*f)*x^12 + 10*(3*b^6*c - 4*a*b^5*d + 5*a^2*b^4*e - 6*a^3*b^3*f
)*x^9 + 60*a^3*b^3*c - 60*a^4*b^2*d + 60*a^5*b*e - 60*a^6*f - 30*(3*a*b^5*c
 - 4*a^2*b^4*d + 5*a^3*b^3*e - 6*a^4*b^2*f)*x^6 - 60*(2*a^2*b^4*c - 3*a^3*b
^3*d + 4*a^4*b^2*e - 5*a^5*b*f)*x^3 + 60*(3*a^3*b^3*c - 4*a^4*b^2*d + 5*a^5
*b*e - 6*a^6*f + (3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)
*log(b*x^3 + a))/(b^8*x^3 + a*b^7)
```

Sympy [A] time = 11.5094, size = 224, normalized size = 1.02

$$-\frac{a^2\left(6a^3f - 5a^2be + 4ab^2d - 3b^3c\right)\log\left(a + bx^3\right)}{3b^7} - \frac{a^6f - a^5be + a^4b^2d - a^3b^3c}{3ab^7 + 3b^8x^3} + \frac{fx^{15}}{15b^2} - \frac{x^{12}\left(2af - be\right)}{12b^3} + \frac{x^9\left(3a^2f -\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a + b*x**3)/(3*b*
*7) - (a**6*f - a**5*b*e + a**4*b**2*d - a**3*b**3*c)/(3*a*b**7 + 3*b**8*x*
*3) + f*x**15/(15*b**2) - x**12*(2*a*f - b*e)/(12*b**3) + x**9*(3*a**2*f -
2*a*b*e + b**2*d)/(9*b**4) - x**6*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**
3*c)/(6*b**5) + x**3*(5*a**4*f - 4*a**3*b*e + 3*a**2*b**2*d - 2*a*b**3*c)/(
3*b**6)
```

Giac [A] time = 1.06618, size = 405, normalized size = 1.84

$$\frac{\left(3\,a^2b^3c - 4\,a^3b^2d - 6\,a^5f + 5\,a^4be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^7} - \frac{3\,a^2b^4cx^3 - 4\,a^3b^3dx^3 - 6\,a^5bfx^3 + 5\,a^4b^2x^3e + 2\,a^3b^3c - 3\,a^4b^2c}{3\left(bx^3 + a\right)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*b^3*c - 4*a^3*b^2*d - 6*a^5*f + 5*a^4*b*e)*log(abs(b*x^3 + a))/b
^7 - 1/3*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 - 6*a^5*b*f*x^3 + 5*a^4*b^2*x^3
*e + 2*a^3*b^3*c - 3*a^4*b^2*d - 5*a^6*f + 4*a^5*b*e)/((b*x^3 + a)*b^7) + 1
```

```
/180*(12*b^8*f*x^15 - 30*a*b^7*f*x^12 + 15*b^8*x^12*e + 20*b^8*d*x^9 + 60*a
^2*b^6*f*x^9 - 40*a*b^7*x^9*e + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 - 120*a^3*b^5
*f*x^6 + 90*a^2*b^6*x^6*e - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 + 300*a^4*b
^4*f*x^3 - 240*a^3*b^5*x^3*e)/b^10
```

3.252 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=180

$$\frac{x^3\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}-\frac{a^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}-$$

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3) + (f*x^12)/(12*b^2)
- (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^6*(a + b*x^3)) - (a*(2*b^3
*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi [A] time = 0.26477, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^3\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}-\frac{a^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^6)/(6*b^4) + ((b*e - 2*a*f)*x^9)/(9*b^3) + (f*x^12)/(12*b^2)
- (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^6*(a + b*x^3)) - (a*(2*b^3
*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int\frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx&=\frac{1}{3}\operatorname{Subst}\left(\int\frac{x^2\left(c+dx+ex^2+fx^3\right)}{(a+bx)^2}\,dx,x,x^3\right)\\&=\frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{b^3c-2ab^2d+3a^2be-4a^3f}{b^5}+\frac{\left(b^2d-2abe+3a^2f\right)x}{b^4}+\frac{(be-2af)x^2}{b^3}\right.\right.\\&=\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^3}{3b^5}+\frac{\left(b^2d-2abe+3a^2f\right)x^6}{6b^4}+\frac{(be-2af)x^9}{9b^3}+\frac{fx^{12}}{12b^2}-\end{aligned}$$

Mathematica [A] time = 0.110585, size = 167, normalized size = 0.93

$$\frac{12bx^3\left(3a^2be-4a^3f-2ab^2d+b^3c\right)+\frac{12a^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a+bx^3}+12a\log\left(a+bx^3\right)\left(-4a^2be+5a^3f+3ab^2d-2b^3c\right)+6b^2x}{36b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*d - 2*a*b*
e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^12 + (12*a^2*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 12*a*(-2*b^3*c + 3*a*b^2*d -
 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3])/(36*b^6)
```

Maple [A] time = 0.013, size = 240, normalized size = 1.3

$$\frac{fx^{12}}{12\,b^2}-\frac{2\,x^9af}{9\,b^3}+\frac{x^9e}{9\,b^2}+\frac{a^2fx^6}{2\,b^4}-\frac{aex^6}{3\,b^3}+\frac{dx^6}{6\,b^2}-\frac{4\,a^3fx^3}{3\,b^5}+\frac{a^2ex^3}{b^4}-\frac{2\,adx^3}{3\,b^3}+\frac{cx^3}{3\,b^2}+\frac{5\,a^4\ln\left(bx^3+a\right)f}{3\,b^6}-\frac{4\,a^3\ln\left(bx^3+a\right)}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/12*f*x^12/b^2-2/9/b^3*x^9*a*f+1/9/b^2*x^9*e+1/2/b^4*x^6*a^2*f-1/3/b^3*x^6
*a*e+1/6/b^2*x^6*d-4/3/b^5*a^3*f*x^3+1/b^4*a^2*e*x^3-2/3/b^3*a*d*x^3+1/3/b^
2*c*x^3+5/3*a^4/b^6*ln(b*x^3+a)*f-4/3*a^3/b^5*ln(b*x^3+a)*e+a^2/b^4*ln(b*x^
3+a)*d-2/3*a/b^3*ln(b*x^3+a)*c+1/3*a^5/b^6/(b*x^3+a)*f-1/3*a^4/b^5/(b*x^3+a
)*e+1/3*a^3/b^4/(b*x^3+a)*d-1/3*a^2/b^3/(b*x^3+a)*c
```

Maxima [A] time = 0.964983, size = 243, normalized size = 1.35

$$-\frac{a^2b^3c-a^3b^2d+a^4be-a^5f}{3\left(b^7x^3+ab^6\right)}+\frac{3\,b^3fx^{12}+4\left(b^3e-2\,ab^2f\right)x^9+6\left(b^3d-2\,ab^2e+3\,a^2bf\right)x^6+12\left(b^3c-2\,ab^2d+3\,a^2be-\right.}{36\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)/(b^7*x^3 + a*b^6) + 1/36*(3*
b^3*f*x^12 + 4*(b^3*e - 2*a*b^2*f)*x^9 + 6*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*
x^6 + 12*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/b^5 - 1/3*(2*a*b^3*
c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*log(b*x^3 + a)/b^6
```

Fricas [A] time = 1.2588, size = 555, normalized size = 3.08

$$\frac{3\,b^5fx^{15}+\left(4\,b^5e-5\,ab^4f\right)x^{12}+2\left(3\,b^5d-4\,ab^4e+5\,a^2b^3f\right)x^9+6\left(2\,b^5c-3\,ab^4d+4\,a^2b^3e-5\,a^3b^2f\right)x^6-12\,a^2b^3c+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*f*x^15 + (4*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 4*a*b^4*e +
5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e - 5*a^3*b^2*f)*x^6
- 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b*e + 12*a^5*f + 12*(a*b^4*c - 2*a^2
*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d + 4*a
^4*b*e - 5*a^5*f + (2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)
*log(b*x^3 + a))/(b^7*x^3 + a*b^6)
```

Sympy [A] time = 11.0927, size = 180, normalized size = 1.

$$\frac{a\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)\log\left(a+bx^3\right)}{3b^6}+\frac{a^5f-a^4be+a^3b^2d-a^2b^3c}{3ab^6+3b^7x^3}+\frac{fx^{12}}{12b^2}-\frac{x^9\left(2af-be\right)}{9b^3}+\frac{x^6\left(3a^2f-2a\right.}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)/(3*b**6)
+ (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x**3)
+ f*x**12/(12*b**2) - x**9*(2*a*f - b*e)/(9*b**3) + x**6*(3*a**2*f - 2*a*b*
e + b**2*d)/(6*b**4) - x**3*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(
3*b**5)
```

Giac [A] time = 1.05787, size = 335, normalized size = 1.86

$$-\frac{\left(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^6}+\frac{2\,ab^4cx^3-3\,a^2b^3dx^3-5\,a^4bfx^3+4\,a^3b^2x^3e+a^2b^3c-2\,a^3b^2d-}{3\left(bx^3+a\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*log(abs(b*x^3 + a))/b^
6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 - 5*a^4*b*f*x^3 + 4*a^3*b^2*x^3*e
+ a^2*b^3*c - 2*a^3*b^2*d - 4*a^5*f + 3*a^4*b*e)/((b*x^3 + a)*b^6) + 1/36*(
3*b^6*f*x^12 - 8*a*b^5*f*x^9 + 4*b^6*x^9*e + 6*b^6*d*x^6 + 18*a^2*b^4*f*x^6
 - 12*a*b^5*x^6*e + 12*b^6*c*x^3 - 24*a*b^5*d*x^3 - 48*a^3*b^3*f*x^3 + 36*a
^2*b^4*x^3*e)/b^8
```

3.253 $\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=140

$$\frac{a\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}+\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{x^6(be-2af)}{6b^3}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (
f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)
) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rubi [A] time = 0.199417, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}+\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{x^6(be-2af)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^6)/(6*b^3) + (
f*x^9)/(9*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*b^5*(a + b*x^3)
) + ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(3*b^5)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-2abe+3a^2f}{b^4}+\frac{(be-2af)x}{b^3}+\frac{fx^2}{b^2}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4(a+bx)^2}\right.\right.\\ &= \frac{\left(b^2d-2abe+3a^2f\right)x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}+\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{3b^5\left(a+bx^3\right)}+\frac{\left(b^3c-\right.}{}\end{aligned}$$

Mathematica [A] time = 0.0984255, size = 129, normalized size = 0.92

$$\frac{\frac{6a\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a+bx^3}+6\log\left(a+bx^3\right)\left(3a^2be-4a^3f-2ab^2d+b^3c\right)+6bx^3\left(3a^2f-2abe+b^2d\right)+3b^2x^6(be-2af)+}{18b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 + 2*b^3*f*x^
9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3) + 6*(b^3*c - 2*a*
b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(18*b^5)
```

Maple [A] time = 0.01, size = 192, normalized size = 1.4

$$\frac{fx^9}{9b^2}-\frac{x^6af}{3b^3}+\frac{ex^6}{6b^2}+\frac{a^2fx^3}{b^4}-\frac{2aex^3}{3b^3}+\frac{dx^3}{3b^2}-\frac{4\ln\left(bx^3+a\right)a^3f}{3b^5}+\frac{\ln\left(bx^3+a\right)a^2e}{b^4}-\frac{2\ln\left(bx^3+a\right)ad}{3b^3}+\frac{\ln\left(bx^3+\right.}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/9*f*x^9/b^2-1/3/b^3*x^6*a*f+1/6/b^2*x^6*e+1/b^4*a^2*f*x^3-2/3/b^3*a*e*x^3
+1/3/b^2*d*x^3-4/3/b^5*ln(b*x^3+a)*a^3*f+1/b^4*ln(b*x^3+a)*a^2*e-2/3/b^3*ln
(b*x^3+a)*a*d+1/3/b^2*ln(b*x^3+a)*c-1/3/b^5*a^4/(b*x^3+a)*f+1/3/b^4*a^3/(b*
x^3+a)*e-1/3/b^3*a^2/(b*x^3+a)*d+1/3/b^2*a/(b*x^3+a)*c
```

Maxima [A] time = 0.958848, size = 186, normalized size = 1.33

$$\frac{ab^3c-a^2b^2d+a^3be-a^4f}{3\left(b^6x^3+ab^5\right)}+\frac{2b^2fx^9+3\left(b^2e-2abf\right)x^6+6\left(b^2d-2abe+3a^2f\right)x^3}{18b^4}+\frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right.}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)/(b^6*x^3 + a*b^5) + 1/18*(2*b^2
*f*x^9 + 3*(b^2*e - 2*a*b*f)*x^6 + 6*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/b^4 +
 1/3*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*log(b*x^3 + a)/b^5
```

Fricas [A] time = 1.30494, size = 432, normalized size = 3.09

$$\frac{2b^4fx^{12}+\left(3b^4e-4ab^3f\right)x^9+3\left(2b^4d-3ab^3e+4a^2b^2f\right)x^6+6ab^3c-6a^2b^2d+6a^3be-6a^4f+6\left(ab^3d-2a^2b^2e\right.}{18\left(b^6x^3+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a*b^3*e + 4
*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e - 6*a^4*f + 6*(a*b^3*
d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c - 2*a^2*b^2*d + 3*a^3*b*e - 4
*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 + a))
/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 10.6441, size = 138, normalized size = 0.99

$$-\frac{a^4f - a^3be + a^2b^2d - ab^3c}{3ab^5 + 3b^6x^3} + \frac{fx^9}{9b^2} - \frac{x^6\left(2af - be\right)}{6b^3} + \frac{x^3\left(3a^2f - 2abe + b^2d\right)}{3b^4} - \frac{\left(4a^3f - 3a^2be + 2ab^2d - b^3c\right)\log(a + b}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*b**6*x**3) + f*
x**9/(9*b**2) - x**6*(2*a*f - b*e)/(6*b**3) + x**3*(3*a**2*f - 2*a*b*e + b*
*2*d)/(3*b**4) - (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)*log(a + b*x*
*3)/(3*b**5)
```

Giac [A] time = 1.06383, size = 293, normalized size = 2.09

$$\frac{\frac{\left(bx^3+a\right)^3\left(2f - \frac{3\left(4abf - b^2e\right)}{\left(bx^3+a\right)b} + \frac{6\left(b^4d + 6a^2b^2f - 3ab^3e\right)}{\left(bx^3+a\right)^2b^2}\right)}{b^4} - \frac{6\left(b^3c - 2ab^2d - 4a^3f + 3a^2be\right)\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2|b|}\right)}{b^4} + \frac{6\left(\frac{ab^6c}{bx^3+a} - \frac{a^2b^5d}{bx^3+a} - \frac{a^4b^3f}{bx^3+a} + \frac{a^3b^4e}{bx^3+a}\right)}{b^7}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/18*((b*x^3 + a)^3*(2*f - 3*(4*a*b*f - b^2*e)/((b*x^3 + a)*b) + 6*(b^4*d +
 6*a^2*b^2*f - 3*a*b^3*e)/((b*x^3 + a)^2*b^2))/b^4 - 6*(b^3*c - 2*a*b^2*d -
 4*a^3*f + 3*a^2*b*e)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 + 6*(a
*b^6*c/(b*x^3 + a) - a^2*b^5*d/(b*x^3 + a) - a^4*b^3*f/(b*x^3 + a) + a^3*b^
4*e/(b*x^3 + a))/b^7)/b
```

3.254 $\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])
/(3*b^4)
```

Rubi [A] time = 0.14512, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1819, 1850}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^6)/(6*b^2) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)/(3*b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])
/(3*b^4)
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^2}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{be-2af}{b^3}+\frac{fx}{b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^2}+\frac{b^2d-2abe+3a^2f}{b^3(a+bx)}\right)dx,x\right.\\
&= \frac{(be-2af)x^3}{3b^3}+\frac{fx^6}{6b^2}-\frac{b^3c-ab^2d+a^2be-a^3f}{3b^4\left(a+bx^3\right)}+\frac{\left(b^2d-2abe+3a^2f\right)\log\left(a+bx^3\right)}{3b^4}
\end{aligned}$$

Mathematica [A] time = 0.0584239, size = 93, normalized size = 0.9

$$\frac{\frac{2(-a^2be+a^3f+ab^2d-b^3c)}{a+bx^3}+2\log\left(a+bx^3\right)\left(3a^2f-2abe+b^2d\right)+2bx^3(be-2af)+b^2fx^6}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/(6*b^4)
```

Maple [A] time = 0.01, size = 142, normalized size = 1.4

$$\frac{fx^6}{6b^2}-\frac{2ax^3f}{3b^3}+\frac{x^3e}{3b^2}+\frac{\ln\left(bx^3+a\right)a^2f}{b^4}-\frac{2\ln\left(bx^3+a\right)ae}{3b^3}+\frac{\ln\left(bx^3+a\right)d}{3b^2}+\frac{a^3f}{3b^4\left(bx^3+a\right)}-\frac{a^2e}{3b^3\left(bx^3+a\right)}+\frac{a\ldots}{3b^2\left(b\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*f*x^6/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+1/b^4*ln(b*x^3+a)*a^2*f-2/3/b^3
*ln(b*x^3+a)*a*e+1/3/b^2*ln(b*x^3+a)*d+1/3/b^4/(b*x^3+a)*a^3*f-1/3/b^3/(b*x
^3+a)*a^2*e+1/3/b^2/(b*x^3+a)*a*d-1/3/b/(b*x^3+a)*c
```

Maxima [A] time = 0.959441, size = 132, normalized size = 1.28

$$-\frac{b^3c-ab^2d+a^2be-a^3f}{3\left(b^5x^3+ab^4\right)}+\frac{bfx^6+2\left(be-2af\right)x^3}{6b^3}+\frac{\left(b^2d-2abe+3a^2f\right)\log\left(bx^3+a\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(b^5*x^3 + a*b^4) + 1/6*(b*f*x^6 +
 2*(b*e - 2*a*f)*x^3)/b^3 + 1/3*(b^2*d - 2*a*b*e + 3*a^2*f)*log(b*x^3 + a)/
b^4
```

Fricas [A] time = 1.13548, size = 305, normalized size = 2.96

$$\frac{b^3fx^9+\left(2b^3e-3ab^2f\right)x^6-2b^3c+2ab^2d-2a^2be+2a^3f+2\left(ab^2e-2a^2bf\right)x^3+2\left(ab^2d-2a^2be+3a^3f+\left(b^3d-2\ldots\right.\right.}{6\left(b^5x^3+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d - 2*a^2*b*
e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d - 2*a^2*b*e + 3*a^3*
```

```
f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] time = 9.22015, size = 97, normalized size = 0.94

$$\frac{a^3f - a^2be + ab^2d - b^3c}{3ab^4 + 3b^5x^3} + \frac{fx^6}{6b^2} - \frac{x^3\left(2af - be\right)}{3b^3} + \frac{\left(3a^2f - 2abe + b^2d\right)\log\left(a + bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] (a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + f*x**6/(
6*b**2) - x**3*(2*a*f - b*e)/(3*b**3) + (3*a**2*f - 2*a*b*e + b**2*d)*log(a
 + b*x**3)/(3*b**4)
```

Giac [B] time = 1.05823, size = 278, normalized size = 2.7

$$-\frac{1}{6}f\left(\frac{\left(bx^3+a\right)^2\left(\frac{6a}{bx^3+a}-1\right)}{b^4} + \frac{6a^2\log\left(\frac{|bx^3+a|}{\left(bx^3+a\right)^2|b|}\right)}{b^4} - \frac{2a^3}{\left(bx^3+a\right)b^4}\right) + \frac{1}{3}\left(\frac{2a\log\left(\frac{|bx^3+a|}{\left(bx^3+a\right)^2|b|}\right)}{b^3} + \frac{bx^3+a}{b^3} - \frac{a^2}{\left(bx^3+a\right)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*log(abs(b*x^3 + a)/
((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) + 1/3*(2*a*log(abs(b
*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3 + a)/b^3 - a^2/((b*x^3 + a)*
b^3))*e - 1/3*d*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 +
 a)*b))/b - 1/3*c/((b*x^3 + a)*b)
```

3.255 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=100

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3ab^3(a+bx^3)}-\frac{\log(a+bx^3)(-a^2be+2a^3f+b^3c)}{3a^2b^3}+\frac{c\log(x)}{a^2}+\frac{fx^3}{3b^2}$$

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3))
 + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3
)
```

Rubi [A] time = 0.125302, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3ab^3(a+bx^3)}-\frac{\log(a+bx^3)(-a^2be+2a^3f+b^3c)}{3a^2b^3}+\frac{c\log(x)}{a^2}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (f*x^3)/(3*b^2) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a*b^3*(a + b*x^3))
 + (c*Log[x])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(3*a^2*b^3
)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^2}+\frac{c}{a^2x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^2}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3(a+bx^3)}+\frac{c\log(x)}{a^2}-\frac{(b^3c-a^2be+2a^3f)\log(a+bx^3)}{3a^2b^3}\end{aligned}$$

Mathematica [A] time = 0.122146, size = 95, normalized size = 0.95

$$\frac{\frac{\frac{a\left(a^2b\left(e+fx^3\right)+a^3(-f)+ab^2\left(fx^6-d\right)+b^3c\right)}{a+bx^3}+\log\left(a+bx^3\right)\left(a^2be-2a^3f-b^3c\right)}{b^3}+3c\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

```
[Out] (3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d + f*x^6)))
/(a + b*x^3) + (-(b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/b^3)/(3*a^2)
```

Maple [A] time = 0.011, size = 125, normalized size = 1.3

$$\frac{fx^3}{3\,b^2}-\frac{2\,a\ln\left(bx^3+a\right)f}{3\,b^3}+\frac{\ln\left(bx^3+a\right)e}{3\,b^2}-\frac{c\ln\left(bx^3+a\right)}{3\,a^2}-\frac{a^2f}{3\,b^3\left(bx^3+a\right)}+\frac{ae}{3\,b^2\left(bx^3+a\right)}-\frac{d}{3\,b\left(bx^3+a\right)}+\frac{c}{3\,a\left(bx^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3*f*x^3/b^2-2/3/b^3*a*ln(b*x^3+a)*f+1/3/b^2*ln(b*x^3+a)*e-1/3*c*ln(b*x^3+
a)/a^2-1/3/b^3*a^2/(b*x^3+a)*f+1/3/b^2*a/(b*x^3+a)*e-1/3/b/(b*x^3+a)*d+1/3/
a/(b*x^3+a)*c+c*ln(x)/a^2
```

Maxima [A] time = 0.95645, size = 135, normalized size = 1.35

$$\frac{fx^3}{3\,b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3\left(ab^4x^3+a^2b^3\right)}+\frac{c\log\left(x^3\right)}{3\,a^2}-\frac{\left(b^3c-a^2be+2\,a^3f\right)\log\left(bx^3+a\right)}{3\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^4*x^3 + a^2*b^
3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(b*x^3 + a)/(a
^2*b^3)
```

Fricas [A] time = 1.43537, size = 293, normalized size = 2.93

$$\frac{a^2b^2fx^6+a^3bfx^3+ab^3c-a^2b^2d+a^3be-a^4f-\left(ab^3c-a^3be+2\,a^4f+\left(b^4c-a^2b^2e+2\,a^3bf\right)x^3\right)\log\left(bx^3+a\right)+3\,(l}{3\left(a^2b^4x^3+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f -
(a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)*log(b*x
```

```
^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^2*b^4*x^3 + a^3*b^3)
```

Sympy [A] time = 33.8253, size = 95, normalized size = 0.95

$$-\frac{a^3f-a^2be+ab^2d-b^3c}{3a^2b^3+3ab^4x^3}+\frac{fx^3}{3b^2}+\frac{c\log(x)}{a^2}-\frac{\left(2a^3f-a^2be+b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

```
[Out] -(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3) + f*
x**3/(3*b**2) + c*log(x)/a**2 - (2*a**3*f - a**2*b*e + b**3*c)*log(a/b + x*
*3)/(3*a**2*b**3)
```

Giac [A] time = 1.07303, size = 169, normalized size = 1.69

$$\frac{fx^3}{3\,b^2}+\frac{c\log(|x|)}{a^2}-\frac{\left(b^3c+2\,a^3f-a^2be\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^2b^3}+\frac{b^4cx^3+2\,a^3bfx^3-a^2b^2x^3e+2\,ab^3c-a^2b^2d+a^4f}{3\left(bx^3+a\right)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*f*x^3/b^2 + c*log(abs(x))/a^2 - 1/3*(b^3*c + 2*a^3*f - a^2*b*e)*log(abs
(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 + 2*a^3*b*f*x^3 - a^2*b^2*x^3*e + 2
*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*b^3)
```

3.256 $\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}$$

```
[Out] -c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^3)
) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3
])/(3*a^3*b^2)
```

Rubi [A] time = 0.14216, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{3a^3b^2}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^2*b^2*(a + b*x^3)
) - ((2*b*c - a*d)*Log[x])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3
])/(3*a^3*b^2)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^2}+\frac{2b^3c-ab^2d+a^3f}{a^3b(a+bx)}\right)dx, x,\right.\\
&= -\frac{c}{3a^2x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2\left(a+bx^3\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(a+bx\right.}{3a^3b^2}
\end{aligned}$$

Mathematica [A] time = 0.0767274, size = 97, normalized size = 0.89

$$\frac{\frac{a\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{b^2}+3\log(x)(ad-2bc)-\frac{ac}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3)
) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3])/
b^2)/(3*a^3)
```

Maple [A] time = 0.014, size = 132, normalized size = 1.2

$$\frac{f\ln\left(bx^3+a\right)}{3b^2}-\frac{d\ln\left(bx^3+a\right)}{3a^2}+\frac{2bc\ln\left(bx^3+a\right)}{3a^3}+\frac{af}{3b^2\left(bx^3+a\right)}-\frac{e}{3b\left(bx^3+a\right)}+\frac{d}{3a\left(bx^3+a\right)}-\frac{bc}{3a^2\left(bx^3+a\right)}-\frac{}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/3*f*ln(b*x^3+a)/b^2-1/3*d*ln(b*x^3+a)/a^2+2/3*b*c*ln(b*x^3+a)/a^3+1/3*a/b
^2/(b*x^3+a)*f-1/3/b/(b*x^3+a)*e+1/3/a/(b*x^3+a)*d-1/3/a^2*b/(b*x^3+a)*c-1/
3*c/x^3/a^2+d*ln(x)/a^2-2*b*c*ln(x)/a^3
```

Maxima [A] time = 0.954806, size = 157, normalized size = 1.44

$$-\frac{ab^2c+\left(2b^3c-ab^2d+a^2be-a^3f\right)x^3}{3\left(a^2b^3x^6+a^3b^2x^3\right)}-\frac{(2bc-ad)\log\left(x^3\right)}{3a^3}+\frac{\left(2b^3c-ab^2d+a^3f\right)\log\left(bx^3+a\right)}{3a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b^2*c + (2*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(a^2*b^3*x^6 + a
^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3
*f)*log(b*x^3 + a)/(a^3*b^2)
```

Fricas [A] time = 1.49415, size = 340, normalized size = 3.12

$$-\frac{a^2b^2c+\left(2ab^3c-a^2b^2d+a^3be-a^4f\right)x^3-\left(\left(2b^4c-ab^3d+a^3bf\right)x^6+\left(2ab^3c-a^2b^2d+a^4f\right)x^3\right)\log\left(bx^3+a\right)+3\left(\left(2b\right.\right.}{3\left(a^3b^3x^6+a^4b^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 - ((2*b^4*c
 - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^4*f)*x^3)*log(b*x^3
```

```
+ a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*c - a^2*b^2*d)*x^3)*log(x))/(a
^3*b^3*x^6 + a^4*b^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.05914, size = 177, normalized size = 1.62

$$-\frac{(2\,bc - ad)\log(|x|)}{a^3} + \frac{\left(2\,b^3c - ab^2d + a^3f\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^3b^2} - \frac{a^2bfx^6 + 4\,b^3cx^3 - 2\,ab^2dx^3 - a^3fx^3 + 2\,a^2bx^3e + 2\,ab^2}{6\left(bx^6 + ax^3\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c - a*d)*log(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(abs(b*
x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*a*b^2*d*x^3 - a^3*
f*x^3 + 2*a^2*b*x^3*e + 2*a*b^2*c)/((b*x^6 + a*x^3)*a^2*b^2)
```

3.257 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$

Optimal. Leaf size=130

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}$$

```
[Out] -c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a
^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*Log[x])/a^4 - ((
3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 0.153838, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{3a^4}+\frac{\log(x)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{2bc-ad}{3a^3x^3}-\frac{c}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -c/(6*a^2*x^6) + (2*b*c - a*d)/(3*a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a
^3*f)/(3*a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*Log[x])/a^4 - ((
3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(3*a^4)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^2}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int\left(\frac{c}{a^2x^3}+\frac{-2bc+ad}{a^3x^2}+\frac{3b^2c-2abd+a^2e}{a^4x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)^2}-\frac{b\left(3b^2c\right.}{a}\right.\right.$$

$$=-\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b\left(a+bx^3\right)}+\frac{\left(3b^2c-2abd+a^2e\right)\log(x)}{a^4}-\frac{\left(3b^2c-2aba\right.}{}$$

Mathematica [A] time = 0.114931, size = 118, normalized size = 0.91

$$-\frac{\frac{2a(-a^2be+a^3f+ab^2d-b^3c)}{b(a+bx^3)}+2\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)-6\log(x)\left(a^2e-2abd+3b^2c\right)+\frac{a^2c}{x^6}+\frac{2a(ad-2bc)}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2*d - a^2*b
*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a^2*e)*Log[x] + 2*(3*
b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/(6*a^4)
```

Maple [A] time = 0.015, size = 167, normalized size = 1.3

$$-\frac{e\ln\left(bx^3+a\right)}{3a^2}+\frac{2\ln\left(bx^3+a\right)bd}{3a^3}-\frac{\ln\left(bx^3+a\right)b^2c}{a^4}-\frac{f}{3b\left(bx^3+a\right)}+\frac{e}{3a\left(bx^3+a\right)}-\frac{bd}{3a^2\left(bx^3+a\right)}+\frac{b^2c}{3a^3\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x)
```

```
[Out] -1/3*e*ln(b*x^3+a)/a^2+2/3/a^3*ln(b*x^3+a)*b*d-1/a^4*ln(b*x^3+a)*b^2*c-1/3/
b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*e-1/3/a^2*b/(b*x^3+a)*d+1/3/a^3*b^2/(b*x^3+a)
*c-1/6*c/a^2/x^6-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c+e*ln(x)/a^2-2/a^3*ln(x)*b*d+
3/a^4*ln(x)*b^2*c
```

Maxima [A] time = 0.979299, size = 186, normalized size = 1.43

$$\frac{2\left(3b^3c-2ab^2d+a^2be-a^3f\right)x^6-a^2bc+\left(3ab^2c-2a^2bd\right)x^3}{6\left(a^3b^2x^9+a^4bx^6\right)}-\frac{\left(3b^2c-2abd+a^2e\right)\log\left(bx^3+a\right)}{3a^4}+\frac{\left(3b^2c-2abd\right.}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*b^3*c - 2*a*b^2*d + a^2*b*e - a^3*f)*x^6 - a^2*b*c + (3*a*b^2*c -
 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2*c - 2*a*b*d + a^2*e
)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(x^3)/a^4
```

Fricas [A] time = 1.42823, size = 431, normalized size = 3.32

$$\frac{2\left(3ab^3c-2a^2b^2d+a^3be-a^4f\right)x^6-a^3bc+\left(3a^2b^2c-2a^3bd\right)x^3-2\left(\left(3b^4c-2ab^3d+a^2b^2e\right)x^9+\left(3ab^3c-2a^2b^2d\right.\right.}{6\left(a^4b^2x^9+a^5bx^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c + (3*a^2*b
^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b^3
*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 + a) + 6*((3*b^4*c - 2*a*b^3*d +
 a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4*b^2
*x^9 + a^5*b*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07027, size = 271, normalized size = 2.08

$$\frac{\left(3\,b^2c - 2\,abd + a^2e\right)\log\left(|x|\right)}{a^4} - \frac{\left(3\,b^3c - 2\,ab^2d + a^2be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^4b} + \frac{3\,b^4cx^3 - 2\,ab^3dx^3 + a^2b^2x^3e + 4\,ab^3c - 3\,a^2b^2d}{3\left(bx^3 + a\right)a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (3*b^2*c - 2*a*b*d + a^2*e)*log(abs(x))/a^4 - 1/3*(3*b^3*c - 2*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/3*(3*b^4*c*x^3 - 2*a*b^3*d*x^3 + a^2
*b^2*x^3*e + 4*a*b^3*c - 3*a^2*b^2*d - a^4*f + 2*a^3*b*e)/((b*x^3 + a)*a^4*
b) - 1/6*(9*b^2*c*x^6 - 6*a*b*d*x^6 + 3*a^2*x^6*e - 4*a*b*c*x^3 + 2*a^2*d*x
^3 + a^2*c)/(a^4*x^6)
```

3.258 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=175

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(2a^2be+a^3(-f)-3ab^2d+4b^3c\right)}{3a^5}-\frac{\log(x)\left(2a^2be+a^3(-f)-3ab^2d+4b\right.}{a^5}$$

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4*b
^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])/a^5 + ((4*b^3*c - 3*a*b^2*d +
 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rubi [A] time = 0.202541, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)\left(2a^2be+a^3(-f)-3ab^2d+4b^3c\right)}{3a^5}-\frac{\log(x)\left(2a^2be+a^3(-f)-3ab^2d+4b\right.}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(6*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*(a + b*x^3)) - ((4*b
^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x])/a^5 + ((4*b^3*c - 3*a*b^2*d +
 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^5)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^2}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^2x^4}+\frac{-2bc+ad}{a^3x^3}+\frac{3b^2c-2abd+a^2e}{a^4x^2}+\frac{-4b^3c+3ab^2d-2a^2be+a^3f}{a^5x}-\right.\right.$$

$$=-\frac{c}{9a^2x^9}+\frac{2bc-ad}{6a^3x^6}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4\left(a+bx^3\right)}-\frac{\left(4b^3c-3ab^2d+2\right.}{a}$$

Mathematica [A] time = 0.10454, size = 160, normalized size = 0.91

$$\frac{\frac{6a(-a^2be+a^3f+ab^2d-b^3c)}{a+bx^3}+6\log\left(a+bx^3\right)\left(2a^2be+a^3(-f)-3ab^2d+4b^3c\right)+18\log(x)\left(-2a^2be+a^3f+3ab^2d-4b^3c\right)-\frac{6a(a}{}}{18a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*a*b*d + a^
2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 18*(
-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log[x] + 6*(4*b^3*c - 3*a*b^2*d +
 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^5)
```

Maple [A] time = 0.018, size = 229, normalized size = 1.3

$$-\frac{\ln\left(bx^3+a\right)f}{3\,a^2}+\frac{2\,b\ln\left(bx^3+a\right)e}{3\,a^3}-\frac{b^2\ln\left(bx^3+a\right)d}{a^4}+\frac{4\,b^3\ln\left(bx^3+a\right)c}{3\,a^5}+\frac{f}{3\,a\left(bx^3+a\right)}-\frac{be}{3\,a^2\left(bx^3+a\right)}+\frac{b^2d}{3\,a^3\left(bx^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*ln(b*x^3+a)*f+2/3*b/a^3*ln(b*x^3+a)*e-b^2/a^4*ln(b*x^3+a)*d+4/3*b^
3/a^5*ln(b*x^3+a)*c+1/3/a/(b*x^3+a)*f-1/3*b/a^2/(b*x^3+a)*e+1/3*b^2/a^3/(b*
x^3+a)*d-1/3*b^3/a^4/(b*x^3+a)*c-1/9*c/a^2/x^9-1/6/a^2/x^6*d+1/3/a^3/x^6*b*
c-1/3/a^2/x^3*e+2/3/a^3/x^3*b*d-1/a^4/x^3*b^2*c+1/a^2*ln(x)*f-2/a^3*ln(x)*b
*e+3/a^4*ln(x)*b^2*d-4/a^5*ln(x)*b^3*c
```

Maxima [A] time = 1.04895, size = 244, normalized size = 1.39

$$-\frac{6\left(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f\right)x^9+3\left(4\,ab^2c-3\,a^2bd+2\,a^3e\right)x^6+2\,a^3c-\left(4\,a^2bc-3\,a^3d\right)x^3}{18\left(a^4bx^{12}+a^5x^9\right)}+\frac{\left(4\,b^3c-3\,ab^2d+2\,a^2\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*x^9 + 3*(4*a*b^2*c - 3*a
^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)*x^3)/(a^4*b*x^12 +
a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(b*x^3 + a)/a^5
 - 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(x^3)/a^5
```

Fricas [A] time = 1.6199, size = 549, normalized size = 3.14

$$-\frac{6\left(4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f\right)x^9+3\left(4\,a^2b^2c-3\,a^3bd+2\,a^4e\right)x^6+2\,a^4c-\left(4\,a^3bc-3\,a^4d\right)x^3-6\left(\left(4\,b^4c-3\,ab^3d+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4*a^2*b^2*c
 - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*a^4*d)*x^3 - 6*((4*b
^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d +
 2*a^3*b*e - a^4*f)*x^9)*log(b*x^3 + a) + 18*((4*b^4*c - 3*a*b^3*d + 2*a^2*
b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9)*
log(x))/(a^5*b*x^12 + a^6*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06707, size = 371, normalized size = 2.12

$$-\frac{\left(4\,b^3c-3\,ab^2d-a^3f+2\,a^2be\right)\log\left(|x|\right)}{a^5}+\frac{\left(4\,b^4c-3\,ab^3d-a^3bf+2\,a^2b^2e\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^5b}-\frac{4\,b^4cx^3-3\,ab^3dx^3-a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b^3*c - 3*a*b^2*d - a^3*f + 2*a^2*b*e)*log(abs(x))/a^5 + 1/3*(4*b^4*c -
 3*a*b^3*d - a^3*b*f + 2*a^2*b^2*e)*log(abs(b*x^3 + a))/(a^5*b) - 1/3*(4*b^
4*c*x^3 - 3*a*b^3*d*x^3 - a^3*b*f*x^3 + 2*a^2*b^2*x^3*e + 5*a*b^3*c - 4*a^2
*b^2*d - 2*a^4*f + 3*a^3*b*e)/((b*x^3 + a)*a^5) + 1/18*(44*b^3*c*x^9 - 33*a
*b^2*d*x^9 - 11*a^3*f*x^9 + 22*a^2*b*x^9*e - 18*a*b^2*c*x^6 + 12*a^2*b*d*x^
6 - 6*a^3*x^6*e + 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)
```

3.259 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=214

$$\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\log\left(a+bx^3\right)\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log}{}$$

```
[Out] -c/(12*a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e)/
(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*a*
b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*
b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rubi [A] time = 0.233997, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\log\left(a+bx^3\right)\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{3a^6}+\frac{b\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```


```
[Out] -c/(12*a^2*x^12) + (2*b*c - a*d)/(9*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*e)/
(6*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f))/(3*a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*a*
b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*a^2*
b*e - 2*a^3*f)*Log[a + b*x^3])/(3*a^6)
```


Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int\left(\frac{c}{a^2x^5}+\frac{-2bc+ad}{a^3x^4}+\frac{3b^2c-2abd+a^2e}{a^4x^3}+\frac{-4b^3c+3ab^2d-2a^2be+a^3f}{a^5x^2}-\right.\right.$$

$$= -\frac{c}{12a^2x^{12}}+\frac{2bc-ad}{9a^3x^9}-\frac{3b^2c-2abd+a^2e}{6a^4x^6}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\frac{b\left(b^3c-ab^2\right.}{3a^5\left(\right.}$$

Mathematica [A] time = 0.202961, size = 198, normalized size = 0.93

$$-\frac{\frac{12ab\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a+bx^3}+\frac{12a\left(-2a^2be+a^3f+3ab^2d-4b^3c\right)}{x^3}+12b\log\left(a+bx^3\right)\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)-36b\log(x)\left(3a\right.}{36a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

```
[Out] -((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c - 2*a*b*d +
 a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^3 + (12*
a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) - 36*b*(5*b^3*c - 4
*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x] + 12*b*(5*b^3*c - 4*a*b^2*d + 3*a^2*
b*e - 2*a^3*f)*Log[a + b*x^3])/(36*a^6)
```

Maple [A] time = 0.02, size = 282, normalized size = 1.3

$$\frac{2\,b\ln\left(bx^3+a\right)f}{3\,a^3}-\frac{b^2\ln\left(bx^3+a\right)e}{a^4}+\frac{4\,b^3\ln\left(bx^3+a\right)d}{3\,a^5}-\frac{5\,b^4\ln\left(bx^3+a\right)c}{3\,a^6}-\frac{fb}{3\,a^2\left(bx^3+a\right)}+\frac{eb^2}{3\,a^3\left(bx^3+a\right)}-\frac{}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x)
```

```
[Out] 2/3*b/a^3*ln(b*x^3+a)*f-b^2/a^4*ln(b*x^3+a)*e+4/3*b^3/a^5*ln(b*x^3+a)*d-5/3
*b^4/a^6*ln(b*x^3+a)*c-1/3*b/a^2/(b*x^3+a)*f+1/3*b^2/a^3/(b*x^3+a)*e-1/3*b^
3/a^4/(b*x^3+a)*d+1/3*b^4/a^5/(b*x^3+a)*c-1/12*c/a^2/x^12-1/9/a^2/x^9*d+2/9
/a^3/x^9*b*c-1/6/a^2/x^6*e+1/3/a^3/x^6*b*d-1/2/a^4/x^6*b^2*c-1/3/a^2/x^3*f+
2/3/a^3/x^3*b*e-1/a^4/x^3*b^2*d+4/3/a^5/x^3*b^3*c-2*b/a^3*ln(x)*f+3*b^2/a^4
*ln(x)*e-4*b^3/a^5*ln(x)*d+5*b^4/a^6*ln(x)*c
```

Maxima [A] time = 1.01741, size = 305, normalized size = 1.43

$$\frac{12\left(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf\right)x^{12}+6\left(5\,ab^3c-4\,a^2b^2d+3\,a^3be-2\,a^4f\right)x^9-2\left(5\,a^2b^2c-4\,a^3bd+3\,a^4e\right)x^6-3}{36\left(a^5bx^{15}+a^6x^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*x^12 + 6*(5*a*b^3*
c - 4*a^2*b^2*d + 3*a^3*b*e - 2*a^4*f)*x^9 - 2*(5*a^2*b^2*c - 4*a^3*b*d + 3
*a^4*e)*x^6 - 3*a^4*c + (5*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b*x^15 + a^6*x^12)
- 1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(b*x^3 + a)/a^6 +
1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(x^3)/a^6
```

Fricas [A] time = 1.60778, size = 670, normalized size = 3.13

$$\frac{12\left(5ab^4c-4a^2b^3d+3a^3b^2e-2a^4bf\right)x^{12}+6\left(5a^2b^3c-4a^3b^2d+3a^4be-2a^5f\right)x^9-2\left(5a^3b^2c-4a^4bd+3a^5e\right)x^6-3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12 + 6*(5*a^
2*b^3*c - 4*a^3*b^2*d + 3*a^4*b*e - 2*a^5*f)*x^9 - 2*(5*a^3*b^2*c - 4*a^4*b
*d + 3*a^5*e)*x^6 - 3*a^5*c + (5*a^4*b*c - 4*a^5*d)*x^3 - 12*((5*b^5*c - 4*
a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^
3*b^2*e - 2*a^4*b*f)*x^12)*log(b*x^3 + a) + 36*((5*b^5*c - 4*a*b^4*d + 3*a^
2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^
4*b*f)*x^12)*log(x))/(a^6*b*x^15 + a^7*x^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07398, size = 447, normalized size = 2.09

$$\frac{\left(5b^4c-4ab^3d-2a^3bf+3a^2b^2e\right)\log\left(|x|\right)}{a^6}-\frac{\left(5b^5c-4ab^4d-2a^3b^2f+3a^2b^3e\right)\log\left(\left|bx^3+a\right|\right)}{3a^6b}+\frac{5b^5cx^3-4ab^4dx^3-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (5*b^4*c - 4*a*b^3*d - 2*a^3*b*f + 3*a^2*b^2*e)*log(abs(x))/a^6 - 1/3*(5*b^
5*c - 4*a*b^4*d - 2*a^3*b^2*f + 3*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^6*b) +
1/3*(5*b^5*c*x^3 - 4*a*b^4*d*x^3 - 2*a^3*b^2*f*x^3 + 3*a^2*b^3*x^3*e + 6*a*
b^4*c - 5*a^2*b^3*d - 3*a^4*b*f + 4*a^3*b^2*e)/((b*x^3 + a)*a^6) - 1/36*(12
5*b^4*c*x^12 - 100*a*b^3*d*x^12 - 50*a^3*b*f*x^12 + 75*a^2*b^2*x^12*e - 48*
a*b^3*c*x^9 + 36*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 24*a^3*b*x^9*e + 18*a^2*b^2
*c*x^6 - 12*a^3*b*d*x^6 + 6*a^4*x^6*e - 8*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4
*c)/(a^6*x^12)
```

3.260 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=369

$$\frac{x^4\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{4b^5}-\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(13a^2be-16a\right.}{18b^{19/3}}$$

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*c - 2*a*b^
2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)
/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) - (a^2*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 1
0*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b
*e - 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^3*c -
 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*b^(19/3))
```

Rubi [A] time = 0.467051, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x^4\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{4b^5}-\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^6\left(a+bx^3\right)}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(13a^2be-16a\right.}{18b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] -((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x)/b^6) + ((b^3*c - 2*a*b^
2*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)
/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) - (a^2*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^6*(a + b*x^3)) - (a^(4/3)*(7*b^3*c - 1
0*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*b^(19/3)) + (a^(4/3)*(7*b^3*c - 10*a*b^2*d + 13*a^2*b
*e - 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) - (a^(4/3)*(7*b^3*c -
 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*b^(19/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^6\left(a+bx^3\right)} - \int \frac{-a^3\left(b^3c-ab^2d+a^2be-a^3f\right)+3a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-3ab\cdots}{\cdots}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^6\left(a+bx^3\right)} - \frac{\int\left(3a^2\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)-3ab\left(b^3c\cdots\right.\right.}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{\left(b^2d-2a\cdots\right.}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{\left(b^2d-2a\cdots\right.}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{\left(b^2d-2a\cdots\right.}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{\left(b^2d-2a\cdots\right.}{\cdots}$$

$$= -\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)x}{b^6} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^4}{4b^5} + \frac{\left(b^2d-2a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.352521, size = 364, normalized size = 0.99

$$\frac{x^4\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{4b^5} + \frac{a^2x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{3b^6\left(a+bx^3\right)} + \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-13a^2be+16a\cdots\right.}{18b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)/(
7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) + (a^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x)/(3*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3*c +
 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^2*b*e
+ 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^3*c + 1
0*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*b^(19/3))
```

Maple [A] time = 0.01, size = 622, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 7/9*a^2/b^4*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))-16/9*a^5/b^7*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)
*x-1))+13/9*a^4/b^6*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))+1/10/b^2*x^10*e+1/7/b^2*x^7*d+1/4/b^2*x^4*c-10/9*a^3/b^5*d/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-7/18*a^2/b^4*c/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*a^4/b^5*x/(b*x^3+a)*
e+1/3*a^3/b^4*x/(b*x^3+a)*d-1/3*a^2/b^3*x/(b*x^3+a)*c+1/3*a^5/b^6*x/(b*x^3+
a)*f+7/9*a^2/b^4*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+13/9*a^4/b^6*e/(1/b*a)
^(2/3)*ln(x+(1/b*a)^(1/3))-13/18*a^4/b^6*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/
3)*x+(1/b*a)^(2/3))-10/9*a^3/b^5*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/9*a^
3/b^5*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+8/9*a^5/b^7*f/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-16/9*a^5/b^7*f/(1/b*a)^(
2/3)*ln(x+(1/b*a)^(1/3))+1/13*f*x^13/b^2-1/5/b^3*x^10*a*f+3/7/b^4*x^7*a^2*f
-2/7/b^3*x^7*a*e-1/b^5*x^4*a^3*f+3/4/b^4*x^4*a^2*e-1/2/b^3*x^4*a*d+5/b^6*a^
4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41347, size = 1145, normalized size = 3.1

$$\frac{1260\, b^5 f x^{16} + 126\left(13\, b^5 e - 16\, a b^4 f\right)x^{13} + 234\left(10\, b^5 d - 13\, a b^4 e + 16\, a^2 b^3 f\right)x^{10} + 585\left(7\, b^5 c - 10\, a b^4 d + 13\, a^2 b^3 e - 16\, a^3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16380*(1260*b^5*f*x^16 + 126*(13*b^5*e - 16*a*b^4*f)*x^13 + 234*(10*b^5*d
 - 13*a*b^4*e + 16*a^2*b^3*f)*x^10 + 585*(7*b^5*c - 10*a*b^4*d + 13*a^2*b^3
*e - 16*a^3*b^2*f)*x^7 - 4095*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16
*a^4*b*f)*x^4 - 1820*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*
a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^
(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(7*a^2*b
^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 1
3*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3)) - 1820*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a
*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x
- (-a/b)^(1/3)) - 5460*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)
*x)/(b^7*x^3 + a*b^6)
```

Sympy [A] time = 10.5078, size = 490, normalized size = 1.33

$$\frac{x\left(a^5 f - a^4 b e + a^3 b^2 d - a^2 b^3 c\right)}{3 a b^6 + 3 b^7 x^3} + \operatorname{RootSum}\left(729 t^3 b^{19} + 4096 a^{13} f^3 - 9984 a^{12} b e f^2 + 7680 a^{11} b^2 d f^2 + 8112 a^{11} b^2 e^2 f - 53\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b**7*x**3)
+ RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**12*b*e*f**2 + 7680*a*
*11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 5376*a**10*b**3*c*f**2 - 12480*a
**10*b**3*d*e*f - 2197*a**10*b**3*e**3 + 8736*a**9*b**4*c*e*f + 4800*a**9*b
**4*d**2*f + 5070*a**9*b**4*d*e**2 - 6720*a**8*b**5*c*d*f - 3549*a**8*b**5*
c*e**2 - 3900*a**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460*a**7*b**6*c*d
*e + 1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6*b**7*c*d**2 +
1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*log(-9*_t*b**6/(1
6*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3*c) + x))) + f*x**13/(13*
b**2) - x**10*(2*a*f - b*e)/(10*b**3) + x**7*(3*a**2*f - 2*a*b*e + b**2*d)/
(7*b**4) - x**4*(4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(4*b**5) + x*
(5*a**4*f - 4*a**3*b*e + 3*a**2*b**2*d - 2*a*b**3*c)/b**6
```

Giac [A] time = 1.07843, size = 609, normalized size = 1.65

$$\frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}ab^3c-10\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d-16\left(-ab^2\right)^{\frac{1}{3}}a^4f+13\left(-ab^2\right)^{\frac{1}{3}}a^3be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^7}-\frac{\left(7a^2b^3c-10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-
a*b^2)^(1/3)*a^4*f + 13*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d - 16*a^5*f
 + 13*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(7*(-
a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d - 16*(-a*b^2)^(1/3)*a^4*
f + 13*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
 - 1/3*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/((b*x^3 + a)*b^6)
+ 1/1820*(140*b^24*f*x^13 - 364*a*b^23*f*x^10 + 182*b^24*x^10*e + 260*b^24*
d*x^7 + 780*a^2*b^22*f*x^7 - 520*a*b^23*x^7*e + 455*b^24*c*x^4 - 910*a*b^23
*d*x^4 - 1820*a^3*b^21*f*x^4 + 1365*a^2*b^22*x^4*e - 3640*a*b^23*c*x + 5460
*a^2*b^22*d*x + 9100*a^4*b^20*f*x - 7280*a^3*b^21*x*e)/b^26
```

3.261 $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=335

$$\frac{x^2\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{2b^5}+\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(11a^2be-14a^3f\right.}{18b^{17/3}}$$

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2)
+ (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*
d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rubi [A] time = 0.705022, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{2b^5}+\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(11a^2be-14a^3f\right.}{18b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/(2*b^5) + ((b^2*d - 2*a*b*e
 + 3*a^2*f)*x^5)/(5*b^4) + ((b*e - 2*a*f)*x^8)/(8*b^3) + (f*x^11)/(11*b^2)
+ (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) + (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(17/3)) + (a^(2/3)*(5*b^3*c - 8*a*b^2*
d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(17/3)) - (a^(2/3
)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(18*b^(17/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x-3ab^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^4-3ab^3\left(b^2\right.}{a+bx^3}}{3ab^6}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{x\left(2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-3ab^3\left(b^2\right.\right.}{a+bx^3}}{3ab^6}$$

$$= \frac{fx^{11}}{11b^2} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{x\left(22a^2b^2\left(b^3c-ab^2d+a^2be-a^3f\right)-33ab^3\left(b^3c-ab^2d+a^2be-a^3\right.\right.}{a+bx^3}}{33ab^7}$$

$$= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \int \frac{x\left(176a^2b^3\left(b^3c-ab^2d+a^2be-a^3f\right)-26\right.}{}$$

$$= \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^5\left(a+bx^3\right)} - \frac{\int \left(-264ab^3\left(b^3c-2ab^2d+3a\right.\right.}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x^2}{2b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3} + \frac{fx^{11}}{11b^2} + \frac{a}{}$$

Mathematica [A] time = 0.165613, size = 319, normalized size = 0.95

$$1980b^{2/3}x^2\left(3a^2be-4a^3f-2ab^2d+b^3c\right) + \frac{1320ab^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a+bx^3} + 220a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-11a^2be\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792*b^(5/3)*(
b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a*f)*x^8 + 360*b^(11/
3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b
*x^3) - 440*Sqrt[3]*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-5*b^3*c + 8*a*b
^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-5*b^
3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(3960*b^(17/3))
```

Maple [B] time = 0.01, size = 584, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -11/9*a^3/b^5*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))+8/9*a^2/b^4*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3
)*x-1))+1/8/b^2*x^8*e+1/5/b^2*x^5*d+1/2/b^2*x^2*c-1/4/b^3*x^8*a*f+3/5/b^4*x
^5*a^2*f-5/9*a/b^3*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1
/3)*x-1))+14/9*a^4/b^6*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a
)^(1/3)*x-1))+5/9*a/b^3*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18*a/b^3*c/(1
/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*a^2/b^3*x^2/(b*x^3+a)
*d+1/3*a/b^2*x^2/(b*x^3+a)*c-14/9*a^4/b^6*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3
))+7/9*a^4/b^6*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+11/9*a
^3/b^5*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-11/18*a^3/b^5*e/(1/b*a)^(1/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-8/9*a^2/b^4*d/(1/b*a)^(1/3)*ln(x+(1/b*a
)^(1/3))+4/9*a^2/b^4*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-
1/3*a^4/b^5*x^2/(b*x^3+a)*f+1/3*a^3/b^4*x^2/(b*x^3+a)*e+1/11*f*x^11/b^2-2/5
/b^3*x^5*a*e-2/b^5*x^2*a^3*f+3/2/b^4*x^2*a^2*e-1/b^3*x^2*a*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29785, size = 1057, normalized size = 3.16

$$\frac{360\, b^4 f x^{14} + 45\left(11\, b^4 e - 14\, a b^3 f\right)x^{11} + 99\left(8\, b^4 d - 11\, a b^3 e + 14\, a^2 b^2 f\right)x^8 + 396\left(5\, b^4 c - 8\, a b^3 d + 11\, a^2 b^2 e - 14\, a^3 b f\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(360*b^4*f*x^14 + 45*(11*b^4*e - 14*a*b^3*f)*x^11 + 99*(8*b^4*d - 11
*a*b^3*e + 14*a^2*b^2*f)*x^8 + 396*(5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14
*a^3*b*f)*x^5 + 660*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*x^2 -
 440*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c -
8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*
sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(5*a*b^3*c - 8*a^2*b^2*d
 + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f
)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/
3)) - 440*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a
```

```
*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2
/b^2)^(2/3)))/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 37.4541, size = 530, normalized size = 1.58

$$-\frac{x^2\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{3ab^5+3b^6x^3}+\mathrm{RootSum}\left(729t^3b^{17}+2744a^{11}f^3-6468a^{10}bef^2+4704a^9b^2df^2+5082a^9b^2e^2f-29\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x**2*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*b**6*x**3)
 + RootSum(729*_t**3*b**17 + 2744*a**11*f**3 - 6468*a**10*b*e*f**2 + 4704*a
**9*b**2*d*f**2 + 5082*a**9*b**2*e**2*f - 2940*a**8*b**3*c*f**2 - 7392*a**8
*b**3*d*e*f - 1331*a**8*b**3*e**3 + 4620*a**7*b**4*c*e*f + 2688*a**7*b**4*d
**2*f + 2904*a**7*b**4*d*e**2 - 3360*a**6*b**5*c*d*f - 1815*a**6*b**5*c*e**
2 - 2112*a**6*b**5*d**2*e + 1050*a**5*b**6*c**2*f + 2640*a**5*b**6*c*d*e +
512*a**5*b**6*d**3 - 825*a**4*b**7*c**2*e - 960*a**4*b**7*c*d**2 + 600*a**3
*b**8*c**2*d - 125*a**2*b**9*c**3, Lambda(_t, _t*log(81*_t**2*b**11/(196*a*
*7*f**2 - 308*a**6*b*e*f + 224*a**5*b**2*d*f + 121*a**5*b**2*e**2 - 140*a**
4*b**3*c*f - 176*a**4*b**3*d*e + 110*a**3*b**4*c*e + 64*a**3*b**4*d**2 - 80
*a**2*b**5*c*d + 25*a*b**6*c**2) + x))) + f*x**11/(11*b**2) - x**8*(2*a*f -
 b*e)/(8*b**3) + x**5*(3*a**2*f - 2*a*b*e + b**2*d)/(5*b**4) - x**2*(4*a**3
*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c)/(2*b**5)
```

Giac [A] time = 1.08749, size = 597, normalized size = 1.78

$$\frac{\left(5\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-8\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11\,a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{2}{3}}b^3c-8\left(-ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) - 14*a^4*f*(-a/b)^(1
/3) + 11*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^(2/3)*a*b^2*d - 14*(
-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 - a^4*f*
x^2 + a^3*b*x^2*e)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a
*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 11*(-a*b^2)^(2/3)*a^2*b*e)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*f*x^11 - 110*
a*b^19*f*x^8 + 55*b^20*x^8*e + 88*b^20*d*x^5 + 264*a^2*b^18*f*x^5 - 176*a*b
^19*x^5*e + 220*b^20*c*x^2 - 440*a*b^19*d*x^2 - 880*a^3*b^17*f*x^2 + 660*a^
2*b^18*x^2*e)/b^22
```

3.262 $\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=328

$$\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{18b^{16/3}}+\frac{x\left(3a^2be-4a^{\cdots}\right.}{}$$

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a
^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(18*b^(16/3))
```

Rubi [A] time = 0.368865, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{18b^{16/3}}+\frac{x\left(3a^2be-4a^{\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^4)/(4*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^10)/(10*b^2) + (a*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*b^(16/3)) - (a^(1/3)*(4*b^3*c - 7*a*b^2*d + 10*a
^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(16/3)) + (a^(1/3)*(4*b^3
*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(18*b^(16/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^5\left(a+bx^3\right)} - \frac{\int \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-3ab^2\left(b^2d-\right.}{a+bx^3}}{3ab^5}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^5\left(a+bx^3\right)} - \frac{\int\left(-3a\left(b^3c-2ab^2d+3a^2be-4a^3f\right)-3ab\left(b^2d-2\right.\right.}{}$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} +$$

$$= \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^4}{4b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^{10}}{10b^2} +$$

Mathematica [A] time = 0.248407, size = 315, normalized size = 0.96

$$\frac{\frac{420a\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a+bx^3} - 70\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-10a^2be+13a^3f+7ab^2d-4b^3c\right)+1260\sqrt[3]{b}x\left(3a^2be\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b^(4/3)*(b^
2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f)*x^7 + 126*b^(10/3)
*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)
 - 140*Sqrt[3]*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b^2*d
- 10*a^2*b*e + 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^3*c +
7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(1260*b^(16/3))
```

Maple [B] time = 0.01, size = 567, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 5/9*a^3/b^5*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/9*a^2/b
^4*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/7/b^2*x^7*e+1/4/b^2*x^4*d+1/b^2*c*
```

```
x+7/9*a^2/b^4*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))+1/10*f*x^10/b^2+1/3*a/b^2*x/(b*x^3+a)*c+13/9*a^4/b^6*f/(1/b*a)^(2/3)*l
n(x+(1/b*a)^(1/3))-13/18*a^4/b^6*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))-10/9*a^3/b^5*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-7/18*a^2/b^4*d
/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9*a/b^3*c/(1/b*a)^(2
/3)*ln(x+(1/b*a)^(1/3))+2/9*a/b^3*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))-1/3*a^4/b^5*x/(b*x^3+a)*f+1/3*a^3/b^4*x/(b*x^3+a)*e-1/3*a^2/b^
3*x/(b*x^3+a)*d-2/b^3*a*d*x-2/7/b^3*x^7*a*f+3/4/b^4*x^4*a^2*f-1/2/b^3*x^4*a
*e-4/b^5*a^3*f*x+3/b^4*a^2*e*x-4/9*a/b^3*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+13/9*a^4/b^6*f/(1/b*a)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-10/9*a^3/b^5*e/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36585, size = 992, normalized size = 3.02

$$\frac{126\,b^4fx^{13}+18\left(10\,b^4e-13\,ab^3f\right)x^{10}+45\left(7\,b^4d-10\,ab^3e+13\,a^2b^2f\right)x^7+315\left(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf\right)x^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(126*b^4*f*x^13 + 18*(10*b^4*e - 13*a*b^3*f)*x^10 + 45*(7*b^4*d - 10
*a*b^3*e + 13*a^2*b^2*f)*x^7 + 315*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13
*a^3*b*f)*x^4 - 140*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*
f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*arct
an(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(4*a*b^3*c - 7*a^2*b
^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3
*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*(4*a*b^
3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b
^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(4*a*b^3*c -
 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*x)/(b^6*x^3 + a*b^5)
```

Sympy [A] time = 10.9149, size = 440, normalized size = 1.34

$$-\frac{x\left(a^4f-a^3be+a^2b^2d-ab^3c\right)}{3ab^5+3b^6x^3}+\mathrm{RootSum}\left(729t^3b^{16}-2197a^{10}f^3+5070a^9bef^2-3549a^8b^2df^2-3900a^8b^2e^2f+2028\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c)/(3*a*b**5 + 3*b**6*x**3) +
RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b*e*f**2 - 3549*a**8*
b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a**7*b**3*c*f**2 + 5460*a**7*b**
3*d*e*f + 1000*a**7*b**3*e**3 - 3120*a**6*b**4*c*e*f - 1911*a**6*b**4*d**2*
f - 2100*a**6*b**4*d*e**2 + 2184*a**5*b**5*c*d*f + 1200*a**5*b**5*c*e**2 +
1470*a**5*b**5*d**2*e - 624*a**4*b**6*c**2*f - 1680*a**4*b**6*c*d*e - 343*a
**4*b**6*d**3 + 480*a**3*b**7*c**2*e + 588*a**3*b**7*c*d**2 - 336*a**2*b**8
*c**2*d + 64*a*b**9*c**3, Lambda(_t, _t*log(9*_t*b**5/(13*a**3*f - 10*a**2*
b*e + 7*a*b**2*d - 4*b**3*c) + x))) + f*x**10/(10*b**2) - x**7*(2*a*f - b*e
)/(7*b**3) + x**4*(3*a**2*f - 2*a*b*e + b**2*d)/(4*b**4) - x*(4*a**3*f - 3*
a**2*b*e + 2*a*b**2*d - b**3*c)/b**5
```

Giac [A] time = 1.0802, size = 532, normalized size = 1.62

$$-\frac{\sqrt{3}\left(4\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-7\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-13\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+10\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^{6}}+\frac{\left(4ab^{3}c-7a^{2}b^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d - 13*(-a*b^
2)^(1/3)*a^3*f + 10*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/b^6 + 1/9*(4*a*b^3*c - 7*a^2*b^2*d - 13*a^4*f + 10*a
^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(4*(-a*b^2)^
(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d - 13*(-a*b^2)^(1/3)*a^3*f + 10*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3*(a*b^
3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x^3 + a)*b^5) + 1/140*(14*b^
18*f*x^10 - 40*a*b^17*f*x^7 + 20*b^18*x^7*e + 35*b^18*d*x^4 + 105*a^2*b^16*
f*x^4 - 70*a*b^17*x^4*e + 140*b^18*c*x - 280*a*b^17*d*x - 560*a^3*b^15*f*x
+ 420*a^2*b^16*x*e)/b^20
```

3.263 $\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=298

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^2be-11a^3f-5ab^2d+2b^3c\right)}{18\sqrt[3]{a}b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^2\right.}{9}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (
f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^
3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*
b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)
) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rubi [A] time = 0.462677, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^2be-11a^3f-5ab^2d+2b^3c\right)}{18\sqrt[3]{a}b^{14/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(8a^2\right.}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/(2*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (
f*x^8)/(8*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^4*(a + b*x^
3)) - ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(14/3)) - ((2*b^3*c - 5*a*
b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b^(14/3)
) + ((2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(14/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)x-3ab^2\left(b^2d-abe+a^2f\right)x^4-3ab^3(be-af)x^7}{a+bx^3}}{3ab^5}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{x\left(-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab^2\left(b^2d-abe+a^2f\right)x^3-3ab^3(be-af)x^6\right)}{a+bx^3}}{3ab^5}$$

$$= \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{x\left(-16ab^2\left(b^3c-ab^2d+a^2be-a^3f\right)-24ab^3\left(b^2d-abe+a^2f\right)x^3-24a\right.}{a+bx^3}}{24ab^6}$$

$$= \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{\int \left(-24ab^2\left(b^2d-2abe+3a^2f\right)x-24ab^3(be-2\right.}{24ab^6}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} + \frac{(2b^3c}{}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{(2b^3c}{}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{(2b^3c}{}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{(2b^3c}{}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x^2}{2b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3b^4\left(a+bx^3\right)} - \frac{(2b^3c}{}$$

Mathematica [A] time = 0.156751, size = 282, normalized size = 0.95

$$\frac{-\frac{120b^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a+bx^3} + \frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^2be-11a^3f-5ab^2d+2b^3c\right)}{\sqrt[3]{a}} + \frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-8a^2be+11a^3f+5ab^2d-2b^3c\right)}{\sqrt[3]{a}} + \frac{40\sqrt{3}}{}}{360b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e - 2*a*f)*x^5
 + 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)
/(a + b*x^3) + (40*Sqrt[3]*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*f)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-2*b^3*c + 5*a*b^
2*d - 8*a^2*b*e + 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(2*b^3*
c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(1/3))/(360*b^(14/3))
```

Maple [B] time = 0.009, size = 529, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/8*f*x^8/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+3/2/b^4*x^2*a^2*f-1/b^3*x^2*a*e
+1/2/b^2*x^2*d+1/3/b^4*x^2/(b*x^3+a)*a^3*f-1/3/b^3*x^2/(b*x^3+a)*a^2*e+1/3/
b^2*x^2/(b*x^3+a)*a*d-1/3/b*x^2/(b*x^3+a)*c+11/9/b^5*a^3*f/(1/b*a)^(1/3)*ln
(x+(1/b*a)^(1/3))-11/18/b^5*a^3*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))-11/9/b^5*a^3*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x-1))-8/9/b^4*a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+4/9/b^4*a^2*
e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+8/9/b^4*a^2*e*3^(1/2)
/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+5/9/b^3*a*d/(1/b*a
)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18/b^3*a*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))-5/9/b^3*a*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))-2/9/b^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*c/(1
/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*c*3^(1/2)/(1/b*a)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7054, size = 2094, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d
- 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3
*e - 11*a^4*b^2*f)*x^2 - 60*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^
2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x
^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a
*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)
*x)/(b*x^3 + a)) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*
b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^
2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^
3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a
*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6), 1/360*(45*a*b^
5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8*a^2*b^4*e +
 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2
*f)*x^2 - 120*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^2*e - 11*a^5*b
*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b
^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(
```

```
1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^
4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6)]
```

Sympy [A] time = 35.7815, size = 484, normalized size = 1.62

$$\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^4+3b^5x^3}+\mathrm{RootSum}\left(729t^3ab^{14}-1331a^9f^3+2904a^8bef^2-1815a^7b^2df^2-2112a^7b^2e^2f+726a^6b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + Roo
tSum(729*_t**3*a*b**14 - 1331*a**9*f**3 + 2904*a**8*b*e*f**2 - 1815*a**7*b*
*2*d*f**2 - 2112*a**7*b**2*e**2*f + 726*a**6*b**3*c*f**2 + 2640*a**6*b**3*d
*e*f + 512*a**6*b**3*e**3 - 1056*a**5*b**4*c*e*f - 825*a**5*b**4*d**2*f - 9
60*a**5*b**4*d*e**2 + 660*a**4*b**5*c*d*f + 384*a**4*b**5*c*e**2 + 600*a**4
*b**5*d**2*e - 132*a**3*b**6*c**2*f - 480*a**3*b**6*c*d*e - 125*a**3*b**6*d
**3 + 96*a**2*b**7*c**2*e + 150*a**2*b**7*c*d**2 - 60*a*b**8*c**2*d + 8*b**
9*c**3, Lambda(_t, _t*log(81*_t**2*a*b**9/(121*a**6*f**2 - 176*a**5*b*e*f +
 110*a**4*b**2*d*f + 64*a**4*b**2*e**2 - 44*a**3*b**3*c*f - 80*a**3*b**3*d*
e + 32*a**2*b**4*c*e + 25*a**2*b**4*d**2 - 20*a*b**5*c*d + 4*b**6*c**2) + x
))) + f*x**8/(8*b**2) - x**5*(2*a*f - b*e)/(5*b**3) + x**2*(3*a**2*f - 2*a*
b*e + b**2*d)/(2*b**4)
```

Giac [A] time = 1.08374, size = 537, normalized size = 1.8

$$-\frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-11\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}-\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx}{3\left(bx^3+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/3) - 11*a^3*f*(-a/b)^(1/3)
 + 8*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4)
- 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*b^4)
 - 1/9*sqrt(3)*(2*(-a*b^2)^(2/3)*b^3*c - 5*(-a*b^2)^(2/3)*a*b^2*d - 11*(-a*
b^2)^(2/3)*a^3*f + 8*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a*b^6) + 1/18*(2*(-a*b^2)^(2/3)*b^3*c - 5*(-a*b^2)
^(2/3)*a*b^2*d - 11*(-a*b^2)^(2/3)*a^3*f + 8*(-a*b^2)^(2/3)*a^2*b*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^6) + 1/40*(5*b^14*f*x^8 - 16*a*b^13
*f*x^5 + 8*b^14*x^5*e + 20*b^14*d*x^2 + 60*a^2*b^12*f*x^2 - 40*a*b^13*x^2*e
)/b^16
```

3.264 $\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=288

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{18a^{2/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(7a}{9}$$

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)
/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((
b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^
2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c
- 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rubi [A] time = 0.32561, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1828, 1887, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{18a^{2/3}b^{13/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(7a}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^4)/(4*b^3) + (f*x^7)
/(7*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^4*(a + b*x^3)) - ((
b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(13/3)) + ((b^3*c - 4*a*b^2*d + 7*a^
2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(13/3)) - ((b^3*c
- 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(2/3)*b^(13/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} - \frac{\int \frac{-a\left(b^3c-ab^2d+a^2be-a^3f\right)-3ab\left(b^2d-abe+a^2f\right)x^3-3ab^2(be-af)x^6-\cdots}{a+bx^3}}{3ab^4}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} - \frac{\int\left(-3a\left(b^2d-2abe+3a^2f\right)-3ab(be-2af)x^3-3a\cdots\right.}{3ab^4}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} + \frac{\left(b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} + \frac{\left(b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} + \frac{\left(b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} + \frac{\left(b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4} + \frac{(be-2af)x^4}{4b^3} + \frac{fx^7}{7b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3b^4\left(a+bx^3\right)} - \frac{\left(b^3c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.157155, size = 277, normalized size = 0.96

$$\frac{-\frac{84\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a+bx^3} + \frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-7a^2be+10a^3f+4ab^2d-b^3c\right)}{a^{2/3}} + \frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{a^{2/3}} + \frac{28\sqrt{3}\cdots}{\cdots}}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (252*b^(1/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^(4/3)*(b*e - 2*a*f)*x^4 +
 36*b^(7/3)*f*x^7 - (84*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a +
 b*x^3) + (28*Sqrt[3]*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - 4*a*b^2*d + 7*
a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + 4*a
*b^2*d - 7*a^2*b*e + 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3))/(252*b^(13/3))
```

Maple [B] time = 0.01, size = 514, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/7*f*x^7/b^2-1/2/b^3*x^4*a*f+1/4/b^2*x^4*e+3/b^4*a^2*f*x-2/b^3*a*e*x+1/b^2
*d*x+1/3/b^4*x/(b*x^3+a)*a^3*f-1/3/b^3*x/(b*x^3+a)*a^2*e+1/3/b^2*x/(b*x^3+a
```

```
)*a*d-1/3/b*x/(b*x^3+a)*c-10/9/b^5*a^3*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+
5/9/b^5*a^3*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-10/9/b^5*
a^3*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+7/9/b
^4*a^2*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-7/18/b^4*a^2*e/(1/b*a)^(2/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/9/b^4*a^2*e/(1/b*a)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-4/9/b^3*a*d/(1/b*a)^(2/3)*ln(x+(1/b*a
)^(1/3))+2/9/b^3*a*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/
9/b^3*a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1
/9/b^2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/18/b^2*c/(1/b*a)^(2/3)*ln(x^2-
(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/b^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44706, size = 2125, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*
b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 - 42*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b
^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10
*a^4*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a
*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sq
rt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e
 - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(
2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c - 4*a
^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3
*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c - 4*a
^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5), 1/252*(36*
a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2*b^4*d - 7*a
^3*b^3*e + 10*a^4*b^2*f)*x^4 + 84*sqrt(1/3)*(a^2*b^4*c - 4*a^3*b^3*d + 7*a^
4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e - 10*a^4*b^2*f)
*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*
b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^
3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3*c
 - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e -
10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b^4*c
 - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5)]
```

Sympy [A] time = 10.1333, size = 398, normalized size = 1.38

$$\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{3ab^4+3b^5x^3}+\mathrm{RootSum}\left(729t^3a^2b^{13}+1000a^9f^3-2100a^8bef^2+1200a^7b^2df^2+1470a^7b^2e^2f-300\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + RootSu
m(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e*f**2 + 1200*a**7*b*
*2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 1680*a**6*b**3*d
*e*f - 343*a**6*b**3*e**3 + 420*a**5*b**4*c*e*f + 480*a**5*b**4*d**2*f + 58
8*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 - 336*a**4*
b**5*d**2*e + 30*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3*b**6*d**3
 - 21*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d - b**9*c**3
, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4*a*b**2*d - b**
3*c) + x))) + f*x**7/(7*b**2) - x**4*(2*a*f - b*e)/(4*b**3) + x*(3*a**2*f -
 2*a*b*e + b**2*d)/b**4
```

Giac [A] time = 1.10032, size = 471, normalized size = 1.64

$$-\frac{\left(b^3c-4\,ab^2d-10\,a^3f+7\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-4\left(-ab^2\right)^{\frac{1}{3}}ab^2d-10\left(-ab^2\right)^{\frac{1}{3}}a^3f+7\right.}{9\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b^3*c - 4*a*b^2*d - 10*a^3*f + 7*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a*b^4) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - 4*(-a*b^2)^(1/3)
*a*b^2*d - 10*(-a*b^2)^(1/3)*a^3*f + 7*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^5) - 1/3*(b^3*c*x - a*b^2*d*
x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*b^4) + 1/18*((-a*b^2)^(1/3)*b^3*c - 4
*(-a*b^2)^(1/3)*a*b^2*d - 10*(-a*b^2)^(1/3)*a^3*f + 7*(-a*b^2)^(1/3)*a^2*b*
e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5) + 1/28*(4*b^12*f*x^7 -
14*a*b^11*f*x^4 + 7*b^12*x^4*e + 28*b^12*d*x + 84*a^2*b^10*f*x - 56*a*b^11*
x*e)/b^14
```

3.265 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} dx$

Optimal. Leaf size=271

$$\frac{x^2 \left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3ab^3 \left(a + bx^3\right)} + \frac{\log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) \left(-5a^2be + 8a^3f + 2ab^2d + b^3c\right)}{18a^{4/3}b^{11/3}} - \frac{\log \left(\sqrt[3]{a} + \sqrt[3]{b}x\right) \left(-5a^2\right.}{9a^4}$$

```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3
)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```

Rubi [A] time = 0.289389, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1828, 1594, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2 \left(a^2be + a^3(-f) - ab^2d + b^3c\right)}{3ab^3 \left(a + bx^3\right)} + \frac{\log \left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) \left(-5a^2be + 8a^3f + 2ab^2d + b^3c\right)}{18a^{4/3}b^{11/3}} - \frac{\log \left(\sqrt[3]{a} + \sqrt[3]{b}x\right) \left(-5a^2\right.}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```


```
[Out] ((b*e - 2*a*f)*x^2)/(2*b^3) + (f*x^5)/(5*b^2) + ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3
)*b^(11/3)) - ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(9*a^(4/3)*b^(11/3)) + ((b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(11/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
```

```
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{-b\left(b^3c+2ab^2d-2a^2be+2a^3f\right)x-3ab^2(be-af)x^4-3ab^3fx^7}{a+bx^3}dx}{3ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{x\left(-b\left(b^3c+2ab^2d-2a^2be+2a^3f\right)-3ab^2(be-af)x^3-3ab^3fx^6\right)}{a+bx^3}dx}{3ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\int \left(-3ab(be-2af)x-3ab^2fx^4+\frac{\left(-b^4c-2ab^3d+5a^2b^2e-8a^3bf\right)}{a+bx^3}\right.}{3ab^4}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} + \frac{\left(b^3c+2ab^2d-5a^2be+8a^3f\right)\int}{3ab^3}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+8a^3f\right)\int}{9a^{4/3}b^{10/3}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+8a^3f\right)\log}{9a^{4/3}b^{11/3}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+8a^3f\right)\log}{9a^{4/3}b^{11/3}}$$

$$= \frac{(be-2af)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3ab^3\left(a+bx^3\right)} - \frac{\left(b^3c+2ab^2d-5a^2be+8a^3f\right)\tan}{3\sqrt{3}a^{4/3}b^{11/3}}$$

Mathematica [A] time = 0.152679, size = 255, normalized size = 0.94

$$\frac{\frac{30b^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a\left(a+bx^3\right)} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^2be+8a^3f+2ab^2d+b^3c\right)}{a^{4/3}} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^2be+8a^3f+2ab^2d+b^3c\right)}{a^{4/3}} - \frac{10\sqrt{3}\tan^{-1}}{}}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(b*e - 2*a*f)*x^2 + 18*b^(5/3)*f*x^5 + (30*b^(2/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 2*a*b^2
*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4
/3) - (10*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/a^(4/3) + (5*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(90*b^(11/3))
```

Maple [B] time = 0.009, size = 495, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/5*f*x^5/b^2-1/b^3*x^2*a*f+1/2/b^2*x^2*e-1/3/b^3*x^2*a^2/(b*x^3+a)*f+1/3/b
^2*x^2*a/(b*x^3+a)*e-1/3*x^2*d/(b*x^3+a)/b+1/3*x^2/a/(b*x^3+a)*c-8/9/b^4*a^
2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+5/9/b^3*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^
(1/3))*e-2/9/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/9/b/a/(1/b*a)^(1/3)*
ln(x+(1/b*a)^(1/3))*c+4/9/b^4*a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))*f-5/18/b^3*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))
*e+1/9/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/18/b/a/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+8/9/b^4*a^2*3^(1/2)/(1
/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-5/9/b^3*a*3^(1/2)/(
1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+2/9/b^2*3^(1/2)/(1
/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/9/b/a*3^(1/2)/(1/
b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48168, size = 1935, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*c
 - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 15*sqrt(1/3)*(a^2*b^4*c +
 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^3
*e + 8*a^4*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b
*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^
(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c + 2*
a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*
b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b^5)
, 1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*
c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 30*sqrt(1/3)*(a^2*b^4*c
+ 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*b^
3*e + 8*a^4*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(a*b^3*c + 2*a^2*b^2*d - 5*a^
3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^3*c +
 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a
^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3 + a^3*b
^5)]
```

Sympy [A] time = 16.8514, size = 461, normalized size = 1.7

$$-\frac{x^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^2b^3+3ab^4x^3}+\operatorname{RootSum}\left(729t^3a^4b^{11}+512a^9f^3-960a^8bef^2+384a^7b^2df^2+600a^7b^2e^2f+192a^6b^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3)
 + RootSum(729*_t**3*a**4*b**11 + 512*a**9*f**3 - 960*a**8*b*e*f**2 + 384*a
**7*b**2*d*f**2 + 600*a**7*b**2*e**2*f + 192*a**6*b**3*c*f**2 - 480*a**6*b*
*3*d*e*f - 125*a**6*b**3*e**3 - 240*a**5*b**4*c*e*f + 96*a**5*b**4*d**2*f +
 150*a**5*b**4*d*e**2 + 96*a**4*b**5*c*d*f + 75*a**4*b**5*c*e**2 - 60*a**4*
b**5*d**2*e + 24*a**3*b**6*c**2*f - 60*a**3*b**6*c*d*e + 8*a**3*b**6*d**3 -
 15*a**2*b**7*c**2*e + 12*a**2*b**7*c*d**2 + 6*a*b**8*c**2*d + b**9*c**3, L
ambda(_t, _t*log(81*_t**2*a**3*b**7/(64*a**6*f**2 - 80*a**5*b*e*f + 32*a**4
*b**2*d*f + 25*a**4*b**2*e**2 + 16*a**3*b**3*c*f - 20*a**3*b**3*d*e - 10*a*
*2*b**4*c*e + 4*a**2*b**4*d**2 + 4*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/
(5*b**2) - x**2*(2*a*f - b*e)/(2*b**3)
```

Giac [A] time = 1.07596, size = 494, normalized size = 1.82

$$-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\left(bx^3+a\right)ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) + 8*a^3*f*(-a/b)^(1/3) -
5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) +
 1/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*a*b^3
) - 1/9*sqrt(3)*((-a*b^2)^(2/3)*b^3*c + 2*(-a*b^2)^(2/3)*a*b^2*d + 8*(-a*b^
2)^(2/3)*a^3*f - 5*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a^2*b^5) + 1/18*((-a*b^2)^(2/3)*b^3*c + 2*(-a*b^2)^(
2/3)*a*b^2*d + 8*(-a*b^2)^(2/3)*a^3*f - 5*(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^5) + 1/10*(2*b^8*f*x^5 - 10*a*b^7*f*
x^2 + 5*b^8*x^2*e)/b^10
```

3.266 $\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=264

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4\right.}{9}$$

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/
3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rubi [A] time = 0.264159, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1858, 1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{18a^{5/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4\right.}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^4)/(4*b^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(3*a*b^3*(a + b*x^3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/
3)) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(9*a^(5/3)*b^(10/3)) - ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{-2b^3c-ab^2d+a^2be-a^3f-3ab(be-af)x^3-3ab^2fx^6}{a+bx^3}dx}{3ab^3}$$

$$= \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} - \frac{\int \frac{4b\left(-2b^3c-ab^2d+a^2be-a^3f\right)-\left(-12a^2b^2f+12ab^2(be-af)\right)x^3}{a+bx^3}dx}{12ab^4}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\int \frac{1}{a+bx}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\int \frac{}{\sqrt[3]{a}+}}{9a^{5/3}b^3}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\log\left(\sqrt[3]{}\right.}{9a^{5/3}b^{10/3}}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\log\left(\sqrt[3]{}\right.}{9a^{5/3}b^{10/3}}$$

$$= \frac{(be-2af)x}{b^3} + \frac{fx^4}{4b^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3ab^3\left(a+bx^3\right)} - \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)\tan^{-1}}{3\sqrt{3}a^{5/3}b^{10/3}}$$

Mathematica [A] time = 0.144892, size = 251, normalized size = 0.95

$$\frac{\frac{12\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a\left(a+bx^3\right)} - \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{a^{5/3}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{a^{5/3}} - \frac{4\sqrt{3}\tan^{-1}}{}}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(2*b^3*c + a*b^2*d - 4
*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) +
(4*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/a^(5
/3) - (2*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2]])/a^(5/3))/(36*b^(10/3))
```

Maple [B] time = 0.009, size = 482, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/4*f*x^4/b^2-2/b^3*a*f*x+1/b^2*x*e-1/3/b^3*x*a^2/(b*x^3+a)*f+1/3/b^2*x*a/(
b*x^3+a)*e-1/3/b*x/(b*x^3+a)*d+1/3*c*x/a/(b*x^3+a)+7/9/b^4*a^2/(1/b*a)^(2/3
```

```
)*ln(x+(1/b*a)^(1/3))*f-4/9/b^3*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/9/b
^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)
^(1/3))-7/18/b^4*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+
2/9/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/18/b^2/(1
/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/9*c/a/b/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/9/b^4*a^2/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-4/9/b^3*a/(1/b*a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+2/9*c/a/b/(1/b*a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52419, size = 1917, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 6*sqrt(1/3)*(2
*a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d -
 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*
(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b
)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*(2*a*b^3*c + a^2*b^2*d
- 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)*
(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3
*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7
*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c - a
^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3
*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c
+ a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d - 4*a^3*b^3*
e + 7*a^4*b^2*f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/
3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c + a^2*b^2
*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^
3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*
b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e
+ 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c
- a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] time = 6.35866, size = 376, normalized size = 1.42

$$-\frac{x\left(a^3f - a^2be + ab^2d - b^3c\right)}{3a^2b^3 + 3ab^4x^3} + \operatorname{RootSum}\left(729t^3a^5b^{10} - 343a^9f^3 + 588a^8bef^2 - 147a^7b^2df^2 - 336a^7b^2e^2f - 294a^6b^3cf\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a**2*b**3 + 3*a*b**4*x**3) +
RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 + 588*a**8*b*e*f**2 - 147*a**7
*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**6*b**3*c*f**2 + 168*a**6*b**3*
d*e*f + 64*a**6*b**3*e**3 + 336*a**5*b**4*c*e*f - 21*a**5*b**4*d**2*f - 48*
a**5*b**4*d*e**2 - 84*a**4*b**5*c*d*f - 96*a**4*b**5*c*e**2 + 12*a**4*b**5*
d**2*e - 84*a**3*b**6*c**2*f + 48*a**3*b**6*c*d*e - a**3*b**6*d**3 + 48*a**
2*b**7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a*b**8*c**2*d - 8*b**9*c**3, Lambda
(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4*a**2*b*e + a*b**2*d + 2*b**3*c) +
x))) + f*x**4/(4*b**2) - x*(2*a*f - b*e)/b**3
```

Giac [A] time = 1.07211, size = 433, normalized size = 1.64

$$-\frac{\left(2b^3c + ab^2d + 7a^3f - 4a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^3} + \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}b^3c + \left(-ab^2\right)^{\frac{1}{3}}ab^2d + 7\left(-ab^2\right)^{\frac{1}{3}}a^3f - 4\left(\right.\right.}{9a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^3*c + a*b^2*d + 7*a^3*f - 4*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^2*b^3) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^3*c + (-a*b^2)^(1/3
)*a*b^2*d + 7*(-a*b^2)^(1/3)*a^3*f - 4*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/3*(b^3*c*x - a*b^2*
d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a*b^3) + 1/18*(2*(-a*b^2)^(1/3)*b^3
*c + (-a*b^2)^(1/3)*a*b^2*d + 7*(-a*b^2)^(1/3)*a^3*f - 4*(-a*b^2)^(1/3)*a^2
*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/4*(b^6*f*x^4 -
 8*a*b^5*f*x + 4*b^6*x*e)/b^8
```

3.267 $\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=265

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^2be+5a^3f-ab^2d+4b^3c\right)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a\right.}{9a}$$

```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/
(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(8/3)) +
((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(
7/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```

Rubi [A] time = 0.253483, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^2be+5a^3f-ab^2d+4b^3c\right)}{18a^{7/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a\right.}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```


```
[Out] -(c/(a^2*x)) + (f*x^2)/(2*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/
(3*a^2*b^2*(a + b*x^3)) + ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(8/3)) +
((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(
7/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b^(8/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+b\left(\frac{b^3c}{a}-b^2d-2abe+2a^2f\right)x^3-3ab^2fx^6}{x^2\left(a+bx^3\right)}dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^2}-3abfx+\frac{b\left(4b^3c-ab^2d-2a^2be+5a^3f\right)x}{a\left(a+bx^3\right)}\right)dx}{3ab^3}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\int \frac{x}{a+bx^3}dx}{3a^2b^2}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{7/3}b^{7/3}} -$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{9a^{7/3}b^{8/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{9a^{7/3}b^{8/3}}$$

$$= -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(4b^3c-ab^2d-2a^2be+5a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}a^{7/3}b^{8/3}}$$

Mathematica [A] time = 0.152978, size = 255, normalized size = 0.96

$$\frac{1}{18}\left(\frac{6x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^2b^2\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^2be+5a^3f-ab^2d+4b^3c\right)}{a^{7/3}b^{8/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(8/3)) + (
2*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(7
/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

Maple [B] time = 0.012, size = 474, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/2*f*x^2/b^2+1/3/b^2*a*x^2/(b*x^3+a)*f-1/3/b*x^2/(b*x^3+a)*e+1/3*d*x^2/a/(
b*x^3+a)-1/3*b/a^2*x^2/(b*x^3+a)*c+5/9/b^3*a*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(
1/3))-5/18/b^3*a*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/
b^3*a*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/9
*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1
/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))+4/9/a^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9
/a^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*c*3^(1/2
)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2*e/(1/b*a)
^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(
1/b*a)^(2/3))+2/9/b^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))-1/a^2*c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48022, size = 1877, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*
b^3*e - 5*a^4*b^2*f)*x^3 + 3*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*
e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*
x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*
b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*
x)/(b*x^3 + a)) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a
*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*
a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/
3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x), 1/18*(9*a^3*b^3*f*x
^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5*a^4*b^2*f)
*x^3 + 6*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^4*b^2*f)*x^4
 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt((a*b^2)^(1/3
)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (
(4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d
- 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (
a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a
*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b*x + (a*b^2
)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x)]
```

Sympy [A] time = 26.6082, size = 457, normalized size = 1.72

$$\frac{-3ab^2c + x^3\left(a^3f - a^2be + ab^2d - 4b^3c\right)}{3a^3b^2x + 3a^2b^3x^4} + \operatorname{RootSum}\left(729t^3a^7b^8 - 125a^9f^3 + 150a^8bef^2 + 75a^7b^2df^2 - 60a^7b^2e^2f - 30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3*a**3*b**2
*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 - 125*a**9*f**3 + 150*
a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7*b**2*e**2*f - 300*a**6*b**3*c
*f**2 - 60*a**6*b**3*d*e*f + 8*a**6*b**3*e**3 + 240*a**5*b**4*c*e*f - 15*a*
*5*b**4*d**2*f + 12*a**5*b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b**5*c
*e**2 + 6*a**4*b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*e + a*
*3*b**6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8*c**2*d
 - 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a**6*f**2 - 20*a*
*5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 40*a**3*b**3*c*f + 4*a**3*
b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*d**2 - 8*a*b**5*c*d + 16*b**6*c**2)
 + x))) + f*x**2/(2*b**2)
```

Giac [A] time = 1.07628, size = 477, normalized size = 1.8

$$\frac{fx^2}{2\,b^2} + \frac{\left(4\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2} - \frac{4\,b^3cx^3 - ab^2dx^3 - a^3fx^3 +}{3\left(bx^4 + ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^2 + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + 5*a^3*f*
(-a/b)^(1/3) - 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x^3 - a^3*f*x^3 + a^2*b*x^3*e +
3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2) + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*b^3*c -
(-a*b^2)^(2/3)*a*b^2*d + 5*(-a*b^2)^(2/3)*a^3*f - 2*(-a*b^2)^(2/3)*a^2*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^4) - 1/18*(4*
(-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + 5*(-a*b^2)^(2/3)*a^3*f - 2*
(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

3.268 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\right.}{}$$

```
[Out] -c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^2
*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(7/3)) - ((5*b^
3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b
^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```

Rubi [A] time = 0.246217, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{18a^{8/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```


```
[Out] -c/(2*a^2*x^2) + (f*x)/b^2 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^2
*b^2*(a + b*x^3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(7/3)) - ((5*b^
3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b
^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(7/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-3ab^2fx^6}{x^3\left(a+bx^3\right)}dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)} - \frac{\int \left(-3abf-\frac{3b^3c}{ax^3}+\frac{b\left(5b^3c-2ab^2d-a^2be+4a^3f\right)}{a\left(a+bx^3\right)}\right)dx}{3ab^3}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\int \frac{1}{a+bx^3}dx}{3a^2b^2}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}d}{9a^{8/3}b^2}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log\left(\sqrt[3]{a}+\right.}{9a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}-\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\log\left(\sqrt[3]{a}+\right.}{9a^{8/3}b^{7/3}}$$

$$= -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^2b^2\left(a+bx^3\right)}+\frac{\left(5b^3c-2ab^2d-a^2be+4a^3f\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}a^{8/3}b^{7/3}}$$

Mathematica [A] time = 0.152077, size = 250, normalized size = 0.96

$$\frac{1}{18}\left(\frac{6x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{a^{8/3}b^{7/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f
)*x))/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(8/3)*b^(7/3)) - (2*
(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^(8/3
)*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

Maple [B] time = 0.011, size = 463, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] f*x/b^2+1/3/b^2*a*x/(b*x^3+a)*f-1/3/b*x/(b*x^3+a)*e+1/3/a*x/(b*x^3+a)*d-1/3
*b/a^2*x/(b*x^3+a)*c-4/9/b^3*a*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/9/b^3*
a*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/b^3*a*f/(1/b*a)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/9/b/a*d/(1/b*a)^
(2/3)*ln(x+(1/b*a)^(1/3))-1/9/b/a*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))+2/9/b/a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^
(1/3)*x-1))-5/9/a^2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*c/(1/b*a)^
(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2*c/(1/b*a)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/9/b^2*e/(1/b*a)^(2/3)*ln(x+(1/
b*a)^(1/3))-1/18/b^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+
1/9/b^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1
/2*c/a^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4632, size = 1952, normalized size = 7.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 + 3*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3
*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^
(1/3)/b))/(b*x^3 + a)) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5
 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*
b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^3*d - a^
2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^
2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2),
 1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a^
4*b^2*e - 8*a^5*b*f)*x^3 - 6*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^3*
e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*
x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b
)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e
 + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^
2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c
 - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*
b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^
5 + a^5*b^3*x^2)]
```

Sympy [A] time = 58.6862, size = 381, normalized size = 1.47

$$\frac{-3ab^2c + x^3\left(2a^3f - 2a^2be + 2ab^2d - 5b^3c\right)}{6a^3b^2x^2 + 6a^2b^3x^5} + \operatorname{RootSum}\left(729t^3a^8b^7 + 64a^9f^3 - 48a^8bef^2 - 96a^7b^2df^2 + 12a^7b^2e^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*a*b**2*c + x**3*(2*a**3*f - 2*a**2*b*e + 2*a*b**2*d - 5*b**3*c))/(6*a**
3*b**2*x**2 + 6*a**2*b**3*x**5) + RootSum(729*_t**3*a**8*b**7 + 64*a**9*f**
3 - 48*a**8*b*e*f**2 - 96*a**7*b**2*d*f**2 + 12*a**7*b**2*e**2*f + 240*a**6
*b**3*c*f**2 + 48*a**6*b**3*d*e*f - a**6*b**3*e**3 - 120*a**5*b**4*c*e*f +
48*a**5*b**4*d**2*f - 6*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f + 15*a**4*b*
*5*c*e**2 - 12*a**4*b**5*d**2*e + 300*a**3*b**6*c**2*f + 60*a**3*b**6*c*d*e
 - 8*a**3*b**6*d**3 - 75*a**2*b**7*c**2*e + 60*a**2*b**7*c*d**2 - 150*a*b**
8*c**2*d + 125*b**9*c**3, Lambda(_t, _t*log(-9*_t*a**3*b**2/(4*a**3*f - a**
2*b*e - 2*a*b**2*d + 5*b**3*c) + x))) + f*x/b**2
```

Giac [A] time = 1.08146, size = 417, normalized size = 1.6

$$\frac{fx}{b^2} + \frac{\left(5b^3c - 2ab^2d + 4a^3f - a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3b^2} - \frac{c}{2a^2x^2} - \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}b^3c - 2\left(-ab^2\right)^{\frac{1}{3}}ab^2d + 4\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 + 1/9*(5*b^3*c - 2*a*b^2*d + 4*a^3*f - a^2*b*e)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2) - 1/9*sqrt(3)*(5*(-a*b^2)^
(1/3)*b^3*c - 2*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2)^(1/3)*a^3*f - (-a*b^2)^
(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^3) - 1/3*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^2*b^2)
 - 1/18*(5*(-a*b^2)^(1/3)*b^3*c - 2*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2)^(1/
3)*a^3*f - (-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^3*b^3)
```

3.269 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2} dx$

Optimal. Leaf size=269

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+\right.}{9a^{10/3}}$$

```
[Out] -c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)*b^(5
/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/(9*a^(10/3)*b^(5/3)) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```

Rubi [A] time = 0.288055, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{18a^{10/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+\right.}{9a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```


```
[Out] -c/(4*a^2*x^4) + (2*b*c - a*d)/(a^3*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x^2)/(3*a^3*b*(a + b*x^3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)*b^(5
/3)) - ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/(9*a^(10/3)*b^(5/3)) + ((7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3)*b^(5/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{b^3c}{a^2}-\frac{b^2d}{a}+be+2af\right)x^6}{x^5\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^5}-\frac{3b^3(-2bc+ad)}{a^2x^2}-\frac{b^2\left(7b^3c-4ab^2d+a^2be+2a^3f\right)x}{a^2\left(a+bx^3\right)}\right)dx}{3ab^3}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}+\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\int \frac{x}{a+bx^3}}{3a^3b}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{}}}{9a^{10/3}b^{4/3}}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\log\left(\sqrt[3]{a}\right.}{9a^{10/3}b^{5/3}}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\log\left(\sqrt[3]{a}\right.}{9a^{10/3}b^{5/3}}$$

$$= -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^3b\left(a+bx^3\right)}-\frac{\left(7b^3c-4ab^2d+a^2be+2a^3f\right)\tan^{-1}\left(\right.}{3\sqrt{3}a^{10/3}b^{5/3}}$$

Mathematica [A] time = 0.168302, size = 255, normalized size = 0.95

$$\frac{-\frac{12\sqrt[3]{a}x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b\left(a+bx^3\right)}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{b^{5/3}}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+2a^3f-4ab^2d+7b^3c\right)}{b^{5/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{}}{}}{\sqrt{3}}\right)}{}}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*c)/x^4 - (36*a^(1/3)*(-2*b*c + a*d))/x - (12*a^(1/3)*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(7*b^3*c -
4*a*b^2*d + a^2*b*e + 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/b^(5/3) - (4*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/b^(5/3) + (2*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(36*a^(10/3))
```

Maple [B] time = 0.013, size = 486, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x)
```

```
[Out] -1/3/b*x^2/(b*x^3+a)*f+1/3/a*x^2/(b*x^3+a)*e-1/3/a^2*b*x^2/(b*x^3+a)*d+1/3/
a^3*b^2*x^2/(b*x^3+a)*c-2/9/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-1/9/a/b
/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+4/9/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))*d-7/9/a^3*b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/9/b^2/(1/b*a)^(1/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/18/a/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*e-2/9/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))*d+7/18/a^3*b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3)
)*c+2/9/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*f+1/9/a/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
e-4/9/a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d
+7/9/a^3*b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
c-1/4*c/a^2/x^4-d/a^2/x+2/a^3/x*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52471, size = 1968, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*
x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 6*sqrt(1/3)*((7*a*b^5*c - 4*a^2*b
^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b^2*
e + 2*a^5*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/
3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e +
 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7*b^4*c -
 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*
e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 +
a^5*b^3*x^4), -1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e
- a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 12*sqrt(1/3)*((7*a*b
^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^
3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 2*((7*b^4*c - 4*a*b^3
*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^
4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
 + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^
2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))
/(a^4*b^4*x^7 + a^5*b^3*x^4)]
```

Sympy [A] time = 138.391, size = 473, normalized size = 1.76

$$\text{RootSum}\left(729t^3a^{10}b^5 + 8a^9f^3 + 12a^8bef^2 - 48a^7b^2df^2 + 6a^7b^2e^2f + 84a^6b^3cf^2 - 48a^6b^3def + a^6b^3e^3 + 84a^5b^4cef + 9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10*b**5 + 8*a**9*f**3 + 12*a**8*b*e*f**2 - 48*a**7*b**
2*d*f**2 + 6*a**7*b**2*e**2*f + 84*a**6*b**3*c*f**2 - 48*a**6*b**3*d*e*f +
a**6*b**3*e**3 + 84*a**5*b**4*c*e*f + 96*a**5*b**4*d**2*f - 12*a**5*b**4*d*
e**2 - 336*a**4*b**5*c*d*f + 21*a**4*b**5*c*e**2 + 48*a**4*b**5*d**2*e + 29
4*a**3*b**6*c**2*f - 168*a**3*b**6*c*d*e - 64*a**3*b**6*d**3 + 147*a**2*b**
7*c**2*e + 336*a**2*b**7*c*d**2 - 588*a*b**8*c**2*d + 343*b**9*c**3, Lambda
(_t, _t*log(81*_t**2*a**7*b**3/(4*a**6*f**2 + 4*a**5*b*e*f - 16*a**4*b**2*d
*f + a**4*b**2*e**2 + 28*a**3*b**3*c*f - 8*a**3*b**3*d*e + 14*a**2*b**4*c*e
 + 16*a**2*b**4*d**2 - 56*a*b**5*c*d + 49*b**6*c**2) + x))) - (3*a**2*b*c +
 x**6*(4*a**3*f - 4*a**2*b*e + 16*a*b**2*d - 28*b**3*c) + x**3*(12*a**2*b*d
 - 21*a*b**2*c))/(12*a**4*b*x**4 + 12*a**3*b**2*x**7)
```

Giac [A] time = 1.08189, size = 483, normalized size = 1.8

$$-\frac{\left(7\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b} + \frac{b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e}{3\left(bx^3 + a\right)a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(7*b^3*c*(-a/b)^(1/3) - 4*a*b^2*d*(-a/b)^(1/3) + 2*a^3*f*(-a/b)^(1/3)
+ a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b) + 1
/3*(b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*a^3*b)
- 1/9*sqrt(3)*(7*(-a*b^2)^(2/3)*b^3*c - 4*(-a*b^2)^(2/3)*a*b^2*d + 2*(-a*b^
2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^4*b^3) + 1/18*(7*(-a*b^2)^(2/3)*b^3*c - 4*(-a*b^2)^(
2/3)*a*b^2*d + 2*(-a*b^2)^(2/3)*a^3*f + (-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^3) + 1/4*(8*b*c*x^3 - 4*a*d*x^3 - a*c)
/(a^3*x^4)
```

3.270 $\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^2b\right.}{9a^{1}}$$

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)*b^
(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(11/3)*b^(4/3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rubi [A] time = 0.27243, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{18a^{11/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^2b\right.}{9a^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(2*a^3*x^2) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*x)/(3*a^3*b*(a + b*x^3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)*b^
(4/3)) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(11/3)*b^(4/3)) - ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3)*b^(4/3))
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}dx = \frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^6(a+bx^3)}dx}{3ab^3}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^6}-\frac{3b^3(-2bc+ad)}{a^2x^3}-\frac{b^2(8b^3c-5ab^2d+2a^2be+a^3f)}{a^2(a+bx^3)}\right)dx}{3ab^3}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{1}{a+b}}{3a^3b}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\int \frac{}{\sqrt[3]{a}}}{9a^{11/3}b}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log(}{9a^{11/3}b^{4/3}}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log(}{9a^{11/3}b^{4/3}}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3a^3b(a+bx^3)}-\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\tan^{-1}}{3\sqrt{3}a^{11/3}b^{4/3}}$$

Mathematica [A] time = 0.162183, size = 253, normalized size = 0.94

$$-\frac{\frac{30a^{2/3}x(-a^2be+a^3f+ab^2d-b^3c)}{b(a+bx^3)}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(2a^2be+a^3f-5ab^2d+8b^3c)}{b^{4/3}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(2a^2be+a^3f-5ab^2d+8b^3c)}{b^{4/3}}-\frac{10\sqrt{3}\tan^{-1}}{}}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*c)/x^5 - (45*a^(2/3)*(-2*b*c + a*d))/x^2 - (30*a^(2/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x))/(b*(a + b*x^3)) - (10*Sqrt[3]*(8*b^3*c
- 5*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/b^(4/3) + (10*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/b^(4/3) - (5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(90*a^(11/3))
```

Maple [B] time = 0.011, size = 477, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x)
```

```
[Out] -1/3/b*x/(b*x^3+a)*f+1/3/a*x/(b*x^3+a)*e-1/3/a^2*b*x/(b*x^3+a)*d+1/3/a^3*b^
2*x/(b*x^3+a)*c+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+2/9/a/b/(1/b*a)
^(2/3)*ln(x+(1/b*a)^(1/3))*e-5/9/a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+8/
9/a^3*b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/9/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))*e+5/18/a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2
/3))*d-4/9/a^3*b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/9/
b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/9/a
/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-5/9/a^
2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+8/9/a^3
*b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/5*c/
a^2/x^5-1/2*d/a^2/x^2+1/a^3/x^2*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55885, size = 1991, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5*
b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 15*sqrt(1/3)*((8*a*b^5*c - 5
*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d + 2*
a^4*b^2*e + a^5*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)
^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*
b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(
a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 10*((8*b^4*
c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a
^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^3*x^8
 + a^6*b^2*x^5), -1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^
4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 30*sqrt(1/3)
*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c -
5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt
(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 5*
((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2
*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x +
(a^2*b)^(1/3)*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 +
(8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^5*b^3*x^8 + a^6*b^2*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08144, size = 429, normalized size = 1.59

$$-\frac{\left(8\,b^3c-5\,ab^2d+a^3f+2\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b}+\frac{\sqrt{3}\left(8\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-5\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^3f+2\,\left(\right.}{9\,a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(8*b^3*c - 5*a*b^2*d + a^3*f + 2*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a^4*b) + 1/9*sqrt(3)*(8*(-a*b^2)^(1/3)*b^3*c - 5*(-a*b^2)^(1/3
)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 2*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) + 1/3*(b^3*c*x - a*b^2*d*
x - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^3*b) + 1/18*(8*(-a*b^2)^(1/3)*b^3*c
 - 5*(-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 2*(-a*b^2)^(1/3)*a^2*b
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^2) + 1/10*(10*b*c*x^3 -
 5*a*d*x^3 - 2*a*c)/(a^3*x^5)
```

3.271 $\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=297

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^2be+a^3(-f)-7ab^2d+10b^3c\right)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4\right.}{}$$

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + ((1
0*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d + 4*
a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^3*
c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(13/3)*b^(2/3))
```

Rubi [A] time = 0.383572, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^2be+a^3(-f)-7ab^2d+10b^3c\right)}{18a^{13/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```


```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(4*a^3*x^4) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^4*(a + b*x^3)) + ((1
0*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(13/3)*b^(2/3)) + ((10*b^3*c - 7*a*b^2*d + 4*
a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(13/3)*b^(2/3)) - ((10*b^3*
c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(13/3)*b^(2/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3(b^2c-abd+a^2e)x^6}{a^2}+\frac{b^3(b^3c-ab^2d+a^2be-a^3f)x^9}{a^3}}{x^8(a+bx^3)}dx}{3ab^3}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)} - \frac{\int \left(-\frac{3b^3c}{ax^8}-\frac{3b^3(-2bc+ad)}{a^2x^5}-\frac{3b^3(3b^2c-2abd+a^2e)}{a^3x^2}-\frac{b^3(-10b^3c+7ab^2d-}{a^3(a+bx^}\right.}{3ab^3}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)}-\frac{(10b^3c-7ab^2d+}{}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)}+\frac{(10b^3c-7ab^2d+}{}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)}+\frac{(10b^3c-7ab^2d+}{}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)}+\frac{(10b^3c-7ab^2d+}{}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)}+\frac{(10b^3c-7ab^2d+}{}$$

Mathematica [A] time = 0.179237, size = 281, normalized size = 0.95

$$\frac{\frac{84\sqrt[3]{a}x^2(-a^2be+a^3f+ab^2d-b^3c)}{a+bx^3}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-4a^2be+a^3f+7ab^2d-10b^3c)}{b^{2/3}}+\frac{28\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(4a^2be+a^3(-f)-7ab^2d+10b^3c)}{b^{2/3}}+\frac{28\sqrt{3}\tan}{}}{252a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

```
[Out] ((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(1/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(10*b^3*c - 7
*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-10*
b^3*c + 7*a*b^2*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/b^(2/3))/(252*a^(13/3))
```

Maple [B] time = 0.015, size = 529, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2/(b*x^3+a)*f-1/3/a^2*x^2/(b*x^3+a)*b*e+1/3/a^3*x^2/(b*x^3+a)*b^2*d
-1/3/a^4*x^2/(b*x^3+a)*b^3*c+4/9/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/
9/a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*e*3^(1/
2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-7/9/a^3*b*d/(1/b
*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/18/a^3*b*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/
3)*x+(1/b*a)^(2/3))+7/9/a^3*b*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))+10/9/a^4*b^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/9/a
^4*b^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-10/9/a^4*b^2*c
*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/9/a*f/b/
(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*f/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(
1/3)*x+(1/b*a)^(2/3))+1/9/a*f*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))-1/7*c/a^2/x^7-1/4/a^2/x^4*d+1/2/a^3/x^4*b*c-e/a^2/x+2/
a^3/x*b*d-3/a^4/x*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46461, size = 2159, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*a
^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*b^
3*c - 7*a^4*b^2*d)*x^3 + 42*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^
3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e - a^5*b*f
)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2
*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)
*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4*c - 7*a*b^
3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e -
 a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*x^10 + a^6*
b^2*x^7), -1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x
^9 + 36*a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(
10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 + 84*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d +
 4*a^3*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e
- a^5*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^
(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e
 - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4
*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d +
4*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*
x^10 + a^6*b^2*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11736, size = 522, normalized size = 1.76

$$\frac{\left(10\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}-\frac{b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e}{3\left(bx^3+a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(10*b^3*c*(-a/b)^(1/3) - 7*a*b^2*d*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3) +
4*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*(
b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x^2*e)/((b*x^3 + a)*a^4) + 1/9*
sqrt(3)*(10*(-a*b^2)^(2/3)*b^3*c - 7*(-a*b^2)^(2/3)*a*b^2*d - (-a*b^2)^(2/3
)*a^3*f + 4*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^5*b^2) - 1/18*(10*(-a*b^2)^(2/3)*b^3*c - 7*(-a*b^2)^(2/3)
*a*b^2*d - (-a*b^2)^(2/3)*a^3*f + 4*(-a*b^2)^(2/3)*a^2*b*e)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^2) - 1/28*(84*b^2*c*x^6 - 56*a*b*d*x^6 + 2
8*a^2*x^6*e - 14*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^4*x^7)
```

3.272 $\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=297

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{18a^{14/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\right.}{}$$

```
[Out] -c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(2
*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) + (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d
+ 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((1
1*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```

Rubi [A] time = 0.369952, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{18a^{14/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```


```
[Out] -c/(8*a^2*x^8) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(2
*a^4*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^4*(a + b*x^3)) + (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(14/3)*b^(1/3)) - ((11*b^3*c - 8*a*b^2*d
+ 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(14/3)*b^(1/3)) + ((1
1*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(18*a^(14/3)*b^(1/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{2b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^9\left(a+bx^3\right)}\,dx}{3ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^9}-\frac{3b^3(-2bc+ad)}{a^2x^6}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^3}-\frac{b^3\left(-11b^3c+8ab^2\right.}{a^3(a+}\right.}{3ab^3}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{\left(11b^3c-8ab^2d\right.}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{\left(11b^3c-8ab^2d\right.}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{\left(11b^3c-8ab^2d\right.}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}-\frac{\left(11b^3c-8ab^2d\right.}{}$$

$$= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)}+\frac{\left(11b^3c-8ab^2d\right.}{}$$

Mathematica [A] time = 0.178107, size = 280, normalized size = 0.94

$$\frac{\frac{120a^{2/3}x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a+bx^3}+\frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{\sqrt[3]{b}}+\frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^2be+2a^3f+8ab^2d-11b^3c\right)}{\sqrt[3]{b}}+\frac{40\sqrt{3}}{}}{360a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

```
[Out] ((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(2/3)*(3*b^
2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a
^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-11*b^3*c
 + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20
*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

Maple [B] time = 0.014, size = 520, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x/(b*x^3+a)*f-1/3/a^2*x/(b*x^3+a)*b*e+1/3/a^3*x/(b*x^3+a)*b^2*d-1/3/a
^4*x/(b*x^3+a)*b^3*c-5/9/a^2*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*e
/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2*e/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+8/9/a^3*b*d/(1/b*a)^(2/
3)*ln(x+(1/b*a)^(1/3))-4/9/a^3*b*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))+8/9/a^3*b*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))-11/9/a^4*b^2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+11/18/a^4*b^2
*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-11/9/a^4*b^2*c/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/9/a*f/b/(1/b*a
)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9/a*f/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+
(1/b*a)^(2/3))+2/9/a*f/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a
)^(1/3)*x-1))-1/8*c/a^2/x^8-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/2/a^2/x^2*e+1/a
^3/x^2*b*d-3/2/a^4/x^2*b^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55454, size = 2182, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^9 + 45
*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(11*a^4*b^2*
c - 8*a^5*b*d)*x^3 + 60*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d + 5*a^3*b^3*e
- 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f
)*x^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 20*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f
)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3
)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((11*b^4*c - 8*a*b^
3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e
 - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^2*x^11 +
a^7*b*x^8), -1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*
f)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(
11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 120*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d +
5*a^3*b^3*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e
 - 2*a^5*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*
x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((11*b^4*c - 8*a*b^3*d
 + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e -
2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a
) + 40*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c
 - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b
)^(2/3)))/(a^6*b^2*x^11 + a^7*b*x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0871, size = 468, normalized size = 1.58

$$\frac{\left(11\,b^3c - 8\,ab^2d - 2\,a^3f + 5\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5} - \frac{\sqrt{3}\left(11\,\left(-ab^2\right)^{\frac{1}{3}}b^3c - 8\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d - 2\,\left(-ab^2\right)^{\frac{1}{3}}a^3f + \right.}{9\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(11*b^3*c - 8*a*b^2*d - 2*a^3*f + 5*a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/a^5 - 1/9*sqrt(3)*(11*(-a*b^2)^(1/3)*b^3*c - 8*(-a*b^2)^(1/3)
*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 1/3*(b^3*c*x - a*b^2*d*x
 - a^3*f*x + a^2*b*x*e)/((b*x^3 + a)*a^4) - 1/18*(11*(-a*b^2)^(1/3)*b^3*c -
 8*(-a*b^2)^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/40*(60*b^2*c*x^6 -
 40*a*b*d*x^6 + 20*a^2*x^6*e - 16*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^4*x
^8)
```

3.273 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx$

Optimal. Leaf size=334

$$\frac{bx^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^2be-4a^3f-10ab^2d+13b^3c\right)}{18a^{16/3}}+\frac{2a^2be+a^3(-f)\cdots}{a^5\cdots}$$

```
[Out] -c/(10*a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/
(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^(1/3)*(13*b^3*c
 - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(3*Sqrt[3]*a^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2
*b*e - 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3*c
 - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*a^(16/3))
```

Rubi [A] time = 0.456739, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{bx^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^2be-4a^3f-10ab^2d+13b^3c\right)}{18a^{16/3}}+\frac{2a^2be+a^3(-f)\cdots}{a^5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```


```
[Out] -c/(10*a^2*x^10) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/
(4*a^4*x^4) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) - (b^(1/3)*(13*b^3*c
 - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(3*Sqrt[3]*a^(16/3)) - (b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2
*b*e - 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(16/3)) + (b^(1/3)*(13*b^3*c
 - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*a^(16/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^5\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{b^4\left(b^3c-\cdots\right.}{\cdots}}{x^{11}\left(a+bx^3\right)}}{3ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^5\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^3c}{ax^{11}}-\frac{3b^3(-2bc+ad)}{a^2x^8}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^5}-\frac{3b^3\left(-4b^3c+3ab^2d-\cdots\right.}{a^4x^2}\cdots\right.}{3ab^3}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\cdots\right.}{3a^5\left(a\cdots\right.}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\cdots\right.}{3a^5\left(a\cdots\right.}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\cdots\right.}{3a^5\left(a\cdots\right.}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\cdots\right.}{3a^5\left(a\cdots\right.}$$

$$= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-ab^2d+\cdots\right.}{3a^5\left(a\cdots\right.}$$

Mathematica [A] time = 0.181351, size = 319, normalized size = 0.96

$$\frac{-\frac{420\sqrt[3]{a}bx^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a+bx^3}+70\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(7a^2be-4a^3f-10ab^2d+13b^3c\right)-\frac{1260\sqrt[3]{a}\left(-2a^2be+a^3f+3ab\cdots\right.}{x}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

```
[Out] ((-126*a^(10/3)*c)/x^10 - (180*a^(7/3)*(-2*b*c + a*d))/x^7 - (315*a^(4/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^(1/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x - (420*a^(1/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*
x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e -
 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-13*b^
3*c + 10*a*b^2*d - 7*a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/
3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(1260*a^(16/3))
```

Maple [A] time = 0.016, size = 575, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x)
```

```
[Out] 1/3*b^2/a^3*x^2/(b*x^3+a)*e-1/10*c/a^2/x^10+10/9*b^2/a^4*d/(1/b*a)^(1/3)*ln
(x+(1/b*a)^(1/3))-13/9*b^3/a^5*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+13/18*b^
3/a^5*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*b/a^2*x^2/(
b*x^3+a)*f-7/9*b/a^3*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+4/9/a^2*f/(1/b*a)^
(1/3)*ln(x+(1/b*a)^(1/3))-2/9/a^2*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))+2/7/a^3/x^7*b*c+1/2/a^3/x^4*b*d-3/4/a^4/x^4*b^2*c+2/a^3/x*b*e-
3/a^4/x*b^2*d+4/a^5/x*b^3*c-1/a^2/x*f-1/7/a^2/x^7*d-1/4/a^2/x^4*e+7/18*b/a^
3*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*b^4/a^5*x^2/(b*
x^3+a)*c-5/9*b^2/a^4*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-
4/9/a^2*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1
/3*b^3/a^4*x^2/(b*x^3+a)*d+7/9*b/a^3*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))-10/9*b^2/a^4*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3
*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+13/9*b^3/a^5*c*3^(1/2)/(1/b*a)^(1/3)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37247, size = 1034, normalized size = 3.1

$$420\left(13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf\right)x^{12}+315\left(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f\right)x^9-45\left(13\,a^2b^2c-10\,a^3bd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(420*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 315*(1
3*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 45*(13*a^2*b^2*c - 10
*a^3*b*d + 7*a^4*e)*x^6 - 126*a^4*c + 18*(13*a^3*b*c - 10*a^4*d)*x^3 + 140*
sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3
*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*arctan(2/3*sqrt(
3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e
- 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)
*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((13*b^4*c
- 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d +
 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a^5*b*x^
13 + a^6*x^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1012, size = 590, normalized size = 1.77

$$-\frac{\left(13\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-10\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6}-\frac{\sqrt{3}\left(13\,\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\,\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) - 4*a^3*b*f*(-a/b)^(1
/3) + 7*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6
 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d - 4*(-a
*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 - a^3*b*f*x
^2 + a^2*b^2*x^2*e)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c - 10*
(-a*b^2)^(2/3)*a*b^2*d - 4*(-a*b^2)^(2/3)*a^3*f + 7*(-a*b^2)^(2/3)*a^2*b*e)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560*b^3*c*x^9 -
420*a*b^2*d*x^9 - 140*a^3*f*x^9 + 280*a^2*b*x^9*e - 105*a*b^2*c*x^6 + 70*a^
2*b*d*x^6 - 35*a^3*x^6*e + 40*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^5*x
^10)
```

3.274 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{2a^5x^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^2be-5a^3f-\right.}{18a^{17/3}}$$

```
[Out] -c/(11*a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e)/
(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^(2/3)*(14*b^3
*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a
^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b^3
*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(17/3))
```

Rubi [A] time = 0.434195, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{2a^5x^2}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(8a^2be-5a^3f-\right.}{18a^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```


```
[Out] -c/(11*a^2*x^11) + (2*b*c - a*d)/(8*a^3*x^8) - (3*b^2*c - 2*a*b*d + a^2*e)/
(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(2*a^5*x^2) + (b*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) - (b^(2/3)*(14*b^3
*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(17/3)) + (b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a
^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(17/3)) - (b^(2/3)*(14*b^3
*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(17/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^5\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{2b^4}{}}{x^{12}\left(a+bx^3\right)}}{3ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^5\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^3c}{ax^{12}}-\frac{3b^3(-2bc+ad)}{a^2x^9}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-\frac{3b^3\left(-4b^3c+3ab^2\right.}{a^4x}\right.}{3ab^3}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3c-ab^2\right.}{3a^5\left(\right.}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3c-ab^2\right.}{3a^5\left(\right.}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3c-ab^2\right.}{3a^5\left(\right.}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3c-ab^2\right.}{3a^5\left(\right.}$$

$$= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b\left(b^3c-ab^2\right.}{3a^5\left(\right.}$$

Mathematica [A] time = 0.187121, size = 317, normalized size = 0.95

$$\frac{-\frac{1320a^{2/3}bx\left(-a^2be+a^3f+ab^2d-b^3c\right)}{a+bx^3}-\frac{1980a^{2/3}\left(-2a^2be+a^3f+3ab^2d-4b^3c\right)}{x^2}+220b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-8a^2be+5a^3f+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

```
[Out] ((-360*a^(11/3)*c)/x^11 - (495*a^(8/3)*(-2*b*c + a*d))/x^8 - (792*a^(5/3)*(
3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^(2/3)*(-4*b^3*c + 3*a*b^2*d - 2*a
^2*b*e + a^3*f))/x^2 - (1320*a^(2/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x)/(a + b*x^3) - 440*Sqrt[3]*b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e
- 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(14*b^
3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2
/3)*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(3960*a^(17/3))
```

Maple [A] time = 0.017, size = 566, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x)
```

```
[Out] -1/11*c/a^2/x^11+14/9*b^3/a^5*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-7/9*b^3/a
^5*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3*b^3/a^4*x/(b*x
^3+a)*d+1/3*b^4/a^5*x/(b*x^3+a)*c-5/9/a^2*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+8/9*b/a^3*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/
3))-4/9*b/a^3*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-11/9*b^
2/a^4*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+11/18*b^2/a^4*d/(1/b*a)^(2/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/2/a^2/x^2*f-1/8/a^2/x^8*d-1/5/a^2/x^5*
e-5/9/a^2*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*f/(1/b*a)^(2/3)*ln(x
^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/4/a^3/x^8*b*c+2/5/a^3/x^5*b*d-3/5/a^4/x
^5*b^2*c+1/a^3/x^2*b*e-3/2/a^4/x^2*b^2*d+2/a^5/x^2*b^3*c+1/3*b^2/a^3*x/(b*x
^3+a)*e-1/3*b/a^2*x/(b*x^3+a)*f+14/9*b^3/a^5*c/(1/b*a)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+8/9*b/a^3*e/(1/b*a)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-11/9*b^2/a^4*d/(1/b*a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38418, size = 1099, normalized size = 3.28

$$\frac{660\left(14\,b^4c - 11\,ab^3d + 8\,a^2b^2e - 5\,a^3bf\right)x^{12} + 396\left(14\,ab^3c - 11\,a^2b^2d + 8\,a^3be - 5\,a^4f\right)x^9 - 99\left(14\,a^2b^2c - 11\,a^3bd + 8\,a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3960*(660*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 396*(1
4*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 99*(14*a^2*b^2*c - 11
*a^3*b*d + 8*a^4*e)*x^6 - 360*a^4*c + 45*(14*a^3*b*c - 11*a^4*d)*x^3 - 440*
sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3
*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((14*b^4*c - 11*a*b^3
*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e
 - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a
^2*(-b^2/a^2)^(2/3)) - 440*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*
f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2
)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b*x^14 + a^6*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08584, size = 528, normalized size = 1.58

$$\frac{\sqrt{3}\left(14\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-11\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-5\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f+8\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^{6}}-\frac{\left(14b^{4}c-11ab^{3}d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^
2)^(1/3)*a^3*f + 8*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/a^6 - 1/9*(14*b^4*c - 11*a*b^3*d - 5*a^3*b*f + 8*a^2*
b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 + 1/18*(14*(-a*b^2)^(1/3
)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 8*(-a*b^2)^(
1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 + 1/3*(b^4*c*x -
 a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^3 + a)*a^5) + 1/440*(880*b^3*c*
x^9 - 660*a*b^2*d*x^9 - 220*a^3*f*x^9 + 440*a^2*b*x^9*e - 264*a*b^2*c*x^6 +
 176*a^2*b*d*x^6 - 88*a^3*x^6*e + 110*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c
)/(a^5*x^11)
```

3.275 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2} dx$

Optimal. Leaf size=375

$$-\frac{b^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{4a^5x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(10a^2be-7a^3f\right.}{18a^{19/3}}$$

```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2*e
)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) - (b*
(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a*b
^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e -
7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c - 13*a
*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(19/3))
```

Rubi [A] time = 0.533546, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{4a^5x^4}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(10a^2be-7a^3f\right.}{18a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```


```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2*e
)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) - (b*
(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13*a*b
^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e -
7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) - (b^(4/3)*(16*b^3*c - 13*a
*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(19/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^6\left(a+bx^3\right)} - \frac{\int \frac{-3b^3c+3b^3\left(\frac{bc}{a}-d\right)x^3-\frac{3b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{3b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{3b^4}{}}{x^{14}\left(a+bx^3\right)}}{3ab^3}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{3a^6\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^3c}{ax^{14}}-\frac{3b^3(-2bc+ad)}{a^2x^{11}}-\frac{3b^3\left(3b^2c-2abd+a^2e\right)}{a^3x^8}-\frac{3b^3\left(-4b^3c+3ab^2\right.}{a^4}\right.}{}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c-4ab^2d\right.}{a}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c-4ab^2d\right.}{a}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c-4ab^2d\right.}{a}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c-4ab^2d\right.}{a}$$

$$= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}-\frac{b\left(5b^3c-4ab^2d\right.}{a}$$

Mathematica [A] time = 0.360017, size = 370, normalized size = 0.99

$$\frac{b^2x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{4a^5x^4}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-10a^2be+7a^3f+\right.}{18a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

```
[Out] -c/(13*a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^2*e
)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) + (b*
(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*c - 13
*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*
a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) + (b^(4/3)*(-16*b^3*c + 13*a*
b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(19/3))
```

Maple [A] time = 0.016, size = 631, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x)
```

```
[Out] 13/18*b^3/a^5*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+16/9*b^
4/a^6*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-8/9*b^4/a^6*c/(1/b*a)^(1/3)*ln(x^
2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/18*b/a^3*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^
(1/3)*x+(1/b*a)^(2/3))+10/9*b^2/a^4*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/9
*b^2/a^4*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-13/9*b^3/a^5
*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-7/9*b/a^3*f/(1/b*a)^(1/3)*ln(x+(1/b*a)
^(1/3))+7/9*b/a^3*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))-10/9*b^2/a^4*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))+13/9*b^3/a^5*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/
b*a)^(1/3)*x-1))-16/9*b^4/a^6*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))-1/13*c/a^2/x^13+1/2/a^3/x^4*b*e-3/4/a^4/x^4*b^2*d+1/a^
5/x^4*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*d-5*b^4/a^6/x*c+1/5/a^3/x
^10*b*c+2/7/a^3/x^7*b*d-3/7/a^4/x^7*b^2*c+1/3*b^2/a^3*x^2/(b*x^3+a)*f-1/3*b
^3/a^4*x^2/(b*x^3+a)*e-1/10/a^2/x^10*d-1/7/a^2/x^7*e-1/4/a^2/x^4*f+1/3*b^4/
a^5*x^2/(b*x^3+a)*d-1/3*b^5/a^6*x^2/(b*x^3+a)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36125, size = 1188, normalized size = 3.17

$$-\frac{5460\left(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f\right)x^{15}+4095\left(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf\right)x^{12}-585\left(16\,a^2b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16380*(5460*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 +
4095*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 585*(16*
a^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 234*(16*a^3*b^2*c -
13*a^4*b*d + 10*a^5*e)*x^6 + 1260*a^5*c - 126*(16*a^4*b*c - 13*a^5*d)*x^3 +
 1820*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 +
(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((16*b^5*c - 13*a*b^4
*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3
*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b
/a)^(1/3)) + 1820*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^1
6 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/
3)*log(b*x + a*(-b/a)^(2/3)))/(a^6*b*x^16 + a^7*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08059, size = 651, normalized size = 1.74

$$\frac{\sqrt{3}\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d-7\left(-ab^2\right)^{\frac{2}{3}}a^3f+10\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^7}+\frac{\left(16b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-13\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d - 7*(-a*b^
2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-a/b)^(1/3) - 13*a*b^4*d*(-a/b)
^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3) + 10*a^2*b^3*(-a/b)^(1/3)*e)*(-a/b)^(1/3)
*log(abs(x - (-a/b)^(1/3)))/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^
2)^(2/3)*a*b^2*d - 7*(-a*b^2)^(2/3)*a^3*f + 10*(-a*b^2)^(2/3)*a^2*b*e)*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*x^2 - a
^3*b^2*f*x^2 + a^2*b^3*x^2*e)/((b*x^3 + a)*a^6) - 1/1820*(9100*b^4*c*x^12 -
 7280*a*b^3*d*x^12 - 3640*a^3*b*f*x^12 + 5460*a^2*b^2*x^12*e - 1820*a*b^3*c
*x^9 + 1365*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 910*a^3*b*x^9*e + 780*a^2*b^2*c
*x^6 - 520*a^3*b*d*x^6 + 260*a^4*x^6*e - 364*a^3*b*c*x^3 + 182*a^4*d*x^3 +
140*a^4*c)/(a^6*x^13)
```

3.276 $\int \frac{x^{14}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=266

$$\frac{x^6\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{6b^6}-\frac{ax^3\left(10a^2be-15a^3f-6ab^2d+3b^3c\right)}{3b^7}+\frac{a^3\left(6a^2be-7a^3f-5ab^2d+4b^3c\right)}{3b^8\left(a+bx^3\right)}-\frac{a^4\left(a\right.}{}$$

```
[Out] -(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/(3*b^7) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*
f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^4) + (f*x^15)/(15*b^3) - (a^4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c
- 5*a*b^2*d + 6*a^2*b*e - 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 1
0*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rubi [A] time = 0.436041, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^6\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{6b^6}-\frac{ax^3\left(10a^2be-15a^3f-6ab^2d+3b^3c\right)}{3b^7}+\frac{a^3\left(6a^2be-7a^3f-5ab^2d+4b^3c\right)}{3b^8\left(a+bx^3\right)}-\frac{a^4\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -(a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/(3*b^7) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(6*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*
f)*x^9)/(9*b^5) + ((b*e - 3*a*f)*x^12)/(12*b^4) + (f*x^15)/(15*b^3) - (a^4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c
- 5*a*b^2*d + 6*a^2*b*e - 7*a^3*f))/(3*b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 1
0*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/(3*b^8)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^{14}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{a\left(-3b^3c+6ab^2d-10a^2be+15a^3f\right)}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6}\right.\right.$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x^3}{3b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^6}{6b^6} + \frac{\left(b^2d-\right.}{}$$

Mathematica [A] time = 0.142904, size = 246, normalized size = 0.92

$$30b^2x^6\left(6a^2be-10a^3f-3ab^2d+b^3c\right)+60abx^3\left(-10a^2be+15a^3f+6ab^2d-3b^3c\right)-\frac{60a^3\left(-6a^2be+7a^3f+5ab^2d-4b^3c\right)}{a+bx^3}+\frac{30a^4\left(-a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b^2*(b^3*c
- 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d - 3*a*b*e + 6*a^2*f
)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*x^15 + (30*a^4*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d - 6*
a^2*b*e + 7*a^3*f))/(a + b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^2*b*e
 - 21*a^3*f)*Log[a + b*x^3])/(180*b^8)
```

Maple [A] time = 0.015, size = 361, normalized size = 1.4

$$\frac{fx^{15}}{15\,b^3}+5\,\frac{a^4fx^3}{b^7}-\frac{10\,a^3ex^3}{3\,b^6}+2\,\frac{a^2dx^3}{b^5}-\frac{acx^3}{b^4}+\frac{a^7f}{6\,b^8\left(bx^3+a\right)^2}-\frac{a^6e}{6\,b^7\left(bx^3+a\right)^2}+\frac{a^5d}{6\,b^6\left(bx^3+a\right)^2}-\frac{a^4c}{6\,b^5\left(bx^3+a\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/15*f*x^15/b^3+5/b^7*a^4*f*x^3-10/3/b^6*a^3*e*x^3+2/b^5*a^2*d*x^3-1/b^4*a*
c*x^3+1/6*a^7/b^8/(b*x^3+a)^2*f-1/6*a^6/b^7/(b*x^3+a)^2*e+1/6*a^5/b^6/(b*x^
3+a)^2*d-1/6*a^4/b^5/(b*x^3+a)^2*c-7/3*a^6/b^8/(b*x^3+a)*f+2*a^5/b^7/(b*x^3
+a)*e-5/3*a^4/b^6/(b*x^3+a)*d+4/3*a^3/b^5/(b*x^3+a)*c-7*a^5/b^8*ln(b*x^3+a)
*f+5*a^4/b^7*ln(b*x^3+a)*e-10/3*a^3/b^6*ln(b*x^3+a)*d+2*a^2/b^5*ln(b*x^3+a)
*c+1/12/b^3*x^12*e+1/9/b^3*x^9*d+1/6/b^3*x^6*c-1/4/b^4*x^12*a*f+2/3/b^5*x^9
*a^2*f-1/3/b^4*x^9*a*e-5/3/b^6*x^6*a^3*f+1/b^5*x^6*a^2*e-1/2/b^4*x^6*a*d
```

Maxima [A] time = 0.954004, size = 371, normalized size = 1.39

$$\frac{7\,a^4b^3c-9\,a^5b^2d+11\,a^6be-13\,a^7f+2\left(4\,a^3b^4c-5\,a^4b^3d+6\,a^5b^2e-7\,a^6bf\right)x^3}{6\left(b^{10}x^6+2\,ab^9x^3+a^2b^8\right)}+\frac{12\,b^4fx^{15}+15\left(b^4e-3\,ab^3f\right)x^{12}+20}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*a^4*b^3*c - 9*a^5*b^2*d + 11*a^6*b*e - 13*a^7*f + 2*(4*a^3*b^4*c - 5
*a^4*b^3*d + 6*a^5*b^2*e - 7*a^6*b*f)*x^3)/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^
8) + 1/180*(12*b^4*f*x^15 + 15*(b^4*e - 3*a*b^3*f)*x^12 + 20*(b^4*d - 3*a*b
^3*e + 6*a^2*b^2*f)*x^9 + 30*(b^4*c - 3*a*b^3*d + 6*a^2*b^2*e - 10*a^3*b*f)
*x^6 - 60*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*x^3)/b^7 + 1/3*
(6*a^2*b^3*c - 10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(b*x^3 + a)/b^8
```

Fricas [A] time = 1.23426, size = 883, normalized size = 3.32

$$\frac{12\,b^7fx^{21}+3\left(5\,b^7e-7\,ab^6f\right)x^{18}+2\left(10\,b^7d-15\,ab^6e+21\,a^2b^5f\right)x^{15}+5\left(6\,b^7c-10\,ab^6d+15\,a^2b^5e-21\,a^3b^4f\right)x^{12}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/180*(12*b^7*f*x^21 + 3*(5*b^7*e - 7*a*b^6*f)*x^18 + 2*(10*b^7*d - 15*a*b^
6*e + 21*a^2*b^5*f)*x^15 + 5*(6*b^7*c - 10*a*b^6*d + 15*a^2*b^5*e - 21*a^3*
b^4*f)*x^12 - 20*(6*a*b^6*c - 10*a^2*b^5*d + 15*a^3*b^4*e - 21*a^4*b^3*f)*x
^9 + 210*a^4*b^3*c - 270*a^5*b^2*d + 330*a^6*b*e - 390*a^7*f - 30*(11*a^2*b
^5*c - 21*a^3*b^4*d + 34*a^4*b^3*e - 50*a^5*b^2*f)*x^6 + 60*(a^3*b^4*c + a^
4*b^3*d - 4*a^5*b^2*e + 8*a^6*b*f)*x^3 + 60*(6*a^4*b^3*c - 10*a^5*b^2*d + 1
5*a^6*b*e - 21*a^7*f + (6*a^2*b^5*c - 10*a^3*b^4*d + 15*a^4*b^3*e - 21*a^5*
b^2*f)*x^6 + 2*(6*a^3*b^4*c - 10*a^4*b^3*d + 15*a^5*b^2*e - 21*a^6*b*f)*x^3
)*log(b*x^3 + a))/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07827, size = 471, normalized size = 1.77

$$\frac{\left(6\,a^2b^3c-10\,a^3b^2d-21\,a^5f+15\,a^4be\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^8}-\frac{18\,a^2b^5cx^6-30\,a^3b^4dx^6-63\,a^5b^2fx^6+45\,a^4b^3x^6e+28\,a^3b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*b^3*c - 10*a^3*b^2*d - 21*a^5*f + 15*a^4*b*e)*log(abs(b*x^3 + a)
)/b^8 - 1/6*(18*a^2*b^5*c*x^6 - 30*a^3*b^4*d*x^6 - 63*a^5*b^2*f*x^6 + 45*a^
4*b^3*x^6*e + 28*a^3*b^4*c*x^3 - 50*a^4*b^3*d*x^3 - 112*a^6*b*f*x^3 + 78*a^
```

```
5*b^2*x^3*e + 11*a^4*b^3*c - 21*a^5*b^2*d - 50*a^7*f + 34*a^6*b*e)/((b*x^3
+ a)^2*b^8) + 1/180*(12*b^12*f*x^15 - 45*a*b^11*f*x^12 + 15*b^12*x^12*e + 2
0*b^12*d*x^9 + 120*a^2*b^10*f*x^9 - 60*a*b^11*x^9*e + 30*b^12*c*x^6 - 90*a*
b^11*d*x^6 - 300*a^3*b^9*f*x^6 + 180*a^2*b^10*x^6*e - 180*a*b^11*c*x^3 + 36
0*a^2*b^10*d*x^3 + 900*a^4*b^8*f*x^3 - 600*a^3*b^9*x^3*e)/b^15
```

3.277 $\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=226

$$\frac{x^3\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{3b^6}-\frac{a^2\left(5a^2be-6a^3f-4ab^2d+3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a\log\left(a+\right.}{}$$

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3)
 + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(
3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3
*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rubi [A] time = 0.330679, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{x^3\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{3b^6}-\frac{a^2\left(5a^2be-6a^3f-4ab^2d+3b^3c\right)}{3b^7\left(a+bx^3\right)}+\frac{a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{a\log\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^6)/(6*b^5) + ((b*e - 3*a*f)*x^9)/(9*b^4) + (f*x^12)/(12*b^3)
 + (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^7*(a + b*x^3)^2) - (a^2*(
3*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(3*b^7*(a + b*x^3)) - (a*(3*b^3
*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/(3*b^7)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\mathrm{Subst}\left(\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\mathrm{Subst}\left(\int \left(\frac{b^3c-3ab^2d+6a^2be-10a^3f}{b^6}+\frac{\left(b^2d-3abe+6a^2f\right)x}{b^5}+\frac{(be-3af)x^2}{b^4}+\right.\right.$$
$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^3}{3b^6}+\frac{\left(b^2d-3abe+6a^2f\right)x^6}{6b^5}+\frac{(be-3af)x^9}{9b^4}+\frac{fx^{12}}{12b^3}+$$

Mathematica [A] time = 0.120015, size = 208, normalized size = 0.92

$$\frac{12bx^3\left(6a^2be-10a^3f-3ab^2d+b^3c\right)+\frac{12a^2\left(-5a^2be+6a^3f+4ab^2d-3b^3c\right)}{a+bx^3}+\frac{6a^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2}+12a\log\left(a+bx^3\right)\left(-10a^2be\right.}{36b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2*d - 3*a*b
*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x^12 + (6*a^3*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 + (12*a^2*(-3*b^3*c + 4*a*b^2*d
 - 5*a^2*b*e + 6*a^3*f))/(a + b*x^3) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*
b*e + 15*a^3*f)*Log[a + b*x^3])/(36*b^7)
```

Maple [A] time = 0.013, size = 313, normalized size = 1.4

$$\frac{fx^{12}}{12\,b^3}-\frac{x^9af}{3\,b^4}+\frac{x^9e}{9\,b^3}+\frac{a^2fx^6}{b^5}-\frac{aex^6}{2\,b^4}+\frac{dx^6}{6\,b^3}-\frac{10\,a^3fx^3}{3\,b^6}+2\,\frac{a^2ex^3}{b^5}-\frac{adx^3}{b^4}+\frac{cx^3}{3\,b^3}-\frac{a^6f}{6\,b^7\left(bx^3+a\right)^2}+\frac{a^5e}{6\,b^6\left(bx^3+a\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/12*f*x^12/b^3-1/3/b^4*x^9*a*f+1/9/b^3*x^9*e+1/b^5*x^6*a^2*f-1/2/b^4*x^6*a
*e+1/6/b^3*x^6*d-10/3/b^6*a^3*f*x^3+2/b^5*a^2*e*x^3-1/b^4*a*d*x^3+1/3/b^3*c
*x^3-1/6*a^6/b^7/(b*x^3+a)^2*f+1/6*a^5/b^6/(b*x^3+a)^2*e-1/6*a^4/b^5/(b*x^3
+a)^2*d+1/6*a^3/b^4/(b*x^3+a)^2*c+5*a^4/b^7*ln(b*x^3+a)*f-10/3*a^3/b^6*ln(b
*x^3+a)*e+2*a^2/b^5*ln(b*x^3+a)*d-a/b^4*ln(b*x^3+a)*c+2*a^5/b^7/(b*x^3+a)*f
-5/3*a^4/b^6/(b*x^3+a)*e+4/3*a^3/b^5/(b*x^3+a)*d-a^2/b^4/(b*x^3+a)*c
```

Maxima [A] time = 0.963488, size = 315, normalized size = 1.39

$$-\frac{5\,a^3b^3c-7\,a^4b^2d+9\,a^5be-11\,a^6f+2\left(3\,a^2b^4c-4\,a^3b^3d+5\,a^4b^2e-6\,a^5bf\right)x^3}{6\left(b^9x^6+2\,ab^8x^3+a^2b^7\right)}+\frac{3\,b^3fx^{12}+4\left(b^3e-3\,ab^2f\right)x^9+6\left(b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(5*a^3*b^3*c - 7*a^4*b^2*d + 9*a^5*b*e - 11*a^6*f + 2*(3*a^2*b^4*c - 4
*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7
) + 1/36*(3*b^3*f*x^12 + 4*(b^3*e - 3*a*b^2*f)*x^9 + 6*(b^3*d - 3*a*b^2*e +
 6*a^2*b*f)*x^6 + 12*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^6 -
1/3*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*log(b*x^3 + a)/b^7
```

Fricas [A] time = 1.34569, size = 765, normalized size = 3.38

$$\frac{3\,b^{6}fx^{18}+2\left(2\,b^{6}e-3\,ab^{5}f\right)x^{15}+\left(6\,b^{6}d-10\,ab^{5}e+15\,a^{2}b^{4}f\right)x^{12}+4\left(3\,b^{6}c-6\,ab^{5}d+10\,a^{2}b^{4}e-15\,a^{3}b^{3}f\right)x^{9}-30\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^6*f*x^18 + 2*(2*b^6*e - 3*a*b^5*f)*x^15 + (6*b^6*d - 10*a*b^5*e +
 15*a^2*b^4*f)*x^12 + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b^4*e - 15*a^3*b^3*f)
*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b*e + 66*a^6*f + 6*(4*a*b^5*c -
 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^3*b^
3*d - a^4*b^2*e + 4*a^5*b*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*a^5*b
*e - 15*a^6*f + (3*a*b^5*c - 6*a^2*b^4*d + 10*a^3*b^3*e - 15*a^4*b^2*f)*x^6
 + 2*(3*a^2*b^4*c - 6*a^3*b^3*d + 10*a^4*b^2*e - 15*a^5*b*f)*x^3)*log(b*x^3
 + a))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09688, size = 402, normalized size = 1.78

$$-\frac{\left(3\,ab^{3}c-6\,a^{2}b^{2}d-15\,a^{4}f+10\,a^{3}be\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,b^{7}}+\frac{9\,ab^{5}cx^{6}-18\,a^{2}b^{4}dx^{6}-45\,a^{4}b^{2}fx^{6}+30\,a^{3}b^{3}x^{6}e+12\,a^{2}b^{4}cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4*f + 10*a^3*b*e)*log(abs(b*x^3 + a))/
b^7 + 1/6*(9*a*b^5*c*x^6 - 18*a^2*b^4*d*x^6 - 45*a^4*b^2*f*x^6 + 30*a^3*b^3
*x^6*e + 12*a^2*b^4*c*x^3 - 28*a^3*b^3*d*x^3 - 78*a^5*b*f*x^3 + 50*a^4*b^2*
x^3*e + 4*a^3*b^3*c - 11*a^4*b^2*d - 34*a^6*f + 21*a^5*b*e)/((b*x^3 + a)^2*
b^7) + 1/36*(3*b^9*f*x^12 - 12*a*b^8*f*x^9 + 4*b^9*x^9*e + 6*b^9*d*x^6 + 36
*a^2*b^7*f*x^6 - 18*a*b^8*x^6*e + 12*b^9*c*x^3 - 36*a*b^8*d*x^3 - 120*a^3*b
^6*f*x^3 + 72*a^2*b^7*x^3*e)/b^12
```

3.278 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

Optimal. Leaf size=186

$$\frac{a(4a^2be-5a^3f-3ab^2d+2b^3c)}{3b^6(a+bx^3)}-\frac{a^2(a^2be+a^3(-f)-ab^2d+b^3c)}{6b^6(a+bx^3)^2}+\frac{\log(a+bx^3)(6a^2be-10a^3f-3ab^2d+b^3c)}{3b^6}+\frac{x^3}{}$$

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (
f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^
3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3))
 + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rubi [A] time = 0.268936, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a(4a^2be-5a^3f-3ab^2d+2b^3c)}{3b^6(a+bx^3)}-\frac{a^2(a^2be+a^3(-f)-ab^2d+b^3c)}{6b^6(a+bx^3)^2}+\frac{\log(a+bx^3)(6a^2be-10a^3f-3ab^2d+b^3c)}{3b^6}+\frac{x^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^6)/(6*b^4) + (
f*x^9)/(9*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*b^6*(a + b*x^
3)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(3*b^6*(a + b*x^3))
 + ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(3*b^6)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx)^3} dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2d-3abe+6a^2f}{b^5}+\frac{(be-3af)x}{b^4}+\frac{fx^2}{b^3}-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5(a+bx)^3}\right.\right.$$

$$= \frac{(b^2d-3abe+6a^2f)x^3}{3b^5}+\frac{(be-3af)x^6}{6b^4}+\frac{fx^9}{9b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{6b^6(a+bx^3)^2}+\frac{a(2b^3}{}$$

Mathematica [A] time = 0.0987472, size = 170, normalized size = 0.91

$$-\frac{-\frac{6a\left(-4a^2be+5a^3f+3ab^2d-2b^3c\right)}{a+bx^3}+\frac{3a^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+6\log\left(a+bx^3\right)\left(6a^2be-10a^3f-3ab^2d+b^3c\right)+6bx^3\left(6a^2f-3ab\right.}{18b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 + 2*b^3*f*x^
9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (6*a*(-2
*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a + b*x^3) + 6*(b^3*c - 3*a*b^2
*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(18*b^6)
```

Maple [A] time = 0.012, size = 266, normalized size = 1.4

$$\frac{fx^9}{9\,b^3}-\frac{x^6af}{2\,b^4}+\frac{ex^6}{6\,b^3}+2\,\frac{a^2fx^3}{b^5}-\frac{aex^3}{b^4}+\frac{dx^3}{3\,b^3}+\frac{a^5f}{6\,b^6\left(bx^3+a\right)^2}-\frac{a^4e}{6\,b^5\left(bx^3+a\right)^2}+\frac{a^3d}{6\,b^4\left(bx^3+a\right)^2}-\frac{a^2c}{6\,b^3\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/9*f*x^9/b^3-1/2/b^4*x^6*a*f+1/6/b^3*x^6*e+2/b^5*a^2*f*x^3-1/b^4*a*e*x^3+1
/3/b^3*d*x^3+1/6/b^6*a^5/(b*x^3+a)^2*f-1/6/b^5*a^4/(b*x^3+a)^2*e+1/6/b^4*a^
3/(b*x^3+a)^2*d-1/6/b^3*a^2/(b*x^3+a)^2*c-10/3/b^6*ln(b*x^3+a)*a^3*f+2/b^5*
ln(b*x^3+a)*a^2*e-1/b^4*ln(b*x^3+a)*a*d+1/3/b^3*ln(b*x^3+a)*c-5/3/b^6*a^4/(
b*x^3+a)*f+4/3/b^5*a^3/(b*x^3+a)*e-1/b^4*a^2/(b*x^3+a)*d+2/3/b^3*a/(b*x^3+a
)*c
```

Maxima [A] time = 0.95037, size = 258, normalized size = 1.39

$$\frac{3\,a^2b^3c-5\,a^3b^2d+7\,a^4be-9\,a^5f+2\left(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf\right)x^3}{6\left(b^8x^6+2\,ab^7x^3+a^2b^6\right)}+\frac{2\,b^2fx^9+3\left(b^2e-3\,abf\right)x^6+6\left(b^2d\right.}{18\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*b^3*c - 5*a^3*b^2*d + 7*a^4*b*e - 9*a^5*f + 2*(2*a*b^4*c - 3*a^2
*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6) +
1/18*(2*b^2*f*x^9 + 3*(b^2*e - 3*a*b*f)*x^6 + 6*(b^2*d - 3*a*b*e + 6*a^2*f)
*x^3)/b^5 + 1/3*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*log(b*x^3 + a)/b
^6
```

Fricas [A] time = 1.29363, size = 635, normalized size = 3.41

$$2\,b^5fx^{15}+\left(3\,b^5e-5\,ab^4f\right)x^{12}+2\left(3\,b^5d-6\,ab^4e+10\,a^2b^3f\right)x^9+3\left(4\,ab^4d-11\,a^2b^3e+21\,a^3b^2f\right)x^6+9\,a^2b^3c-15\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*f*x^15 + (3*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 6*a*b^4*e +
10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^3*b^2*f)*x^6 + 9*a^2
*b^3*c - 15*a^3*b^2*d + 21*a^4*b*e - 27*a^5*f + 6*(2*a*b^4*c - 2*a^2*b^3*d
+ a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^5*c - 3*a*b^4*d + 6*a^2*b^3*e - 10*a^3*b
^2*f)*x^6 + a^2*b^3*c - 3*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a*b^4*c - 3
*a^2*b^3*d + 6*a^3*b^2*e - 10*a^4*b*f)*x^3)*log(b*x^3 + a))/(b^8*x^6 + 2*a*
b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07923, size = 319, normalized size = 1.72

$$\frac{\left(b^3c - 3\,ab^2d - 10\,a^3f + 6\,a^2be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^6} - \frac{3\,b^5cx^6 - 9\,ab^4dx^6 - 30\,a^3b^2fx^6 + 18\,a^2b^3x^6e + 2\,ab^4cx^3 - 12\,a^2b^3dx^3 -}{6\left(bx^3 + a\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^3*c - 3*a*b^2*d - 10*a^3*f + 6*a^2*b*e)*log(abs(b*x^3 + a))/b^6 - 1/
6*(3*b^5*c*x^6 - 9*a*b^4*d*x^6 - 30*a^3*b^2*f*x^6 + 18*a^2*b^3*x^6*e + 2*a*
b^4*c*x^3 - 12*a^2*b^3*d*x^3 - 50*a^4*b*f*x^3 + 28*a^3*b^2*x^3*e - 4*a^3*b^
2*d - 21*a^5*f + 11*a^4*b*e)/((b*x^3 + a)^2*b^6) + 1/18*(2*b^6*f*x^9 - 9*a*
b^5*f*x^6 + 3*b^6*x^6*e + 6*b^6*d*x^3 + 36*a^2*b^4*f*x^3 - 18*a*b^5*x^3*e)/
b^9
```

3.279 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=146

$$-\frac{3a^2be-4a^3f-2ab^2d+b^3c}{3b^5\left(a+bx^3\right)}+\frac{a\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{x^3(be-3af)}{3b^4}$$

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3
*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b
^5)
```

Rubi [A] time = 0.200995, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{3a^2be-4a^3f-2ab^2d+b^3c}{3b^5\left(a+bx^3\right)}+\frac{a\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{x^3(be-3af)}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^3)/(3*b^4) + (f*x^6)/(6*b^3) + (a*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f))/(6*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3
*f)/(3*b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/(3*b
^5)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{(a+bx)^3}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\text{Subst}\left(\int \left(\frac{be-3af}{b^4}+\frac{fx}{b^3}+\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4(a+bx)^3}+\frac{b^3c-2ab^2d+3a^2be}{b^4(a+bx)^2}\right.\right.$$

$$=\frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{6b^5\left(a+bx^3\right)^2}-\frac{b^3c-2ab^2d+3a^2be-4a^3f}{3b^5\left(a+bx^3\right)}+$$

Mathematica [A] time = 0.0737187, size = 145, normalized size = 0.99

$$\frac{a^2b^2\left(3d-4ex^3-11fx^6\right)+2\left(a+bx^3\right)^2\log\left(a+bx^3\right)\left(6a^2f-3abe+b^2d\right)+a^3b\left(2fx^3-5e\right)+7a^4f-ab^3\left(c-4x^3\left(d+e\right.\right.}{6b^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f*x^6) + b^
4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d + e*x^3 - f*x^6)) + 2*
(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*x^3
)^2)
```

Maple [A] time = 0.013, size = 213, normalized size = 1.5

$$\frac{fx^6}{6\,b^3}-\frac{ax^3f}{b^4}+\frac{x^3e}{3\,b^3}-\frac{a^4f}{6\,b^5\left(bx^3+a\right)^2}+\frac{a^3e}{6\,b^4\left(bx^3+a\right)^2}-\frac{a^2d}{6\,b^3\left(bx^3+a\right)^2}+\frac{ac}{6\,b^2\left(bx^3+a\right)^2}+2\,\frac{\ln\left(bx^3+a\right)a^2f}{b^5}-\frac{\ln\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*f*x^6/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e-1/6/b^5*a^4/(b*x^3+a)^2*f+1/6/b^4
*a^3/(b*x^3+a)^2*e-1/6/b^3*a^2/(b*x^3+a)^2*d+1/6/b^2*a/(b*x^3+a)^2*c+2/b^5*
ln(b*x^3+a)*a^2*f-1/b^4*ln(b*x^3+a)*a*e+1/3/b^3*ln(b*x^3+a)*d+4/3/b^5/(b*x^
3+a)*a^3*f-1/b^4/(b*x^3+a)*a^2*e+2/3/b^3/(b*x^3+a)*a*d-1/3/b^2/(b*x^3+a)*c
```

Maxima [A] time = 0.952873, size = 198, normalized size = 1.36

$$-\frac{ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+2\left(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf\right)x^3}{6\left(b^7x^6+2\,ab^6x^3+a^2b^5\right)}+\frac{bfx^6+2\left(be-3\,af\right)x^3}{6\,b^4}+\frac{\left(b^2d-3\,abe+6\,a^2f\right.}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + 2*(b^4*c - 2*a*b^3*d +
3*a^2*b^2*e - 4*a^3*b*f)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5) + 1/6*(b*f*
x^6 + 2*(b*e - 3*a*f)*x^3)/b^4 + 1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*log(b*x^3
+ a)/b^5
```

Fricas [A] time = 1.25994, size = 470, normalized size = 3.22

$$\frac{b^4fx^{12}+2\left(b^4e-2\,ab^3f\right)x^9+\left(4\,ab^3e-11\,a^2b^2f\right)x^6-ab^3c+3\,a^2b^2d-5\,a^3be+7\,a^4f-2\left(b^4c-2\,ab^3d+2\,a^2b^2e-a^3bf\right)}{6\left(b^7x^6+2\,ab^6x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*f*x^12 + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2*b^2*f)*x^
6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^4*c - 2*a*b^3*d + 2*
a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^6 + a^2*b
^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)*log
(b*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08806, size = 197, normalized size = 1.35

$$\frac{\left(b^2d + 6\,a^2f - 3\,abe\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^5} + \frac{b^3fx^6 - 6\,ab^2fx^3 + 2\,b^3x^3e}{6\,b^6} - \frac{ab^3c - 3\,a^2b^2d - 7\,a^4f + 5\,a^3be + 2\left(b^4c - 2\,ab^3d\right.}{6\left(bx^3 + a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*d + 6*a^2*f - 3*a*b*e)*log(abs(b*x^3 + a))/b^5 + 1/6*(b^3*f*x^6 -
6*a*b^2*f*x^3 + 2*b^3*x^3*e)/b^6 - 1/6*(a*b^3*c - 3*a^2*b^2*d - 7*a^4*f + 5
*a^3*b*e + 2*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2*b^2*e)*x^3)/((b*x^3 + a
)^2*b^5)
```

3.280 $\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=109

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6b^4\left(a+bx^3\right)^2}-\frac{3a^2f-2abe+b^2d}{3b^4\left(a+bx^3\right)}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}+\frac{fx^3}{3b^3}$$

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2)
 - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a +
 b*x^3])/(3*b^4)
```

Rubi [A] time = 0.151546, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1819, 1850}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6b^4\left(a+bx^3\right)^2}-\frac{3a^2f-2abe+b^2d}{3b^4\left(a+bx^3\right)}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (f*x^3)/(3*b^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*b^4*(a + b*x^3)^2)
 - (b^2*d - 2*a*b*e + 3*a^2*f)/(3*b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a +
 b*x^3])/(3*b^4)
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{(a+bx)^3} dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{f}{b^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3(a+bx)^3}+\frac{b^2d-2abe+3a^2f}{b^3(a+bx)^2}+\frac{be-3af}{b^3(a+bx)}\right) dx, x, x^3\right)\\ &= \frac{fx^3}{3b^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6b^4\left(a+bx^3\right)^2}-\frac{b^2d-2abe+3a^2f}{3b^4\left(a+bx^3\right)}+\frac{(be-3af)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.0541032, size = 105, normalized size = 0.96

$$\frac{a^2b\left(3e-4fx^3\right)-5a^3f+ab^2\left(-d+4ex^3+4fx^6\right)+2\left(a+bx^3\right)^2(be-3af)\log\left(a+bx^3\right)-b^3\left(c+2dx^3-2fx^9\right)}{6b^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6) - b^3*(c
 + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^
4*(a + b*x^3)^2)
```

Maple [A] time = 0.01, size = 156, normalized size = 1.4

$$\frac{fx^3}{3\,b^3}+\frac{a^3f}{6\,b^4\left(bx^3+a\right)^2}-\frac{a^2e}{6\,b^3\left(bx^3+a\right)^2}+\frac{ad}{6\,b^2\left(bx^3+a\right)^2}-\frac{c}{6\,b\left(bx^3+a\right)^2}-\frac{\ln\left(bx^3+a\right)af}{b^4}+\frac{\ln\left(bx^3+a\right)e}{3\,b^3}-\frac{a}{b^4\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*f*x^3/b^3+1/6/b^4/(b*x^3+a)^2*a^3*f-1/6/b^3/(b*x^3+a)^2*a^2*e+1/6/b^2/(
b*x^3+a)^2*a*d-1/6/b/(b*x^3+a)^2*c-1/b^4*ln(b*x^3+a)*a*f+1/3/b^3*ln(b*x^3+a
)*e-1/b^4/(b*x^3+a)*a^2*f+2/3/b^3/(b*x^3+a)*a*e-1/3/b^2/(b*x^3+a)*d
```

Maxima [A] time = 0.951215, size = 147, normalized size = 1.35

$$\frac{fx^3}{3\,b^3}-\frac{b^3c+ab^2d-3\,a^2be+5\,a^3f+2\left(b^3d-2\,ab^2e+3\,a^2bf\right)x^3}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}+\frac{\left(be-3\,af\right)\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*a
*b^2*e + 3*a^2*b*f)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*(b*e - 3*a
*f)*log(b*x^3 + a)/b^4
```

Fricas [A] time = 1.165, size = 329, normalized size = 3.02

$$\frac{2\,b^3fx^9+4\,ab^2fx^6-b^3c-ab^2d+3\,a^2be-5\,a^3f-2\left(b^3d-2\,ab^2e+2\,a^2bf\right)x^3+2\left(\left(b^3e-3\,ab^2f\right)x^6+a^2be-3\,a^3f+\right.}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

[Out] 1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f - 2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a*b^2*f)*x^6 + a^2*b*e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.07817, size = 135, normalized size = 1.24

$$\frac{fx^3}{3\,b^3} - \frac{\left(3\,af - be\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^4} - \frac{b^3c + ab^2d + 5\,a^3f + 2\left(b^3d + 3\,a^2bf - 2\,ab^2e\right)x^3 - 3\,a^2be}{6\left(bx^3 + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/3*f*x^3/b^3 - 1/3*(3*a*f - b*e)*log(abs(b*x^3 + a))/b^4 - 1/6*(b^3*c + a*b^2*d + 5*a^3*f + 2*(b^3*d + 3*a^2*b*f - 2*a*b^2*e)*x^3 - 3*a^2*b*e)/((b*x^3 + a)^2*b^4)

3.281 $\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=114

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6ab^3\left(a+bx^3\right)^2}+\frac{-a^2be+2a^3f+b^3c}{3a^2b^3\left(a+bx^3\right)}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}$$

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*
b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*
Log[a + b*x^3])/3
```

Rubi [A] time = 0.15477, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6ab^3\left(a+bx^3\right)^2}+\frac{-a^2be+2a^3f+b^3c}{3a^2b^3\left(a+bx^3\right)}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a*b^3*(a + b*x^3)^2) + (b^3*c - a^2*
b*e + 2*a^3*f)/(3*a^2*b^3*(a + b*x^3)) + (c*Log[x])/a^3 - ((c/a^3 - f/b^3)*
Log[a + b*x^3])/3
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x(a+bx)^3} dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2(a+bx)^3}+\frac{-b^3c+a^2be-2a^3f}{a^2b^2(a+bx)^2}+\frac{-b^3c+a^3f}{a^3b^2(a+bx)}\right) d\right.\\
&= \frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3\left(a+bx^3\right)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)
\end{aligned}$$

Mathematica [A] time = 0.105827, size = 104, normalized size = 0.91

$$\frac{\frac{\frac{a\left(-a^2b^2\left(d+2ex^3\right)-a^3b\left(e-4fx^3\right)+3a^4f+3ab^3c+2b^4cx^3\right)}{\left(a+bx^3\right)^2}+2\left(a^3f-b^3c\right)\log\left(a+bx^3\right)}{b^3}+6c\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

```
[Out] (6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d + 2*e*x^3
) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) + a^3*f)*Log[a + b*x^
3])/b^3)/(6*a^3)
```

Maple [A] time = 0.015, size = 147, normalized size = 1.3

$$-\frac{a^2f}{6\,b^3\left(bx^3+a\right)^2}+\frac{ae}{6\,b^2\left(bx^3+a\right)^2}-\frac{d}{6\,b\left(bx^3+a\right)^2}+\frac{c}{6\,a\left(bx^3+a\right)^2}+\frac{\ln\left(bx^3+a\right)f}{3\,b^3}-\frac{c\ln\left(bx^3+a\right)}{3\,a^3}+\frac{2\,af}{3\,b^3\left(bx^3+a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^3+a)^2*f+1/6*a/b^2/(b*x^3+a)^2*e-1/6/b/(b*x^3+a)^2*d+1/6/
a/(b*x^3+a)^2*c+1/3/b^3*ln(b*x^3+a)*f-1/3*c*ln(b*x^3+a)/a^3+2/3*a/b^3/(b*x^
3+a)*f-1/3/b^2/(b*x^3+a)*e+1/3/a^2/(b*x^3+a)*c+c*ln(x)/a^3
```

Maxima [A] time = 0.953155, size = 174, normalized size = 1.53

$$\frac{3\,ab^3c-a^2b^2d-a^3be+3\,a^4f+2\left(b^4c-a^2b^2e+2\,a^3bf\right)x^3}{6\left(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3\right)}+\frac{c\log\left(x^3\right)}{3\,a^3}-\frac{\left(b^3c-a^3f\right)\log\left(bx^3+a\right)}{3\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f + 2*(b^4*c - a^2*b^2*e + 2*a
^3*b*f)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) + 1/3*c*log(x^3)/a^3 -
 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)
```

Fricas [A] time = 1.36771, size = 374, normalized size = 3.28

$$\frac{3\,a^2b^3c-a^3b^2d-a^4be+3\,a^5f+2\left(ab^4c-a^3b^2e+2\,a^4bf\right)x^3-2\left(\left(b^5c-a^3b^2f\right)x^6+a^2b^3c-a^5f+2\left(ab^4c-a^4bf\right)x^3\right)\log}{6\left(a^3b^5x^6+2\,a^4b^4x^3+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a^3*b^2*e +
 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*c - a^5*f + 2*(a*b^4
*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*x^6 + 2*a*b^4*c*x^3 + a^2*b^3*
c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07024, size = 173, normalized size = 1.52

$$\frac{c\log\left(|x|\right)}{a^3} - \frac{\left(b^3c - a^3f\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^3b^3} + \frac{3\,b^4cx^6 - 3\,a^3bfx^6 + 8\,ab^3cx^3 - 2\,a^4fx^3 - 2\,a^3bx^3e + 6\,a^2b^2c - a^3bd - a^4e}{6\left(bx^3 + a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*log(abs(b*x^3 + a))/(a^3*b^3) + 1/6
*(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^4*f*x^3 - 2*a^3*b*x^3*e
 + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^2*a^3*b^2)
```

3.282 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=134

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^2b^2\left(a+bx^3\right)^2}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}+\frac{(3bc-ad)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{c}{3a^3x^3}$$

```
[Out] -c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^3)
^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d)*
Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 0.171251, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^2b^2\left(a+bx^3\right)^2}-\frac{a^3f-ab^2d+2b^3c}{3a^3b^2\left(a+bx^3\right)}+\frac{(3bc-ad)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3bc-ad)}{a^4}-\frac{c}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^2*b^2*(a + b*x^3)
^2) - (2*b^3*c - a*b^2*d + a^3*f)/(3*a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d)*
Log[x])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/(3*a^4)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^2}+\frac{-3bc+ad}{a^4x}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx)^3}+\frac{2b^3c-ab^2d+a^3f}{a^3b(a+bx)^2}-\frac{b(-3bc\right.\right.\\ &= -\frac{c}{3a^3x^3}-\frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2\left(a+bx^3\right)^2}-\frac{2b^3c-ab^2d+a^3f}{3a^3b^2\left(a+bx^3\right)}-\frac{(3bc-ad)\log(x)}{a^4}+\frac{(3bc-ad)\log}{3a^4}\end{aligned}$$

Mathematica [A] time = 0.0914598, size = 121, normalized size = 0.9

$$\frac{\frac{a^2(-a^2be+a^3f+ab^2d-b^3c)}{b^2(a+bx^3)^2}-\frac{2a(a^3f-ab^2d+2b^3c)}{b^2(a+bx^3)}+2(3bc-ad)\log\left(a+bx^3\right)+6\log(x)(ad-3bc)-\frac{2ac}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^
3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x^3)) + 6*(-3*b*c + a
*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/(6*a^4)
```

Maple [A] time = 0.014, size = 163, normalized size = 1.2

$$\frac{af}{6\,b^2\left(bx^3+a\right)^2}-\frac{e}{6\,b\left(bx^3+a\right)^2}+\frac{d}{6\,a\left(bx^3+a\right)^2}-\frac{bc}{6\,a^2\left(bx^3+a\right)^2}-\frac{d\ln\left(bx^3+a\right)}{3\,a^3}+\frac{bc\ln\left(bx^3+a\right)}{a^4}-\frac{f}{3\,b^2\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] 1/6*a/b^2/(b*x^3+a)^2*f-1/6/b/(b*x^3+a)^2*e+1/6/a/(b*x^3+a)^2*d-1/6/a^2*b/(
b*x^3+a)^2*c-1/3*d*ln(b*x^3+a)/a^3+b*c*ln(b*x^3+a)/a^4-1/3/b^2/(b*x^3+a)*f+
1/3/a^2/(b*x^3+a)*d-2/3/a^3*b/(b*x^3+a)*c-1/3*c/a^3/x^3+d*ln(x)/a^3-3*b*c*l
n(x)/a^4
```

Maxima [A] time = 0.961867, size = 194, normalized size = 1.45

$$-\frac{2\left(3\,b^4c-ab^3d+a^3bf\right)x^6+2\,a^2b^2c+\left(9\,ab^3c-3\,a^2b^2d+a^3be+a^4f\right)x^3}{6\left(a^3b^4x^9+2\,a^4b^3x^6+a^5b^2x^3\right)}+\frac{(3\,bc-ad)\log\left(bx^3+a\right)}{3\,a^4}-\frac{(3\,bc-ad)\log}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^3*c - 3*a^
2*b^2*d + a^3*b*e + a^4*f)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^3*x^6 + a^5*b^2*x^3)
 + 1/3*(3*b*c - a*d)*log(b*x^3 + a)/a^4 - 1/3*(3*b*c - a*d)*log(x^3)/a^4
```

Fricas [A] time = 1.39977, size = 497, normalized size = 3.71

$$-\frac{2\left(3\,ab^4c-a^2b^3d+a^4bf\right)x^6+2\,a^3b^2c+\left(9\,a^2b^3c-3\,a^3b^2d+a^4be+a^5f\right)x^3-2\left(\left(3\,b^5c-ab^4d\right)x^9+2\left(3\,ab^4c-a^2b^3d\right.\right.}{6\left(a^4b^4x^9+2\,a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*a^2*b^3*c
- 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*
b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*log(b*x^3 + a) + 6*
((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a
^3*b^2*d)*x^3)*log(x))/(a^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09671, size = 234, normalized size = 1.75

$$-\frac{(3\,bc-ad)\log(|x|)}{a^4}+\frac{\left(3\,b^2c-abd\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{3\,bcx^3-adx^3-ac}{3\,a^4x^3}-\frac{9\,b^5cx^6-3\,ab^4dx^6+22\,ab^4cx^3-8\,a^2b^3dx^3+}{6\left(bx^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(3*b*c - a*d)*log(abs(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*log(abs(b*x^3 + a))/
(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6*(9*b^5*c*x^6 - 3*
a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3 + 2*a^4*b*f*x^3 + 14*a^2*b^3
*c - 6*a^3*b^2*d + a^5*f + a^4*b*e)/((b*x^3 + a)^2*a^4*b^2)
```

3.283 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx$

Optimal. Leaf size=163

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^3b\left(a+bx^3\right)^2}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+$$

```
[Out] -c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a
^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b*x
^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a^5 - ((6*b^2*c - 3*a*b*d + a^2
*e)*Log[a + b*x^3])/(3*a^5)
```

Rubi [A] time = 0.199532, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^3b\left(a+bx^3\right)^2}+\frac{a^2e-2abd+3b^2c}{3a^4\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{3a^5}+\frac{\log(x)\left(a^2e-3abd+6b^2c\right)}{a^5}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -c/(6*a^3*x^6) + (3*b*c - a*d)/(3*a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a
^3*f)/(6*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(3*a^4*(a + b*x
^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x])/a^5 - ((6*b^2*c - 3*a*b*d + a^2
*e)*Log[a + b*x^3])/(3*a^5)
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{a^3x^3}+\frac{-3bc+ad}{a^4x^2}+\frac{6b^2c-3abd+a^2e}{a^5x}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx)^3}-\frac{b(3\right.\right.\\ &= -\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b\left(a+bx^3\right)^2}+\frac{3b^2c-2abd+a^2e}{3a^4\left(a+bx^3\right)}+\frac{\left(6b^2c-3abd+a^2\right.}{a^5}\end{aligned}$$

Mathematica [A] time = 0.106458, size = 149, normalized size = 0.91

$$\frac{\frac{a^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b\left(a+bx^3\right)^2}+\frac{2a\left(a^2e-2abd+3b^2c\right)}{a+bx^3}-2\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)+6\log(x)\left(a^2e-3abd+6b^2c\right)-\frac{a^2c}{x^6}-\frac{2a(ad}{x}}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d + a^2*e))/(a + b*x^
3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2*(6*b^2*c - 3*a*b*d + a^2*e)*L
og[a + b*x^3])/(6*a^5)
```

Maple [A] time = 0.017, size = 213, normalized size = 1.3

$$-\frac{f}{6\,b\left(bx^3+a\right)^2}+\frac{e}{6\,a\left(bx^3+a\right)^2}-\frac{bd}{6\,a^2\left(bx^3+a\right)^2}+\frac{b^2c}{6\,a^3\left(bx^3+a\right)^2}-\frac{e\ln\left(bx^3+a\right)}{3\,a^3}+\frac{\ln\left(bx^3+a\right)bd}{a^4}-2\,\frac{\ln\left(bx^3+a\right)b}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x)
```

```
[Out] -1/6/b/(b*x^3+a)^2*f+1/6/a/(b*x^3+a)^2*e-1/6/a^2*b/(b*x^3+a)^2*d+1/6/a^3*b^
2/(b*x^3+a)^2*c-1/3*e*ln(b*x^3+a)/a^3+1/a^4*ln(b*x^3+a)*b*d-2/a^5*ln(b*x^3+
a)*b^2*c+1/3/a^2/(b*x^3+a)*e-2/3/a^3/(b*x^3+a)*b*d+1/a^4/(b*x^3+a)*b^2*c-1/
6*c/a^3/x^6-1/3/a^3/x^3*d+1/a^4/x^3*b*c+e*ln(x)/a^3-3/a^4*ln(x)*b*d+6/a^5*l
n(x)*b^2*c
```

Maxima [A] time = 0.980084, size = 246, normalized size = 1.51

$$\frac{2\left(6\,b^4c-3\,ab^3d+a^2b^2e\right)x^9+\left(18\,ab^3c-9\,a^2b^2d+3\,a^3be-a^4f\right)x^6-a^3bc+2\left(2\,a^2b^2c-a^3bd\right)x^3}{6\left(a^4b^3x^{12}+2\,a^5b^2x^9+a^6bx^6\right)}-\frac{\left(6\,b^2c-3\,abd+a^2e\right)}{3\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^2*b^2*d +
3*a^3*b*e - a^4*f)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3*b*d)*x^3)/(a^4*b^3*
x^12 + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(b*x
^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(x^3)/a^5
```

Fricas [B] time = 1.31011, size = 652, normalized size = 4.

$$\frac{2\left(6\,ab^4c-3\,a^2b^3d+a^3b^2e\right)x^9+\left(18\,a^2b^3c-9\,a^3b^2d+3\,a^4be-a^5f\right)x^6-a^4bc+2\left(2\,a^3b^2c-a^4bd\right)x^3-2\left(\left(6\,b^5c-3\,ab^4d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c - 9*a^3*b^
2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c - a^4*b*d)*x^3 - 2*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(b*x^3 + a) + 6*
((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*
b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(x))/(a^5*b^3*x^
12 + 2*a^6*b^2*x^9 + a^7*b*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07307, size = 255, normalized size = 1.56

$$\frac{\left(6\,b^2c-3\,abd+a^2e\right)\log\left(|x|\right)}{a^5}-\frac{\left(6\,b^3c-3\,ab^2d+a^2be\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^5b}+\frac{12\,b^4cx^9-6\,ab^3dx^9+2\,a^2b^2x^9e+18\,ab^3cx^6\cdots}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (6*b^2*c - 3*a*b*d + a^2*e)*log(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b^2*d + a^
2*b*e)*log(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 - 6*a*b^3*d*x^9 + 2*
a^2*b^2*x^9*e + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^6 - a^4*f*x^6 + 3*a^3*b*x^6*
e + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 - a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b)
```

3.284 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$

Optimal. Leaf size=218

$$-\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(3a^2be+a^3(-f)-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(}{}$$

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (4*
b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c - 6
*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*
b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```

Rubi [A] time = 0.263711, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$-\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5\left(a+bx^3\right)}-\frac{a^2be+a^3(-f)-ab^2d+b^3c}{6a^4\left(a+bx^3\right)^2}+\frac{\log\left(a+bx^3\right)\left(3a^2be+a^3(-f)-6ab^2d+10b^3c\right)}{3a^6}-\frac{\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```


```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(6*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(6*a^4*(a + b*x^3)^2) - (4*
b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*(a + b*x^3)) - ((10*b^3*c - 6
*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3*a^2*
b*e - a^3*f)*Log[a + b*x^3])/(3*a^6)
```


Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx)^3}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{c}{a^3x^4} + \frac{-3bc+ad}{a^4x^3} + \frac{6b^2c-3abd+a^2e}{a^5x^2} + \frac{-10b^3c+6ab^2d-3a^2be+a^3f}{a^6x} \right.\right.$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{6a^4x^6} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6a^4\left(a+bx^3\right)^2} - \frac{4b^3c-3ab^2d+2a}{3a^5\left(a+bx\right.}$$

Mathematica [A] time = 0.137317, size = 200, normalized size = 0.92

$$\frac{\frac{3a^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2} + \frac{6a\left(-2a^2be+a^3f+3ab^2d-4b^3c\right)}{a+bx^3} + 6\log\left(a+bx^3\right)\left(3a^2be+a^3(-f)-6ab^2d+10b^3c\right) + 18\log(x)\left(-3a^2l\right.}{18a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*a*b*d + a^
2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 +
(6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-10*b^3*
c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3*a^2
*b*e - a^3*f)*Log[a + b*x^3])/(18*a^6)
```

Maple [A] time = 0.02, size = 293, normalized size = 1.3

$$\frac{f}{6a\left(bx^3+a\right)^2} - \frac{be}{6a^2\left(bx^3+a\right)^2} + \frac{b^2d}{6a^3\left(bx^3+a\right)^2} - \frac{b^3c}{6a^4\left(bx^3+a\right)^2} - \frac{\ln\left(bx^3+a\right)f}{3a^3} + \frac{b\ln\left(bx^3+a\right)e}{a^4} - 2\frac{b^2\ln\left(bx^3\right.}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a/(b*x^3+a)^2*f-1/6*b/a^2/(b*x^3+a)^2*e+1/6*b^2/a^3/(b*x^3+a)^2*d-1/6*b
^3/a^4/(b*x^3+a)^2*c-1/3/a^3*ln(b*x^3+a)*f+b/a^4*ln(b*x^3+a)*e-2*b^2/a^5*ln
(b*x^3+a)*d+10/3*b^3/a^6*ln(b*x^3+a)*c+1/3/a^2/(b*x^3+a)*f-2/3*b/a^3/(b*x^3
+a)*e+b^2/a^4/(b*x^3+a)*d-4/3*b^3/a^5/(b*x^3+a)*c-1/9*c/a^3/x^9-1/6/a^3/x^6
*d+1/2/a^4/x^6*b*c-1/3/a^3/x^3*e+1/a^4/x^3*b*d-2/a^5/x^3*b^2*c+1/a^3*ln(x)*
f-3/a^4*ln(x)*b*e+6/a^5*ln(x)*b^2*d-10/a^6*ln(x)*b^3*c
```

Maxima [A] time = 0.980614, size = 313, normalized size = 1.44

$$-\frac{6\left(10b^4c-6ab^3d+3a^2b^2e-a^3bf\right)x^{12} + 9\left(10ab^3c-6a^2b^2d+3a^3be-a^4f\right)x^9 + 2\left(10a^2b^2c-6a^3bd+3a^4e\right)x^6 + }{18\left(a^5b^2x^{15}+2a^6bx^{12}+a^7x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(6*(10*b^4*c - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^12 + 9*(10*a*b^3*
c - 6*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^9 + 2*(10*a^2*b^2*c - 6*a^3*b*d + 3*
a^4*e)*x^6 + 2*a^4*c - (5*a^3*b*c - 3*a^4*d)*x^3)/(a^5*b^2*x^15 + 2*a^6*b*x
^12 + a^7*x^9) + 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(b*x^3 +
 a)/a^6 - 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(x^3)/a^6
```

Fricas [A] time = 1.44256, size = 837, normalized size = 3.84

$$-\frac{6\left(10\,ab^4c-6\,a^2b^3d+3\,a^3b^2e-a^4bf\right)x^{12}+9\left(10\,a^2b^3c-6\,a^3b^2d+3\,a^4be-a^5f\right)x^9+2\left(10\,a^3b^2c-6\,a^4bd+3\,a^5e\right)x^6+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(6*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + 9*(10*a^
2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9 + 2*(10*a^3*b^2*c - 6*a^4*b*
d + 3*a^5*e)*x^6 + 2*a^5*c - (5*a^4*b*c - 3*a^5*d)*x^3 - 6*((10*b^5*c - 6*a
*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^
3*b^2*e - a^4*b*f)*x^12 + (10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*
x^9)*log(b*x^3 + a) + 18*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*
x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + (10*a^2*
b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(x))/(a^6*b^2*x^15 + 2*a^7
*b*x^12 + a^8*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1005, size = 437, normalized size = 2.

$$-\frac{\left(10\,b^3c-6\,ab^2d-a^3f+3\,a^2be\right)\log\left(|x|\right)}{a^6}+\frac{\left(10\,b^4c-6\,ab^3d-a^3bf+3\,a^2b^2e\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^6b}-\frac{30\,b^5cx^6-18\,ab^4dx^6-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -(10*b^3*c - 6*a*b^2*d - a^3*f + 3*a^2*b*e)*log(abs(x))/a^6 + 1/3*(10*b^4*c
 - 6*a*b^3*d - a^3*b*f + 3*a^2*b^2*e)*log(abs(b*x^3 + a))/(a^6*b) - 1/6*(30
*b^5*c*x^6 - 18*a*b^4*d*x^6 - 3*a^3*b^2*f*x^6 + 9*a^2*b^3*x^6*e + 68*a*b^4*
c*x^3 - 42*a^2*b^3*d*x^3 - 8*a^4*b*f*x^3 + 22*a^3*b^2*x^3*e + 39*a^2*b^3*c
- 25*a^3*b^2*d - 6*a^5*f + 14*a^4*b*e)/((b*x^3 + a)^2*a^6) + 1/18*(110*b^3*
c*x^9 - 66*a*b^2*d*x^9 - 11*a^3*f*x^9 + 33*a^2*b*x^9*e - 36*a*b^2*c*x^6 + 1
8*a^2*b*d*x^6 - 6*a^3*x^6*e + 9*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^6*x
^9)
```

3.285 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3}\,dx$

Optimal. Leaf size=258

$$\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b\log\left(a+bx\right.}{}$$

```
[Out] -c/(12*a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e)/
(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c - 4
*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*
b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^
2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)
```

Rubi [A] time = 0.304493, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1821, 1620}

$$\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{3a^6\left(a+bx^3\right)}+\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b\log\left(a+bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```


```
[Out] -c/(12*a^3*x^12) + (3*b*c - a*d)/(9*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*e)/
(6*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(6*a^5*(a + b*x^3)^2) + (b*(5*b^3*c - 4
*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(3*a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*a*
b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x])/a^7 - (b*(15*b^3*c - 10*a*b^2*d + 6*a^
2*b*e - 3*a^3*f)*Log[a + b*x^3])/(3*a^7)
```


Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5(a+bx)^3}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\,\text{Subst}\left(\int \left(\frac{c}{a^3x^5}+\frac{-3bc+ad}{a^4x^4}+\frac{6b^2c-3abd+a^2e}{a^5x^3}+\frac{-10b^3c+6ab^2d-3a^2be+a^3f}{a^6x^2}-\frac{b}{}\right.\right.$$

$$=-\frac{c}{12a^3x^{12}}+\frac{3bc-ad}{9a^4x^9}-\frac{6b^2c-3abd+a^2e}{6a^5x^6}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b\left(b^3c-ab^2d-\right.}{6a^5\left(a+\right.}$$

Mathematica [A] time = 0.179807, size = 238, normalized size = 0.92

$$-\frac{\frac{a\left(a^3b^2x^6\left(15c+40dx^3-108ex^6+36fx^9\right)-12a^2b^3x^9\left(5c-15dx^3+6ex^6\right)-2a^4bx^3\left(3c+5dx^3+12ex^6-27fx^9\right)+a^5\left(3c+4dx^3+6ex^6+12fx^9\right)+30ab^4x^{12}\left(4dx^3-9c\right)-180b^5cx\right.}{x^{12}\left(a+bx^3\right)^2}}{36a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

```
[Out] (-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*b^3*x^9*(5
*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 + 12*e*x^6 - 27*f*x^9
) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^3*b^2*x^6*(15*c + 40*d*x^3
 - 108*e*x^6 + 36*f*x^9)))/(x^12*(a + b*x^3)^2)) + 36*b*(15*b^3*c - 10*a*b^
2*d + 6*a^2*b*e - 3*a^3*f)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*
e + 3*a^3*f)*Log[a + b*x^3])/(36*a^7)
```

Maple [A] time = 0.019, size = 349, normalized size = 1.4

$$-\frac{d}{9\,a^3x^9}-\frac{e}{6\,a^3x^6}-\frac{f}{3\,a^3x^3}-\frac{c}{12\,a^3x^{12}}-2\,\frac{b^2\ln\left(bx^3+a\right)e}{a^5}+\frac{10\,b^3\ln\left(bx^3+a\right)d}{3\,a^6}+\frac{bc}{3\,a^4x^9}+\frac{bd}{2\,a^4x^6}-\frac{b^2c}{a^5x^6}+\frac{be}{a^4x^3}-2\,\frac{b}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x)
```

```
[Out] -1/9/a^3/x^9*d-1/6/a^3/x^6*e-1/3/a^3/x^3*f-1/12*c/a^3/x^12-2*b^2/a^5*ln(b*x
^3+a)*e+10/3*b^3/a^6*ln(b*x^3+a)*d+1/3/a^4/x^9*b*c+1/2/a^4/x^6*b*d-1/a^5/x^
6*b^2*c+1/a^4/x^3*b*e-2/a^5/x^3*b^2*d+10/3/a^6/x^3*b^3*c-3*b/a^4*ln(x)*f+6*
b^2/a^5*ln(x)*e-10*b^3/a^6*ln(x)*d+15*b^4/a^7*ln(x)*c-1/6*b/a^2/(b*x^3+a)^2
*f+1/6*b^2/a^3/(b*x^3+a)^2*e-1/6*b^3/a^4/(b*x^3+a)^2*d+1/6*b^4/a^5/(b*x^3+a
)^2*c+b/a^4*ln(b*x^3+a)*f+5/3*b^4/a^6/(b*x^3+a)*c-2/3*b/a^3/(b*x^3+a)*f-5*b
^4/a^7*ln(b*x^3+a)*c+b^2/a^4/(b*x^3+a)*e-4/3*b^3/a^5/(b*x^3+a)*d
```

Maxima [A] time = 0.988422, size = 378, normalized size = 1.47

$$\frac{12\left(15\,b^5c-10\,ab^4d+6\,a^2b^3e-3\,a^3b^2f\right)x^{15}+18\left(15\,ab^4c-10\,a^2b^3d+6\,a^3b^2e-3\,a^4bf\right)x^{12}+4\left(15\,a^2b^3c-10\,a^3b^2d+6\right.}{36\left(a^6b^2x^{18}+2\,a^7bx^{15}+a^8x^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(12*(15*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*x^15 + 18*(15*
a*b^4*c - 10*a^2*b^3*d + 6*a^3*b^2*e - 3*a^4*b*f)*x^12 + 4*(15*a^2*b^3*c -
10*a^3*b^2*d + 6*a^4*b*e - 3*a^5*f)*x^9 - (15*a^3*b^2*c - 10*a^4*b*d + 6*a^
5*e)*x^6 - 3*a^5*c + 2*(3*a^4*b*c - 2*a^5*d)*x^3)/(a^6*b^2*x^18 + 2*a^7*b*x
^15 + a^8*x^12) - 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log
(b*x^3 + a)/a^7 + 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log
(x^3)/a^7
```

Fricas [A] time = 1.63972, size = 984, normalized size = 3.81

$$\frac{12\left(15\,ab^5c-10\,a^2b^4d+6\,a^3b^3e-3\,a^4b^2f\right)x^{15}+18\left(15\,a^2b^4c-10\,a^3b^3d+6\,a^4b^2e-3\,a^5bf\right)x^{12}+4\left(15\,a^3b^3c-10\,a^4b^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + 18*
(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^12 + 4*(15*a^3*b^
3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9 - 3*a^6*c - (15*a^4*b^2*c - 1
0*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c - 10
*a*b^5*d + 6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d +
 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2
*e - 3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d + 6*a^2*b
^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a
^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^
12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15 + a^9*x^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07742, size = 513, normalized size = 1.99

$$\frac{\left(15\,b^4c-10\,ab^3d-3\,a^3bf+6\,a^2b^2e\right)\log\left(|x|\right)}{a^7}-\frac{\left(15\,b^5c-10\,ab^4d-3\,a^3b^2f+6\,a^2b^3e\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^7b}+\frac{45\,b^6cx^6-30\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (15*b^4*c - 10*a*b^3*d - 3*a^3*b*f + 6*a^2*b^2*e)*log(abs(x))/a^7 - 1/3*(15
*b^5*c - 10*a*b^4*d - 3*a^3*b^2*f + 6*a^2*b^3*e)*log(abs(b*x^3 + a))/(a^7*b
```

```
) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 - 9*a^3*b^3*f*x^6 + 18*a^2*b^4*x^6*e
 + 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3 - 22*a^4*b^2*f*x^3 + 42*a^3*b^3*x^3*e
 + 56*a^2*b^4*c - 39*a^3*b^3*d - 14*a^5*b*f + 25*a^4*b^2*e)/((b*x^3 + a)^2*
a^7) - 1/36*(375*b^4*c*x^12 - 250*a*b^3*d*x^12 - 75*a^3*b*f*x^12 + 150*a^2*
b^2*x^12*e - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 + 12*a^4*f*x^9 - 36*a^3*b*x
^9*e + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*x^6*e - 12*a^3*b*c*x^3 + 4
*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

3.286 $\int \frac{x^{12}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=416

$$\frac{x^4\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{4b^6}-\frac{a^2x\left(31a^2be-37a^3f-25ab^2d+19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\ldots$$

```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^3*c - 3*a*
b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c -
 25*a*b^2*d + 31*a^2*b*e - 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35
*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*a*b^2
*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) - (a^
(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Rubi [A] time = 0.742162, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{x^4\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{4b^6}-\frac{a^2x\left(31a^2be-37a^3f-25ab^2d+19b^3c\right)}{18b^7\left(a+bx^3\right)}+\frac{a^3x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] -((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x)/b^7) + ((b^3*c - 3*a*
b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) - (a^2*(19*b^3*c -
 25*a*b^2*d + 31*a^2*b*e - 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) - (a^(4/3)*(35
*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(22/3)) + (a^(4/3)*(35*b^3*c - 65*a*b^2
*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) - (a^
(4/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{12}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = \frac{a^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^7\left(a+bx^3\right)^2} - \frac{\int a^4\left(b^3c-ab^2d+a^2be-a^3f\right)-6a^3b\left(b^3c-ab^2d+a^2be-a^3f\right)x^3+6a^2b^2(}{}$$

$$= \frac{a^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^7\left(a+bx^3\right)^2} - \frac{a^2\left(19b^3c-25ab^2d+31a^2be-37a^3f\right)x}{18b^7\left(a+bx^3\right)} + \frac{\int 2a^4b^6}{}$$

$$= \frac{a^3\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^7\left(a+bx^3\right)^2} - \frac{a^2\left(19b^3c-25ab^2d+31a^2be-37a^3f\right)x}{18b^7\left(a+bx^3\right)} + \frac{\int\left(-18\right.}{}$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} + \frac{\left(b^2d\right.}{}$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} + \frac{\left(b^2d\right.}{}$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} + \frac{\left(b^2d\right.}{}$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} + \frac{\left(b^2d\right.}{}$$

$$= -\frac{a\left(3b^3c-6ab^2d+10a^2be-15a^3f\right)x}{b^7} + \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^4}{4b^6} + \frac{\left(b^2d\right.}{}$$

Mathematica [A] time = 0.482441, size = 411, normalized size = 0.99

$$\frac{x^4\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{4b^6} + \frac{a^2x\left(-31a^2be+37a^3f+25ab^2d-19b^3c\right)}{18b^7\left(a+bx^3\right)} + \frac{a^3x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2} + \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^7
)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*c +
25*a*b^2*d - 31*a^2*b*e + 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)*(-35
*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b^2*d -
104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(22/3)) + (a^(4/3)
*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*b^(22/3))
```

Maple [A] time = 0.013, size = 706, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -19/18*a^2/b^3/(b*x^3+a)^2*x^4*c+17/9*a^6/b^7/(b*x^3+a)^2*f*x-14/9*a^5/b^6/
(b*x^3+a)^2*e*x+11/9*a^4/b^5/(b*x^3+a)^2*d*x-8/9*a^3/b^4/(b*x^3+a)^2*c*x+1/
13*f*x^13/b^3+37/18*a^5/b^6/(b*x^3+a)^2*x^4*f-31/18*a^4/b^5/(b*x^3+a)^2*x^4
*e+25/18*a^3/b^4/(b*x^3+a)^2*x^4*d+104/27*a^4/b^7*e/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))-35/54*a^2/b^5*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/
3))-65/27*a^3/b^6*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+65/54*a^3/b^6*d/(1/b*
a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-152/27*a^5/b^8*f/(1/b*a)^(2/
3)*ln(x+(1/b*a)^(1/3))+76/27*a^5/b^8*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))+35/27*a^2/b^5*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-52/27*a^4
/b^7*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3/10/b^4*x^10*a*
f+6/7/b^5*x^7*a^2*f-3/7/b^4*x^7*a*e-5/2/b^6*x^4*a^3*f+3/2/b^5*x^4*a^2*e-3/4
/b^4*x^4*a*d+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x+1/10/b
^3*x^10*e+1/7/b^3*x^7*d+1/4/b^3*x^4*c-152/27*a^5/b^8*f/(1/b*a)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+104/27*a^4/b^7*e/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*a^3/b^6*d/(1/b*a)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+35/27*a^2/b^5*c/(1/b
*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32608, size = 1589, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/49140*(3780*b^6*f*x^19 + 378*(13*b^6*e - 19*a*b^5*f)*x^16 + 108*(65*b^6*d
 - 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 351*(35*b^6*c - 65*a*b^5*d + 104*a^2
*b^4*e - 152*a^3*b^3*f)*x^10 - 3510*(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^
3*e - 152*a^4*b^2*f)*x^7 - 9555*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*
e - 152*a^5*b*f)*x^4 - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*
b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*
f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*
(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a
^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^
3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3)) - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*
a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2
*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1
```

```
/3)*log(x - (-a/b)^(1/3)) - 5460*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e
 - 152*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09094, size = 675, normalized size = 1.62

$$\frac{\sqrt{3}\left(35\left(-ab^{2}\right)^{\frac{1}{3}}ab^{3}c-65\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{2}d-152\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}f+104\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^{8}}-\frac{\left(35a^{2}b^{3}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152
*(-a*b^2)^(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/27*(35*a^2*b^3*c - 65*a^3*b^2*d - 15
2*a^5*f + 104*a^4*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/
54*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d - 152*(-a*b^2)^
(1/3)*a^4*f + 104*(-a*b^2)^(1/3)*a^3*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/b^8 - 1/18*(19*a^2*b^4*c*x^4 - 25*a^3*b^3*d*x^4 - 37*a^5*b*f*x^4 +
31*a^4*b^2*x^4*e + 16*a^3*b^3*c*x - 22*a^4*b^2*d*x - 34*a^6*f*x + 28*a^5*b*
x*e)/((b*x^3 + a)^2*b^7) + 1/1820*(140*b^36*f*x^13 - 546*a*b^35*f*x^10 + 18
2*b^36*x^10*e + 260*b^36*d*x^7 + 1560*a^2*b^34*f*x^7 - 780*a*b^35*x^7*e + 4
55*b^36*c*x^4 - 1365*a*b^35*d*x^4 - 4550*a^3*b^33*f*x^4 + 2730*a^2*b^34*x^4
*e - 5460*a*b^35*c*x + 10920*a^2*b^34*d*x + 27300*a^4*b^32*f*x - 18200*a^3*
b^33*x*e)/b^39
```

3.287 $\int \frac{x^{10}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=384

$$\frac{x^2\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{2b^6}+\frac{ax^2\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)}-\frac{a^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}-\frac{a^{2/3}\,\mathrm{lo}}{}$$

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a
*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) +
(a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c
 - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20
/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Rubi [A] time = 1.05046, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{2b^6}+\frac{ax^2\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)}-\frac{a^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}-\frac{a^{2/3}\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) + (a
*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3)) +
(a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(20/3)) + (a^(2/3)*(20*b^3*c
 - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20
/3)) - (a^(2/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{10}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} - \frac{\int \frac{-2a^3b\left(b^3c-ab^2d+a^2be-a^3f\right)x+6a^2b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^4-6}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(-2a^3b\left(b^3c-ab^2d+a^2be-a^3f\right)+6a^2b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-\right.}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-16a^3f\right)x^2}{9b^6\left(a+bx^3\right)} + \frac{\int \frac{-2a^3b^7(}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-16a^3f\right)x^2}{9b^6\left(a+bx^3\right)} + \frac{\int \frac{x(-2a^3b}{}}{}$$

$$= \frac{fx^{11}}{11b^3} - \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-16a^3f\right)x^2}{9b^6\left(a+bx^3\right)} + \int$$

$$= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-\right.}{9b^6\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} - \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(7b^3c-10ab^2d+13a^2be-\right.}{9b^6\left(a+bx^3\right)}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} -$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} -$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} -$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} -$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x^2}{2b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^5}{5b^5} + \frac{(be-3af)x^8}{8b^4} + \frac{fx^{11}}{11b^3} -$$

Mathematica [A] time = 0.402336, size = 380, normalized size = 0.99

$$\frac{x^2\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{2b^6} + \frac{ax^2\left(13a^2be-16a^3f-10ab^2d+7b^3c\right)}{9b^6\left(a+bx^3\right)} + \frac{a^2x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{6b^6\left(a+bx^3\right)^2} + \frac{a^{2/3}\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b*
e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^3)
 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2) +
 (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x^3))
 - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*b^3*c
+ 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(20/
3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

Maple [B] time = 0.013, size = 668, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -17/18*a^3/b^4/(b*x^3+a)^2*x^2*d+11/18*a^2/b^3/(b*x^3+a)^2*x^2*c+7/9*a/b^2/
(b*x^3+a)^2*x^5*c-29/18*a^5/b^6/(b*x^3+a)^2*x^2*f+23/18*a^4/b^5/(b*x^3+a)^2
*x^2*e-10/27*a/b^4*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-11
9/27*a^4/b^7*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+119/54*a^4/b^7*f/(1/b*a)^(
1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+77/27*a^3/b^6*e/(1/b*a)^(1/3)*ln
(x+(1/b*a)^(1/3))-77/54*a^3/b^6*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))-44/27*a^2/b^5*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-16/9*a^4/b^5/(
b*x^3+a)^2*x^5*f+13/9*a^3/b^4/(b*x^3+a)^2*x^5*e-10/9*a^2/b^3/(b*x^3+a)^2*x^
5*d-77/27*a^3/b^6*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))+1/11*f*x^11/b^3+3/b^5*x^2*a^2*e-3/2/b^4*x^2*a*d-3/8/b^4*x^8*a*f+6/
5/b^5*x^5*a^2*f-3/5/b^4*x^5*a*e-5/b^6*x^2*a^3*f+22/27*a^2/b^5*d/(1/b*a)^(1/
3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+20/27*a/b^4*c/(1/b*a)^(1/3)*ln(x+(
1/b*a)^(1/3))+1/8/b^3*x^8*e+1/5/b^3*x^5*d+1/2/b^3*x^2*c+44/27*a^2/b^5*d*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-20/27*a/b^4*c*
3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+119/27*a^4/
b^7*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35236, size = 1496, normalized size = 3.9

$$1080\, b^5 f x^{17} + 135\left(11\, b^5 e - 17\, a b^4 f\right)x^{14} + 54\left(44\, b^5 d - 77\, a b^4 e + 119\, a^2 b^3 f\right)x^{11} + 297\left(20\, b^5 c - 44\, a b^4 d + 77\, a^2 b^3 e - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11880*(1080*b^5*f*x^17 + 135*(11*b^5*e - 17*a*b^4*f)*x^14 + 54*(44*b^5*d
- 77*a*b^4*e + 119*a^2*b^3*f)*x^11 + 297*(20*b^5*c - 44*a*b^4*d + 77*a^2*b^
3*e - 119*a^3*b^2*f)*x^8 + 1056*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e -
 119*a^4*b*f)*x^5 + 660*(20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5
*f)*x^2 - 440*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*
f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4
*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*((20*b^5*c - 44
*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d
+ 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 11
9*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2
/b^2)^(1/3)) - 440*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*
x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c
- 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x
+ b*(-a^2/b^2)^(2/3)))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09179, size = 663, normalized size = 1.73

$$\frac{\left(20\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 44\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 119\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 77\,a^3b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^6} + \frac{\sqrt{3}\left(20\left(-ab^2\right)^{\frac{2}{3}}b^3c - 44\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(20*a*b^3*c*(-a/b)^(1/3) - 44*a^2*b^2*d*(-a/b)^(1/3) - 119*a^4*f*(-a/b
)^(1/3) + 77*a^3*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^6) + 1/27*sqrt(3)*(20*(-a*b^2)^(2/3)*b^3*c - 44*(-a*b^2)^(2/3)*a*b^2*d
 - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/54*(20*(-a*b^2)^(2/3)*b^3*c - 4
4*(-a*b^2)^(2/3)*a*b^2*d - 119*(-a*b^2)^(2/3)*a^3*f + 77*(-a*b^2)^(2/3)*a^2
*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^8 + 1/18*(14*a*b^4*c*x^5 -
 20*a^2*b^3*d*x^5 - 32*a^4*b*f*x^5 + 26*a^3*b^2*x^5*e + 11*a^2*b^3*c*x^2 -
17*a^3*b^2*d*x^2 - 29*a^5*f*x^2 + 23*a^4*b*x^2*e)/((b*x^3 + a)^2*b^6) + 1/4
40*(40*b^30*f*x^11 - 165*a*b^29*f*x^8 + 55*b^30*x^8*e + 88*b^30*d*x^5 + 528
*a^2*b^28*f*x^5 - 264*a*b^29*x^5*e + 220*b^30*c*x^2 - 660*a*b^29*d*x^2 - 22
00*a^3*b^27*f*x^2 + 1320*a^2*b^28*x^2*e)/b^33
```

3.288 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=375

$$\frac{ax\left(25a^2be-31a^3f-19ab^2d+13b^3c\right)}{18b^6\left(a+bx^3\right)}-\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(65a^2be\right.}{54b^{19/3}}$$

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3
*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)
*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*
b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (
a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```

Rubi [A] time = 0.606352, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$\frac{ax\left(25a^2be-31a^3f-19ab^2d+13b^3c\right)}{18b^6\left(a+bx^3\right)}-\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(65a^2be\right.}{54b^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^4)/(4*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^10)/(10*b^3) - (a^
2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^6*(a + b*x^3)^2) + (a*(13*b^3
*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(18*b^6*(a + b*x^3)) + (a^(1/3)
*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(19/3)) - (a^(1/3)*(14*b^3*c - 35*a*
b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*b^(19/3)) + (
a^(1/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(19/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
```

```
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^6\left(a+bx^3\right)^2} - \frac{\int \frac{-a^3\left(b^3c-ab^2d+a^2be-a^3f\right)+6a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-6ab^2}{}}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(13b^3c-19ab^2d+25a^2be-31a^3f\right)x}{18b^6\left(a+bx^3\right)} + \int \frac{-2a^3b}{}$$

$$= -\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^6\left(a+bx^3\right)^2} + \frac{a\left(13b^3c-19ab^2d+25a^2be-31a^3f\right)x}{18b^6\left(a+bx^3\right)} + \frac{\int\left(18a\right.}{}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3}$$

$$= \frac{\left(b^3c-3ab^2d+6a^2be-10a^3f\right)x}{b^6} + \frac{\left(b^2d-3abe+6a^2f\right)x^4}{4b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^{10}}{10b^3}$$

Mathematica [A] time = 0.37398, size = 362, normalized size = 0.97

$$\frac{\frac{210a\sqrt[3]{b}x\left(25a^2be-31a^3f-19ab^2d+13b^3c\right)}{a+bx^3} + \frac{630a^2\sqrt[3]{b}x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2} - 70\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-65a^2be+104a^3f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*b^(4/3)*(b
^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*f)*x^7 + 378*b^(10/3
)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a +
b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x
)/(a + b*x^3) - 140*Sqrt[3]*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e +
104*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-14*b
^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a
^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*b^(19/3))
```

Maple [A] time = 0.012, size = 651, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -3/7/b^4*x^7*a*f+3/2/b^5*x^4*a^2*f-3/4/b^4*x^4*a*e-10/b^6*a^3*f*x+6/b^5*a^2
*e*x-3/b^4*a*d*x-19/18*a^2/b^3/(b*x^3+a)^2*x^4*d+1/10*f*x^10/b^3-14/9*a^5/b
^6/(b*x^3+a)^2*f*x+11/9*a^4/b^5/(b*x^3+a)^2*e*x-8/9*a^3/b^4/(b*x^3+a)^2*d*x
+5/9*a^2/b^3/(b*x^3+a)^2*c*x+104/27*a^4/b^7*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1
/3))-65/27*a^3/b^6*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+65/54*a^3/b^6*e/(1/b
*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+35/27*a^2/b^5*d/(1/b*a)^(2/
3)*ln(x+(1/b*a)^(1/3))-35/54*a^2/b^5*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))-14/27*a/b^4*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-31/18*a^4/b
^5/(b*x^3+a)^2*x^4*f+25/18*a^3/b^4/(b*x^3+a)^2*x^4*e-52/27*a^4/b^7*f/(1/b*a
)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/7/b^3*x^7*e+1/4/b^3*x^4*d+1
/b^3*c*x+7/27*a/b^4*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1
3/18*a/b^2/(b*x^3+a)^2*x^4*c+104/27*a^4/b^7*f/(1/b*a)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*a^3/b^6*e/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+35/27*a^2/b^5*d/(1/b*a)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-14/27*a/b^4*c/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44258, size = 1424, normalized size = 3.8

$$\frac{378\,b^5 f x^{16} + 108\left(5\,b^5 e - 8\,ab^4 f\right)x^{13} + 27\left(35\,b^5 d - 65\,ab^4 e + 104\,a^2 b^3 f\right)x^{10} + 270\left(14\,b^5 c - 35\,ab^4 d + 65\,a^2 b^3 e - 104\,a^3 b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(378*b^5*f*x^16 + 108*(5*b^5*e - 8*a*b^4*f)*x^13 + 27*(35*b^5*d - 65
*a*b^4*e + 104*a^2*b^3*f)*x^10 + 270*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e
- 104*a^3*b^2*f)*x^7 + 735*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*
a^4*b*f)*x^4 - 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3
*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*
a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*arcta
n(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*((14*b^5*c - 35*a*b^4
*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a
^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*
b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*((14*b^5
*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3
*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2
```

```
*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(14*a^2*b^3*c
 - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b
^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08754, size = 598, normalized size = 1.59

$$-\frac{\sqrt{3}\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-35\left(-ab^2\right)^{\frac{1}{3}}ab^2d-104\left(-ab^2\right)^{\frac{1}{3}}a^3f+65\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^7}+\frac{\left(14ab^3c-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d - 104*(-
a*b^2)^(1/3)*a^3*f + 65*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/27*(14*a*b^3*c - 35*a^2*b^2*d - 104*a^4*
f + 65*a^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) - 1/54*(14*
(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d - 104*(-a*b^2)^(1/3)*a^3*f
 + 65*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7
+ 1/18*(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 - 31*a^4*b*f*x^4 + 25*a^3*b^2*x^4
*e + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x - 28*a^5*f*x + 22*a^4*b*x*e)/((b*x^3 +
 a)^2*b^6) + 1/140*(14*b^27*f*x^10 - 60*a*b^26*f*x^7 + 20*b^27*x^7*e + 35*b
^27*d*x^4 + 210*a^2*b^25*f*x^4 - 105*a*b^26*x^4*e + 140*b^27*c*x - 420*a*b^
26*d*x - 1400*a^3*b^24*f*x + 840*a^2*b^25*x*e)/b^30
```

3.289 $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=345

$$-\frac{x^2\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{9b^5\left(a+bx^3\right)}+\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^2be-77a^3\right.}{54\sqrt[3]{a}b^{17/3}}$$

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (
f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*
x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b
*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c -
 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*
b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```

Rubi [A] time = 0.763222, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1836, 1488, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{9b^5\left(a+bx^3\right)}+\frac{ax^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^2be-77a^3\right.}{54\sqrt[3]{a}b^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/(2*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (
f*x^8)/(8*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*
x^3)^2) - ((4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(9*b^5*(a + b
*x^3)) - ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(17/3)) - ((5*b^3*c -
 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*
b^(17/3)) + ((5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(17/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
```

```
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\int \frac{2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)x-6ab^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^4-6ab^3\left(b^2 \cdots\right.}{\left(a+bx^3\right)^2}}{6ab^6}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(2a^2b\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-6ab^3\left(b^2 \cdots\right.\right.}{\left(a+bx^3\right)^2}}{6ab^6}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)} + \int \frac{2a^2b^6\left(5b^3c-11a \cdots\right.}{\cdots}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)} + \int \frac{x\left(2a^2b^6\left(5b^3c-1 \cdots\right.\right.}{\cdots}$$

$$= \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)} + \int \frac{x\left(16a^2 \cdots\right.}{\cdots}$$

$$= \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3c-7ab^2d+10a^2be-13a^3f\right)x^2}{9b^5\left(a+bx^3\right)} + \frac{\int \left(144a \cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x^2}{2b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^8}{8b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^5\left(a+bx^3\right)^2} - \frac{\left(4b^3 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.223697, size = 329, normalized size = 0.95

$$\frac{-\frac{120b^{2/3}x^2\left(10a^2be-13a^3f-7ab^2d+4b^3c\right)}{a+bx^3} + \frac{180ab^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2} + \frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(44a^2be-77a^3f-20ab^2d+5b^3c\right)}{\sqrt[3]{a}} + \frac{40\log\left(\sqrt[3]{\cdots}\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e - 3*a*f)*x^
5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^
```

```
3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 7
7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-5*b^3
*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3)
+ (20*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))
```

Maple [B] time = 0.013, size = 611, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -3/2/b^4*x^2*a*e-4/9/b/(b*x^3+a)^2*x^5*c-5/27/b^3*c/(1/b*a)^(1/3)*ln(x+(1/b
*a)^(1/3))+5/54/b^3*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3
/5/b^4*x^5*a*f+3/b^5*x^2*a^2*f+1/8*f*x^8/b^3-5/18/b^2/(b*x^3+a)^2*x^2*a*c+1
1/18/b^3/(b*x^3+a)^2*x^2*a^2*d+1/5/b^3*x^5*e+1/2/b^3*x^2*d-17/18/b^4/(b*x^3
+a)^2*x^2*a^3*e+23/18/b^5/(b*x^3+a)^2*x^2*a^4*f+13/9/b^4/(b*x^3+a)^2*x^5*f*
a^3-10/9/b^3/(b*x^3+a)^2*x^5*a^2*e+7/9/b^2/(b*x^3+a)^2*x^5*a*d+5/27/b^3*c*3
^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-10/27/b^4*a*
d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+77/27/b^6*a^3*f/(1/b*
a)^(1/3)*ln(x+(1/b*a)^(1/3))-77/54/b^6*a^3*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(
1/3)*x+(1/b*a)^(2/3))-44/27/b^5*a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+22/
27/b^5*a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+20/27/b^4*
a*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-77/27/b^6*a^3*f*3^(1/2)/(1/b*a)^(1/3)
*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+44/27/b^5*a^2*e*3^(1/2)/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-20/27/b^4*a*d*3^(1/2)/(1/b*
a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.48138, size = 2915, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^
6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*
a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3
*e - 77*a^5*b^2*f)*x^2 - 60*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*
```

```
b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*
f)*x^6 + 2*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*
sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2
)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/
(b*x^3 + a)) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6
 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a
^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b
^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e -
 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2
*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*
log(b*x + (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7), 1/1080*(13
5*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b^6*d - 44*a^
2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e -
 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5
*b^2*f)*x^2 - 120*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*a^5*b^2*e - 77
*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)*x^3)*sqrt((a*b^
2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)
/b) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*
b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d
+ 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)
*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^4
*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b*x +
 (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09243, size = 601, normalized size = 1.74

$$-\frac{\left(5\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-77\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5}-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{2}{3}}b^3c-20\left(-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(5*b^3*c*(-a/b)^(1/3) - 20*a*b^2*d*(-a/b)^(1/3) - 77*a^3*f*(-a/b)^(1/
3) + 44*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^
5) - 1/27*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 20*(-a*b^2)^(2/3)*a*b^2*d - 77*
(-a*b^2)^(2/3)*a^3*f + 44*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^7) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 -
 26*a^3*b*f*x^5 + 20*a^2*b^2*x^5*e + 5*a*b^3*c*x^2 - 11*a^2*b^2*d*x^2 - 23*
```

```
a^4*f*x^2 + 17*a^3*b*x^2*e)/((b*x^3 + a)^2*b^5) + 1/54*(5*(-a*b^2)^(2/3)*b^
3*c - 20*(-a*b^2)^(2/3)*a*b^2*d - 77*(-a*b^2)^(2/3)*a^3*f + 44*(-a*b^2)^(2/
3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^7) + 1/40*(5*b^21
*f*x^8 - 24*a*b^20*f*x^5 + 8*b^21*x^5*e + 20*b^21*d*x^2 + 120*a^2*b^19*f*x^
2 - 60*a*b^20*x^2*e)/b^24
```

3.290 $\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=336

$$-\frac{x\left(19a^2be-25a^3f-13ab^2d+7b^3c\right)}{18b^5\left(a+bx^3\right)}+\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-65a^3f\right.}{54a^{2/3}b^{16/3}}$$

```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)
/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2)
- ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) -
 ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^
2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)
) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```

Rubi [A] time = 0.508736, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1828, 1858, 1887, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(19a^2be-25a^3f-13ab^2d+7b^3c\right)}{18b^5\left(a+bx^3\right)}+\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-65a^3f\right.}{54a^{2/3}b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b^2*d - 3*a*b*e + 6*a^2*f)*x)/b^5 + ((b*e - 3*a*f)*x^4)/(4*b^4) + (f*x^7)
/(7*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2)
- ((7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(18*b^5*(a + b*x^3)) -
 ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3
)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(16/3)) + ((2*b^3*c - 14*a*b^
2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(16/3)
) - ((2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(54*a^(2/3)*b^(16/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\int \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab\left(b^3c-ab^2d+a^2be-a^3f\right)x^3-6ab^2\left(b^2d-abe\cdots\right.}{\left(a+bx^3\right)^2}}{6ab^5}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c-13ab^2d+19a^2be-25a^3f\right)x}{18b^5\left(a+bx^3\right)} + \frac{\int \frac{2a^2b^4\left(2b^3c-5ab^2\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c-13ab^2d+19a^2be-25a^3f\right)x}{18b^5\left(a+bx^3\right)} + \frac{\int \left(18a^2b^4\left(b^2d\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c\cdots\right.}{\cdots}$$

$$= \frac{\left(b^2d-3abe+6a^2f\right)x}{b^5} + \frac{(be-3af)x^4}{4b^4} + \frac{fx^7}{7b^3} + \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^5\left(a+bx^3\right)^2} - \frac{\left(7b^3c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.313184, size = 323, normalized size = 0.96

$$\frac{-\frac{42\sqrt[3]{b}x\left(19a^2be-25a^3f-13ab^2d+7b^3c\right)}{a+bx^3} + \frac{126a\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2} + \frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-35a^2be+65a^3f+14ab^2d-2b^3c\right)}{a^{2/3}} + \frac{28\log\left(\sqrt[3]{a}\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3*a*f)*x^4
+ 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)
/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)
*x)/(a + b*x^3) + (28*Sqrt[3]*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*
f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(2*b^3*c - 14
*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(
-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

Maple [B] time = 0.012, size = 596, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] -14/27/b^4*a*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+7/27/b^4*a*d/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/27/b^3*c/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+13/18/b^2/(b*x^3+a)^2*x^4*a*d-1/27/b
^3*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3/4/b^4*x^4*a*f+6/
b^5*a^2*f*x-3/b^4*a*e*x-7/18/b/(b*x^3+a)^2*x^4*c+2/27/b^3*c/(1/b*a)^(2/3)*l
n(x+(1/b*a)^(1/3))+1/7*f*x^7/b^3+1/4/b^3*x^4*e+1/b^3*d*x+35/27/b^5*a^2*e/(1
/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-35/54/b^5*a^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))-2/9/b^2/(b*x^3+a)^2*a*c*x+25/18/b^4/(b*x^3+a)^2*x^
4*a^3*f-19/18/b^3/(b*x^3+a)^2*x^4*a^2*e-65/27/b^6*a^3*f/(1/b*a)^(2/3)*ln(x+
(1/b*a)^(1/3))+65/54/b^6*a^3*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))+5/9/b^3/(b*x^3+a)^2*a^2*d*x+11/9/b^5/(b*x^3+a)^2*a^4*f*x-8/9/b^4/(b
*x^3+a)^2*a^3*e*x-65/27/b^6*a^3*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(1/b*a)^(1/3)*x-1))+35/27/b^5*a^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))-14/27/b^4*a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.46143, size = 2986, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14*
a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*b^
4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 - 42*sqrt(1/3)*(2*a^3*b^4*c - 14*a^4
*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3*b^4
*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*
a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*
x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*
e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f
+ 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2
/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*b^5*c - 14*
a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 3
5*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*
b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*a^3*b^4*c - 14
*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a
^4*b^6), 1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 +
```

```
 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 1
4*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 + 84*sqrt(1/3)*(2*a^3*b^4*c
- 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35
*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3
*e - 65*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b
)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*((2*b^5*c -
 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d
 + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*
a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3
)*a) + 28*((2*b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2
*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d
 + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3
)) - 84*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^
8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10782, size = 539, normalized size = 1.6

$$-\frac{\left(2\,b^{3}c-14\,ab^{2}d-65\,a^{3}f+35\,a^{2}be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{5}}+\frac{\sqrt{3}\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{3}c-14\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}d-65\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}f\right.}{27\,ab^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b^3*c - 14*a*b^2*d - 65*a^3*f + 35*a^2*b*e)*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^5) + 1/27*sqrt(3)*(2*(-a*b^2)^(1/3)*b^3*c - 14*(-a*b
^2)^(1/3)*a*b^2*d - 65*(-a*b^2)^(1/3)*a^3*f + 35*(-a*b^2)^(1/3)*a^2*b*e)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^6) + 1/54*(2*(-a*b
^2)^(1/3)*b^3*c - 14*(-a*b^2)^(1/3)*a*b^2*d - 65*(-a*b^2)^(1/3)*a^3*f + 35*
(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^6) -
1/18*(7*b^4*c*x^4 - 13*a*b^3*d*x^4 - 25*a^3*b*f*x^4 + 19*a^2*b^2*x^4*e + 4*
a*b^3*c*x - 10*a^2*b^2*d*x - 22*a^4*f*x + 16*a^3*b*x*e)/((b*x^3 + a)^2*b^5)
 + 1/28*(4*b^18*f*x^7 - 21*a*b^17*f*x^4 + 7*b^18*x^4*e + 28*b^18*d*x + 168*
a^2*b^16*f*x - 84*a*b^17*x*e)/b^21
```

3.291 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=316

$$\frac{x^2\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{9ab^4\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-20a^2be+44a^3\right.}{54a^{4/3}b^{14/3}}$$

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10
*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3
)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))
```

Rubi [A] time = 0.504646, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1851, 1594, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{9ab^4\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-20a^2be+44a^3\right.}{54a^{4/3}b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x^2)/(2*b^4) + (f*x^5)/(5*b^3) - ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*x^2)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10
*a^3*f)*x^2)/(9*a*b^4*(a + b*x^3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3
)*b^(14/3)) - ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(4/3)*b^(14/3)) + ((b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(14/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1851

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} - \frac{\int \frac{-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)x-6ab^2\left(b^2d-abe+a^2f\right)x^4-6ab^3(be-a\cdots}{\left(a+bx^3\right)^2}}{6ab^5}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(-2ab\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab^2\left(b^2d-abe+a^2f\right)x^3-6ab^3(be-\cdots}{\left(a+bx^3\right)^2}}{6ab^5}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} + \int \frac{2ab^5\left(b^3c+5ab^2\cdots}{\cdots}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} + \int \frac{x\left(2ab^5\left(b^3c+5a\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\right)x^2}{9ab^4\left(a+bx^3\right)} + \frac{\int\left(18a^2b^5(be\cdots\right.}{\cdots}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\cdots\right.}{9ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\cdots\right.}{9ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\cdots\right.}{9ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\cdots\right.}{9ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x^2}{2b^4} + \frac{fx^5}{5b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-4ab^2d+7a^2be-10a^3f\cdots\right.}{9ab^4\left(a+bx^3\right)}$$

Mathematica [A] time = 0.205988, size = 300, normalized size = 0.95

$$\frac{\frac{30b^{2/3}x^2\left(7a^2be-10a^3f-4ab^2d+b^3c\right)}{a\left(a+bx^3\right)} - \frac{45b^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-20a^2be+44a^3f+5ab^2d+b^3c\right)}{a^{4/3}} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\cdots}}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/3)*(b^3*c - 4*a*b^2*
d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 5*a*b
^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/
a^(4/3) - (10*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/a^(4/3) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/
```

```
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))
```

Maple [B] time = 0.011, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/5*f*x^5/b^3-3/2/b^4*x^2*a*f+1/2/b^3*x^2*e-10/9/b^3/(b*x^3+a)^2*a^2*x^5*f+
7/9/b^2/(b*x^3+a)^2*a*x^5*e-4/9/b/(b*x^3+a)^2*x^5*d+1/9/(b*x^3+a)^2/a*x^5*c
-17/18/b^4/(b*x^3+a)^2*x^2*a^3*f+11/18/b^3/(b*x^3+a)^2*x^2*a^2*e-5/18/b^2/(
b*x^3+a)^2*x^2*a*d-1/18/b/(b*x^3+a)^2*x^2*c-44/27/b^5*a^2/(1/b*a)^(1/3)*ln(
x+(1/b*a)^(1/3))*f+20/27/b^4*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e-5/27/b^3
/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/27/b^2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^
(1/3))*c+22/27/b^5*a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*
f-10/27/b^4*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+5/54/b^
3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/54/b^2/a/(1/b*a)^
(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+44/27/b^5*a^2*3^(1/2)/(1/b*a)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-20/27/b^4*a*3^(1/2)/(1/b
*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+5/27/b^3*3^(1/2)/(1/b
*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/27/b^2/a*3^(1/2)/(1
/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5351, size = 2723, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^
6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*
a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 15*sqrt(1/3)*(a^3*b^4*c + 5*
a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4
*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5
*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^
3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c
```

```
 + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2
 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b
^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f
+ 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6), 1/270*
(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b^6*c - 2
0*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c + 5*a^3*b^4
*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 30*sqrt(1/3)*(a^3*b^4*c + 5*a^4*b^3
*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3*b^4*e + 44
*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)
*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqr
t(-(-a*b^2)^(1/3)/a)/b) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e + 44*a^3*b^2
*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(a*b^4*c + 5*
a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-
a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e
+ 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2*(
a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*log(
b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09331, size = 558, normalized size = 1.77

$$-\frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c+5\left(-ab^2\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(b^3*c*(-a/b)^(1/3) + 5*a*b^2*d*(-a/b)^(1/3) + 44*a^3*f*(-a/b)^(1/3)
- 20*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4
) - 1/27*sqrt(3)*((-a*b^2)^(2/3)*b^3*c + 5*(-a*b^2)^(2/3)*a*b^2*d + 44*(-a*
b^2)^(2/3)*a^3*f - 20*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^6) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 - 20
*a^3*b*f*x^5 + 14*a^2*b^2*x^5*e - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 - 17*a^4*f*
x^2 + 11*a^3*b*x^2*e)/((b*x^3 + a)^2*a*b^4) + 1/54*((-a*b^2)^(2/3)*b^3*c +
5*(-a*b^2)^(2/3)*a*b^2*d + 44*(-a*b^2)^(2/3)*a^3*f - 20*(-a*b^2)^(2/3)*a^2*
b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^6) + 1/10*(2*b^12*f*x^
5 - 15*a*b^11*f*x^2 + 5*b^12*x^2*e)/b^15
```

3.292 $\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=307

$$\frac{x\left(13a^2be-19a^3f-7ab^2d+b^3c\right)}{18ab^4\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^2be+35a^3f+2\right.}{54a^{5/3}b^{13/3}}$$

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)
*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/
3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```

Rubi [A] time = 0.411702, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1858, 1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(13a^2be-19a^3f-7ab^2d+b^3c\right)}{18ab^4\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^2be+35a^3f+2\right.}{54a^{5/3}b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^4)/(4*b^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*x)/(6*b^4*(a + b*x^3)^2) + ((b^3*c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)
*x)/(18*a*b^4*(a + b*x^3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(13/
3)) + ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/(27*a^(5/3)*b^(13/3)) - ((b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(13/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
```

```
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} - \frac{\int \frac{-a\left(b^3c-ab^2d+a^2be-a^3f\right)-6ab\left(b^2d-abe+a^2f\right)x^3-6ab^2(be-af)x^6-6ab^3}{\left(a+bx^3\right)^2}}{6ab^4}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)} + \int \frac{2ab^3\left(b^3c+2ab^2d-5a^2\right.}{}$$

$$= \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)} + \int \frac{8ab^4\left(b^3c+2ab\right.}{}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^4}{4b^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6b^4\left(a+bx^3\right)^2} + \frac{\left(b^3c-7ab^2d+13a^2be-19a^3f\right)x}{18ab^4\left(a+bx^3\right)}$$

Mathematica [A] time = 0.201663, size = 294, normalized size = 0.96

$$\frac{\frac{6\sqrt[3]{b}x\left(13a^2be-19a^3f-7ab^2d+b^3c\right)}{a\left(a+bx^3\right)} - \frac{18\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\left(a+bx^3\right)^2} - \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^2be+35a^3f+2ab^2d+b^3c\right)}{a^{5/3}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-14a^2\right.}{a^5}}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b^3*c - 7*a*b^2*d + 1
3*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(b^3*c + 2*a*b^2*d -
14*a^2*b*e + 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3)
 + (4*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/a^(5/3) - (2*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

Maple [B] time = 0.011, size = 561, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/4*f*x^4/b^3-3/b^4*a*f*x+1/b^3*x*e-19/18/b^3/(b*x^3+a)^2*x^4*a^2*f+13/18/b
^2/(b*x^3+a)^2*x^4*a*e-7/18/b/(b*x^3+a)^2*x^4*d+1/18/(b*x^3+a)^2/a*x^4*c-8/
9/b^4/(b*x^3+a)^2*a^3*f*x+5/9/b^3/(b*x^3+a)^2*a^2*e*x-2/9/b^2/(b*x^3+a)^2*a
*d*x-1/9/b/(b*x^3+a)^2*c*x+35/27/b^5*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*
f-14/27/b^4*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+2/27/b^3/(1/b*a)^(2/3)*ln
(x+(1/b*a)^(1/3))*d+1/27/b^2/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-35/54/b^
5*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+7/27/b^4*a/(1/b
*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/27/b^3/(1/b*a)^(2/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/54/b^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*
a)^(1/3)*x+(1/b*a)^(2/3))*c+35/27/b^5*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-14/27/b^4*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+2/27/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+1/27/b^2/a/(1/b*a)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60752, size = 2716, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(27*a^3*b^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b
^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(a^3*
b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d -
14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*
e + 35*a^5*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*
e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2
*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*
a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a
^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(
2/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*
e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1/108*(27*a^3*b
^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^5*c - 14*a^3*b^
4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 12*sqrt(1/3)*(a^3*b^4*c + 2*a^4*b
^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d - 14*a^3*b^4*e +
35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b^3*e + 35*a^5*b^2*
f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)
```

```
^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*((b^5*c + 2*a*b^4*d - 14*a^2*b^3*e
 + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^5*f + 2*
(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b^4*d - 14*a
^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a^
5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e
 + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09215, size = 495, normalized size = 1.61

$$-\frac{\left(b^3c+2\,ab^2d+35\,a^3f-14\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c+2\left(-ab^2\right)^{\frac{1}{3}}ab^2d+35\left(-ab^2\right)^{\frac{1}{3}}a^3f-14\right.}{27\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(b^3*c + 2*a*b^2*d + 35*a^3*f - 14*a^2*b*e)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^2*b^4) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b^3*c + 2*(-a*b^2)^
(1/3)*a*b^2*d + 35*(-a*b^2)^(1/3)*a^3*f - 14*(-a*b^2)^(1/3)*a^2*b*e)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^5) + 1/54*((-a*b^2)^
(1/3)*b^3*c + 2*(-a*b^2)^(1/3)*a*b^2*d + 35*(-a*b^2)^(1/3)*a^3*f - 14*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^5) + 1/1
8*(b^4*c*x^4 - 7*a*b^3*d*x^4 - 19*a^3*b*f*x^4 + 13*a^2*b^2*x^4*e - 2*a*b^3*
c*x - 4*a^2*b^2*d*x - 16*a^4*f*x + 10*a^3*b*x*e)/((b*x^3 + a)^2*a*b^4) + 1/
4*(b^9*f*x^4 - 12*a*b^8*f*x + 4*b^9*x*e)/b^12
```

3.293 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=301

$$\frac{x^2 \left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{9a^2b^3 \left(a+bx^3\right)}+\frac{x^2 \left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6ab^3 \left(a+bx^3\right)^2}+\frac{\log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) \left(5a^2be-20a^3f\right.}{54a^{7/3}b^{11/3}}$$

```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b
*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a +
b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3
)) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```

Rubi [A] time = 0.368046, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1828, 1594, 1482, 459, 292, 31, 634, 617, 204, 628}

$$\frac{x^2 \left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{9a^2b^3 \left(a+bx^3\right)}+\frac{x^2 \left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6ab^3 \left(a+bx^3\right)^2}+\frac{\log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) \left(5a^2be-20a^3f\right.}{54a^{7/3}b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```


```
[Out] (f*x^2)/(2*b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b
*x^3)^2) + ((2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(9*a^2*b^3*(a +
b*x^3)) - ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(11/3)) - ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(11/3
)) + ((2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(11/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^
2 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m
- Mod[m, n])/n)*(q + 1)), x] + Dist[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
)), Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*e^(2*p +
 (m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n])*(a + b*x^n + c*x^(2*n))^p - (
-d)^((m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod[m, n] + 1) +
e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGt
Q[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} - \frac{\int \frac{-2b\left(2b^3c+ab^2d-a^2be+a^3f\right)x-6ab^2(be-af)x^4-6ab^3fx^7}{\left(a+bx^3\right)^2}dx}{6ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} - \frac{\int \frac{x\left(-2b\left(2b^3c+ab^2d-a^2be+a^3f\right)-6ab^2(be-af)x^3-6ab^3fx^6\right)}{\left(a+bx^3\right)^2}dx}{6ab^4}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} + \frac{\int \frac{x\left(2b^3\left(\frac{2b^3c}{a}+b^2d+5a\right.\right.}{a}}{1}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} + \frac{\left(2b^3c+ab\right.}{}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c+ab\right.}{}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c+ab\right.}{}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c+ab\right.}{}$$

$$= \frac{fx^2}{2b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(2b^3c+ab^2d-4a^2be+7a^3f\right)x^2}{9a^2b^3\left(a+bx^3\right)} - \frac{\left(2b^3c+ab\right.}{}$$

Mathematica [A] time = 0.200122, size = 284, normalized size = 0.94

$$\frac{\frac{6b^{2/3}x^2\left(-4a^2be+7a^3f+ab^2d+2b^3c\right)}{a^2\left(a+bx^3\right)} + \frac{9b^{2/3}x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a\left(a+bx^3\right)^2} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be-20a^3f+ab^2d+2b^3c\right)}{a^{7/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^2b\right.}{a^7}}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a*
(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/
(a^2*(a + b*x^3)) - (2*Sqrt[3]*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (2*(2*b^3*c + a*b^2*d
 + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) + ((2*b^3*c + a*
b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/a^(7/3))/(54*b^(11/3))
```

Maple [B] time = 0.011, size = 550, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*f*x^2/b^3+7/9/b^2/(b*x^3+a)^2*a*x^5*f-4/9/b/(b*x^3+a)^2*x^5*e+1/9/(b*x^
3+a)^2/a*x^5*d+2/9*b/(b*x^3+a)^2/a^2*x^5*c+11/18/b^3/(b*x^3+a)^2*a^2*x^2*f-
5/18/b^2/(b*x^3+a)^2*a*x^2*e-1/18/b/(b*x^3+a)^2*x^2*d+7/18/(b*x^3+a)^2/a*x^
2*c+20/27/b^4*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-5/27/b^3/(1/b*a)^(1/3)*
ln(x+(1/b*a)^(1/3))*e-1/27/b^2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-2/27/b
/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-10/27/b^4*a/(1/b*a)^(1/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*e+1/54/b^2/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))*d+1/27/b/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-
20/27/b^4*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*f+5/27/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*e+1/27/b^2/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))*d+2/27/b/a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49314, size = 2538, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
3*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3
- a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(
a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((2*b^5*c + a*b^4*d + 5
*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*a
^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(2
/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4*
d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e -
 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^
2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5),
 1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*b
^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2 -
6*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b^6*
c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*b^4*
```

```
d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3
)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + ((2*b^5*c + a*b^4*d +
5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e - 20*
a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*c + a*b^4
*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*e
- 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(a*b
^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09405, size = 522, normalized size = 1.73

$$\frac{fx^2}{2\,b^3} - \frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 20\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3} - \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}b^3c + \left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/b^3 - 1/27*(2*b^3*c*(-a/b)^(1/3) + a*b^2*d*(-a/b)^(1/3) - 20*a^3*
f*(-a/b)^(1/3) + 5*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/(a^3*b^3) - 1/27*sqrt(3)*(2*(-a*b^2)^(2/3)*b^3*c + (-a*b^2)^(2/3)*a*b
^2*d - 20*(-a*b^2)^(2/3)*a^3*f + 5*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^5) + 1/18*(4*b^4*c*x^5 + 2*a*b
^3*d*x^5 + 14*a^3*b*f*x^5 - 8*a^2*b^2*x^5*e + 7*a*b^3*c*x^2 - a^2*b^2*d*x^2
 + 11*a^4*f*x^2 - 5*a^3*b*x^2*e)/((b*x^3 + a)^2*a^2*b^3) + 1/54*(2*(-a*b^2)
^(2/3)*b^3*c + (-a*b^2)^(2/3)*a*b^2*d - 20*(-a*b^2)^(2/3)*a^3*f + 5*(-a*b^2
)^(2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^5)
```

3.294 $\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=292

$$\frac{x\left(-7a^2be+13a^3f+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be-14a^3f+ab^2\right.}{54a^{8/3}b^{10/3}}$$

```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2)
 + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3))
- ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d +
2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5
*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))
```

Rubi [A] time = 0.307193, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1858, 1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(-7a^2be+13a^3f+ab^2d+5b^3c\right)}{18a^2b^3\left(a+bx^3\right)}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be-14a^3f+ab^2\right.}{54a^{8/3}b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```


```
[Out] (f*x)/b^3 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2)
 + ((5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(18*a^2*b^3*(a + b*x^3))
- ((5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(10/3)) + ((5*b^3*c + a*b^2*d +
2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(10/3)) - ((5
*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/(54*a^(8/3)*b^(10/3))
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} - \frac{\int \frac{-5b^3c-ab^2d+a^2be-a^3f-6ab(be-af)x^3-6ab^2fx^6}{\left(a+bx^3\right)^2}dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\int \frac{2b^2\left(5b^3c+ab^2d+2a^2be-5a^3f\right)+\cdots}{a+bx^3}\cdots}{18a^2b^5}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+ab^2d+2a^2\cdots\right.}{9\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+ab^2d+2a^2\cdots\right.}{2\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+ab^2d+2a^2\cdots\right.}{2\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} + \frac{\left(5b^3c+ab^2d+2a^2\cdots\right.}{2\cdots}$$

$$= \frac{fx}{b^3} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6ab^3\left(a+bx^3\right)^2} + \frac{\left(5b^3c+ab^2d-7a^2be+13a^3f\right)x}{18a^2b^3\left(a+bx^3\right)} - \frac{\left(5b^3c+ab^2d+2a^2\cdots\right.}{9\cdots}$$

Mathematica [A] time = 0.189997, size = 279, normalized size = 0.96

$$\frac{\frac{3\sqrt[3]{b}x\left(-7a^2be+13a^3f+ab^2d+5b^3c\right)}{a^2\left(a+bx^3\right)} + \frac{9\sqrt[3]{b}x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a\left(a+bx^3\right)^2} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be-14a^3f+ab^2d+5b^3c\right)}{a^{8/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^2be-14a\cdots\right.}{a^{8/3}}\cdots}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*f*x + (9*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a +
 b*x^3)^2) + (3*b^(1/3)*(5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(a^2*
(a + b*x^3)) - (2*Sqrt[3]*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) + (2*(5*b^3*c + a*b^2*d + 2*
a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - ((5*b^3*c + a*b^2*d
 + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^
(8/3))/(54*b^(10/3))
```

Maple [B] time = 0.011, size = 539, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x)
```

```
[Out] f*x/b^3+13/18/b^2/(b*x^3+a)^2*x^4*a*f-7/18/b/(b*x^3+a)^2*x^4*e+1/18/(b*x^3+
a)^2/a*x^4*d+5/18*b/(b*x^3+a)^2/a^2*x^4*c+5/9/b^3/(b*x^3+a)^2*a^2*f*x-2/9/b
^2/(b*x^3+a)^2*a*e*x-1/9/b/(b*x^3+a)^2*d*x+4/9*c/a*x/(b*x^3+a)^2-14/27/b^4*
a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+2/27/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(
1/3))*e+1/27/b^2/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+5/27*c/a^2/b/(1/b*a)
^(2/3)*ln(x+(1/b*a)^(1/3))+7/27/b^4*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+
(1/b*a)^(2/3))*f-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))*e-1/54/b^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-5/54*
c/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-14/27/b^4*a/(1/
b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/27/b^3/(1/
b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/27/b^2/a/(
1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+5/27*c/a^2
/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49873, size = 2587, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^5
*b^2*f)*x^4 - 3*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6*b
*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2*b^
5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*l
og((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2
*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((5*
b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d
 + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b
*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a
^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e -
14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 6*(4*a^3*b^4*
c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*a^5*b^5*x^3 +
 a^6*b^4), 1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*
e + 49*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e
- 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2
*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt(-(-a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-
a^2*b)^(1/3)/b)/a^2) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^
6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3
*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(
2/3)*x - (-a^2*b)^(1/3)*a) + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c +
```

```
a^2*b^3*d + 2*a^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^
2*b)^(2/3)) + 6*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^
4*b^6*x^6 + 2*a^5*b^5*x^3 + a^6*b^4)]
```

Sympy [A] time = 170.751, size = 418, normalized size = 1.43

$$\frac{x^4\left(13a^3bf-7a^2b^2e+ab^3d+5b^4c\right)+x\left(10a^4f-4a^3be-2a^2b^2d+8ab^3c\right)}{18a^4b^3+36a^3b^4x^3+18a^2b^5x^6}+\mathrm{RootSum}\left(19683t^3a^8b^{10}+2744a^9f^3-117\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(13*a**3*b*f - 7*a**2*b**2*e + a*b**3*d + 5*b**4*c) + x*(10*a**4*f -
4*a**3*b*e - 2*a**2*b**2*d + 8*a*b**3*c))/(18*a**4*b**3 + 36*a**3*b**4*x**3
 + 18*a**2*b**5*x**6) + RootSum(19683*_t**3*a**8*b**10 + 2744*a**9*f**3 - 1
176*a**8*b*e*f**2 - 588*a**7*b**2*d*f**2 + 168*a**7*b**2*e**2*f - 2940*a**6
*b**3*c*f**2 + 168*a**6*b**3*d*e*f - 8*a**6*b**3*e**3 + 840*a**5*b**4*c*e*f
 + 42*a**5*b**4*d**2*f - 12*a**5*b**4*d*e**2 + 420*a**4*b**5*c*d*f - 60*a**
4*b**5*c*e**2 - 6*a**4*b**5*d**2*e + 1050*a**3*b**6*c**2*f - 60*a**3*b**6*c
*d*e - a**3*b**6*d**3 - 150*a**2*b**7*c**2*e - 15*a**2*b**7*c*d**2 - 75*a*b
**8*c**2*d - 125*b**9*c**3, Lambda(_t, _t*log(-27*_t*a**3*b**3/(14*a**3*f -
 2*a**2*b*e - a*b**2*d - 5*b**3*c) + x))) + f*x/b**3
```

Giac [A] time = 1.09674, size = 463, normalized size = 1.59

$$\frac{fx}{b^3}-\frac{\left(5\,b^3c+ab^2d-14\,a^3f+2\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3}+\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^3c+\left(-ab^2\right)^{\frac{1}{3}}ab^2d-14\,\left(-ab^2\right)^{\frac{1}{3}}a^3f+\right.}{27\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 - 1/27*(5*b^3*c + a*b^2*d - 14*a^3*f + 2*a^2*b*e)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b^3*c + (
-a*b^2)^(1/3)*a*b^2*d - 14*(-a*b^2)^(1/3)*a^3*f + 2*(-a*b^2)^(1/3)*a^2*b*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^4) + 1/54*(5*
(-a*b^2)^(1/3)*b^3*c + (-a*b^2)^(1/3)*a*b^2*d - 14*(-a*b^2)^(1/3)*a^3*f + 2
*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
 + 1/18*(5*b^4*c*x^4 + a*b^3*d*x^4 + 13*a^3*b*f*x^4 - 7*a^2*b^2*x^4*e + 8*a
*b^3*c*x - 2*a^2*b^2*d*x + 10*a^4*f*x - 4*a^3*b*x*e)/((b*x^3 + a)^2*a^2*b^3
)
```

3.295 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

Optimal. Leaf size=303

$$-\frac{x^2\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{9a^3b^2\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be-5a^3f\right.}{54a^{10/3}b^{8/3}}$$

```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*
x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b
*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*
a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)
) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```

Rubi [A] time = 0.339039, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1829, 1484, 453, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(-a^2be+4a^3f-2ab^2d+5b^3c\right)}{9a^3b^2\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be-5a^3f\right.}{54a^{10/3}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```


```
[Out] -(c/(a^3*x)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^2*b^2*(a + b*
x^3)^2) - ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(9*a^3*b^2*(a + b
*x^3)) + ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(8/3)) + ((14*b^3*c - 2*
a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(8/3)
) - ((14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(8/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
```

```
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{\int \frac{-6b^3c+2b\left(\frac{2b^3c}{a}-2b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^2(a+bx^3)^2}dx}{6ab^3}$$

$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{9a^3b^2(a+bx^3)} + \frac{\int \frac{18ab^5c-2b^3(5b^3c-2ab^2}{x^2(a+bx}}{18a^3b}$$

$$= -\frac{c}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{9a^3b^2(a+bx^3)} - (14b^3c-2ab^2$$

$$= -\frac{c}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{9a^3b^2(a+bx^3)} + (14b^3c-2ab^2$$

$$= -\frac{c}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{9a^3b^2(a+bx^3)} + (14b^3c-2ab^2$$

$$= -\frac{c}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{9a^3b^2(a+bx^3)} + (14b^3c-2ab^2$$

$$= -\frac{c}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^2b^2(a+bx^3)^2} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{9a^3b^2(a+bx^3)} + (14b^3c-2ab^2$$

Mathematica [A] time = 0.222532, size = 286, normalized size = 0.94

$$\frac{-\frac{6\sqrt[3]{a}x^2(-a^2be+4a^3f-2ab^2d+5b^3c)}{b^2(a+bx^3)} + \frac{9a^{4/3}x^2(-a^2be+a^3f+ab^2d-b^3c)}{b^2(a+bx^3)^2} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^2be+5a^3f+2ab^2d-14b^3c)}{b^{8/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^2be}{b^8}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)
/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)
*x^2)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^
3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(8/3) - (2*(-14*b^3*c +
 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(8/3) + ((-14*b
^3*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(8/3))/(54*a^(10/3))
```

Maple [B] time = 0.014, size = 547, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -4/9/(b*x^3+a)^2/b*x^5*f+1/9/a/(b*x^3+a)^2*x^5*e+2/9/a^2/(b*x^3+a)^2*b*x^5*
d-5/9/a^3/(b*x^3+a)^2*b^2*x^5*c-5/18*a/(b*x^3+a)^2/b^2*x^2*f-1/18/(b*x^3+a)
^2/b*x^2*e+7/18*d/a*x^2/(b*x^3+a)^2-13/18/a^2/(b*x^3+a)^2*b*x^2*c-5/27/b^3/
(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-1/27/a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(
1/3))*e-2/27*d/a^2/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+14/27/a^3/(1/b*a)^(1
/3)*ln(x+(1/b*a)^(1/3))*c+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))*f+1/54/a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3)
)*e+1/27*d/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-7/27/a
^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+5/27/b^3*3^(1/2)/(
1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/27/a/b^2*3^(1/2)
/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+2/27*d/a^2*3^(1/
2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-14/27/a^3*3^(1
/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-c/a^3/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.50544, size = 2591, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e + 4*a^4*b^3*
f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 3*s
qrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(14*
a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + (14*a^3*b^4*c - 2*
a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3
 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt
((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((14*b^5*c - 2*a*b^
4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*
e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-
a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((14*b^
5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*
d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*
a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^6*x^7 + 2*a^5*b^
5*x^4 + a^6*b^4*x), -1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3
*b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a
^5*b^2*f)*x^3 + 6*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*
b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 +
 (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
+ ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2
*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4
*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^
2)^(2/3)) - 2*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14
```

```
*a*b^4*c - 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3
*b^2*d - a^4*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a
^4*b^6*x^7 + 2*a^5*b^5*x^4 + a^6*b^4*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11206, size = 525, normalized size = 1.73

$$-\frac{c}{a^3x} + \frac{\left(14\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2} + \frac{\sqrt{3}\left(14\,\left(-ab^2\right)^{\frac{2}{3}}b^3c - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -c/(a^3*x) + 1/27*(14*b^3*c*(-a/b)^(1/3) - 2*a*b^2*d*(-a/b)^(1/3) - 5*a^3*f
*(-a/b)^(1/3) - a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^4*b^2) + 1/27*sqrt(3)*(14*(-a*b^2)^(2/3)*b^3*c - 2*(-a*b^2)^(2/3)*a*b
^2*d - 5*(-a*b^2)^(2/3)*a^3*f - (-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^4) - 1/18*(10*b^4*c*x^5 - 4*a*b^3
*d*x^5 + 8*a^3*b*f*x^5 - 2*a^2*b^2*x^5*e + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*x^2
 + 5*a^4*f*x^2 + a^3*b*x^2*e)/((b*x^3 + a)^2*a^3*b^2) - 1/54*(14*(-a*b^2)^(
2/3)*b^3*c - 2*(-a*b^2)^(2/3)*a*b^2*d - 5*(-a*b^2)^(2/3)*a^3*f - (-a*b^2)^(
2/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^4)
```

3.296 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=301

$$-\frac{x\left(-a^2be+7a^3f-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be-2a^3f-5a\right.}{54a^{11/3}b^{7/3}}$$

```
[Out] -c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a + b*
x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a + b
*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*
a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)
) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rubi [A] time = 0.329154, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1829, 1484, 453, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(-a^2be+7a^3f-5ab^2d+11b^3c\right)}{18a^3b^2\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^2be-2a^3f-5a\right.}{54a^{11/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^2*b^2*(a + b*
x^3)^2) - ((11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(18*a^3*b^2*(a + b
*x^3)) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(7/3)) - ((20*b^3*c - 5*
a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(7/3)
) + ((20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(7/3))
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
```

```
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+b\left(\frac{5b^3c}{a}-5b^2d-abe+a^2f\right)x^3-6ab^2fx^6}{x^3\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} + \frac{\int \frac{18ab^5c-2b^3\left(11b^3c-5ab^2d-a^2\right.}{x^3\left(a+bx^3\right)}}{18a^3b^5}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \left(20b^3c-5ab^2d\right.$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \frac{\left(20b^3c-5ab^2d\right.}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \left(20b^3c-5ab^2d\right.$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} - \left(20b^3c-5ab^2d\right.$$

$$= -\frac{c}{2a^3x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2} - \frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)} + \frac{\left(20b^3c-5ab^2d\right.}{}$$

Mathematica [A] time = 0.213489, size = 283, normalized size = 0.94

$$\frac{\frac{9a^{5/3}x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b^2\left(a+bx^3\right)^2} - \frac{3a^{2/3}x\left(-a^2be+7a^3f-5ab^2d+11b^3c\right)}{b^2\left(a+bx^3\right)} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^2be+2a^3f+5ab^2d-20b^3c\right)}{b^{7/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^2be+2a^3f\right.}{b^{7/3}}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)
/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f
)*x)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3
*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(7/3) + (2*(-20*b^3*c +
5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(7/3) - ((-20*b^
3*c + 5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(7/3))/(54*a^(11/3))
```

Maple [B] time = 0.013, size = 539, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -7/18/(b*x^3+a)^2/b*x^4*f+1/18/a/(b*x^3+a)^2*x^4*e+5/18/a^2/(b*x^3+a)^2*b*x
^4*d-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c-2/9*a/(b*x^3+a)^2/b^2*x*f-1/9/(b*x^3+a
)^2/b*x*e+4/9/a/(b*x^3+a)^2*x*d-7/9/a^2/(b*x^3+a)^2*b*x*c+2/27/b^3/(1/b*a)^
(2/3)*ln(x+(1/b*a)^(1/3))*f+1/27/a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+
5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-20/27/a^3/(1/b*a)^(2/3)*ln(x
+(1/b*a)^(1/3))*c-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/
3))*f-1/54/a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-5/54
/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+10/27/a^3/(1/b
*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+2/27/b^3/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/27/a/b^2/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+5/27/a^2/b/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-20/27/a^3/(1/b*a)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/2*c/a^3/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84402, size = 2618, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^
2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 3*s
qrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*
a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*
a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x
^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x -
 (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((20*b^5*c - 5*a*b
^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2
*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^
5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*
d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*
a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*
b^4*x^5 + a^7*b^3*x^2), -1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d
 - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e
 + 2*a^6*b*f)*x^3 + 6*sqrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*
a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x
^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt((a^2*b)
^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*
b)^(1/3)/b)/a^2) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 +
2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c -
5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(
2/3)*x + (a^2*b)^(1/3)*a) + 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^
```

```
2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a
^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x +
(a^2*b)^(2/3)))/(a^5*b^5*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)

[Out] Timed out
```

Giac [A] time = 1.084, size = 486, normalized size = 1.61

$$\frac{\left(20\,b^3c-5\,ab^2d-2\,a^3f-a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2}-\frac{\sqrt{3}\left(20\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-5\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}a^3f-\left(-a\right.\right.}{27\,a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/27*(20*b^3*c - 5*a*b^2*d - 2*a^3*f - a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^4*b^2) - 1/27*sqrt(3)*(20*(-a*b^2)^(1/3)*b^3*c - 5*(-a*b^2)
^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f - (-a*b^2)^(1/3)*a^2*b*e)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^3) - 1/54*(20*(-a*b^2)^
(1/3)*b^3*c - 5*(-a*b^2)^(1/3)*a*b^2*d - 2*(-a*b^2)^(1/3)*a^3*f - (-a*b^2)^
(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^3) - 1/18*(2
0*b^4*c*x^6 - 5*a*b^3*d*x^6 + 7*a^3*b*f*x^6 - a^2*b^2*x^6*e + 32*a*b^3*c*x^
3 - 8*a^2*b^2*d*x^3 + 4*a^4*f*x^3 + 2*a^3*b*x^3*e + 9*a^2*b^2*c)/((b*x^4 +
a*x)^2*a^3*b^2)
```

3.297 $\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=317

$$\frac{x^2\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{9a^4b\left(a+bx^3\right)}+\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be+a^3f-14\ldots\right.}{54a^{13/3}b^{5/3}}$$

```
[Out] -c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f
)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(
5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```

Rubi [A] time = 0.369523, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1829, 1484, 1488, 292, 31, 634, 617, 204, 628}

$$\frac{x^2\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{9a^4b\left(a+bx^3\right)}+\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be+a^3f-14\ldots\right.}{54a^{13/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```


```
[Out] -c/(4*a^3*x^4) + (3*b*c - a*d)/(a^4*x) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f
)*x^2)/(9*a^4*b*(a + b*x^3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(13/3)*b^(
5/3)) - ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(27*a^(13/3)*b^(5/3)) + ((35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3)*b^(5/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
```

```
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-2b^2\left(\frac{2b^3c}{a^2}-\frac{2b^2d}{a}+2be+af\right)x^6}{x^5\left(a+bx^3\right)^2}dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \frac{\int \frac{-18a^2b^5c+18ab^5(2bc-ad}{\cdots}}{\cdots}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \frac{\int\left(-\frac{18ab^5c}{x^5}+\frac{18b^5(3bc-}{x^2}\cdots\right.}{\cdots}$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} + \cdots$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \cdots$$

$$= -\frac{c}{4a^3x^4} + \frac{3bc-ad}{a^4x} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(8b^3c-5ab^2d+2a^2be+a^3f\right)x^2}{9a^4b\left(a+bx^3\right)} - \cdots$$

Mathematica [A] time = 0.232386, size = 303, normalized size = 0.96

$$\frac{\frac{12\sqrt[3]{a}x^2\left(2a^2be+a^3f-5ab^2d+8b^3c\right)}{b\left(a+bx^3\right)} - \frac{18a^{4/3}x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b\left(a+bx^3\right)^2} + \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^2be+a^3f-14ab^2d+35b^3c\right)}{b^{5/3}} - \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^2\cdots\right.}{\cdots}}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/3)*(-(b^3*
c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2) + (12*a^(1/3)*(8*b^3
*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(35*b
^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/b^(5/3) - (4*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3))
```

Maple [B] time = 0.013, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x)
```

```
[Out] 1/9/a/(b*x^3+a)^2*x^5*f+2/9/a^2/(b*x^3+a)^2*x^5*b*e-5/9/a^3/(b*x^3+a)^2*x^5
*b^2*d+8/9/a^4/(b*x^3+a)^2*x^5*b^3*c-1/18/(b*x^3+a)^2/b*x^2*f+7/18/a/(b*x^3
+a)^2*x^2*e-13/18/a^2/(b*x^3+a)^2*b*x^2*d+19/18/a^3/(b*x^3+a)^2*b^2*x^2*c-1
/27/a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-2/27/a^2/b/(1/b*a)^(1/3)*ln(x
+(1/b*a)^(1/3))*e+14/27/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-35/27/a^4*b
/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/54/a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*f+1/27/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+
(1/b*a)^(2/3))*e-7/27/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))*d+35/54/a^4*b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/27
/a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/
27/a^2/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-
14/27/a^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d
+35/27/a^4*b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)
)*c-1/4*c/a^3/x^4-d/a^3/x+3/a^4/x*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07189, size = 2773, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*a
^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^
6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 6*sqrt(1/3)*((35*a*b^6*c - 14*a^2*
b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d + 2*
a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*e + a
^6*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b
*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^3*e + a^
3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b*f)*x^7 +
 (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c - 14*a*b^4*d
+ 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*
e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^6*b^4*x^7 + a^
7*b^3*x^4), 1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)
*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^
5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 12*sqrt(1/3)*((35*a*b^6
```

```
*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^
3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a
^5*b^2*e + a^6*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x +
(-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*((35*b^5*c - 14*a*b^4*d + 2*
a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e +
a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b
^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c
 - 14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*
d + 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e +
 a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^
6*b^4*x^7 + a^7*b^3*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10135, size = 547, normalized size = 1.73

$$-\frac{\left(35\,b^{3}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,ab^{2}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{3}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^{2}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{5}b}-\frac{\sqrt{3}\left(35\left(-ab^{2}\right)^{\frac{2}{3}}b^{3}c-14\,\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(35*b^3*c*(-a/b)^(1/3) - 14*a*b^2*d*(-a/b)^(1/3) + a^3*f*(-a/b)^(1/3)
 + 2*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b)
- 1/27*sqrt(3)*(35*(-a*b^2)^(2/3)*b^3*c - 14*(-a*b^2)^(2/3)*a*b^2*d + (-a*b
^2)^(2/3)*a^3*f + 2*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(a^5*b^3) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 2*
a^3*b*f*x^5 + 4*a^2*b^2*x^5*e + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 - a^4*f*x
^2 + 7*a^3*b*x^2*e)/((b*x^3 + a)^2*a^4*b) + 1/54*(35*(-a*b^2)^(2/3)*b^3*c -
 14*(-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)*a^3*f + 2*(-a*b^2)^(2/3)*a^2*b*
e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^3) + 1/4*(12*b*c*x^3 - 4
*a*d*x^3 - a*c)/(a^4*x^4)
```

3.298 $\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=316

$$\frac{x\left(5a^2be+a^3f-11ab^2d+17b^3c\right)}{18a^4b\left(a+bx^3\right)}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be+a^3f-20ab^2\right.}{54a^{14/3}b^{4/3}}$$

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a
^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)*
b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/
3)*x]])/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(54*a^(14/3)*b^(4/3))
```

Rubi [A] time = 0.368028, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1829, 1484, 1488, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(5a^2be+a^3f-11ab^2d+17b^3c\right)}{18a^4b\left(a+bx^3\right)}+\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be+a^3f-20ab^2\right.}{54a^{14/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```


```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(2*a^4*x^2) + ((b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((17*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a
^3*f)*x)/(18*a^4*b*(a + b*x^3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3
*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)*
b^(4/3)) + ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/
3)*x]])/(27*a^(14/3)*b^(4/3)) - ((44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(54*a^(14/3)*b^(4/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1484

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1)), x] + Dist[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1)), Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*(-d)^(-((m
```

```
 - Mod[m, n])/n) + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d*(Mod[m, n]
+ 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}dx = \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-b^2\left(\frac{5b^3c}{a^2}-\frac{5b^2d}{a}+5be+af\right)x^6}{x^6\left(a+bx^3\right)^2}dx}{6ab^3}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{\int \frac{-18a^2b^5c+18ab^5(2bc-ad)x^3-}{x^6(}}{1}$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{\int\left(-\frac{18ab^5c}{x^6}+\frac{18b^5(3bc-ad)}{x^3}\right.}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} + \frac{(4}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} + \frac{(4}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} + \frac{(4}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} + \frac{(4}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{2a^4x^2} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^3b\left(a+bx^3\right)^2} + \frac{\left(17b^3c-11ab^2d+5a^2be+a^3f\right)x}{18a^4b\left(a+bx^3\right)} - \frac{(4}{}$$

Mathematica [A] time = 0.233369, size = 299, normalized size = 0.95

$$\frac{-\frac{45a^{5/3}x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{b\left(a+bx^3\right)^2}+\frac{15a^{2/3}x\left(5a^2be+a^3f-11ab^2d+17b^3c\right)}{b\left(a+bx^3\right)}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^2be+a^3f-20ab^2d+44b^3c\right)}{b^{4/3}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(5a^2}{b}}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(5/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)^2) + (15*a^(2/3)*(17*b^
3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(44*
b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(4/3) + (10*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14/3))
```

Maple [B] time = 0.017, size = 566, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x)
```

```
[Out] 1/18/a/(b*x^3+a)^2*x^4*f+5/18/a^2/(b*x^3+a)^2*x^4*b*e-11/18/a^3/(b*x^3+a)^2
*x^4*b^2*d+17/18/a^4/(b*x^3+a)^2*x^4*b^3*c-1/9/(b*x^3+a)^2/b*x*f+4/9/a/(b*x
^3+a)^2*x*e-7/9/a^2/(b*x^3+a)^2*b*x*d+10/9/a^3/(b*x^3+a)^2*b^2*x*c+1/27/a/b
^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a
)^(1/3))*e-20/27/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+44/27/a^4*b/(1/b*a
)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/54/a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*f-5/54/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))*e+10/27/a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-2
2/27/a^4*b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/27/a/b^2
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+5/27/a^2
/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-20/27/
a^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+44/27
/a^4*b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/
5*c/a^3/x^5-1/2*d/a^3/x^2+3/2/a^4/x^2*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83024, size = 2785, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 54
*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^
6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 15*sqrt(1/3)*((44*a*b^6*c - 20*a^
2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^3*b^4*d +
5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a^5*b^2*e +
 a^6*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x
- a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(
a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a
^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8
+ (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*((44*b^5*c - 20*a*b^4*d +
 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e
 + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^11 + 2*a^7*b^3*x^8 + a^8
*b^2*x^5), 1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f
)*x^9 - 54*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 30*sqrt(1/3)*((44*a*b^6
```

```
*c - 20*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^
3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a
^5*b^2*e + a^6*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*((44*b^5*c - 20*a
*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a
^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)
*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
(44*b^5*c - 20*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20
*a^2*b^3*d + 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*
a^4*b*e + a^5*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^
11 + 2*a^7*b^3*x^8 + a^8*b^2*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09964, size = 491, normalized size = 1.55

$$-\frac{\left(44\,b^3c-20\,ab^2d+a^3f+5\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b}+\frac{\sqrt{3}\left(44\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-20\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^3f+5\right.}{27\,a^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(44*b^3*c - 20*a*b^2*d + a^3*f + 5*a^2*b*e)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^5*b) + 1/27*sqrt(3)*(44*(-a*b^2)^(1/3)*b^3*c - 20*(-a*b^2
)^(1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b^2) + 1/54*(44*(-a*b^2)
^(1/3)*b^3*c - 20*(-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^3*f + 5*(-a*b^2
)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b^2) + 1/18*
(17*b^4*c*x^4 - 11*a*b^3*d*x^4 + a^3*b*f*x^4 + 5*a^2*b^2*x^4*e + 20*a*b^3*c
*x - 14*a^2*b^2*d*x - 2*a^4*f*x + 8*a^3*b*x*e)/((b*x^3 + a)^2*a^4*b) + 1/10
*(15*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^4*x^5)
```

3.299 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{x^2\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{9a^5\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(14a^2be-2a^3f\right.}{54a^{16/3}b^{2/3}}$$

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) - (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + ((6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d
 + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) -
((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rubi [A] time = 0.56931, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2\left(5a^2be-2a^3f-8ab^2d+11b^3c\right)}{9a^5\left(a+bx^3\right)}-\frac{x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(14a^2be-2a^3f\right.}{54a^{16/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(4*a^4*x^4) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^4*(a + b*x^3)^2) - (
(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(9*a^5*(a + b*x^3)) + ((6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(16/3)*b^(2/3)) + ((65*b^3*c - 35*a*b^2*d
 + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(16/3)*b^(2/3)) -
((65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(54*a^(16/3)*b^(2/3))
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{4b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^8\left(a+bx^3\right)^2}\,dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)} + \frac{\int \frac{18b^6c-18b^6\left(\frac{2bc}{a}-d\right)\ldots}{\ldots}}{\ldots}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2} - \frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)x^2}{9a^5\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^6c}{ax^8}+\frac{18b^6(-3b\ldots}{a^2x\ldots}\right.}{\ldots}$$

$$= -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2}-\frac{\left(11b^3c-8ab^2\ldots\right.}{9a^5\ldots}$$

$$= -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2}-\frac{\left(11b^3c-8ab^2\ldots\right.}{9a^5\ldots}$$

$$= -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2}-\frac{\left(11b^3c-8ab^2\ldots\right.}{9a^5\ldots}$$

$$= -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2}-\frac{\left(11b^3c-8ab^2\ldots\right.}{9a^5\ldots}$$

$$= -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^4\left(a+bx^3\right)^2}-\frac{\left(11b^3c-8ab^2\ldots\right.}{9a^5\ldots}$$

Mathematica [A] time = 0.23793, size = 328, normalized size = 0.96

$$\frac{\frac{126a^{4/3}x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}+\frac{84\sqrt[3]{a}x^2\left(-5a^2be+2a^3f+8ab^2d-11b^3c\right)}{a+bx^3}+\frac{14\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-14a^2be+2a^3f+35ab^2d-65b^3c\right)}{b^{2/3}}+\frac{28\log\left(\sqrt[3]{\ldots}\right.}{\ldots}}{756\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

```
[Out] ((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a^(1/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b*e
+ 2*a^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a^2*b
*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(6
5*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2
/3) + (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))
```

Maple [B] time = 0.02, size = 611, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x)
```

```
[Out] 2/9/a^2/(b*x^3+a)^2*x^5*b*f-5/9/a^3/(b*x^3+a)^2*x^5*e*b^2-e/a^3/x-1/7*c/a^3
/x^7+8/9/a^4/(b*x^3+a)^2*x^5*d*b^3-11/9/a^5/(b*x^3+a)^2*x^5*c*b^4-13/18/a^2
/(b*x^3+a)^2*x^2*b*e+19/18/a^3/(b*x^3+a)^2*x^2*b^2*d-25/18/a^4/(b*x^3+a)^2*
x^2*b^3*c-14/27/a^3*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))-35/27/a^4*b*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+35/54/a^4*b*d/(1
/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+65/27/a^5*b^2*c/(1/b*a)^(
1/3)*ln(x+(1/b*a)^(1/3))-65/54/a^5*b^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))-2/27/a^2*f/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/27/a^2*f
/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+7/18/a/(b*x^3+a)^2*x
^2*f+14/27/a^3*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-7/27/a^3*e/(1/b*a)^(1/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+3/4/a^4/x^4*b*c+3/a^4/x*b*d-6/a^5/x*
b^2*c-1/4/a^3/x^4*d+35/27/a^4*b*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*
(2/(1/b*a)^(1/3)*x-1))-65/27/a^5*b^2*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))+2/27/a^2*f*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3
^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76082, size = 3031, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^12 +
 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108*a
^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*a^4
*b^3*c - 7*a^5*b^2*d)*x^3 + 42*sqrt(1/3)*((65*a*b^6*c - 35*a^2*b^5*d + 14*a
^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*
e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e - 2*a^6
*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
 + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*
b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*
a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*
x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(
2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((65*b^5*c - 3
5*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d
 + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b
*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^6*b^4*x^13
+ 2*a^7*b^3*x^10 + a^8*b^2*x^7), -1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14
```

```
*a^3*b^4*e - 2*a^4*b^3*f)*x^12 + 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*
b^3*e - 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d
+ 14*a^5*b^2*e)*x^6 - 27*(13*a^4*b^3*c - 7*a^5*b^2*d)*x^3 + 84*sqrt(1/3)*((
65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^
5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*
a^4*b^3*d + 14*a^5*b^2*e - 2*a^6*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(s
qrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((65*b^5*
c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*
b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*
a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (
-a*b^2)^(2/3)) - 28*((65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x
^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a
^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x
 - (-a*b^2)^(1/3)))/(a^6*b^4*x^13 + 2*a^7*b^3*x^10 + a^8*b^2*x^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09876, size = 586, normalized size = 1.71

$$\frac{\left(65\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 35\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 14\,a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6} + \frac{\sqrt{3}\left(65\left(-ab^2\right)^{\frac{2}{3}}b^3c - 35\,(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(-a/b)^(1/3) - 2*a^3*f*(-a/b)^(1/3
) + 14*a^2*b*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 +
1/27*sqrt(3)*(65*(-a*b^2)^(2/3)*b^3*c - 35*(-a*b^2)^(2/3)*a*b^2*d - 2*(-a*b
^2)^(2/3)*a^3*f + 14*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a^6*b^2) - 1/18*(22*b^4*c*x^5 - 16*a*b^3*d*x^5 - 4
*a^3*b*f*x^5 + 10*a^2*b^2*x^5*e + 25*a*b^3*c*x^2 - 19*a^2*b^2*d*x^2 - 7*a^4
*f*x^2 + 13*a^3*b*x^2*e)/((b*x^3 + a)^2*a^5) - 1/54*(65*(-a*b^2)^(2/3)*b^3*
c - 35*(-a*b^2)^(2/3)*a*b^2*d - 2*(-a*b^2)^(2/3)*a^3*f + 14*(-a*b^2)^(2/3)*
a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b^2) - 1/28*(168*b^2
*c*x^6 - 84*a*b*d*x^6 + 28*a^2*x^6*e - 21*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c
)/(a^5*x^7)
```

3.300 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx$

Optimal. Leaf size=341

$$-\frac{x\left(11a^2be-5a^3f-17ab^2d+23b^3c\right)}{18a^5\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(20a^2be-5a^3f-\right.}{54a^{17/3}\sqrt[3]{b}}$$

```
[Out] -c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(2
*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2) -
 ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3)) +
((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^
2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3))
 + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```

Rubi [A] time = 0.54676, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(11a^2be-5a^3f-17ab^2d+23b^3c\right)}{18a^5\left(a+bx^3\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(20a^2be-5a^3f-\right.}{54a^{17/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```


```
[Out] -c/(8*a^3*x^8) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(2
*a^5*x^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^4*(a + b*x^3)^2) -
 ((23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(18*a^5*(a + b*x^3)) +
((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(17/3)*b^(1/3)) - ((77*b^3*c - 44*a*b^
2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(17/3)*b^(1/3))
 + ((77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(17/3)*b^(1/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{5b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^9\left(a+bx^3\right)^2}dx}{6ab^3}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x}{18a^5\left(a+bx^3\right)} + \frac{\int \frac{18b^6c-18b^6\left(\frac{2bc}{a}-d\right)x^3+ \cdots}{\cdots}}{\cdots}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+11a^2be-5a^3f\right)x}{18a^5\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^6c}{ax^9} + \frac{18b^6(-3bc+\cdots}{a^2x^6} \cdots\right.}{\cdots}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+\cdots\right.}{18a^5\left(a\cdots\right.}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+\cdots\right.}{18a^5\left(a\cdots\right.}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+\cdots\right.}{18a^5\left(a\cdots\right.}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+\cdots\right.}{18a^5\left(a\cdots\right.}$$

$$= -\frac{c}{8a^3x^8} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{2a^5x^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^4\left(a+bx^3\right)^2} - \frac{\left(23b^3c-17ab^2d+\cdots\right.}{18a^5\left(a\cdots\right.}$$

Mathematica [A] time = 0.235911, size = 324, normalized size = 0.95

$$\frac{\frac{180a^{5/3}x\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2} + \frac{60a^{2/3}x\left(-11a^2be+5a^3f+17ab^2d-23b^3c\right)}{a+bx^3} + \frac{20\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(20a^2be-5a^3f-44ab^2d+77b^3c\right)}{\sqrt[3]{b}} + \frac{40\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\cdots}}{1080a^{1\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

```
[Out] ((-135*a^(8/3)*c)/x^8 - (216*a^(5/3)*(-3*b*c + a*d))/x^5 - (540*a^(2/3)*(6*
b^2*c - 3*a*b*d + a^2*e))/x^2 + (180*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x)/(a + b*x^3)^2 + (60*a^(2/3)*(-23*b^3*c + 17*a*b^2*d - 11*a^2*b*
e + 5*a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(77*b^3*c - 44*a*b^2*d + 20*a^2*b
*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-
77*b^3*c + 44*a*b^2*d - 20*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(
1/3) + (20*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(1080*a^(17/3))
```

Maple [B] time = 0.017, size = 603, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x)
```

```
[Out] -1/8*c/a^3/x^8-23/18/a^5/(b*x^3+a)^2*x^4*b^4*c-7/9/a^2/(b*x^3+a)^2*b*e*x+4/
9/a/(b*x^3+a)^2*f*x-20/27/a^3*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+10/27/a^3
*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+3/5/a^4/x^5*b*c+3/2/
a^4/x^2*b*d-3/a^5/x^2*b^2*c-1/5/a^3/x^5*d-1/2/a^3/x^2*e+10/9/a^3/(b*x^3+a)^
2*b^2*d*x-13/9/a^4/(b*x^3+a)^2*b^3*c*x+77/54/a^5*b^2*c/(1/b*a)^(2/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+5/18/a^2/(b*x^3+a)^2*x^4*b*f-11/18/a^3/(b*x
^3+a)^2*x^4*b^2*e+17/18/a^4/(b*x^3+a)^2*x^4*b^3*d+5/27/a^2*f/b/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))-5/54/a^2*f/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))-20/27/a^3*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))+44/27/a^4*b*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-22/27/a^4*b*d/
(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-77/27/a^5*b^2*c/(1/b*a)
^(2/3)*ln(x+(1/b*a)^(1/3))+44/27/a^4*b*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(1/b*a)^(1/3)*x-1))-77/27/a^5*b^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+5/27/a^2*f/b/(1/b*a)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77692, size = 3013, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a^5*b^2*f)*x^1
2 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^9 + 135*a
^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x^6 - 54*(7*a^5*b^2*
c - 4*a^6*b*d)*x^3 + 60*sqrt(1/3)*((77*a*b^6*c - 44*a^2*b^5*d + 20*a^3*b^4*
e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a
^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x
^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 20*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*
f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (
77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((77*b^5*c - 44*a*b^4*d +
 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b
^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*
f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2
*x^11 + a^9*b*x^8), -1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e
```

```
 - 5*a^5*b^2*f)*x^12 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a
^6*b*f)*x^9 + 135*a^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x
^6 - 54*(7*a^5*b^2*c - 4*a^6*b*d)*x^3 + 120*sqrt(1/3)*((77*a*b^6*c - 44*a^2
*b^5*d + 20*a^3*b^4*e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d
+ 20*a^4*b^3*e - 5*a^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*
b^2*e - 5*a^6*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((77*b^5*c - 44*a
*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d +
20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e
- 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 40*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77
*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c -
44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)
^(2/3)))/(a^7*b^3*x^14 + 2*a^8*b^2*x^11 + a^9*b*x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09785, size = 532, normalized size = 1.56

$$\frac{\left(77\,b^3c-44\,ab^2d-5\,a^3f+20\,a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6}-\frac{\sqrt{3}\left(77\,\left(-ab^2\right)^{\frac{1}{3}}b^3c-44\,\left(-ab^2\right)^{\frac{1}{3}}ab^2d-5\,\left(-ab^2\right)^{\frac{1}{3}}a^3f\cdots\right.}{27\,a^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*(77*b^3*c - 44*a*b^2*d - 5*a^3*f + 20*a^2*b*e)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/a^6 - 1/27*sqrt(3)*(77*(-a*b^2)^(1/3)*b^3*c - 44*(-a*b^2)^
(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b^2)^(1/3)*a^2*b*e)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) - 1/54*(77*(-a*b^2)^
(1/3)*b^3*c - 44*(-a*b^2)^(1/3)*a*b^2*d - 5*(-a*b^2)^(1/3)*a^3*f + 20*(-a*b
^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) - 1/18*
(23*b^4*c*x^4 - 17*a*b^3*d*x^4 - 5*a^3*b*f*x^4 + 11*a^2*b^2*x^4*e + 26*a*b^
3*c*x - 20*a^2*b^2*d*x - 8*a^4*f*x + 14*a^3*b*x*e)/((b*x^3 + a)^2*a^5) - 1/
40*(120*b^2*c*x^6 - 60*a*b*d*x^6 + 20*a^2*x^6*e - 24*a*b*c*x^3 + 8*a^2*d*x^
3 + 5*a^2*c)/(a^5*x^8)
```

3.301 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=381

$$\frac{bx^2\left(8a^2be-5a^3f-11ab^2d+14b^3c\right)}{9a^6\left(a+bx^3\right)}+\frac{bx^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-\right.}{54a^{19/3}}$$

```
[Out] -c/(10*a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c -
11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(104*
b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/
(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d
 + 35*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(27*a^(19/3)) + (b^(1/3
)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rubi [A] time = 0.713572, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$\frac{bx^2\left(8a^2be-5a^3f-11ab^2d+14b^3c\right)}{9a^6\left(a+bx^3\right)}+\frac{bx^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-\right.}{54a^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] -c/(10*a^3*x^10) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(4*a^5*x^4) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + (b*(14*b^3*c -
11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(9*a^6*(a + b*x^3)) - (b^(1/3)*(104*
b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/
(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(19/3)) - (b^(1/3)*(104*b^3*c - 65*a*b^2*d
 + 35*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(27*a^(19/3)) + (b^(1/3
)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(19/3))
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\cdots}{x^{11}\left(a+bx^3\right)^2}}{6ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(14b^3c-11ab^2d+8a^2be-5a^3f\right)x^2}{9a^6\left(a+bx^3\right)} + \frac{\int \frac{18b^7c-18b^7\left(\frac{2bc}{a}\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(14b^3c-11ab^2d+8a^2be-5a^3f\right)x^2}{9a^6\left(a+bx^3\right)} + \frac{\int\left(\frac{18b^7c}{ax^{11}}+\frac{18b^7\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2\cdots\right.}{6a^5\cdots}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2\cdots\right.}{6a^5\cdots}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2\cdots\right.}{6a^5\cdots}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2\cdots\right.}{6a^5\cdots}$$

$$= -\frac{c}{10a^3x^{10}} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{4a^5x^4} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2\cdots\right.}{6a^5\cdots}$$

Mathematica [A] time = 0.392011, size = 366, normalized size = 0.96

$$\frac{-\frac{630a^{4/3}bx^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{\left(a+bx^3\right)^2}-\frac{420\sqrt[3]{a}bx^2\left(-8a^2be+5a^3f+11ab^2d-14b^3c\right)}{a+bx^3}+70\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(35a^2be-14a^3f\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

```
[Out] ((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945*a^(4/3)*(
6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^3*c + 6*a*b^2*d - 3*
a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)
*x^2)/(a + b*x^3)^2 - (420*a^(1/3)*b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e +
5*a^3*f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 3
5*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(
1/3)*(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3
)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*a^(19/3))
```

Maple [A] time = 0.019, size = 659, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x)
```

```
[Out] -104/27*b^3/a^6*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-6/a^5/x*b^2*d+10/a^6/x*
b^3*c-25/18*b^3/a^4/(b*x^3+a)^2*x^2*d+65/27*b^2/a^5*d/(1/b*a)^(1/3)*ln(x+(1
/b*a)^(1/3))-65/54*b^2/a^5*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))-1/10*c/a^3/x^10-11/9*b^4/a^5/(b*x^3+a)^2*x^5*d+14/9*b^5/a^6/(b*x^3+a)
^2*x^5*c+14/27/a^3*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-7/27/a^3*f/(1/b*a)^(
1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+3/7/a^4/x^7*b*c+3/4/a^4/x^4*b*d-
3/2/a^5/x^4*b^2*c+3/a^4/x*b*e-1/7/a^3/x^7*d-1/4/a^3/x^4*e-1/a^3/x*f+35/27*b
/a^4*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/2
7*b^2/a^5*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
+104/27*b^3/a^6*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)
*x-1))-5/9*b^2/a^3/(b*x^3+a)^2*x^5*f+8/9*b^3/a^4/(b*x^3+a)^2*x^5*e+52/27*b^
3/a^6*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-14/27/a^3*f*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-35/27*b/a^4*e/
(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+35/54*b/a^4*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))-13/18*b/a^2/(b*x^3+a)^2*x^2*f+19/18*b^2/a^3/(b*x^3
+a)^2*x^2*e+31/18*b^4/a^5/(b*x^3+a)^2*x^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61915, size = 1472, normalized size = 3.86

$$\frac{420\left(104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f\right)x^{15}+735\left(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf\right)x^{12}+270\left(104\,a^2b^3c-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3780*(420*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 + 7
35*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 270*(104
*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 27*(104*a^3*b^2*c
- 65*a^4*b*d + 35*a^5*e)*x^6 - 378*a^5*c + 108*(8*a^4*b*c - 5*a^5*d)*x^3 +
140*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 +
2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*
b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((104*b^5*c - 65*a*b^4*d + 35*a^2
*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e
- 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)
*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((104
*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c -
```

```
65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^
2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))/(a
^6*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09807, size = 656, normalized size = 1.72

$$-\frac{\left(104\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-65\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+35\,a^2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7}-\frac{\sqrt{3}\left(104\,\left(-ab^2\right)^{\frac{2}{3}}b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(104*b^4*c*(-a/b)^(1/3) - 65*a*b^3*d*(-a/b)^(1/3) - 14*a^3*b*f*(-a/b)
^(1/3) + 35*a^2*b^2*(-a/b)^(1/3)*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/a^7 - 1/27*sqrt(3)*(104*(-a*b^2)^(2/3)*b^3*c - 65*(-a*b^2)^(2/3)*a*b^2*d -
 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^7*b) + 1/54*(104*(-a*b^2)^(2/3)*b^3*c -
 65*(-a*b^2)^(2/3)*a*b^2*d - 14*(-a*b^2)^(2/3)*a^3*f + 35*(-a*b^2)^(2/3)*a^
2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^7*b) + 1/18*(28*b^5*c*x^
5 - 22*a*b^4*d*x^5 - 10*a^3*b^2*f*x^5 + 16*a^2*b^3*x^5*e + 31*a*b^4*c*x^2 -
 25*a^2*b^3*d*x^2 - 13*a^4*b*f*x^2 + 19*a^3*b^2*x^2*e)/((b*x^3 + a)^2*a^6)
+ 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9 - 140*a^3*f*x^9 + 420*a^2*b*x^9*e
 - 210*a*b^2*c*x^6 + 105*a^2*b*d*x^6 - 35*a^3*x^6*e + 60*a^2*b*c*x^3 - 20*a
^3*d*x^3 - 14*a^3*c)/(a^6*x^10)
```

3.302 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=380

$$\frac{bx\left(17a^2be-11a^3f-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}-\frac{b^{2/3}\log(\cdot}{}$$

```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c
- 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*(1
19*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^
2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) - (b^(
2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Rubi [A] time = 0.668966, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 200, 31, 634, 617, 204, 628}

$$\frac{bx\left(17a^2be-11a^3f-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{2a^6x^2}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}-\frac{b^{2/3}\log(\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```


```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c
- 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) - (b^(2/3)*(1
19*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^
2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) - (b^(
2/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^5\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}-\frac{5b^4\left(b^3c-\right.}{}}{x^{12}\left(a+bx^3\right)^2}}{6ab^3}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(29b^3c-23ab^2d+17a^2be-11a^3f\right)x}{18a^6\left(a+bx^3\right)} + \frac{\int \frac{18b^7c-18b^7\left(\frac{2bc}{a}-d\right)}{}}{}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^5\left(a+bx^3\right)^2} + \frac{b\left(29b^3c-23ab^2d+17a^2be-11a^3f\right)x}{18a^6\left(a+bx^3\right)} + \frac{\int\left(\frac{18b^7c}{ax^{12}}+\frac{18b^7(-3}{a^2x}\right.}{}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b^3c-ab^2d+\right.}{6a^5\left(a\right.}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b^3c-ab^2d+\right.}{6a^5\left(a\right.}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b^3c-ab^2d+\right.}{6a^5\left(a\right.}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b^3c-ab^2d+\right.}{6a^5\left(a\right.}$$

$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b\left(b^3c-ab^2d+\right.}{6a^5\left(a\right.}$$

Mathematica [A] time = 0.428823, size = 376, normalized size = 0.99

$$\frac{bx\left(17a^2be-11a^3f-23ab^2d+29b^3c\right)}{18a^6\left(a+bx^3\right)} + \frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{2a^6x^2} + \frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2} + \frac{b^{2/3}\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

```
[Out] -c/(11*a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) + (b*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b^3*c
- 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) + (b^(2/3)*(-
119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a*b^2*d +
 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) + (b^(2/3)*
(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

Maple [A] time = 0.018, size = 651, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x)
```

```
[Out] 3/5/a^4/x^5*b*d-6/5/a^5/x^5*b^2*c+3/2/a^4/x^2*b*e-3/a^5/x^2*b^2*d+5/a^6/x^2
*b^3*c-20/27/a^3*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+10/27/a^3*f/(1/b*a)^(2
/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+3/8/a^4/x^8*b*c-11/18*b^2/a^3/(b*
x^3+a)^2*x^4*f+17/18*b^3/a^4/(b*x^3+a)^2*x^4*e-1/11*c/a^3/x^11-23/18*b^4/a^
5/(b*x^3+a)^2*x^4*d+29/18*b^5/a^6/(b*x^3+a)^2*x^4*c-7/9*b/a^2/(b*x^3+a)^2*f
*x-20/27/a^3*f/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-
1))+44/27*b/a^4*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-22/27*b/a^4*e/(1/b*a)^(
2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-77/27*b^2/a^5*d/(1/b*a)^(2/3)*ln
(x+(1/b*a)^(1/3))+77/54*b^2/a^5*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))+119/27*b^3/a^6*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-119/54*b^3/a^
6*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+10/9*b^2/a^3/(b*x^3
+a)^2*e*x-13/9*b^3/a^4/(b*x^3+a)^2*d*x+16/9*b^4/a^5/(b*x^3+a)^2*c*x-1/8/a^3
/x^8*d-1/5/a^3/x^5*e-1/2/a^3/x^2*f-77/27*b^2/a^5*d/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+119/27*b^3/a^6*c/(1/b*a)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+44/27*b/a^4*e/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62499, size = 1543, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11880*(660*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 +
1056*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 297*(1
19*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 54*(119*a^3*b^2*
c - 77*a^4*b*d + 44*a^5*e)*x^6 - 1080*a^5*c + 135*(17*a^4*b*c - 11*a^5*d)*x
^3 - 440*sqrt(3)*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^
17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119
*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((119*b^5*c
 - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2
*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d +
44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2
```

```
)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3
*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20
*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^1
1)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^6*b^2*x^17 + 2*a^7*b*
x^14 + a^8*x^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07959, size = 594, normalized size = 1.56

$$\frac{\sqrt{3}\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d-20\left(-ab^2\right)^{\frac{1}{3}}a^3f+44\left(-ab^2\right)^{\frac{1}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^7}-\frac{\left(119b^4c-77ab^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a
*b^2)^(1/3)*a^3*f + 44*(-a*b^2)^(1/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4*c - 77*a*b^3*d - 20*a^3*b*f +
 44*a^2*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 + 1/54*(119*(-a*
b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d - 20*(-a*b^2)^(1/3)*a^3*f + 44
*(-a*b^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 + 1/1
8*(29*b^5*c*x^4 - 23*a*b^4*d*x^4 - 11*a^3*b^2*f*x^4 + 17*a^2*b^3*x^4*e + 32
*a*b^4*c*x - 26*a^2*b^3*d*x - 14*a^4*b*f*x + 20*a^3*b^2*x*e)/((b*x^3 + a)^2
*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 - 220*a^3*f*x^9 + 660*a^2*
b*x^9*e - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^6 - 88*a^3*x^6*e + 165*a^2*b*c*x^
3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^6*x^11)
```

3.303 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=424

$$-\frac{b^2x^2\left(11a^2be-8a^3f-14ab^2d+17b^3c\right)}{9a^7\left(a+bx^3\right)}-\frac{b^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^4}{}$$

```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2*e
)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) - (b
*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*
b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^3*
c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d +
 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) - (b^(4/3)*
(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Rubi [A] time = 0.853138, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1829, 1834, 292, 31, 634, 617, 204, 628}

$$-\frac{b^2x^2\left(11a^2be-8a^3f-14ab^2d+17b^3c\right)}{9a^7\left(a+bx^3\right)}-\frac{b^2x^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{4a^6x^4}-\frac{b^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```


```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2*e
)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) - (b
*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) - (b^2*(17*b^3*c - 14*a*
b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152*b^3*
c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d +
 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) - (b^(4/3)*
(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^3c+6b^3\left(\frac{bc}{a}-d\right)x^3-\frac{6b^3\left(b^2c-abd+a^2e\right)x^6}{a^2}+\frac{6b^3\left(b^3c-ab^2d+a^2be-a^3f\right)x^9}{a^3}}{x^{14}\left(a+bx^3\right)^2}}{6ab^3}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{b^2\left(17b^3c-14ab^2d+11a^2be-8a^3f\right)x^2}{9a^7\left(a+bx^3\right)} + \frac{\int \frac{18b^8c-18b}{}}{}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{6a^6\left(a+bx^3\right)^2} - \frac{b^2\left(17b^3c-14ab^2d+11a^2be-8a^3f\right)x^2}{9a^7\left(a+bx^3\right)} + \frac{\int\left(\frac{18b^8c}{ax^{14}}+\right.}{}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^3c-1\right.}{}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^3c-1\right.}{}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^3c-1\right.}{}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^3c-1\right.}{}$$

$$= -\frac{c}{13a^3x^{13}} + \frac{3bc-ad}{10a^4x^{10}} - \frac{6b^2c-3abd+a^2e}{7a^5x^7} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} - \frac{b\left(15b^3c-1\right.}{}$$

Mathematica [A] time = 0.465972, size = 419, normalized size = 0.99

$$\frac{b^2x^2\left(-11a^2be+8a^3f+14ab^2d-17b^3c\right)}{9a^7\left(a+bx^3\right)} + \frac{b^2x^2\left(-a^2be+a^3f+ab^2d-b^3c\right)}{6a^6\left(a+bx^3\right)^2} + \frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{4a^6x^4} + \frac{b^{4/3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

```
[Out] -c/(13*a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^2*e
)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4) + (b
*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) + (b^2*(-17*b^3*c +
14*a*b^2*d - 11*a^2*b*e + 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/3)*(152
*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a*b^2*d +
65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(22/3)) + (b^(4/3)*(
-152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(54*a^(22/3))
```

Maple [A] time = 0.022, size = 716, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x)
```

```
[Out] 8/9*b^3/a^4/(b*x^3+a)^2*x^5*f-11/9*b^4/a^5/(b*x^3+a)^2*x^5*e+14/9*b^5/a^6/(
b*x^3+a)^2*x^5*d+3/10/a^4/x^10*b*c+3/7/a^4/x^7*b*d-6/7/a^5/x^7*b^2*c+3/4/a^
4/x^4*b*e-3/2/a^5/x^4*b^2*d+5/2/a^6/x^4*b^3*c+3*b/a^4/x*f-6*b^2/a^5/x*e+10*
b^3/a^6/x*d-15*b^4/a^7/x*c+104/27*b^3/a^6*d*3^(1/2)/(1/b*a)^(1/3)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-152/27*b^4/a^7*c*3^(1/2)/(1/b*a)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/13*c/a^3/x^13-1/10/a^3/x^10*d-1/7/
a^3/x^7*e-1/4/a^3/x^4*f-104/27*b^3/a^6*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+
52/27*b^3/a^6*d/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+152/27*
b^4/a^7*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-76/27*b^4/a^7*c/(1/b*a)^(1/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-35/27*b/a^4*f/(1/b*a)^(1/3)*ln(x+(1/b*
a)^(1/3))+35/54*b/a^4*f/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))
+65/27*b^2/a^5*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-65/54*b^2/a^5*e/(1/b*a)^
(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-37/18*b^5/a^6/(b*x^3+a)^2*x^2*c
-25/18*b^3/a^4/(b*x^3+a)^2*x^2*e+31/18*b^4/a^5/(b*x^3+a)^2*x^2*d-17/9*b^6/a
^7/(b*x^3+a)^2*x^5*c+19/18*b^2/a^3/(b*x^3+a)^2*x^2*f+35/27*b/a^4*f*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-65/27*b^2/a^5*e*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68566, size = 1638, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/49140*(5460*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^18
 + 9555*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15 +
3510*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 - 351
*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 3780*a^6*c +
 108*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 378*(19*a^5*b*c - 13*a^
6*d)*x^3 + 1820*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b
^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*
x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-
b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((152*b^6
*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104
```

```
*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b
^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 1820*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e -
35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4
*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*
x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^7*b^2*x^19 + 2*a^8*b*x^16
+ a^9*x^13)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1053, size = 717, normalized size = 1.69

$$\frac{\sqrt{3}\left(152\left(-ab^2\right)^{\frac{2}{3}}b^3c-104\left(-ab^2\right)^{\frac{2}{3}}ab^2d-35\left(-ab^2\right)^{\frac{2}{3}}a^3f+65\left(-ab^2\right)^{\frac{2}{3}}a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^8}+\frac{\left(152b^5c\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2*d - 35*(-
a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^2*b*e)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/a^8 + 1/27*(152*b^5*c*(-a/b)^(1/3) - 104*a*b^4*d
*(-a/b)^(1/3) - 35*a^3*b^2*f*(-a/b)^(1/3) + 65*a^2*b^3*(-a/b)^(1/3)*e)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^8 - 1/54*(152*(-a*b^2)^(2/3)*b^3*c -
104*(-a*b^2)^(2/3)*a*b^2*d - 35*(-a*b^2)^(2/3)*a^3*f + 65*(-a*b^2)^(2/3)*a^
2*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^8 - 1/18*(34*b^6*c*x^5 -
28*a*b^5*d*x^5 - 16*a^3*b^3*f*x^5 + 22*a^2*b^4*x^5*e + 37*a*b^5*c*x^2 - 31*
a^2*b^4*d*x^2 - 19*a^4*b^2*f*x^2 + 25*a^3*b^3*x^2*e)/((b*x^3 + a)^2*a^7) -
1/1820*(27300*b^4*c*x^12 - 18200*a*b^3*d*x^12 - 5460*a^3*b*f*x^12 + 10920*a
^2*b^2*x^12*e - 4550*a*b^3*c*x^9 + 2730*a^2*b^2*d*x^9 + 455*a^4*f*x^9 - 136
5*a^3*b*x^9*e + 1560*a^2*b^2*c*x^6 - 780*a^3*b*d*x^6 + 260*a^4*x^6*e - 546*
a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^7*x^13)
```

3.304 $\int \frac{(1-x)x^4}{1+x^3} dx$

Optimal. Leaf size=54

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[
1 - x + x^2]/6
```

Rubi [A] time = 0.0719371, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1887, 1874, 31, 634, 618, 204, 628}

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[
1 - x + x^2]/6
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x^4}{1+x^3}\,dx &= \int \left(x-x^2+\frac{(-1+x)x}{1+x^3}\right)dx\\ &= \frac{x^2}{2}-\frac{x^3}{3}+\int \frac{(-1+x)x}{1+x^3}\,dx\\ &= \frac{x^2}{2}-\frac{x^3}{3}+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= \frac{x^2}{2}-\frac{x^3}{3}+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= \frac{x^2}{2}-\frac{x^3}{3}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{x^2}{2}-\frac{x^3}{3}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0139, size = 59, normalized size = 1.09

$$\frac{1}{6}\left(-2x^3+3x^2-\log\left(x^2-x+1\right)+2\log\left(x^3+1\right)+2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^4)/(1 + x^3),x]
```

```
[Out] (3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[
1 - x + x^2] + 2*Log[1 + x^3])/6
```

Maple [A] time = 0.005, size = 45, normalized size = 0.8

$$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^4/(x^3+1),x)
```

```
[Out] -1/3*x^3+1/2*x^2+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+2/
3*ln(1+x)
```

Maxima [A] time = 1.4146, size = 59, normalized size = 1.09

$$-\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

Fricas [A] time = 1.55536, size = 140, normalized size = 2.59

$$-\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] time = 0.1289, size = 53, normalized size = 0.98

$$-\frac{x^3}{3} + \frac{x^2}{2} + \frac{2\log\left(x+1\right)}{3} + \frac{\log\left(x^2-x+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**4/(x**3+1),x)
```

```
[Out] -x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.05003, size = 61, normalized size = 1.13

$$-\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^4/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^
2 - x + 1) + 2/3*log(abs(x + 1))
```

3.305 $\int \frac{(1-x)x^3}{1+x^3} dx$

Optimal. Leaf size=30

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0400098, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1887, 1860, 31, 628}

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^3)/(1 + x^3),x]
```

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x^3}{1+x^3}\,dx &= \int \left(1-x-\frac{1-x}{1+x^3}\right)dx\\ &= x-\frac{x^2}{2}-\int \frac{1-x}{1+x^3}\,dx\\ &= x-\frac{x^2}{2}-\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1}{1+x}\,dx\\ &= x-\frac{x^2}{2}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0043203, size = 30, normalized size = 1.

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^3)/(1 + x^3),x]
```

```
[Out] x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^3/(x^3+1),x)
```

```
[Out] x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.42281, size = 32, normalized size = 1.07

$$-\frac{1}{2}x^2+x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Fricas [A] time = 1.51494, size = 73, normalized size = 2.43

$$-\frac{1}{2}x^2+x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Sympy [A] time = 0.089936, size = 24, normalized size = 0.8

$$-\frac{x^2}{2} + x - \frac{2\log(x+1)}{3} + \frac{\log\left(x^2 - x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**3/(x**3+1),x)
```

```
[Out] -x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3
```

Giac [A] time = 1.07455, size = 34, normalized size = 1.13

$$-\frac{1}{2}x^2 + x + \frac{1}{3}\log\left(x^2 - x + 1\right) - \frac{2}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^3/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

3.306 $\int \frac{(1-x)x^2}{1+x^3}\,dx$

Optimal. Leaf size=44

$$\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1 - x + x^2
]/6
```

Rubi [A] time = 0.0598778, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1887, 1874, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1 - x + x^2
]/6
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)x^2}{1+x^3}\,dx &= \int \left(-1+\frac{1+x^2}{1+x^3}\right)dx \\
&= -x+\int \frac{1+x^2}{1+x^3}\,dx \\
&= -x+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx+\frac{2}{3}\int \frac{1}{1+x}\,dx \\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx \\
&= -x+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -x+\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0086687, size = 53, normalized size = 1.2

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x^3+1\right)-x+\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x^2)/(1 + x^3),x]
```

```
[Out] -x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/6
 + Log[1 + x^3]/3
```

Maple [A] time = 0.003, size = 38, normalized size = 0.9

$$-x+\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x^2/(x^3+1),x)
```

```
[Out] -x+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+2/3*ln(1+x)
```

Maxima [A] time = 1.41157, size = 50, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

Fricas [A] time = 1.49866, size = 117, normalized size = 2.66

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(x + 1)
```

Sympy [A] time = 0.123329, size = 44, normalized size = 1.

$$-x+\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x**2/(x**3+1),x)
```

```
[Out] -x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sq
rt(3)/3)/3
```

Giac [A] time = 1.06137, size = 51, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3*
log(abs(x + 1))
```

3.307 $\int \frac{(1-x)x}{1+x^3} \, dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 - x + x^2]/
6
```

Rubi [A] time = 0.0418063, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1874, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - (2*Log[1 + x])/3 - Log[1 - x + x^2]/
6
```

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)x}{1+x^3}\,dx &= \frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx - \frac{2}{3}\int \frac{1}{1+x}\,dx \\
&= -\frac{2}{3}\log(1+x) - \frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2}{3}\log(1+x) - \frac{1}{6}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1+x) - \frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0072435, size = 50, normalized size = 1.22

$$\frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(x^3+1\right) - \frac{1}{3}\log(x+1) + \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - Lo
g[1 + x^3]/3
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-\frac{\ln\left(x^2-x+1\right)}{6} + \frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - \frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(x^3+1),x)
```

```
[Out] -1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*ln(1+x)
```

Maxima [A] time = 1.41225, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

Fricas [A] time = 1.54614, size = 112, normalized size = 2.73

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
x + 1)
```

Sympy [A] time = 0.131201, size = 42, normalized size = 1.02

$$-\frac{2\log(x+1)}{3}-\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3
)/3)/3
```

Giac [A] time = 1.05767, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log(
abs(x + 1))
```

3.308 $\int \frac{1-x}{x(1+x^3)} \, dx$

Optimal. Leaf size=42

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x +
 x^2]/6
```

Rubi [A] time = 0.0493821, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1834, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x +
 x^2]/6
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x}{x\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x}-\frac{2}{3(1+x)}+\frac{-1-x}{3\left(1-x+x^2\right)}\right)dx\\
&= \log(x)-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1-x}{1-x+x^2}\,dx\\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= \log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0076564, size = 53, normalized size = 1.26

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x^3+1\right)+\log(x)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x]/3 + Log[1 - x +
 x^2]/6 - Log[1 + x^3]/3
```

Maple [A] time = 0.006, size = 37, normalized size = 0.9

$$-\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\ln(x)-\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x/(x^3+1),x)
```

```
[Out] -1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+ln(x)-2/3*ln(1+x)
```

Maxima [A] time = 1.42337, size = 49, normalized size = 1.17

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

Fricas [A] time = 1.4933, size = 126, normalized size = 3.

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1) + log(x)
```

Sympy [A] time = 0.175426, size = 46, normalized size = 1.1

$$\log\left(x\right)-\frac{2\log\left(x+1\right)}{3}-\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3
```

Giac [A] time = 1.08784, size = 51, normalized size = 1.21

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(abs(x + 1)) + log(abs(x))
```

3.309 $\int \frac{1-x}{x^2(1+x^3)}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 + x])/3 + L
og[1 - x + x^2]/6
```

Rubi [A] time = 0.049683, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1834, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 + x])/3 + L
og[1 - x + x^2]/6
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{3(1+x)}+\frac{-2+x}{3\left(1-x+x^2\right)}\right)dx\\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-2+x}{1-x+x^2}\,dx\\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{x}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{1}{x}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0153023, size = 60, normalized size = 1.22

$$-\frac{1}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x^3+1\right)-\frac{1}{x}-\log(x)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^2*(1 + x^3)),x]
```

```
[Out] -x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x]/3 - Log[
1 - x + x^2]/6 + Log[1 + x^3]/3
```

Maple [A] time = 0.008, size = 44, normalized size = 0.9

$$\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)-x^{-1}-\ln(x)+\frac{2\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x^2/(x^3+1),x)
```

```
[Out] 1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/x-ln(x)+2/3*ln(1+
x)
```

Maxima [A] time = 1.47203, size = 58, normalized size = 1.18

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(x + 1) - log(x)
```

Fricas [A] time = 1.53385, size = 144, normalized size = 2.94

$$-\frac{2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x\log\left(x^2-x+1\right)-4x\log\left(x+1\right)+6x\log\left(x\right)+6}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - x*log(x^2 - x + 1) - 4*x*
log(x + 1) + 6*x*log(x) + 6)/x
```

Sympy [A] time = 0.194682, size = 49, normalized size = 1.

$$-\log\left(x\right)+\frac{2\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x**2/(x**3+1),x)
```

```
[Out] -log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3 - 1/x
```

Giac [A] time = 1.06451, size = 61, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(|x+1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) + 2
/3*log(abs(x + 1)) - log(abs(x))
```

3.310 $\int \frac{1-x}{x^3(1+x^3)}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

```
[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rubi [A] time = 0.0326286, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1834, 628}

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(x^3*(1 + x^3)),x]
```

```
[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{x^3\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{x^3}-\frac{1}{x^2}-\frac{2}{3(1+x)}+\frac{-1+2x}{3\left(1-x+x^2\right)}\right)dx\\ &= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.003987, size = 32, normalized size = 1.

$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(x^3*(1 + x^3)),x]
```

```
[Out] -1/(2*x^2) + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$-\frac{1}{2x^2}+x^{-1}-\frac{2\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/x^3/(x^3+1),x)
```

```
[Out] -1/2/x^2+1/x-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Maxima [A] time = 1.40062, size = 38, normalized size = 1.19

$$\frac{2x-1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

Fricas [A] time = 1.42257, size = 85, normalized size = 2.66

$$\frac{2x^2\log\left(x^2-x+1\right)-4x^2\log(x+1)+6x-3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2*log(x^2 - x + 1) - 4*x^2*log(x + 1) + 6*x - 3)/x^2
```

Sympy [A] time = 0.109497, size = 27, normalized size = 0.84

$$-\frac{2\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{3}+\frac{2x-1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x**3/(x**3+1),x)
```

```
[Out] -2*log(x + 1)/3 + log(x**2 - x + 1)/3 + (2*x - 1)/(2*x**2)
```

Giac [A] time = 1.04714, size = 39, normalized size = 1.22

$$\frac{2x-1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/x^3/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

3.311 $\int \frac{x(1+2x)}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 - x + x^2])/
6
```

Rubi [A] time = 0.0410974, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1874, 31, 634, 618, 204, 628}

$$\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x))/(1 + x^3),x]
```


```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x]/3 + (5*Log[1 - x + x^2])/
6
```


Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+2x)}{1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{1+x}\,dx + \frac{1}{3}\int \frac{-1+5x}{1-x+x^2}\,dx\\ &= \frac{1}{3}\log(1+x) + \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx + \frac{5}{6}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{3}\log(1+x) + \frac{5}{6}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1+x) + \frac{5}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0073541, size = 47, normalized size = 1.15

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)+4\log\left(x^3+1\right)-2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x + x^2] + 4
*Log[1 + x^3])/6
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$\frac{5\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln(1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+2*x)/(x^3+1),x)
```

```
[Out] 5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*ln(1+x)
```

Maxima [A] time = 1.40721, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Fricas [A] time = 1.54763, size = 112, normalized size = 2.73

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

Sympy [A] time = 0.138427, size = 42, normalized size = 1.02

$$\frac{\log\left(x+1\right)}{3}+\frac{5\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x**3+1),x)
```

```
[Out] log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)
/3)/3
```

Giac [A] time = 1.06795, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log(
abs(x + 1))
```

3.312 $\int \frac{x(1+2x)}{1-x^3} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0395694, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1875, 31, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x^2]/2
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+2x)}{1-x^3}\,dx &= \frac{1}{3}\int \frac{-3-3x}{1+x+x^2}\,dx+\int \frac{1}{1-x}\,dx\\ &= -\log(1-x)-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.010297, size = 53, normalized size = 1.36

$$\frac{1}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(1-x^3\right)-\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x))/(1 - x^3),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6 -
(2*Log[1 - x^3])/3
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$-\ln(-1+x)-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+2*x)/(-x^3+1),x)
```

```
[Out] -ln(-1+x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.40839, size = 43, normalized size = 1.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
 1)
```

Fricas [A] time = 1.50109, size = 108, normalized size = 2.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x -
 1)
```

Sympy [A] time = 0.12768, size = 41, normalized size = 1.05

$$-\log\left(x-1\right)-\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x**3+1),x)
```

```
[Out] -log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/3
```

Giac [A] time = 1.05645, size = 45, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{2}\log\left(x^2+x+1\right)-\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)/(-x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(abs
(x - 1))
```

3.313 $\int x^2 (c + dx + ex^2) (a + bx^3) \, dx$

Optimal. Leaf size=55

$$\frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8$$

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Rubi [A] time = 0.0554037, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$\frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 (c + dx + ex^2) (a + bx^3) \, dx &= \int (acx^2 + adx^3 + aex^4 + bcx^5 + bdx^6 + bex^7) \, dx \\ &= \frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8\end{aligned}$$

Mathematica [A] time = 0.0032462, size = 55, normalized size = 1.

$$\frac{1}{3}acx^3 + \frac{1}{4}adx^4 + \frac{1}{5}aex^5 + \frac{1}{6}bcx^6 + \frac{1}{7}bdx^7 + \frac{1}{8}bex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e*
x^8)/8
```

Maple [A] time = 0.002, size = 44, normalized size = 0.8

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bcx^6}{6} + \frac{bdx^7}{7} + \frac{bex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

Maxima [A] time = 0.928077, size = 58, normalized size = 1.05

$$\frac{1}{8}\,bex^{8} + \frac{1}{7}\,bdx^{7} + \frac{1}{6}\,bcx^{6} + \frac{1}{5}\,aex^{5} + \frac{1}{4}\,adx^{4} + \frac{1}{3}\,acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

Fricas [A] time = 1.29515, size = 112, normalized size = 2.04

$$\frac{1}{8}x^{8}eb + \frac{1}{7}x^{7}db + \frac{1}{6}x^{6}cb + \frac{1}{5}x^{5}ea + \frac{1}{4}x^{4}da + \frac{1}{3}x^{3}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*b + 1/7*x^7*d*b + 1/6*x^6*c*b + 1/5*x^5*e*a + 1/4*x^4*d*a + 1/3*x
^3*c*a
```

Sympy [A] time = 0.063593, size = 49, normalized size = 0.89

$$\frac{acx^{3}}{3} + \frac{adx^{4}}{4} + \frac{aex^{5}}{5} + \frac{bcx^{6}}{6} + \frac{bdx^{7}}{7} + \frac{bex^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 + b*e*x**8/8
```

Giac [A] time = 1.06384, size = 61, normalized size = 1.11

$$\frac{1}{8}\,bx^{8}e + \frac{1}{7}\,bdx^{7} + \frac{1}{6}\,bcx^{6} + \frac{1}{5}\,ax^{5}e + \frac{1}{4}\,adx^{4} + \frac{1}{3}\,acx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*e + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*x^5*e + 1/4*a*d*x^4 + 1/3*a
*c*x^3
```

3.314 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Rubi [A] time = 0.0376349, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1628}

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx&=\int\left(acx+adx^2+aex^3+bcx^4+bdx^5+bex^6\right)\,dx\\&=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7\end{aligned}$$

Mathematica [A] time = 0.0022814, size = 55, normalized size = 1.

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e*
x^7)/7
```

Maple [A] time = 0.001, size = 44, normalized size = 0.8

$$\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

Maxima [A] time = 0.945712, size = 58, normalized size = 1.05

$$\frac{1}{7}\,bex^7 + \frac{1}{6}\,bdx^6 + \frac{1}{5}\,bcx^5 + \frac{1}{4}\,aex^4 + \frac{1}{3}\,adx^3 + \frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

Fricas [A] time = 1.28433, size = 112, normalized size = 2.04

$$\frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4ea + \frac{1}{3}x^3da + \frac{1}{2}x^2ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*e*a + 1/3*x^3*d*a + 1/2*x
^2*c*a
```

Sympy [A] time = 0.063504, size = 49, normalized size = 0.89

$$\frac{acx^2}{2} + \frac{adx^3}{3} + \frac{aex^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/7
```

Giac [A] time = 1.06715, size = 61, normalized size = 1.11

$$\frac{1}{7}\,bx^7e + \frac{1}{6}\,bdx^6 + \frac{1}{5}\,bcx^5 + \frac{1}{4}\,ax^4e + \frac{1}{3}\,adx^3 + \frac{1}{2}\,acx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*x^4*e + 1/3*a*d*x^3 + 1/2*a
*c*x^2
```

3.315 $\int (c+dx+ex^2)(a+bx^3)\,dx$

Optimal. Leaf size=50

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Rubi [A] time = 0.0245263, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2)(a+bx^3)\,dx &= \int (ac+adx+aex^2+bcx^3+bdx^4+bex^5)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6\end{aligned}$$

Mathematica [A] time = 0.0018268, size = 50, normalized size = 1.

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/6
```

Maple [A] time = 0.002, size = 41, normalized size = 0.8

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{bcx^4}{4}+\frac{bdx^5}{5}+\frac{bex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

Maxima [A] time = 0.950306, size = 54, normalized size = 1.08

$$\frac{1}{6}\,bex^{6}+\frac{1}{5}\,bdx^{5}+\frac{1}{4}\,bcx^{4}+\frac{1}{3}\,aex^{3}+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A] time = 1.28266, size = 104, normalized size = 2.08

$$\frac{1}{6}x^{6}eb+\frac{1}{5}x^{5}db+\frac{1}{4}x^{4}cb+\frac{1}{3}x^{3}ea+\frac{1}{2}x^{2}da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*b + 1/5*x^5*d*b + 1/4*x^4*c*b + 1/3*x^3*e*a + 1/2*x^2*d*a + x*c*a
```

Sympy [A] time = 0.064803, size = 46, normalized size = 0.92

$$acx+\frac{adx^{2}}{2}+\frac{aex^{3}}{3}+\frac{bcx^{4}}{4}+\frac{bdx^{5}}{5}+\frac{bex^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x**6/6
```

Giac [A] time = 1.04892, size = 57, normalized size = 1.14

$$\frac{1}{6}\,bx^{6}e+\frac{1}{5}\,bdx^{5}+\frac{1}{4}\,bcx^{4}+\frac{1}{3}\,ax^{3}e+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6*e + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^2 + a*c*x
```

3.316 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx$

Optimal. Leaf size=46

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Rubi [A] time = 0.0245177, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx &= \int \left(ad + \frac{ac}{x} + aex + bcx^2 + bdx^3 + bex^4\right) dx\\ &= adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)\end{aligned}$$

Mathematica [A] time = 0.0041212, size = 46, normalized size = 1.

$$ac\log(x) + adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

Maple [A] time = 0.003, size = 39, normalized size = 0.9

$$adx + \frac{aex^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4} + \frac{bex^5}{5} + ac\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x,x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

Maxima [A] time = 0.949336, size = 51, normalized size = 1.11

$$\frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="maxima")
```

```
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

Fricas [A] time = 1.49063, size = 103, normalized size = 2.24

$$\frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="fricas")
```

```
[Out] 1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

Sympy [A] time = 0.272387, size = 44, normalized size = 0.96

$$ac\log(x) + adx + \frac{aex^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4} + \frac{bex^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*c*x**3/3 + b*d*x**4/4 + b*e*x**5/5
```

Giac [A] time = 1.08311, size = 55, normalized size = 1.2

$$\frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}ax^2e + adx + ac\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*e + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*x^2*e + a*d*x + a*c*log(abs
(x))
```

3.317 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Rubi [A] time = 0.0350813, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx &= \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+bcx+bdx^2+bex^3\right)dx\\ &= -\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)\end{aligned}$$

Mathematica [A] time = 0.0048538, size = 44, normalized size = 1.

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

Maple [A] time = 0.004, size = 39, normalized size = 0.9

$$-\frac{ac}{x}+aex+\frac{bcx^2}{2}+\frac{bdx^3}{3}+\frac{bex^4}{4}+ad\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^2,x)
```

```
[Out] -a*c/x+a*e*x+1/2*b*c*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

Maxima [A] time = 0.943359, size = 51, normalized size = 1.16

$$\frac{1}{4}bex^4 + \frac{1}{3}bdx^3 + \frac{1}{2}bcx^2 + aex + ad\log(x) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(x) - a*c/x
```

Fricas [A] time = 1.43856, size = 113, normalized size = 2.57

$$\frac{3\,bex^5 + 4\,bdx^4 + 6\,bcx^3 + 12\,aex^2 + 12\,adx\log(x) - 12\,ac}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*e*x^5 + 4*b*d*x^4 + 6*b*c*x^3 + 12*a*e*x^2 + 12*a*d*x*log(x) - 12
*a*c)/x
```

Sympy [A] time = 0.296316, size = 41, normalized size = 0.93

$$-\frac{ac}{x} + ad\log(x) + aex + \frac{bcx^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*c*x**2/2 + b*d*x**3/3 + b*e*x**4/4
```

Giac [A] time = 1.06735, size = 55, normalized size = 1.25

$$\frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{2}bcx^2 + axe + ad\log(|x|) - \frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*x*e + a*d*log(abs(x)) - a*c/x
```

3.318 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx$

Optimal. Leaf size=44

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Rubi [A] time = 0.0341978, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1628}

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
```

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx &= \int \left(bc+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+bdx+bex^2\right)dx\\ &= -\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3+ae\log(x)\end{aligned}$$

Mathematica [A] time = 0.0040648, size = 44, normalized size = 1.

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
```

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

Maple [A] time = 0.005, size = 39, normalized size = 0.9

$$-\frac{ac}{2\,x^2}-\frac{ad}{x}+bcx+\frac{bdx^2}{2}+\frac{bex^3}{3}+ae\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)/x^3,x)
```

```
[Out] -1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

Maxima [A] time = 0.937409, size = 51, normalized size = 1.16

$$\frac{1}{3}\,bex^3 + \frac{1}{2}\,bdx^2 + bcx + ae\log\left(x\right) - \frac{2\,adx + ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*log(x) - 1/2*(2*a*d*x + a*c)/x^2
```

Fricas [A] time = 1.50428, size = 111, normalized size = 2.52

$$\frac{2\,bex^5 + 3\,bdx^4 + 6\,bcx^3 + 6\,aex^2\log\left(x\right) - 6\,adx - 3\,ac}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*e*x^5 + 3*b*d*x^4 + 6*b*c*x^3 + 6*a*e*x^2*log(x) - 6*a*d*x - 3*a*c
)/x^2
```

Sympy [A] time = 0.365596, size = 42, normalized size = 0.95

$$ae\log\left(x\right) + bcx + \frac{bdx^2}{2} + \frac{bex^3}{3} - \frac{ac + 2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)/x**3,x)
```

```
[Out] a*e*log(x) + b*c*x + b*d*x**2/2 + b*e*x**3/3 - (a*c + 2*a*d*x)/(2*x**2)
```

Giac [A] time = 1.06201, size = 55, normalized size = 1.25

$$\frac{1}{3}\,bx^3e + \frac{1}{2}\,bdx^2 + bcx + ae\log\left(|x|\right) - \frac{2\,adx + ac}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*e + 1/2*b*d*x^2 + b*c*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c)/x
^2
```

3.319 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^
10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.0589412, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1582, 1850}

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x^
10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx &= \frac{c\left(a+bx^3\right)^3}{9b}+\int \left(a+bx^3\right)^2\left(-cx^2+x^2\left(c+dx+ex^2\right)\right)\,dx\\
&= \frac{c\left(a+bx^3\right)^3}{9b}+\int \left(a^2dx^3+a^2ex^4+2abdx^6+2abex^7+b^2dx^9+b^2ex^{10}\right)\,dx\\
&= \frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{c\left(a+bx^3\right)^3}{9b}
\end{aligned}$$

Mathematica [A] time = 0.0035515, size = 97, normalized size = 1.18

$$\frac{1}{3}a^2cx^3+\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{3}abcx^6+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (2*a*b*d*x^
7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11
```

Maple [A] time = 0.002, size = 80, normalized size = 1.

$$\frac{b^2ex^{11}}{11}+\frac{b^2dx^{10}}{10}+\frac{b^2cx^9}{9}+\frac{abex^8}{4}+\frac{2\,abdx^7}{7}+\frac{abcx^6}{3}+\frac{a^2ex^5}{5}+\frac{a^2dx^4}{4}+\frac{a^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*e*x^8+2/7*a*b*d*x^7+1
/3*a*b*c*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3
```

Maxima [A] time = 0.938983, size = 107, normalized size = 1.3

$$\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

Fricas [A] time = 1.22065, size = 198, normalized size = 2.41

$$\frac{1}{11}x^{11}eb^2+\frac{1}{10}x^{10}db^2+\frac{1}{9}x^9cb^2+\frac{1}{4}x^8eba+\frac{2}{7}x^7dba+\frac{1}{3}x^6cba+\frac{1}{5}x^5ea^2+\frac{1}{4}x^4da^2+\frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x^8*e*b*a + 2/7*x^7
*d*b*a + 1/3*x^6*c*b*a + 1/5*x^5*e*a^2 + 1/4*x^4*d*a^2 + 1/3*x^3*c*a^2
```

Sympy [A] time = 0.07362, size = 92, normalized size = 1.12

$$\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{abcx^6}{3}+\frac{2abdx^7}{7}+\frac{abex^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*a*b*d*x**7
/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11
```

Giac [A] time = 1.08003, size = 111, normalized size = 1.35

$$\frac{1}{11}\,b^2x^{11}e+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abx^8e+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2x^5e+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*x^8*e + 2/7*a*b
*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.320 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (2*a*b*e*x^7)/7 + (b^2*c*
x^8)/8 + (b^2*e*x^10)/10 + (d*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.0510239, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1582, 1850}

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (2*a*b*e*x^7)/7 + (b^2*c*
x^8)/8 + (b^2*e*x^10)/10 + (d*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx&=\frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a+bx^3\right)^2\left(-dx^2+x\left(c+dx+ex^2\right)\right)dx\\&=\frac{d\left(a+bx^3\right)^3}{9b}+\int\left(a^2cx+a^2ex^3+2abcx^4+2abex^6+b^2cx^7+b^2ex^9\right)dx\\&=\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d\left(a+bx^3\right)^3}{9b}\end{aligned}$$

Mathematica [A] time = 0.0030289, size = 97, normalized size = 1.18

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x^
6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9 + (b^2*e*x^10)/10
```

Maple [A] time = 0., size = 80, normalized size = 1.

$$\frac{b^2ex^{10}}{10}+\frac{b^2dx^9}{9}+\frac{b^2cx^8}{8}+\frac{2\,abex^7}{7}+\frac{abdx^6}{3}+\frac{2\,abcx^5}{5}+\frac{a^2ex^4}{4}+\frac{a^2dx^3}{3}+\frac{a^2cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/10*b^2*e*x^10+1/9*b^2*d*x^9+1/8*b^2*c*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5
*a*b*c*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2
```

Maxima [A] time = 0.932844, size = 107, normalized size = 1.3

$$\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

Fricas [A] time = 1.31605, size = 196, normalized size = 2.39

$$\frac{1}{10}x^{10}eb^2+\frac{1}{9}x^9db^2+\frac{1}{8}x^8cb^2+\frac{2}{7}x^7eba+\frac{1}{3}x^6dba+\frac{2}{5}x^5cba+\frac{1}{4}x^4ea^2+\frac{1}{3}x^3da^2+\frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*b^2 + 1/9*x^9*d*b^2 + 1/8*x^8*c*b^2 + 2/7*x^7*e*b*a + 1/3*x^6*d
*b*a + 2/5*x^5*c*b*a + 1/4*x^4*e*a^2 + 1/3*x^3*d*a^2 + 1/2*x^2*c*a^2
```

Sympy [A] time = 0.071493, size = 94, normalized size = 1.15

$$\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{b^2cx^8}{8}+\frac{b^2dx^9}{9}+\frac{b^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**6
/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b**2*e*x**10/10
```

Giac [A] time = 1.0604, size = 111, normalized size = 1.35

$$\frac{1}{10}\,b^2x^{10}e + \frac{1}{9}\,b^2dx^9 + \frac{1}{8}\,b^2cx^8 + \frac{2}{7}\,abx^7e + \frac{1}{3}\,abdx^6 + \frac{2}{5}\,abcx^5 + \frac{1}{4}\,a^2x^4e + \frac{1}{3}\,a^2dx^3 + \frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10*e + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d
*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.321 $\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2 dx$

Optimal. Leaf size=77

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.0610721, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1582, 1850}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7 +
 (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^2 dx &= \frac{e\left(a+bx^3\right)^3}{9b}+\int (c+dx)\left(a+bx^3\right)^2 dx\\ &= \frac{e\left(a+bx^3\right)^3}{9b}+\int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right) dx\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e\left(a+bx^3\right)^3}{9b}\end{aligned}$$

Mathematica [A] time = 0.0027036, size = 92, normalized size = 1.19

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 +
 (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e*x^9)/9
```

Maple [A] time = 0.001, size = 77, normalized size = 1.

$$\frac{b^2ex^9}{9}+\frac{b^2dx^8}{8}+\frac{b^2cx^7}{7}+\frac{abex^6}{3}+\frac{2\,abdx^5}{5}+\frac{abcx^4}{2}+\frac{a^2ex^3}{3}+\frac{a^2dx^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2,x)
```

```
[Out] 1/9*b^2*e*x^9+1/8*b^2*d*x^8+1/7*b^2*c*x^7+1/3*a*b*e*x^6+2/5*a*b*d*x^5+1/2*a
*b*c*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Maxima [A] time = 0.939354, size = 103, normalized size = 1.34

$$\frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A] time = 1.36238, size = 185, normalized size = 2.4

$$\frac{1}{9}x^9eb^2+\frac{1}{8}x^8db^2+\frac{1}{7}x^7cb^2+\frac{1}{3}x^6eba+\frac{2}{5}x^5dba+\frac{1}{2}x^4cba+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e*b^2 + 1/8*x^8*d*b^2 + 1/7*x^7*c*b^2 + 1/3*x^6*e*b*a + 2/5*x^5*d*b
*a + 1/2*x^4*c*b*a + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.097755, size = 88, normalized size = 1.14

$$a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{abex^6}{3}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}+\frac{b^2ex^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 +
a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x**9/9
```

Giac [A] time = 1.06257, size = 107, normalized size = 1.39

$$\frac{1}{9}b^2x^9e + \frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{1}{3}abx^6e + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9*e + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*x^6*e + 2/5*a*b*d*x
^5 + 1/2*a*b*c*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

3.322 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx$

Optimal. Leaf size=88

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Rubi [A] time = 0.0518618, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx &= \int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+2abdx^3+2abex^4+b^2cx^5+b^2dx^6+b^2ex^7\right)dx\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8+a^2c\log(\end{aligned}$$

Mathematica [A] time = 0.0076852, size = 88, normalized size = 1.

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

Maple [A] time = 0.001, size = 75, normalized size = 0.9

$$a^2dx + \frac{a^2ex^2}{2} + \frac{2\,abcx^3}{3} + \frac{abdx^4}{2} + \frac{2\,abex^5}{5} + \frac{b^2cx^6}{6} + \frac{b^2dx^7}{7} + \frac{b^2ex^8}{8} + a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x,x)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*x
^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

Maxima [A] time = 0.948685, size = 100, normalized size = 1.14

$$\frac{1}{8}\,b^2ex^8 + \frac{1}{7}\,b^2dx^7 + \frac{1}{6}\,b^2cx^6 + \frac{2}{5}\,abex^5 + \frac{1}{2}\,abdx^4 + \frac{2}{3}\,abcx^3 + \frac{1}{2}\,a^2ex^2 + a^2dx + a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

Fricas [A] time = 1.49492, size = 184, normalized size = 2.09

$$\frac{1}{8}\,b^2ex^8 + \frac{1}{7}\,b^2dx^7 + \frac{1}{6}\,b^2cx^6 + \frac{2}{5}\,abex^5 + \frac{1}{2}\,abdx^4 + \frac{2}{3}\,abcx^3 + \frac{1}{2}\,a^2ex^2 + a^2dx + a^2c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

Sympy [A] time = 0.352949, size = 88, normalized size = 1.

$$a^2c\log(x) + a^2dx + \frac{a^2ex^2}{2} + \frac{2abcx^3}{3} + \frac{abdx^4}{2} + \frac{2abex^5}{5} + \frac{b^2cx^6}{6} + \frac{b^2dx^7}{7} + \frac{b^2ex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + 2*a*b*c*x**3/3 + a*b*d*x**4/2 +
2*a*b*e*x**5/5 + b**2*c*x**6/6 + b**2*d*x**7/7 + b**2*e*x**8/8
```

Giac [A] time = 1.05081, size = 105, normalized size = 1.19

$$\frac{1}{8}\,b^2x^8e + \frac{1}{7}\,b^2dx^7 + \frac{1}{6}\,b^2cx^6 + \frac{2}{5}\,abx^5e + \frac{1}{2}\,abdx^4 + \frac{2}{3}\,abcx^3 + \frac{1}{2}\,a^2x^2e + a^2dx + a^2c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8*e + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*x^5*e + 1/2*a*b*d*x
^4 + 2/3*a*b*c*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))
```

3.323 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} dx$

Optimal. Leaf size=83

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Rubi [A] time = 0.0629493, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx &= \int \left(a^2e+\frac{a^2c}{x^2}+\frac{a^2d}{x}+2abcx+2abdx^2+2abex^3+b^2cx^4+b^2dx^5+b^2ex^6\right)dx\\ &= -\frac{a^2c}{x}+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7+a^2d\log(x\end{aligned}$$

Mathematica [A] time = 0.0077615, size = 83, normalized size = 1.

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^2
*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

Maple [A] time = 0.005, size = 74, normalized size = 0.9

$$-\frac{a^2c}{x}+a^2ex+abc x^2+\frac{2\,abdx^3}{3}+\frac{abex^4}{2}+\frac{b^2cx^5}{5}+\frac{b^2dx^6}{6}+\frac{b^2ex^7}{7}+a^2d\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x)
```

```
[Out] -a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b^
2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

Maxima [A] time = 0.948298, size = 99, normalized size = 1.19

$$\frac{1}{7}\,b^2ex^7+\frac{1}{6}\,b^2dx^6+\frac{1}{5}\,b^2cx^5+\frac{1}{2}\,abex^4+\frac{2}{3}\,abdx^3+abcx^2+a^2ex+a^2d\log\left(x\right)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(x) - a^2*c/x
```

Fricas [A] time = 1.53432, size = 204, normalized size = 2.46

$$\frac{30\,b^2ex^8+35\,b^2dx^7+42\,b^2cx^6+105\,abex^5+140\,abdx^4+210\,abcx^3+210\,a^2ex^2+210\,a^2dx\log\left(x\right)-210\,a^2c}{210\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/210*(30*b^2*e*x^8 + 35*b^2*d*x^7 + 42*b^2*c*x^6 + 105*a*b*e*x^5 + 140*a*b
*d*x^4 + 210*a*b*c*x^3 + 210*a^2*e*x^2 + 210*a^2*d*x*log(x) - 210*a^2*c)/x
```

Sympy [A] time = 0.339739, size = 82, normalized size = 0.99

$$-\frac{a^2c}{x}+a^2d\log\left(x\right)+a^2ex+abcx^2+\frac{2abdx^3}{3}+\frac{abex^4}{2}+\frac{b^2cx^5}{5}+\frac{b^2dx^6}{6}+\frac{b^2ex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + a*b*c*x**2 + 2*a*b*d*x**3/3 + a*b*e*
x**4/2 + b**2*c*x**5/5 + b**2*d*x**6/6 + b**2*e*x**7/7
```

Giac [A] time = 1.05151, size = 104, normalized size = 1.25

$$\frac{1}{7}\,b^2x^7e+\frac{1}{6}\,b^2dx^6+\frac{1}{5}\,b^2cx^5+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+abcx^2+a^2xe+a^2d\log\left(|x|\right)-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7*e + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*x^4*e + 2/3*a*b*d*x
^3 + a*b*c*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x
```

3.324 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Rubi [A] time = 0.0641961, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx &= \int \left(2abc+\frac{a^2c}{x^3}+\frac{a^2d}{x^2}+\frac{a^2e}{x}+2abdx+2abex^2+b^2cx^3+b^2dx^4+b^2ex^5\right)dx\\ &= -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a^2e\log(x)\end{aligned}$$

Mathematica [A] time = 0.0084296, size = 84, normalized size = 1.

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (b
^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

Maple [A] time = 0.006, size = 75, normalized size = 0.9

$$-\frac{a^2c}{2\,x^2}-\frac{a^2d}{x}+2\,abcx+abd x^2+\frac{2\,abe x^3}{3}+\frac{b^2c x^4}{4}+\frac{b^2d x^5}{5}+\frac{b^2e x^6}{6}+a^2e\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x)
```

```
[Out] -1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+a*b*d*x^2+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5*
b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

Maxima [A] time = 0.943366, size = 100, normalized size = 1.19

$$\frac{1}{6}\,b^2ex^6+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abex^3+abdx^2+2\,abcx+a^2e\log(x)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(x) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

Fricas [A] time = 1.46572, size = 198, normalized size = 2.36

$$\frac{10\,b^2ex^8+12\,b^2dx^7+15\,b^2cx^6+40\,abex^5+60\,abdx^4+120\,abcx^3+60\,a^2ex^2\log(x)-60\,a^2dx-30\,a^2c}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*e*x^8 + 12*b^2*d*x^7 + 15*b^2*c*x^6 + 40*a*b*e*x^5 + 60*a*b*d*
x^4 + 120*a*b*c*x^3 + 60*a^2*e*x^2*log(x) - 60*a^2*d*x - 30*a^2*c)/x^2
```

Sympy [A] time = 0.394683, size = 85, normalized size = 1.01

$$a^2e\log(x)+2abcx+abdx^2+\frac{2abex^3}{3}+\frac{b^2cx^4}{4}+\frac{b^2dx^5}{5}+\frac{b^2ex^6}{6}-\frac{a^2c+2a^2dx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**3,x)
```

```
[Out] a**2*e*log(x) + 2*a*b*c*x + a*b*d*x**2 + 2*a*b*e*x**3/3 + b**2*c*x**4/4 + b
**2*d*x**5/5 + b**2*e*x**6/6 - (a**2*c + 2*a**2*d*x)/(2*x**2)
```

Giac [A] time = 1.05293, size = 105, normalized size = 1.25

$$\frac{1}{6}\,b^2x^6e+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abx^3e+abdx^2+2\,abcx+a^2e\log(|x|)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6*e + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*x^3*e + a*b*d*x^2 +
 2*a*b*c*x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

3.325 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^3 dx$

Optimal. Leaf size=110

$$\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*
a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
+ (c*(a + b*x^3)^4)/(12*b)
```

Rubi [A] time = 0.0787603, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1582, 1850}

$$\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3*
a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
+ (c*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^3 dx &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a+bx^3\right)^3 \left(-cx^2+x^2\left(c+dx+ex^2\right)\right) dx \\
&= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a^3dx^3+a^3ex^4+3a^2bdx^6+3a^2bex^7+3ab^2dx^9+3ab^2ex^{10}+b\right. \\
&= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3dx^{13}+
\end{aligned}$$

Mathematica [A] time = 0.0038531, size = 139, normalized size = 1.26

$$\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 + (3*a^2*b*
d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a
*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
```

Maple [A] time = 0.001, size = 116, normalized size = 1.1

$$\frac{b^3ex^{14}}{14}+\frac{b^3dx^{13}}{13}+\frac{b^3cx^{12}}{12}+\frac{3\,ab^2ex^{11}}{11}+\frac{3\,ab^2dx^{10}}{10}+\frac{ab^2cx^9}{3}+\frac{3\,a^2bex^8}{8}+\frac{3\,a^2bdx^7}{7}+\frac{a^2bcx^6}{2}+\frac{a^3ex^5}{5}+\frac{a^3dx^4}{4}+\frac{a^3cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] 1/14*b^3*e*x^14+1/13*b^3*d*x^13+1/12*b^3*c*x^12+3/11*a*b^2*e*x^11+3/10*a*b^
2*d*x^10+1/3*a*b^2*c*x^9+3/8*a^2*b*e*x^8+3/7*a^2*b*d*x^7+1/2*a^2*b*c*x^6+1/
5*a^3*e*x^5+1/4*a^3*d*x^4+1/3*a^3*c*x^3
```

Maxima [A] time = 0.941417, size = 155, normalized size = 1.41

$$\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

Fricas [A] time = 1.30698, size = 288, normalized size = 2.62

$$\frac{1}{14}x^{14}eb^3+\frac{1}{13}x^{13}db^3+\frac{1}{12}x^{12}cb^3+\frac{3}{11}x^{11}eb^2a+\frac{3}{10}x^{10}db^2a+\frac{1}{3}x^9cb^2a+\frac{3}{8}x^8eba^2+\frac{3}{7}x^7dba^2+\frac{1}{2}x^6cba^2+\frac{1}{5}x^5ea^3+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*e*b^3 + 1/13*x^13*d*b^3 + 1/12*x^12*c*b^3 + 3/11*x^11*e*b^2*a + 3
/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3/8*x^8*e*b*a^2 + 3/7*x^7*d*b*a^2 + 1/
2*x^6*c*b*a^2 + 1/5*x^5*e*a^3 + 1/4*x^4*d*a^3 + 1/3*x^3*c*a^3
```

Sympy [A] time = 0.075398, size = 138, normalized size = 1.25

$$\frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{a^2bcx^6}{2} + \frac{3a^2bdx^7}{7} + \frac{3a^2bex^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{b^3cx^{12}}{12} + \frac{b^3dx^{13}}{13} + \frac{b^3ex}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 + 3*a**2*b*
d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*a*
b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x**13/13 + b**3*e*x**14/14
```

Giac [A] time = 1.05876, size = 161, normalized size = 1.46

$$\frac{1}{14}\,b^3x^{14}e + \frac{1}{13}\,b^3dx^{13} + \frac{1}{12}\,b^3cx^{12} + \frac{3}{11}\,ab^2x^{11}e + \frac{3}{10}\,ab^2dx^{10} + \frac{1}{3}\,ab^2cx^9 + \frac{3}{8}\,a^2bx^8e + \frac{3}{7}\,a^2bdx^7 + \frac{1}{2}\,a^2bcx^6 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*x^11*e + 3
/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/
2*a^2*b*c*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.326 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=110

$$\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*
a*b^2*c*x^8)/8 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*e*x^13)/13 +
(d*(a + b*x^3)^4)/(12*b)
```

Rubi [A] time = 0.0693723, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1582, 1850}

$$\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3*
a*b^2*c*x^8)/8 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*e*x^13)/13 +
(d*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^3\,dx&=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a+bx^3\right)^3\left(-dx^2+x\left(c+dx+ex^2\right)\right)dx\\&=\frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a^3cx+a^3ex^3+3a^2bcx^4+3a^2bex^6+3ab^2cx^7+3ab^2ex^9+b^3cx^{10}\right.\\&=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3e\end{aligned}$$

Mathematica [A] time = 0.0036847, size = 139, normalized size = 1.26

$$\frac{3}{5}a^2bcx^5 + \frac{1}{2}a^2bdx^6 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^3cx^2 + \frac{1}{3}a^3dx^3 + \frac{1}{4}a^3ex^4 + \frac{3}{8}ab^2cx^8 + \frac{1}{3}ab^2dx^9 + \frac{3}{10}ab^2ex^{10} + \frac{1}{11}b^3cx^{11} + \frac{1}{12}b^3d$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b*
d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b^2*d*x^9)/3 + (3*a*b
^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12)/12 + (b^3*e*x^13)/13
```

Maple [A] time = 0.001, size = 116, normalized size = 1.1

$$\frac{b^3ex^{13}}{13} + \frac{b^3dx^{12}}{12} + \frac{b^3cx^{11}}{11} + \frac{3\,ab^2ex^{10}}{10} + \frac{ab^2dx^9}{3} + \frac{3\,ab^2cx^8}{8} + \frac{3\,a^2bex^7}{7} + \frac{a^2bdx^6}{2} + \frac{3\,a^2bcx^5}{5} + \frac{a^3ex^4}{4} + \frac{a^3dx^3}{3} + \frac{a^3c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] 1/13*b^3*e*x^13+1/12*b^3*d*x^12+1/11*b^3*c*x^11+3/10*a*b^2*e*x^10+1/3*a*b^2
*d*x^9+3/8*a*b^2*c*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*
a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

Maxima [A] time = 0.919483, size = 155, normalized size = 1.41

$$\frac{1}{13}\,b^3ex^{13} + \frac{1}{12}\,b^3dx^{12} + \frac{1}{11}\,b^3cx^{11} + \frac{3}{10}\,ab^2ex^{10} + \frac{1}{3}\,ab^2dx^9 + \frac{3}{8}\,ab^2cx^8 + \frac{3}{7}\,a^2bex^7 + \frac{1}{2}\,a^2bdx^6 + \frac{3}{5}\,a^2bcx^5 + \frac{1}{4}\,a^3ex^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

Fricas [A] time = 1.26776, size = 285, normalized size = 2.59

$$\frac{1}{13}x^{13}eb^3 + \frac{1}{12}x^{12}db^3 + \frac{1}{11}x^{11}cb^3 + \frac{3}{10}x^{10}eb^2a + \frac{1}{3}x^9db^2a + \frac{3}{8}x^8cb^2a + \frac{3}{7}x^7eba^2 + \frac{1}{2}x^6dba^2 + \frac{3}{5}x^5cba^2 + \frac{1}{4}x^4ea^3 + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e*b^3 + 1/12*x^12*d*b^3 + 1/11*x^11*c*b^3 + 3/10*x^10*e*b^2*a + 1
/3*x^9*d*b^2*a + 3/8*x^8*c*b^2*a + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*
x^5*c*b*a^2 + 1/4*x^4*e*a^3 + 1/3*x^3*d*a^3 + 1/2*x^2*c*a^3
```

Sympy [A] time = 0.076247, size = 138, normalized size = 1.25

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b*
d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b**2*d*x**9/3 + 3*a*b*
*2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**12/12 + b**3*e*x**13/13
```

Giac [A] time = 1.08196, size = 161, normalized size = 1.46

$$\frac{1}{13}b^3x^{13}e+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2x^{10}e+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bx^7e+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3x^4e+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*x^10*e + 1
/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*
a^2*b*c*x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.327 $\int \left(c+dx+ex^2\right) \left(a+bx^3\right)^3 \, dx$

Optimal. Leaf size=105

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (e*(a +
b*x^3)^4)/(12*b)
```

Rubi [A] time = 0.0969183, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1582, 1850}

$$\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*
c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (e*(a +
b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^3 \, dx &= \frac{e\left(a+bx^3\right)^4}{12b}+\int (c+dx)\left(a+bx^3\right)^3 \, dx\\
&= \frac{e\left(a+bx^3\right)^4}{12b}+\int \left(a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b^3cx^9+\right.\\
&= a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{1}
\end{aligned}$$

Mathematica [A] time = 0.0036741, size = 134, normalized size = 1.28

$$\frac{3}{4}a^2bcx^4 + \frac{3}{5}a^2bdx^5 + \frac{1}{2}a^2bex^6 + a^3cx + \frac{1}{2}a^3dx^2 + \frac{1}{3}a^3ex^3 + \frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{3}ab^2ex^9 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11} + \frac{1}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^
5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (a*b^2*e*x
^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (b^3*e*x^12)/12
```

Maple [A] time = 0.001, size = 113, normalized size = 1.1

$$\frac{b^3ex^{12}}{12} + \frac{b^3dx^{11}}{11} + \frac{b^3cx^{10}}{10} + \frac{ab^2ex^9}{3} + \frac{3\,ab^2dx^8}{8} + \frac{3\,ab^2cx^7}{7} + \frac{a^2bex^6}{2} + \frac{3\,a^2bdx^5}{5} + \frac{3\,a^2bcx^4}{4} + \frac{a^3ex^3}{3} + \frac{a^3dx^2}{2} + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3,x)
```

```
[Out] 1/12*b^3*e*x^12+1/11*b^3*d*x^11+1/10*b^3*c*x^10+1/3*a*b^2*e*x^9+3/8*a*b^2*d
*x^8+3/7*a*b^2*c*x^7+1/2*a^2*b*e*x^6+3/5*a^2*b*d*x^5+3/4*a^2*b*c*x^4+1/3*a^
3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 0.93441, size = 151, normalized size = 1.44

$$\frac{1}{12}\,b^3ex^{12} + \frac{1}{11}\,b^3dx^{11} + \frac{1}{10}\,b^3cx^{10} + \frac{1}{3}\,ab^2ex^9 + \frac{3}{8}\,ab^2dx^8 + \frac{3}{7}\,ab^2cx^7 + \frac{1}{2}\,a^2bex^6 + \frac{3}{5}\,a^2bdx^5 + \frac{3}{4}\,a^2bcx^4 + \frac{1}{3}\,a^3ex^3 + \frac{1}{2}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A] time = 1.28377, size = 274, normalized size = 2.61

$$\frac{1}{12}x^{12}eb^3 + \frac{1}{11}x^{11}db^3 + \frac{1}{10}x^{10}cb^3 + \frac{1}{3}x^9eb^2a + \frac{3}{8}x^8db^2a + \frac{3}{7}x^7cb^2a + \frac{1}{2}x^6eba^2 + \frac{3}{5}x^5dba^2 + \frac{3}{4}x^4cba^2 + \frac{1}{3}x^3ea^3 + \frac{1}{2}x^2da^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e*b^3 + 1/11*x^11*d*b^3 + 1/10*x^10*c*b^3 + 1/3*x^9*e*b^2*a + 3/8
*x^8*d*b^2*a + 3/7*x^7*c*b^2*a + 1/2*x^6*e*b*a^2 + 3/5*x^5*d*b*a^2 + 3/4*x^
4*c*b*a^2 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.075057, size = 134, normalized size = 1.28

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{3a^2bcx^4}{4} + \frac{3a^2bdx^5}{5} + \frac{a^2bex^6}{2} + \frac{3ab^2cx^7}{7} + \frac{3ab^2dx^8}{8} + \frac{ab^2ex^9}{3} + \frac{b^3cx^{10}}{10} + \frac{b^3dx^{11}}{11} + \frac{b^3ex^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x
**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + a*b**2*e*
x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 + b**3*e*x**12/12
```

Giac [A] time = 1.08264, size = 157, normalized size = 1.5

$$\frac{1}{12}b^3x^{12}e + \frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{1}{3}ab^2x^9e + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{1}{2}a^2bx^6e + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{3}a^3x^3e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*e + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*x^9*e + 3/8
*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*x^6*e + 3/5*a^2*b*d*x^5 + 3/4*a^
2*b*c*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

3.328 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x} \, dx$

Optimal. Leaf size=127

$$a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}$$

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Rubi [A] time = 0.0736885, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx &= \int \left(a^3d+\frac{a^3c}{x}+a^3ex+3a^2bcx^2+3a^2bdx^3+3a^2bex^4+3ab^2cx^5+3ab^2dx^6+3ab^2ex^7\right.\\ &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b\end{aligned}$$

Mathematica [A] time = 0.0087516, size = 127, normalized size = 1.

$$a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5)
/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9)/
9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

Maple [A] time = 0.001, size = 110, normalized size = 0.9

$$a^3dx+\frac{a^3ex^2}{2}+a^2bcx^3+\frac{3\,a^2bdx^4}{4}+\frac{3\,a^2bex^5}{5}+\frac{ab^2cx^6}{2}+\frac{3\,ab^2dx^7}{7}+\frac{3\,ab^2ex^8}{8}+\frac{b^3cx^9}{9}+\frac{b^3dx^{10}}{10}+\frac{b^3ex^{11}}{11}+a^3c\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x,x)
```

```
[Out] a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^2
*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11*b
^3*e*x^11+a^3*c*ln(x)
```

Maxima [A] time = 0.952839, size = 147, normalized size = 1.16

$$\frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

Fricas [A] time = 1.49194, size = 265, normalized size = 2.09

$$\frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

Sympy [A] time = 0.375604, size = 131, normalized size = 1.03

$$a^3c\log{(x)}+a^3dx+\frac{a^3ex^2}{2}+a^2bcx^3+\frac{3a^2bdx^4}{4}+\frac{3a^2bex^5}{5}+\frac{ab^2cx^6}{2}+\frac{3ab^2dx^7}{7}+\frac{3ab^2ex^8}{8}+\frac{b^3cx^9}{9}+\frac{b^3dx^{10}}{10}+\frac{b^3ex^1}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + a**2*b*c*x**3 + 3*a**2*b*d*x**4/
4 + 3*a**2*b*e*x**5/5 + a*b**2*c*x**6/2 + 3*a*b**2*d*x**7/7 + 3*a*b**2*e*x*
```

```
*8/8 + b**3*c*x**9/9 + b**3*d*x**10/10 + b**3*e*x**11/11
```

Giac [A] time = 1.07317, size = 154, normalized size = 1.21

$$\frac{1}{11}b^3x^{11}e+\frac{1}{10}b^3dx^{10}+\frac{1}{9}b^3cx^9+\frac{3}{8}ab^2x^8e+\frac{3}{7}ab^2dx^7+\frac{1}{2}ab^2cx^6+\frac{3}{5}a^2bx^5e+\frac{3}{4}a^2bdx^4+a^2bcx^3+\frac{1}{2}a^3x^2e+a^3dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*x^8*e + 3/7*a
*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*x^5*e + 3/4*a^2*b*d*x^4 + a^2*b*c*
x^3 + 1/2*a^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))
```

3.329 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} \, dx$

Optimal. Leaf size=125

$$\frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 - \frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3e$$

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Rubi [A] time = 0.0922203, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 - \frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3e$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2} \, dx = \int \left(a^3e + \frac{a^3c}{x^2} + \frac{a^3d}{x} + 3a^2bcx + 3a^2bdx^2 + 3a^2bex^3 + 3ab^2cx^4 + 3ab^2dx^5 + 3ab^2ex^6\right.$$
$$= -\frac{a^3c}{x} + a^3ex + \frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b$$

Mathematica [A] time = 0.0083931, size = 125, normalized size = 1.

$$\frac{3}{2}a^2bcx^2 + a^2bdx^3 + \frac{3}{4}a^2bex^4 - \frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{3}{5}ab^2cx^5 + \frac{1}{2}ab^2dx^6 + \frac{3}{7}ab^2ex^7 + \frac{1}{8}b^3cx^8 + \frac{1}{9}b^3dx^9 + \frac{1}{10}b^3e$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)/
4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)/8
 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

Maple [A] time = 0.006, size = 110, normalized size = 0.9

$$-\frac{a^3c}{x}+a^3ex+\frac{3\,a^2bc x^2}{2}+a^2bdx^3+\frac{3\,a^2bex^4}{4}+\frac{3\,ab^2cx^5}{5}+\frac{ab^2dx^6}{2}+\frac{3\,ab^2ex^7}{7}+\frac{b^3cx^8}{8}+\frac{b^3dx^9}{9}+\frac{b^3ex^{10}}{10}+a^3d\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x)
```

```
[Out] -a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x^
5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*x^
10+a^3*d*ln(x)
```

Maxima [A] time = 0.940401, size = 147, normalized size = 1.18

$$\frac{1}{10}\,b^3ex^{10}+\frac{1}{9}\,b^3dx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{7}\,ab^2ex^7+\frac{1}{2}\,ab^2dx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{4}\,a^2bex^4+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+a^3ex+a^3d\log(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*e*x + a^3*d*log(x) - a^3*c/x
```

Fricas [A] time = 1.45317, size = 305, normalized size = 2.44

$$\frac{252\,b^3ex^{11}+280\,b^3dx^{10}+315\,b^3cx^9+1080\,ab^2ex^8+1260\,ab^2dx^7+1512\,ab^2cx^6+1890\,a^2bex^5+2520\,a^2bdx^4+3780\,a^2}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^3*e*x^11 + 280*b^3*d*x^10 + 315*b^3*c*x^9 + 1080*a*b^2*e*x^8
+ 1260*a*b^2*d*x^7 + 1512*a*b^2*c*x^6 + 1890*a^2*b*e*x^5 + 2520*a^2*b*d*x^4
 + 3780*a^2*b*c*x^3 + 2520*a^3*e*x^2 + 2520*a^3*d*x*log(x) - 2520*a^3*c)/x
```

Sympy [A] time = 0.39922, size = 128, normalized size = 1.02

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3a^2bcx^2}{2}+a^2bdx^3+\frac{3a^2bex^4}{4}+\frac{3ab^2cx^5}{5}+\frac{ab^2dx^6}{2}+\frac{3ab^2ex^7}{7}+\frac{b^3cx^8}{8}+\frac{b^3dx^9}{9}+\frac{b^3ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + 3*a**2*b*c*x**2/2 + a**2*b*d*x**3 +
3*a**2*b*e*x**4/4 + 3*a*b**2*c*x**5/5 + a*b**2*d*x**6/2 + 3*a*b**2*e*x**7/7
```

```
+ b**3*c*x**8/8 + b**3*d*x**9/9 + b**3*e*x**10/10
```

Giac [A] time = 1.05804, size = 154, normalized size = 1.23

$$\frac{1}{10}\,b^3x^{10}e+\frac{1}{9}\,b^3dx^9+\frac{1}{8}\,b^3cx^8+\frac{3}{7}\,ab^2x^7e+\frac{1}{2}\,ab^2dx^6+\frac{3}{5}\,ab^2cx^5+\frac{3}{4}\,a^2bx^4e+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+a^3xe+a^3d\log(|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*x^7*e + 1/2*a*b
^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 3/2*a^2*b*c*x^
2 + a^3*x*e + a^3*d*log(abs(x)) - a^3*c/x
```

3.330 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx$

Optimal. Leaf size=126

$$3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Rubi [A] time = 0.0864589, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx+ex^2\right)\left(a+bx^3\right)^3}{x^3}\,dx &= \int \left(3a^2bc+\frac{a^3c}{x^3}+\frac{a^3d}{x^2}+\frac{a^3e}{x}+3a^2bdx+3a^2bex^2+3ab^2cx^3+3ab^2dx^4+3ab^2ex^5+b^3\right.\\ &= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7\end{aligned}$$

Mathematica [A] time = 0.0090364, size = 126, normalized size = 1.

$$3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x^
3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)/7
 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

Maple [A] time = 0.006, size = 111, normalized size = 0.9

$$-\frac{a^{3}c}{2\,x^{2}}-\frac{a^{3}d}{x}+3\,a^{2}bcx+\frac{3\,a^{2}bd x^{2}}{2}+a^{2}bex^{3}+\frac{3\,ab^{2}cx^{4}}{4}+\frac{3\,ab^{2}dx^{5}}{5}+\frac{ab^{2}ex^{6}}{2}+\frac{b^{3}cx^{7}}{7}+\frac{b^{3}dx^{8}}{8}+\frac{b^{3}ex^{9}}{9}+a^{3}e\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x)
```

```
[Out] -1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c*
x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e*x
^9+a^3*e*ln(x)
```

Maxima [A] time = 0.935902, size = 149, normalized size = 1.18

$$\frac{1}{9}\,b^{3}ex^{9}+\frac{1}{8}\,b^{3}dx^{8}+\frac{1}{7}\,b^{3}cx^{7}+\frac{1}{2}\,ab^{2}ex^{6}+\frac{3}{5}\,ab^{2}dx^{5}+\frac{3}{4}\,ab^{2}cx^{4}+a^{2}bex^{3}+\frac{3}{2}\,a^{2}bdx^{2}+3\,a^{2}bcx+a^{3}e\log\left(x\right)-\frac{2\,a^{3}dx}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*e*x^6 + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*e*x^3 + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(x) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Fricas [A] time = 1.46691, size = 308, normalized size = 2.44

$$\frac{280\,b^{3}ex^{11}+315\,b^{3}dx^{10}+360\,b^{3}cx^{9}+1260\,ab^{2}ex^{8}+1512\,ab^{2}dx^{7}+1890\,ab^{2}cx^{6}+2520\,a^{2}bex^{5}+3780\,a^{2}bdx^{4}+756}{2520\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*b^3*e*x^11 + 315*b^3*d*x^10 + 360*b^3*c*x^9 + 1260*a*b^2*e*x^8
+ 1512*a*b^2*d*x^7 + 1890*a*b^2*c*x^6 + 2520*a^2*b*e*x^5 + 3780*a^2*b*d*x^4
 + 7560*a^2*b*c*x^3 + 2520*a^3*e*x^2*log(x) - 2520*a^3*d*x - 1260*a^3*c)/x^
2
```

Sympy [A] time = 0.460926, size = 129, normalized size = 1.02

$$a^{3}e\log\left(x\right)+3a^{2}bcx+\frac{3a^{2}bdx^{2}}{2}+a^{2}bex^{3}+\frac{3ab^{2}cx^{4}}{4}+\frac{3ab^{2}dx^{5}}{5}+\frac{ab^{2}ex^{6}}{2}+\frac{b^{3}cx^{7}}{7}+\frac{b^{3}dx^{8}}{8}+\frac{b^{3}ex^{9}}{9}-\frac{a^{3}c+2a^{3}dx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**3,x)
```

```
[Out] a**3*e*log(x) + 3*a**2*b*c*x + 3*a**2*b*d*x**2/2 + a**2*b*e*x**3 + 3*a*b**2
*c*x**4/4 + 3*a*b**2*d*x**5/5 + a*b**2*e*x**6/2 + b**3*c*x**7/7 + b**3*d*x*
```

```
*8/8 + b**3*e*x**9/9 - (a**3*c + 2*a**3*d*x)/(2*x**2)
```

Giac [A] time = 1.0553, size = 155, normalized size = 1.23

$$\frac{1}{9}b^3x^9e + \frac{1}{8}b^3dx^8 + \frac{1}{7}b^3cx^7 + \frac{1}{2}ab^2x^6e + \frac{3}{5}ab^2dx^5 + \frac{3}{4}ab^2cx^4 + a^2bx^3e + \frac{3}{2}a^2bdx^2 + 3\,a^2bcx + a^3e\log\left(|x|\right) - \frac{2\,a^3dx +}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*e + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*x^6*e + 3/5*a*b^2
*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*x^3*e + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x + a^
3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

3.331 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^4 dx$

Optimal. Leaf size=138

$$\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c\left(a+bx^3\right)^5}{15b}+\frac{1}{16}b^4dx$$

```
[Out] (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^
2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*e*
x^14)/7 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17 + (c*(a + b*x^3)^5)/(15*b)
```

Rubi [A] time = 0.0985253, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1582, 1850}

$$\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c\left(a+bx^3\right)^5}{15b}+\frac{1}{16}b^4dx$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a^
2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*e*
x^14)/7 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17 + (c*(a + b*x^3)^5)/(15*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4 dx &= \frac{c\left(a+bx^3\right)^5}{15b}+\int \left(a+bx^3\right)^4\left(-cx^2+x^2\left(c+dx+ex^2\right)\right) dx\\
&= \frac{c\left(a+bx^3\right)^5}{15b}+\int \left(a^4dx^3+a^4ex^4+4a^3bdx^6+4a^3bex^7+6a^2b^2dx^9+6a^2b^2ex^{10}+\right.\\
&= \frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}
\end{aligned}$$

Mathematica [A] time = 0.0048061, size = 181, normalized size = 1.31

$$\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{2}{3}a^3bcx^6+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{1}{3}a^4cx^3+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{1}{3}ab^3cx^{12}+\frac{4}{13}ab$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3 + (4*a^3*
b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 +
 (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*
e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17
```

Maple [A] time = 0.001, size = 152, normalized size = 1.1

$$\frac{b^4ex^{17}}{17}+\frac{b^4dx^{16}}{16}+\frac{b^4cx^{15}}{15}+\frac{2\,ab^3ex^{14}}{7}+\frac{4\,ab^3dx^{13}}{13}+\frac{ab^3cx^{12}}{3}+\frac{6\,a^2b^2ex^{11}}{11}+\frac{3\,a^2b^2dx^{10}}{5}+\frac{2\,a^2b^2cx^9}{3}+\frac{a^3bex^8}{2}+\frac{4\,a^3bc}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/17*b^4*e*x^17+1/16*b^4*d*x^16+1/15*b^4*c*x^15+2/7*a*b^3*e*x^14+4/13*a*b^3
*d*x^13+1/3*a*b^3*c*x^12+6/11*a^2*b^2*e*x^11+3/5*a^2*b^2*d*x^10+2/3*a^2*b^2
*c*x^9+1/2*a^3*b*e*x^8+4/7*a^3*b*d*x^7+2/3*a^3*b*c*x^6+1/5*a^4*e*x^5+1/4*a^
4*d*x^4+1/3*a^4*c*x^3
```

Maxima [A] time = 0.949499, size = 204, normalized size = 1.48

$$\frac{1}{17}\,b^4ex^{17}+\frac{1}{16}\,b^4dx^{16}+\frac{1}{15}\,b^4cx^{15}+\frac{2}{7}\,ab^3ex^{14}+\frac{4}{13}\,ab^3dx^{13}+\frac{1}{3}\,ab^3cx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

Fricas [A] time = 1.30364, size = 373, normalized size = 2.7

$$\frac{1}{17}x^{17}eb^4+\frac{1}{16}x^{16}db^4+\frac{1}{15}x^{15}cb^4+\frac{2}{7}x^{14}eb^3a+\frac{4}{13}x^{13}db^3a+\frac{1}{3}x^{12}cb^3a+\frac{6}{11}x^{11}eb^2a^2+\frac{3}{5}x^{10}db^2a^2+\frac{2}{3}x^9cb^2a^2+\frac{1}{2}x^8eba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*e*b^4 + 1/16*x^16*d*b^4 + 1/15*x^15*c*b^4 + 2/7*x^14*e*b^3*a + 4/
13*x^13*d*b^3*a + 1/3*x^12*c*b^3*a + 6/11*x^11*e*b^2*a^2 + 3/5*x^10*d*b^2*a
```

```
^2 + 2/3*x^9*c*b^2*a^2 + 1/2*x^8*e*b*a^3 + 4/7*x^7*d*b*a^3 + 2/3*x^6*c*b*a^
3 + 1/5*x^5*e*a^4 + 1/4*x^4*d*a^4 + 1/3*x^3*c*a^4
```

Sympy [A] time = 0.086843, size = 184, normalized size = 1.33

$$\frac{a^4cx^3}{3} + \frac{a^4dx^4}{4} + \frac{a^4ex^5}{5} + \frac{2a^3bcx^6}{3} + \frac{4a^3bdx^7}{7} + \frac{a^3bex^8}{2} + \frac{2a^2b^2cx^9}{3} + \frac{3a^2b^2dx^{10}}{5} + \frac{6a^2b^2ex^{11}}{11} + \frac{ab^3cx^{12}}{3} + \frac{4ab^3dx^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**3/3 + a**4*d*x**4/4 + a**4*e*x**5/5 + 2*a**3*b*c*x**6/3 + 4*a**3*
b*d*x**7/7 + a**3*b*e*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10/5
 + 6*a**2*b**2*e*x**11/11 + a*b**3*c*x**12/3 + 4*a*b**3*d*x**13/13 + 2*a*b*
*3*e*x**14/7 + b**4*c*x**15/15 + b**4*d*x**16/16 + b**4*e*x**17/17
```

Giac [A] time = 1.06244, size = 211, normalized size = 1.53

$$\frac{1}{17}\,b^4x^{17}e + \frac{1}{16}\,b^4dx^{16} + \frac{1}{15}\,b^4cx^{15} + \frac{2}{7}\,ab^3x^{14}e + \frac{4}{13}\,ab^3dx^{13} + \frac{1}{3}\,ab^3cx^{12} + \frac{6}{11}\,a^2b^2x^{11}e + \frac{3}{5}\,a^2b^2dx^{10} + \frac{2}{3}\,a^2b^2cx^9 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/17*b^4*x^17*e + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*x^14*e + 4/
13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^
10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*x^8*e + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c*x^
6 + 1/5*a^4*x^5*e + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

3.332 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx$

Optimal. Leaf size=138

$$\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{d\left(a+bx^3\right)^5}{15b}+\frac{1}{14}b^4cx^{14}+$$

```
[Out] (a^4*c*x^2)/2 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (4*a^3*b*e*x^7)/7 + (3*
a^2*b^2*c*x^8)/4 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (4*a*b^3*e*
x^13)/13 + (b^4*c*x^14)/14 + (b^4*e*x^16)/16 + (d*(a + b*x^3)^5)/(15*b)
```

Rubi [A] time = 0.0934948, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1582, 1850}

$$\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{d\left(a+bx^3\right)^5}{15b}+\frac{1}{14}b^4cx^{14}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (4*a^3*b*e*x^7)/7 + (3*
a^2*b^2*c*x^8)/4 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (4*a*b^3*e*
x^13)/13 + (b^4*c*x^14)/14 + (b^4*e*x^16)/16 + (d*(a + b*x^3)^5)/(15*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx &= \frac{d\left(a+bx^3\right)^5}{15b}+\int \left(a+bx^3\right)^4\left(-dx^2+x\left(c+dx+ex^2\right)\right)\,dx\\ &= \frac{d\left(a+bx^3\right)^5}{15b}+\int \left(a^4cx+a^4ex^3+4a^3bcx^4+4a^3bex^6+6a^2b^2cx^7+6a^2b^2ex^9+4ab^3c\right.\\ &= \frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}a\end{aligned}$$

Mathematica [A] time = 0.0042877, size = 181, normalized size = 1.31

$$\frac{3}{4}a^2b^2cx^8+\frac{2}{3}a^2b^2dx^9+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^4cx^2+\frac{1}{3}a^4dx^3+\frac{1}{4}a^4ex^4+\frac{4}{11}ab^3cx^{11}+\frac{1}{3}a$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] (a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (2*a^3*
b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3
+ (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (a*b^3*d*x^12)/3 + (4*a*b^3*
e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*d*x^15)/15 + (b^4*e*x^16)/16
```

Maple [A] time = 0.002, size = 152, normalized size = 1.1

$$\frac{b^4ex^{16}}{16}+\frac{b^4dx^{15}}{15}+\frac{b^4cx^{14}}{14}+\frac{4\,ab^3ex^{13}}{13}+\frac{ab^3dx^{12}}{3}+\frac{4\,ab^3cx^{11}}{11}+\frac{3\,a^2b^2ex^{10}}{5}+\frac{2\,a^2b^2dx^9}{3}+\frac{3\,a^2b^2cx^8}{4}+\frac{4\,a^3bex^7}{7}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/16*b^4*e*x^16+1/15*b^4*d*x^15+1/14*b^4*c*x^14+4/13*a*b^3*e*x^13+1/3*a*b^3
*d*x^12+4/11*a*b^3*c*x^11+3/5*a^2*b^2*e*x^10+2/3*a^2*b^2*d*x^9+3/4*a^2*b^2*
c*x^8+4/7*a^3*b*e*x^7+2/3*a^3*b*d*x^6+4/5*a^3*b*c*x^5+1/4*a^4*e*x^4+1/3*a^4
*d*x^3+1/2*a^4*c*x^2
```

Maxima [A] time = 0.939755, size = 204, normalized size = 1.48

$$\frac{1}{16}\,b^4ex^{16}+\frac{1}{15}\,b^4dx^{15}+\frac{1}{14}\,b^4cx^{14}+\frac{4}{13}\,ab^3ex^{13}+\frac{1}{3}\,ab^3dx^{12}+\frac{4}{11}\,ab^3cx^{11}+\frac{3}{5}\,a^2b^2ex^{10}+\frac{2}{3}\,a^2b^2dx^9+\frac{3}{4}\,a^2b^2cx^8+\frac{4}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

Fricas [A] time = 1.30116, size = 371, normalized size = 2.69

$$\frac{1}{16}x^{16}eb^4+\frac{1}{15}x^{15}db^4+\frac{1}{14}x^{14}cb^4+\frac{4}{13}x^{13}eb^3a+\frac{1}{3}x^{12}db^3a+\frac{4}{11}x^{11}cb^3a+\frac{3}{5}x^{10}eb^2a^2+\frac{2}{3}x^9db^2a^2+\frac{3}{4}x^8cb^2a^2+\frac{4}{7}x^7e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e*b^4 + 1/15*x^15*d*b^4 + 1/14*x^14*c*b^4 + 4/13*x^13*e*b^3*a + 1
/3*x^12*d*b^3*a + 4/11*x^11*c*b^3*a + 3/5*x^10*e*b^2*a^2 + 2/3*x^9*d*b^2*a^
```

```
2 + 3/4*x^8*c*b^2*a^2 + 4/7*x^7*e*b*a^3 + 2/3*x^6*d*b*a^3 + 4/5*x^5*c*b*a^3
 + 1/4*x^4*e*a^4 + 1/3*x^3*d*a^4 + 1/2*x^2*c*a^4
```

Sympy [A] time = 0.087054, size = 185, normalized size = 1.34

$$\frac{a^4cx^2}{2} + \frac{a^4dx^3}{3} + \frac{a^4ex^4}{4} + \frac{4a^3bcx^5}{5} + \frac{2a^3bdx^6}{3} + \frac{4a^3bex^7}{7} + \frac{3a^2b^2cx^8}{4} + \frac{2a^2b^2dx^9}{3} + \frac{3a^2b^2ex^{10}}{5} + \frac{4ab^3cx^{11}}{11} + \frac{ab^3dx^{12}}{3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x**2/2 + a**4*d*x**3/3 + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5 + 2*a**3*
b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 + 2*a**2*b**2*d*x**9/
3 + 3*a**2*b**2*e*x**10/5 + 4*a*b**3*c*x**11/11 + a*b**3*d*x**12/3 + 4*a*b*
*3*e*x**13/13 + b**4*c*x**14/14 + b**4*d*x**15/15 + b**4*e*x**16/16
```

Giac [A] time = 1.05148, size = 211, normalized size = 1.53

$$\frac{1}{16}\,b^4x^{16}e + \frac{1}{15}\,b^4dx^{15} + \frac{1}{14}\,b^4cx^{14} + \frac{4}{13}\,ab^3x^{13}e + \frac{1}{3}\,ab^3dx^{12} + \frac{4}{11}\,ab^3cx^{11} + \frac{3}{5}\,a^2b^2x^{10}e + \frac{2}{3}\,a^2b^2dx^9 + \frac{3}{4}\,a^2b^2cx^8 + \frac{4}{7}\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*b^4*x^16*e + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*x^13*e + 1
/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*x^10*e + 2/3*a^2*b^2*d*x^
9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*x^5
 + 1/4*a^4*x^4*e + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

3.333 $\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx$

Optimal. Leaf size=130

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{e\left(a+bx^3\right)^5}{15b}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}$$

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14 + (e*(a + b*x^3)^5)/(15*b)
```

Rubi [A] time = 0.144909, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1582, 1850}

$$\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+a^3bcx^4+\frac{4}{5}a^3bdx^5+a^4cx+\frac{1}{2}a^4dx^2+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{e\left(a+bx^3\right)^5}{15b}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```


```
[Out] a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x^
7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (b^
4*c*x^13)/13 + (b^4*d*x^14)/14 + (e*(a + b*x^3)^5)/(15*b)
```


Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx &= \frac{e\left(a+bx^3\right)^5}{15b}+\int (c+dx)\left(a+bx^3\right)^4\,dx\\
&= \frac{e\left(a+bx^3\right)^5}{15b}+\int \left(a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7+4ab^3c\right.\\
&= a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx
\end{aligned}$$

Mathematica [A] time = 0.0038972, size = 173, normalized size = 1.33

$$\frac{6}{7}a^2b^2cx^7 + \frac{3}{4}a^2b^2dx^8 + \frac{2}{3}a^2b^2ex^9 + a^3bcx^4 + \frac{4}{5}a^3bdx^5 + \frac{2}{3}a^3bex^6 + a^4cx + \frac{1}{2}a^4dx^2 + \frac{1}{3}a^4ex^3 + \frac{2}{5}ab^3cx^{10} + \frac{4}{11}ab^3dx^{11} +$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 +
 (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b^2
*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (a*b^3*e*x^12)/3 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (b^4*e*x^15)/15
```

Maple [A] time = 0.001, size = 148, normalized size = 1.1

$$\frac{b^4ex^{15}}{15} + \frac{b^4dx^{14}}{14} + \frac{b^4cx^{13}}{13} + \frac{ab^3ex^{12}}{3} + \frac{4\,ab^3dx^{11}}{11} + \frac{2\,ab^3cx^{10}}{5} + \frac{2\,a^2b^2ex^9}{3} + \frac{3\,a^2b^2dx^8}{4} + \frac{6\,a^2b^2cx^7}{7} + \frac{2\,a^3bex^6}{3} + \frac{4\,a^3bd}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4,x)
```

```
[Out] 1/15*b^4*e*x^15+1/14*b^4*d*x^14+1/13*b^4*c*x^13+1/3*a*b^3*e*x^12+4/11*a*b^3
*d*x^11+2/5*a*b^3*c*x^10+2/3*a^2*b^2*e*x^9+3/4*a^2*b^2*d*x^8+6/7*a^2*b^2*c*
x^7+2/3*a^3*b*e*x^6+4/5*a^3*b*d*x^5+a^3*b*c*x^4+1/3*a^4*e*x^3+1/2*a^4*d*x^2
+a^4*c*x
```

Maxima [A] time = 0.935401, size = 198, normalized size = 1.52

$$\frac{1}{15}\,b^4ex^{15} + \frac{1}{14}\,b^4dx^{14} + \frac{1}{13}\,b^4cx^{13} + \frac{1}{3}\,ab^3ex^{12} + \frac{4}{11}\,ab^3dx^{11} + \frac{2}{5}\,ab^3cx^{10} + \frac{2}{3}\,a^2b^2ex^9 + \frac{3}{4}\,a^2b^2dx^8 + \frac{6}{7}\,a^2b^2cx^7 + \frac{2}{3}\,a^3be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Fricas [A] time = 1.28289, size = 355, normalized size = 2.73

$$\frac{1}{15}x^{15}eb^4 + \frac{1}{14}x^{14}db^4 + \frac{1}{13}x^{13}cb^4 + \frac{1}{3}x^{12}eb^3a + \frac{4}{11}x^{11}db^3a + \frac{2}{5}x^{10}cb^3a + \frac{2}{3}x^9eb^2a^2 + \frac{3}{4}x^8db^2a^2 + \frac{6}{7}x^7cb^2a^2 + \frac{2}{3}x^6eba^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e*b^4 + 1/14*x^14*d*b^4 + 1/13*x^13*c*b^4 + 1/3*x^12*e*b^3*a + 4/
11*x^11*d*b^3*a + 2/5*x^10*c*b^3*a + 2/3*x^9*e*b^2*a^2 + 3/4*x^8*d*b^2*a^2
```

```
+ 6/7*x^7*c*b^2*a^2 + 2/3*x^6*e*b*a^3 + 4/5*x^5*d*b*a^3 + x^4*c*b*a^3 + 1/3
*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^4
```

Sympy [A] time = 0.083989, size = 178, normalized size = 1.37

$$a^4cx + \frac{a^4dx^2}{2} + \frac{a^4ex^3}{3} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{2a^3bex^6}{3} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2a^2b^2ex^9}{3} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/
5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a**
2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + a*b**3*e*x**12
/3 + b**4*c*x**13/13 + b**4*d*x**14/14 + b**4*e*x**15/15
```

Giac [A] time = 1.06435, size = 205, normalized size = 1.58

$$\frac{1}{15}b^4x^{15}e + \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{1}{3}ab^3x^{12}e + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{2}{3}a^2b^2x^9e + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{2}{3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15*e + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*x^12*e + 4/
11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*x^9*e + 3/4*a^2*b^2*d*x^8
+ 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*x^6*e + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1/3
*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

3.334 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx$

Optimal. Leaf size=166

$$a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^4c\log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} +$$

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Rubi [A] time = 0.108534, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^4c\log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} +$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx &= \int \left(a^4d + \frac{a^4c}{x} + a^4ex + 4a^3bcx^2 + 4a^3bdx^3 + 4a^3bex^4 + 6a^2b^2cx^5 + 6a^2b^2dx^6 + 6a^2b^2e\right.\\ &= a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{9}\end{aligned}$$

Mathematica [A] time = 0.0088974, size = 166, normalized size = 1.

$$a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^4c\log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} +$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5)
/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3*c
*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (b^4
```

```
*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

Maple [A] time = 0.003, size = 145, normalized size = 0.9

$$a^4dx+\frac{a^4ex^2}{2}+\frac{4\,a^3bcx^3}{3}+a^3bdx^4+\frac{4\,a^3bex^5}{5}+a^2b^2cx^6+\frac{6\,a^2b^2dx^7}{7}+\frac{3\,a^2b^2ex^8}{4}+\frac{4\,ab^3cx^9}{9}+\frac{2\,ab^3dx^{10}}{5}+\frac{4\,ab^3ex^1}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x,x)
```

```
[Out] a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*c
*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10+4
/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*ln(x
)
```

Maxima [A] time = 0.936197, size = 194, normalized size = 1.17

$$\frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10}+\frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

Fricas [A] time = 1.46817, size = 347, normalized size = 2.09

$$\frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10}+\frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="fricas")
```

```
[Out] 1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
e*x^2 + a^4*d*x + a^4*c*log(x)
```

Sympy [A] time = 0.456358, size = 175, normalized size = 1.05

$$a^4c\log{(x)}+a^4dx+\frac{a^4ex^2}{2}+\frac{4a^3bcx^3}{3}+a^3bdx^4+\frac{4a^3bex^5}{5}+a^2b^2cx^6+\frac{6a^2b^2dx^7}{7}+\frac{3a^2b^2ex^8}{4}+\frac{4ab^3cx^9}{9}+\frac{2ab^3dx^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x,x)
```

```
[Out] a**4*c*log(x) + a**4*d*x + a**4*e*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x**
4 + 4*a**3*b*e*x**5/5 + a**2*b**2*c*x**6 + 6*a**2*b**2*d*x**7/7 + 3*a**2*b*
*2*e*x**8/4 + 4*a*b**3*c*x**9/9 + 2*a*b**3*d*x**10/5 + 4*a*b**3*e*x**11/11
+ b**4*c*x**12/12 + b**4*d*x**13/13 + b**4*e*x**14/14
```

Giac [A] time = 1.06076, size = 203, normalized size = 1.22

$$\frac{1}{14}b^4x^{14}e + \frac{1}{13}b^4dx^{13} + \frac{1}{12}b^4cx^{12} + \frac{4}{11}ab^3x^{11}e + \frac{2}{5}ab^3dx^{10} + \frac{4}{9}ab^3cx^9 + \frac{3}{4}a^2b^2x^8e + \frac{6}{7}a^2b^2dx^7 + a^2b^2cx^6 + \frac{4}{5}a^3bx^5e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14*e + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*x^11*e + 2
/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*x^8*e + 6/7*a^2*b^2*d*x^7 +
 a^2*b^2*c*x^6 + 4/5*a^3*b*x^5*e + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a^4*
x^2*e + a^4*d*x + a^4*c*log(abs(x))
```

3.335 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx$

Optimal. Leaf size=162

$$\frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 - \frac{a^4c}{x} + a^4d\log(x) + a^4ex + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}$$

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Rubi [A] time = 0.13332, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 - \frac{a^4c}{x} + a^4d\log(x) + a^4ex + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx = \int \left(a^4e + \frac{a^4c}{x^2} + \frac{a^4d}{x} + 4a^3bcx + 4a^3bdx^2 + 4a^3bex^3 + 6a^2b^2cx^4 + 6a^2b^2dx^5 + 6a^2b^2\right.$$
$$= -\frac{a^4c}{x} + a^4ex + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 + \frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + \frac{1}{2}$$

Mathematica [A] time = 0.0087761, size = 162, normalized size = 1.

$$\frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 - \frac{a^4c}{x} + a^4d\log(x) + a^4ex + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

```
[Out] -((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
(6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)/2
```

```
 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^12)/
12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

Maple [A] time = 0.006, size = 145, normalized size = 0.9

$$-\frac{a^4c}{x} + a^4ex + 2\,a^3bcx^2 + \frac{4\,a^3bdx^3}{3} + a^3bex^4 + \frac{6\,a^2b^2cx^5}{5} + a^2b^2dx^6 + \frac{6\,a^2b^2ex^7}{7} + \frac{ab^3cx^8}{2} + \frac{4\,ab^3dx^9}{9} + \frac{2\,ab^3ex^{10}}{5} + \frac{b^4c}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x)
```

```
[Out] -a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x^
5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b^3
*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

Maxima [A] time = 0.941723, size = 194, normalized size = 1.2

$$\frac{1}{13}\,b^4ex^{13} + \frac{1}{12}\,b^4dx^{12} + \frac{1}{11}\,b^4cx^{11} + \frac{2}{5}\,ab^3ex^{10} + \frac{4}{9}\,ab^3dx^9 + \frac{1}{2}\,ab^3cx^8 + \frac{6}{7}\,a^2b^2ex^7 + a^2b^2dx^6 + \frac{6}{5}\,a^2b^2cx^5 + a^3bex^4 + \frac{4}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x + a^
4*d*log(x) - a^4*c/x
```

Fricas [A] time = 1.4897, size = 433, normalized size = 2.67

$$\frac{13860\,b^4ex^{14} + 15015\,b^4dx^{13} + 16380\,b^4cx^{12} + 72072\,ab^3ex^{11} + 80080\,ab^3dx^{10} + 90090\,ab^3cx^9 + 154440\,a^2b^2ex^8 + 18018}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="fricas")
```

```
[Out] 1/180180*(13860*b^4*e*x^14 + 15015*b^4*d*x^13 + 16380*b^4*c*x^12 + 72072*a*
b^3*e*x^11 + 80080*a*b^3*d*x^10 + 90090*a*b^3*c*x^9 + 154440*a^2*b^2*e*x^8
+ 180180*a^2*b^2*d*x^7 + 216216*a^2*b^2*c*x^6 + 180180*a^3*b*e*x^5 + 240240
*a^3*b*d*x^4 + 360360*a^3*b*c*x^3 + 180180*a^4*e*x^2 + 180180*a^4*d*x*log(x
) - 180180*a^4*c)/x
```

Sympy [A] time = 0.47288, size = 168, normalized size = 1.04

$$-\frac{a^4c}{x} + a^4d\log{(x)} + a^4ex + 2a^3bcx^2 + \frac{4a^3bdx^3}{3} + a^3bex^4 + \frac{6a^2b^2cx^5}{5} + a^2b^2dx^6 + \frac{6a^2b^2ex^7}{7} + \frac{ab^3cx^8}{2} + \frac{4ab^3dx^9}{9} + \frac{2ab^3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**2,x)
```

```
[Out] -a**4*c/x + a**4*d*log(x) + a**4*e*x + 2*a**3*b*c*x**2 + 4*a**3*b*d*x**3/3
+ a**3*b*e*x**4 + 6*a**2*b**2*c*x**5/5 + a**2*b**2*d*x**6 + 6*a**2*b**2*e*x
**7/7 + a*b**3*c*x**8/2 + 4*a*b**3*d*x**9/9 + 2*a*b**3*e*x**10/5 + b**4*c*x
**11/11 + b**4*d*x**12/12 + b**4*e*x**13/13
```

Giac [A] time = 1.0459, size = 203, normalized size = 1.25

$$\frac{1}{13}\,b^4x^{13}e + \frac{1}{12}\,b^4dx^{12} + \frac{1}{11}\,b^4cx^{11} + \frac{2}{5}\,ab^3x^{10}e + \frac{4}{9}\,ab^3dx^9 + \frac{1}{2}\,ab^3cx^8 + \frac{6}{7}\,a^2b^2x^7e + a^2b^2dx^6 + \frac{6}{5}\,a^2b^2cx^5 + a^3bx^4e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13*e + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*x^10*e + 4/
9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*x^7*e + a^2*b^2*d*x^6 + 6/5*a
^2*b^2*c*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*x*e + a^
4*d*log(abs(x)) - a^4*c/x
```

3.336 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx$

Optimal. Leaf size=166

$$\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3e$$

```
[Out] -(a^4*c)/(2*x^2) - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Rubi [A] time = 0.124775, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3e$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -(a^4*c)/(2*x^2) - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2\right)\left(a+bx^3\right)^4}{x^3}\,dx=\int \left(4a^3bc+\frac{a^4c}{x^3}+\frac{a^4d}{x^2}+\frac{a^4e}{x}+4a^3bdx+4a^3bex^2+6a^2b^2cx^3+6a^2b^2dx^4+6a^2b^2ex^5+\right.$$

$$=-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3$$

Mathematica [A] time = 0.0089017, size = 166, normalized size = 1.

$$\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3e$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

```
[Out] -(a^4*c)/(2*x^2) - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^3
)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^3*
```

```
c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*d*x
^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

Maple [A] time = 0.006, size = 147, normalized size = 0.9

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4a^3bex^3}{3}+\frac{3a^2b^2cx^4}{2}+\frac{6a^2b^2dx^5}{5}+a^2b^2ex^6+\frac{4ab^3cx^7}{7}+\frac{ab^3dx^8}{2}+\frac{4ab^3ex^9}{9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x)
```

```
[Out] -1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b^
2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4/9
*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

Maxima [A] time = 0.949211, size = 197, normalized size = 1.19

$$\frac{1}{12}b^4ex^{12}+\frac{1}{11}b^4dx^{11}+\frac{1}{10}b^4cx^{10}+\frac{4}{9}ab^3ex^9+\frac{1}{2}ab^3dx^8+\frac{4}{7}ab^3cx^7+a^2b^2ex^6+\frac{6}{5}a^2b^2dx^5+\frac{3}{2}a^2b^2cx^4+\frac{4}{3}a^3bex^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*e*x^12 + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*e*x^9 + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(x)
- 1/2*(2*a^4*d*x + a^4*c)/x^2
```

Fricas [A] time = 1.52702, size = 413, normalized size = 2.49

$$\frac{1155b^4ex^{14}+1260b^4dx^{13}+1386b^4cx^{12}+6160ab^3ex^{11}+6930ab^3dx^{10}+7920ab^3cx^9+13860a^2b^2ex^8+16632a^2b^2}{1386}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/13860*(1155*b^4*e*x^14 + 1260*b^4*d*x^13 + 1386*b^4*c*x^12 + 6160*a*b^3*e
*x^11 + 6930*a*b^3*d*x^10 + 7920*a*b^3*c*x^9 + 13860*a^2*b^2*e*x^8 + 16632*
a^2*b^2*d*x^7 + 20790*a^2*b^2*c*x^6 + 18480*a^3*b*e*x^5 + 27720*a^3*b*d*x^4
 + 55440*a^3*b*c*x^3 + 13860*a^4*e*x^2*log(x) - 13860*a^4*d*x - 6930*a^4*c)
/x^2
```

Sympy [A] time = 0.558303, size = 173, normalized size = 1.04

$$a^4e\log(x)+4a^3bcx+2a^3bdx^2+\frac{4a^3bex^3}{3}+\frac{3a^2b^2cx^4}{2}+\frac{6a^2b^2dx^5}{5}+a^2b^2ex^6+\frac{4ab^3cx^7}{7}+\frac{ab^3dx^8}{2}+\frac{4ab^3ex^9}{9}+\frac{b^4cx^1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**3,x)
```

```
[Out] a**4*e*log(x) + 4*a**3*b*c*x + 2*a**3*b*d*x**2 + 4*a**3*b*e*x**3/3 + 3*a**2
*b**2*c*x**4/2 + 6*a**2*b**2*d*x**5/5 + a**2*b**2*e*x**6 + 4*a*b**3*c*x**7/
7 + a*b**3*d*x**8/2 + 4*a*b**3*e*x**9/9 + b**4*c*x**10/10 + b**4*d*x**11/11
 + b**4*e*x**12/12 - (a**4*c + 2*a**4*d*x)/(2*x**2)
```

Giac [A] time = 1.05512, size = 205, normalized size = 1.23

$$\frac{1}{12}\,b^4x^{12}e+\frac{1}{11}\,b^4dx^{11}+\frac{1}{10}\,b^4cx^{10}+\frac{4}{9}\,ab^3x^9e+\frac{1}{2}\,ab^3dx^8+\frac{4}{7}\,ab^3cx^7+a^2b^2x^6e+\frac{6}{5}\,a^2b^2dx^5+\frac{3}{2}\,a^2b^2cx^4+\frac{4}{3}\,a^3bx^3e+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12*e + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*x^9*e + 1/2
*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*x^6*e + 6/5*a^2*b^2*d*x^5 + 3/2*a^
2*b^2*c*x^4 + 4/3*a^3*b*x^3*e + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(abs
(x)) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```

3.337 $\int \frac{x^3(c+dx+ex^2)}{a+bx^3} dx$

Optimal. Leaf size=205

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-$$

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.261464, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(
1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/
3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx+ex^2\right)}{a+bx^3}\,dx &= \int \left(\frac{c}{b}+\frac{dx}{b}+\frac{ex^2}{b}-\frac{ac+adx+aex^2}{b\left(a+bx^3\right)}\right)dx\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int \frac{ac+adx+aex^2}{a+bx^3}\,dx}{b}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\int \frac{ac+adx}{a+bx^3}\,dx}{b}-\frac{(ae)\int \frac{x^2}{a+bx^3}\,dx}{b}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}c+a^{4/3}d\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}c+a^{4/3}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int}{3b}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}-\frac{\left(a^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\right)\int}{2b^{4/3}}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}\\
&= \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,(}{}
\end{aligned}
$$

Mathematica [A] time = 0.104917, size = 191, normalized size = 0.93

$$\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(a^{2/3}d-\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a
^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(-(a^(1/3
)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^(1/
3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*e*Lo
g[a + b*x^3])/(6*b^2)
```

Maple [A] time = 0.003, size = 231, normalized size = 1.1

$$\frac{ex^3}{3b}+\frac{dx^2}{2b}+\frac{cx}{b}-\frac{ac}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ac}{6b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a\sqrt{3}c}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*e*x^3/b+1/2*d*x^2/b+c*x/b-1/3*a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c
+1/6*a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3*a/b^2/
(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3/b^2/(
1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a*d-1/6/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(
1/3)*x+(1/b*a)^(2/3))*a*d-1/3/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*
(2/(1/b*a)^(1/3)*x-1))*a*d-1/3*a*e*ln(b*x^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.92844, size = 10954, normalized size = 53.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*e*x^3 + 18*b*d*x^2 - 2*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2*log(1/36*((-I*sqrt(3) + 1)*
(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 +
a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6
 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*
b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d
+ 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sq
rt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/5
4*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^
3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^
2) + (b^2*c^3 + a*b*d^3)*x) + 36*b*c*x + (((-I*sqrt(3) + 1)*(a^2*e^2/b^4 -
(a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 +
1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d
*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3
+ a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e
^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2 + 3*sqrt(1/3)*b^2*s
qrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1
/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*
e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
 a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
+ 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c
*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^
4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)
/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2
*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/
18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e
)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*
e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d
 + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*
b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*
b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)
*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^
3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*c^3 + a*b*d^3)*x + 1/1
2*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/
27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/
b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*
d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b
^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*e)*sqrt(-(((-I*sqrt(3)
 + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*
c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a
^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e
^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/
54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*
b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a
^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
```

```
- 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^
(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2)/b^4)) + (((-I*sqrt(3
) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b
*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 +
a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1
/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b
^2 - 3*sqrt(1/3)*b^2*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2
*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d
+ a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6
)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*
c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2
/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a
/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3
- 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*
(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3
+ a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*
b*c*d + 36*a^2*e^2)/b^4) - 18*a*e)*log(-1/36*((-I*sqrt(3) + 1)*(a^2*e^2/b^4
 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5
 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*
c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c
^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^
3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)^2*b^4*d - 2*a*b*c*d^
2 + a*b*c^2*e - a^2*d*e^2 - 1/6*(b^3*c^2 - 2*a*b^2*d*e)*((-I*sqrt(3) + 1)*(
a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a
*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
- (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6
+ 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b
^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2) + 2*(b^2*
c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*
d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a
*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2
*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3
)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d
^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^4*d - 6*b^3*c^2 - 6*a*b^2*d*
e)*sqrt(-(((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*
a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
 a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 6*a*e/b^2)^2*b^4 - 12*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d +
a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c
*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/
b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/
b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 -
 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*a*b^2*e + 144*a*b*c*d + 36*a^2*e^2
)/b^4)))/b^2
```

Sympy [A] time = 1.59667, size = 178, normalized size = 0.87

$$\operatorname{RootSum}\left(27t^3b^6 + 27t^2ab^4e + t\left(9a^2b^2e^2 + 9ab^3cd\right) + a^3e^3 + 3a^2bcde - a^2bd^3 + ab^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^4d + 6tab}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2 + 9*a*b**3
*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b**2*c**3, Lambda(_t,
_t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e - 3*_t*b**3*c**2 + a**2*d*e**2
 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3)))) + c*x/b + d*x**2/(2
*b) + e*x**3/(3*b)
```

Giac [A] time = 1.08081, size = 302, normalized size = 1.47

$$-\frac{ae\log\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{2\,b^2x^3e+3\,b^2dx^2+6\,b^2cx}{6\,b^3}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2c-\left(-ab^2\right)^{\frac{2}{3}}abd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^4}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*a*e*log(abs(b*x^3 + a))/b^2 + 1/6*(2*b^2*x^3*e + 3*b^2*d*x^2 + 6*b^2*c
*x)/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^2*c - (-a*b^2)^(2/3)*a*b*d)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/6*((-a*b^2)^(1
/3)*a*b^2*c + (-a*b^2)^(2/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/(a*b^4) + 1/3*(a*b^6*d*(-a/b)^(1/3) + a*b^6*c)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a*b^7)
```

3.338 $\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+$$

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d
- a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)
*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) +
(c*Log[a + b*x^3])/(3*b)
```

Rubi [A] time = 0.247859, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```


```
[Out] (d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*d
- a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1/3)
*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)) +
(c*Log[a + b*x^3])/(3*b)
```


Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{a+bx^3}\,dx = \int \left(\frac{d}{b}+\frac{ex}{b}-\frac{ad+aex-bcx^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{b}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{b}+c\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{c\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{a}\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx}}}{3b}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{3b^{4/3}}+\frac{c\log\left(a+bx^3\right)}{3b}-\frac{\left(a^{2/3}\left(\sqrt[3]{b}d+\sqrt[3]{ae}\right)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{2b^{4/3}}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{c\log}{}$$

$$= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{ae}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{ae}}{\sqrt[3]{b}}\right.}{}$$

Mathematica [A] time = 0.0885497, size = 184, normalized size = 0.95

$$\frac{-\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2b^{2/3}c\log\left(a+bx^3\right)+2\sqrt{3}\sqrt[3]{a}}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^(1/3)*b^(1/3)*d) + a
^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3])/(
6*b^(5/3))
```

Maple [A] time = 0.003, size = 221, normalized size = 1.2

$$\frac{ex^2}{2b}+\frac{dx}{b}-\frac{ad}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ad}{6b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a\sqrt{3}d}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/2*e*x^2/b+d*x/b-1/3/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a*d+1/6/b^2/(1/
b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a*d-1/3/b^2/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*d+1/3/b^2*a/(1/b*a)^(1/
3)*ln(x+(1/b*a)^(1/3))*e-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))*e-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))*e+1/3*c*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.57806, size = 9513, normalized size = 49.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*e*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e
)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b*log(1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 +
 a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d
*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/b^5)^(1/3) - 2*c/b)^2*b^3*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 + 1/2*(b^2*d
^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/
b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^
3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + (b*d^3 + a*e^3)*x) + 1
2*d*x + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2
*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3
/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3
 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2
 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2
/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
 a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e
^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b
^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 +
 (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*
d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) -
 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3
 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3
*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3
- 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*
(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d
^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 +
 a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/
b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)) + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^
2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3
+ a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*
e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*
```

```
b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2
+ a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqr
t(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^
2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*
d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3) + 6*c)*log(-
1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/
b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3
- 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e - b*c*d^2 - b*c^2*e - 2*a*d*
e^2 - 1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 -
(b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)
*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)
*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b
^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + 2*(b*d
^3 + a*e^3)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (
b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*
a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^
5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^3*e - 2
*b^2*d^2 + 2*b^2*c*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*
c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/
b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
+ (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3 + 4*(
2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 -
 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(
b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b^2*c + 4*b*c^2 + 16*a*d*e)/b^3)))/b
```

Sympy [A] time = 1.50061, size = 150, normalized size = 0.78

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c + t\left(9ab^2de + 9b^3c^2\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^3e - 6tb^2ce - 3t\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b**3*c**2) -
 a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*log(x + (9*_
t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2 + 2*a*d*e**2 + b*c**2*e + b*c*
d**2)/(a*e**3 + b*d**3)))) + d*x/b + e*x**2/(2*b)
```

Giac [A] time = 1.07844, size = 285, normalized size = 1.48

$$\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,b} + \frac{bx^2e + 2\,bdx}{2\,b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2d - \left(-ab^2\right)^{\frac{2}{3}}abe\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ab^2d + \left(-ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c*log(abs(b*x^3 + a))/b + 1/2*(b*x^2*e + 2*b*d*x)/b^2 - 1/3*sqrt(3)*((-
a*b^2)^(1/3)*a*b^2*d - (-a*b^2)^(2/3)*a*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/6*((-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(2
/3)*a*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) + 1/3*(a*b^4*(-
a/b)^(1/3)*e + a*b^4*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

3.339 $\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$

Optimal. Leaf size=183

$$\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}+\frac{d\,\mathrm{lo}}{}$$

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])
/(3*b)
```

Rubi [A] time = 0.225588, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}+\frac{d\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x^3])
/(3*b)
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{a+bx^3}\,dx = \int \left(\frac{e}{b}-\frac{ae-bcx-bdx^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{ex}{b}-\frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{b}$$

$$= \frac{ex}{b}-\frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{b}+d\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{ex}{b}+\frac{d\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}bc+2a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}bc-a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b}$$

$$= \frac{ex}{b}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b}+$$

$$= \frac{ex}{b}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}+\frac{d\log\left(a+\right.}{3b}$$

$$= \frac{ex}{b}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/}\right.}{6\sqrt[3]{a}}$$

Mathematica [A] time = 0.0558075, size = 200, normalized size = 1.09

$$-\frac{\left(a^{4/3}\left(-\sqrt[3]{b}\right)e-a^{2/3}bc\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{5/3}}+\frac{\left(a^{4/3}\left(-\sqrt[3]{b}\right)e-a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3ab^{5/3}}+\frac{\left(a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\text{t}}{\sqrt{3}ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

```
[Out] (e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b^(5/3)) + ((-(a^(2/3)*b*c) - a^(4/3)*b^(1
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a*b^(5/3)) - ((-(a^(2/3)*b*c) - a^(4/3)
*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(5/3)) +
 (d*Log[a + b*x^3])/(3*b)
```

Maple [A] time = 0.003, size = 209, normalized size = 1.1

$$\frac{ex}{b}-\frac{ae}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ae}{6b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a\sqrt{3}e}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{3b}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] e*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b^2*a/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^
(1/3))*c+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b*
3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*d*ln(
b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.56091, size = 9754, normalized size = 53.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3
/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b
^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d
^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*log(-1/4*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c - a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*
e^2 - 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^
2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^
3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d
/b) - (b^2*c^3 - a^2*e^3)*x) - 12*e*x - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3)
- 2*d/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2
- (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b
)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*
d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e
)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e
)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c
+ a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^
2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c
^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2*c^3 - a^2*e^3)*x + 3/4*sqrt(
1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^
3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4)
 - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/
b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^
4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*a*b^3*c + 2*a*b^2*c*d + 2*
a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^
2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^
3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d
^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*
c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2)) - ((2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2
- c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^
```

```
2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c
^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*
d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*
e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e
)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^
2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d
^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3)
- 2*d/b)*b*d + 4*d^2 - 16*c*e)/b^2) + 6*d)*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4)
)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^
4))^(1/3) - 2*d/b)^2*a*b^3*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 + 1/2*(2
*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c
*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sq
rt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b) - 2*(b^2
*c^3 - a^2*e^3)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2
 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 -
 (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3
 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/
b)*a*b^3*c + 2*a*b^2*c*d + 2*a^2*b*e^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1
/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^
(1/3) - 2*d/b)^2*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 -
c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*d + 4
*d^2 - 16*c*e)/b^2)))/b
```

Sympy [A] time = 1.65329, size = 160, normalized size = 0.87

$$\text{RootSum}\left(27t^3ab^4 - 27t^2ab^3d + t\left(-9ab^2ce + 9ab^2d^2\right) + a^2e^3 + 3abcde - abd^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{-9t^2ab^3c - 3ta\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e + 9*a*b**2*
d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3, Lambda(_t, _t*log(x
 + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*_t*a*b**2*c*d + a**2*d*e**2 +
2*a*b*c**2*e - a*b*c*d**2)/(a**2*e**3 - b**2*c**3)))) + e*x/b
```

Giac [A] time = 1.07542, size = 257, normalized size = 1.4

$$\frac{xe}{b}+\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,b}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae+\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right.\right.}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] x*e/b + 1/3*d*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*e + (-a
*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2
) - 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a*b^2) - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*log
(abs(x - (-a/b)^(1/3)))/(a*b^3)
```

3.340 $\int \frac{c+dx+ex^2}{a+bx^3}\,dx$

Optimal. Leaf size=177

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{e\log(a}{3b}$$

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b
)
```

Rubi [A] time = 0.132435, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{e\log(a}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] -(((b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(2/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b
)
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{a+bx^3}\,dx &= e\int \frac{x^2}{a+bx^3}\,dx+\int \frac{c+dx}{a+bx^3}\,dx\\
&= \frac{e\log\left(a+bx^3\right)}{3b}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{2/3}}+\frac{1}{2}\left(\frac{c}{\sqrt[3]{a}}+\frac{d}{\sqrt[3]{b}}\right.\\
&= \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{2/3}}+\frac{e\log\left(a+bx^3\right)}{3b}+\frac{\left(\sqrt[3]{b}c+\right.}{}\\
&= -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{6a^{2/3}b^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0916627, size = 176, normalized size = 0.99

$$
\frac{-\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{ta}}{6ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1/3
) + b^(1/3)*x] - b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*e*Log[a + b*x^3])/(6*a*b)
```

Maple [A] time = 0.003, size = 200, normalized size = 1.1

$$\frac{c}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{3b}\ln\left(x+\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*c/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*c/b/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*c/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(1/b*a)^(1/3)*x-1))-1/3*d/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6*d/b/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*d*3^(1/2)/b/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*e*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.51533, size = 10437, normalized size = 58.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2)
)/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b
^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2
*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)
*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^
2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/
b)^2*a^2*b^2*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 - 1/2*(a*b^2*c^2 - 2*a
^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2
))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (
```

```
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b
) + (b^2*c^3 + a*b*d^3)*x) - ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b
*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) +
 (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2
*b^3))^(1/3) - 2*e/b)*b + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*
e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d
 + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b
^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*
e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a
^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
 a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1
/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e - a^2*d*e^2 + 1/2*(a*b^2
*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e
^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^
2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/
3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2*b^2*d + 2*a*b^2*c^2 + 2*
a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2
)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*
b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*
d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3)
 - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a
*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(
a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3
+ a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(
1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))) - ((2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b
*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*b - 3*sqrt(1/3)*b*sqrt(-((2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3
 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*
(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3
/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 +
```

```
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e +
16*b*c*d + 4*a*e^2)/(a*b^2)) + 6*e)*log(-1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*
b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*
e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a^2*b^2*d - 2*a*b*c*d^2 + a*b*c^2*e
- a^2*d*e^2 + 1/2*(a*b^2*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b
^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e
^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b) + 2*(b^2*c^3 + a*b*d^3)*x - 3/4*sqrt(1
/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2
*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(
3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2
) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a^2
*b^2*d + 2*a*b^2*c^2 + 2*a^2*b*d*e)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3
) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(
a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2
)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d
*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)^2*a*b^2 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a
*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a
*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2*e/b)*a*b*e + 16*b*c*d + 4*a*e^2)/(a*b^2))
))/b
```

Sympy [A] time = 1.27179, size = 160, normalized size = 0.9

$$\text{RootSum}\left(27t^3a^2b^3 - 27t^2a^2b^2e + t\left(9a^2be^2 + 9ab^2cd\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2b^2d - 6ta\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*e**2 + 9*a
*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*
log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*e + 3*_t*a*b**2*c**2 + a**2*d*
e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3))))
```

Giac [A] time = 1.07264, size = 246, normalized size = 1.39

$$\frac{e\log\left(\left|bx^3 + a\right|\right)}{3b} - \frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}} + bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*e*log(abs(b*x^3 + a))/b - 1/3*(b*d*(-a/b)^(1/3) + b*c)*(-a/b)^(1/3)*log
(abs(x - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^
(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/
6*((-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^2*b^4)
```

3.341 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{c\log(a}{3a}$$

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a
 + b*x^3])/(3*a)
```

Rubi [A] time = 0.206411, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{c\log(a}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] -(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*Log[a
 + b*x^3])/(3*a)
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx &= \int \left(\frac{c}{ax}+\frac{ad+aex-bcx^2}{a\left(a+bx^3\right)}\right)dx\\
&= \frac{c\log(x)}{a}+\frac{\int \frac{ad+aex-bcx^2}{a+bx^3}\,dx}{a}\\
&= \frac{c\log(x)}{a}+\frac{\int \frac{ad+aex}{a+bx^3}\,dx}{a}-\frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a}\\
&= \frac{c\log(x)}{a}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}}\\
&= \frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{1}{2}\left(\frac{d}{\sqrt[3]{a}}+\frac{e}{\sqrt[3]{b}}\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\\
&= \frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{c\log\left(a+bx^3\right)}{3a}+\\
&= -\frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{c\log(x)}{a}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{}\right.}{6a^{2/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.088848, size = 176, normalized size = 0.96

$$\frac{\left(a^{2/3}e-\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2b^{2/3}c\log\left(a+bx^3\right)-2\sqrt{3}\sqrt[3]{a}\left(\cdots\right.}{6ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a
^(1/3) + b^(1/3)*x] + (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*c*Log[a + b*x^3])/(6*a*b^(2/3))
```

Maple [A] time = 0.005, size = 207, normalized size = 1.1

$$\frac{d}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{3b}\ln\left(x+\sqrt[3]{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*d+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))*d-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b/(1/
b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/3/b*3^(1/2)/(1/b*a)^(1
/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*c*ln(b*x^3+a)/a+c*ln(x)
/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.6854, size = 10657, normalized size = 57.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a
^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)
/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3
) + 6*c/a)*a*log(1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))
/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^
2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b
*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^3*b^2))^(1/3) + 6*c/a)^2*a^2*b*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 - 1/6*(a*
b*d^2 + 2*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*
b^2))^(1/3) + 6*c/a) + (b*d^3 + a*e^3)*x) - (((-I*sqrt(3) + 1)*(c^2/a^2 - (
b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1
/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a + 3*sqrt(1/3)*a*sqrt(-(((-
I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b
*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2
*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^
3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(
3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*c)*log(-1/36*((-I
*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*
c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2*
a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2*a*b*c*e)*((-I*sq
rt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2
 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*
e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a) + 2*(
b*d^3 + a*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*
d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b
^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 - 6*a*b*c*e)*sqrt(
-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/
18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c
^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c
/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/
27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2
) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*
sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
```

```
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b))) - (((-I*sqrt(3)
+ 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d
*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 +
1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a - 3*sqrt(
1/3)*a*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*
c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a
*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))
^(1/3) + 6*c/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(
a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) +
1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)) - 18*
c)*log(-1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c
^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) -
1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*
e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^
(1/3) + 6*c/a)^2*a^2*b*e - b*c*d^2 - b*c^2*e - 2*a*d*e^2 + 1/6*(a*b*d^2 + 2
*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/5
4*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3
)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/
3) + 6*c/a) + 2*(b*d^3 + a*e^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(c^2/
a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3
*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^
2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a^2*b*e + 6*a*b*d^2 -
 6*a*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-
1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b
^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(
I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^
3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*
b^2))^(1/3) + 6*c/a)^2*a^2*b - 12*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d
*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3
+ a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^
2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*
b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*
d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)*a*b*c + 36*b*c^2 + 144*a*d*e)/(a^2*b)))
 - 36*c*log(x))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08962, size = 267, normalized size = 1.45

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{c\log\left(\left|x\right|\right)}{a}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+a^2bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\right.}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a + c*log(abs(x))/a + 1/3*sqrt(3)*((-a*b^2)^(1/3
)*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(a*b^2) - 1/3*(a^2*b*(-a/b)^(1/3)*e + a^2*b*d)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b) + 1/6*((-a*b^2)^(1/3)*a*b^3*d + (-a*b^2)^(2/3)*a*b
^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.342 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=192

$$-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}-d$$

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a + ((b^(2/3)*c + a^(2
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(1/3)) - ((b^(2/3)*c + a^(2/3
)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) -
(d*Log[a + b*x^3])/(3*a)
```

Rubi [A] time = 0.213757, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}-d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a + ((b^(2/3)*c + a^(2
/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(1/3)) - ((b^(2/3)*c + a^(2/3
)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)) -
(d*Log[a + b*x^3])/(3*a)
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^2}+\frac{d}{ax}+\frac{ae-bcx-bdx^2}{a\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int \frac{ae-bcx-bdx^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\int \frac{ae-bcx}{a+bx^3}\,dx}{a}-\frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3\right)}{3a}+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}bc+2a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}bc-a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\cdots}}{3a^{4/3}}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2\cdots}}}{2a}$$

$$= -\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}-\frac{d\cdots}{}$$

$$= -\frac{c}{ax}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c+a^{2/3}\cdots\right.}{}$$

Mathematica [A] time = 0.238511, size = 184, normalized size = 0.96

$$-\frac{\frac{\left(a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{2\left(a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}a^{2/3}\left(a^{2/3}e-b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+2ad\log\left(a+bx^3\right)+}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -((6*a*c)/x + (2*Sqrt[3]*a^(2/3)*(-(b^(2/3)*c) + a^(2/3)*e)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 6*a*d*Log[x] - (2*(a^(2/3)*b^(2/3)*
c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((a^(2/3)*b^(2/3)*c + a^
(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*a*d*Lo
g[a + b*x^3])/(6*a^2)
```

Maple [A] time = 0.007, size = 216, normalized size = 1.1

$$\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}e}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{3a}\ln\left(x+\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*e+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))*e+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6/a/(1/
b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/a*3^(1/2)/(1/b*a)^(1
/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3*d*ln(b*x^3+a)/a+d*ln(x)
/a-c/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.23121, size = 10066, normalized size = 52.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/
a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*
b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x*log(-1/36
*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
 c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54
*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/
18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*
b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c - a*b*c*d^2
 + 2*a*b*c^2*e + a^2*d*e^2 + 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*
(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3
)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^
3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3
- a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - (b^2*c^3 - a^2*e^3)*x) - 36*d*x*log(x)
 - (((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^
2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1
/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 +
 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x - 3*sqrt(1/3)*
a*x*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1
/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4
*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^
3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*
((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 -
c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*
(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/1
8*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b
) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e
)/a^2) - 18*d*x)*log(1/36*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1
/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (
d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d
/a)^2*a^3*b*c + a*b*c*d^2 - 2*a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^
3*e^2)*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*
(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
- 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^
3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)
/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 -
a^2*e^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/
(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
6*d/a)*a^3*b*c - 6*a^2*b*c*d - 6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2
 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c
*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 +
 a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))
^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2)) - (((-I*sqrt(3) + 1)*(d^2/a^2
 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3
```

```
 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54
*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^
3)/(a^4*b))^(1/3) + 6*d/a)*a*x + 3*sqrt(1/3)*a*x*sqrt(-(((-I*sqrt(3) + 1)*(
d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(
b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)
/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3
 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 -
 a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (
d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^
2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(
a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 - 144*c*e)/a^2) - 18*d*x)*log(1/36*((-I
*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)
*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2
*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d
^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) -
1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c + a*b*c*d^2 - 2*
a*b*c^2*e - a^2*d*e^2 - 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sqrt(3) + 1)*(d^2/
a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^
4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1
/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2
*e^3)/(a^4*b))^(1/3) + 6*d/a) - 2*(b^2*c^3 - a^2*e^3)*x - 1/12*sqrt(1/3)*((
(-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c
*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(
b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18
*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
 - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a^3*b*c - 6*a^2*b*c*d -
 6*a^3*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3
/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(
-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3
*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a
^2 - 12*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18
*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
 - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a
^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*d + 36*d^2 -
 144*c*e)/a^2)) + 36*c)/(a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08301, size = 279, normalized size = 1.45

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{d\log\left(|x|\right)}{a}-\frac{c}{ax}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2be\right)\left(-\frac{a}{b}\right.}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a + d*log(abs(x))/a - c/(a*x) + 1/6*((-a*b^2)^(1
/3)*a*e - (-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b
) + 1/3*(a*b^2*c*(-a/b)^(1/3) - a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/(a^3*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^2*e + (-a*b^2)^(2/3)*b^2*c)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3)
```

3.343 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=203

$$\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-$$

```
[Out] -c/(2*a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (e*Log[x])/a - (b^(1/
3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(2/3)
*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(5/3)) - (e*Log[a + b*x^3])/(3*a)
```

Rubi [A] time = 0.193687, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```


```
[Out] -c/(2*a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (e*Log[x])/a - (b^(1/
3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(2/3)
*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(5/3)) - (e*Log[a + b*x^3])/(3*a)
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)}\,dx = \int \left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}-\frac{b\left(c+dx+ex^2\right)}{a\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx+ex^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b\int \frac{c+dx}{a+bx^3}\,dx}{a}-\frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}-\frac{\left(b\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \cdots}{3a^{5/3}}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}+\frac{\left(\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\right)\int \cdots}{6a^{5/3}}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\cdots\right)}{6a^{5/3}}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{e\log(x)}{a}-\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\cdots$$

Mathematica [A] time = 0.177999, size = 192, normalized size = 0.95

$$\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(a^{2/3}d-\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*Log[x] + 2*b^(1/3)*(
-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/
3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] -
2*a*e*Log[a + b*x^3])/(6*a^2)
```

Maple [A] time = 0.005, size = 225, normalized size = 1.1

$$-\frac{c}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{6a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d}{3a}\ln\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] -1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c+1/6/a/(1/b*a)^(2/3)*ln(x^2-(1/b*
a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(1/b*a)^(1/3)*x-1))*c+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-1/6/a/(1
/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/3/a*3^(1/2)/(1/b*a)^(
1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3*e*ln(b*x^3+a)/a-d/a/x+
e*ln(x)/a-1/2*c/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.21964, size = 9917, normalized size = 48.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

[Out] -1/36*(2*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 +
1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 +
a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^
3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2*log(1/36*((-I*s
qrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d +
a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3
- 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d + 2*a*b*c*d^2 - a*b*c^2*e +
a^2*d*e^2 + 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d
+ a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 +
a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) +
9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3
+ a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3)
+ 6*e/a) + (b^2*c^3 + a*b*d^3)*x) - 36*e*x^2*log(x) + 36*d*x - (((-I*sqrt(
3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^
2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 + 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sq
rt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a
*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2
- (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*
(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)
^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36
*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b
*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18
*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e
^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c
^2*e - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 -
(b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b
*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(
1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54
*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5
)^(1/3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x + 1/12*sqrt(1/3)*(((-I*sqrt(3) +
1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I
sqrt(3) + 1)(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d
+ a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^
3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c
*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 -
(d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/
a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a
^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 +
1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/a^5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) - (((-I*sqrt(3) +
1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e
/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*

```
e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2 - 3*sqrt(1/3)*a*x^2*sqrt(-(((-I*sqrt(3
) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2
)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*
e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2 - (
b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c
^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(
b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^
(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3) - 18*e*x^2)*log(-1/36*((-
I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d
 + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d
^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*
c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 -
 (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d - 2*a*b*c*d^2 + a*b*c^2*e
 - a^2*d*e^2 - 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*
c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3
 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3)
 + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*
c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1
/3) + 6*e/a) + 2*(b^2*c^3 + a*b*d^3)*x - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*
(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4
 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/
a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/a^5)^(1/3) + 6*e/a)*a^4*d - 6*a^2*b*c^2 - 6*a^3*d*e)*sqrt(-(((-I*sqr
t(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*
e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d +
 a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3
 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^3 - 12*((-I*sqrt(3) + 1)*(e^2/a^2
- (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(
b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^
(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/5
4*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^
5)^(1/3) + 6*e/a)*a^2*e + 144*b*c*d + 36*a*e^2)/a^3)) + 18*c)/(a*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07828, size = 285, normalized size = 1.4

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{e\log\left(\left|x\right|\right)}{a}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b}+\frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a + e*log(abs(x))/a - 1/6*((-a*b^2)^(1/3)*b*c +
(-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a
*b^2*d*(-a/b)^(1/3) + a*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3
*b) - 1/2*(2*d*x + c)/(a*x^2) - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b
^2)^(2/3)*a*b^2*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^3*b^3)
```

3.344 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2} dx$

Optimal. Leaf size=190

$$-\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}-\frac{c+\ldots}{3b(\ldots}$$

```
[Out] -(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(5/3)) + ((b
^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(5/3)) - ((d
 - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.16616, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1823, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}}-\frac{\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}-\frac{c+\ldots}{3b(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] -(c + d*x + e*x^2)/(3*b*(a + b*x^3)) - ((b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(5/3)) + ((b
^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(5/3)) - ((d
 - (2*a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(2/3)*b^(4/3))
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\int \frac{d+2ex}{a+bx^3}\,dx}{3b}$$

$$= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d+2\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}d+2\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b^{4/3}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}+\frac{\left(\frac{\sqrt[3]{b}d}{\sqrt[3]{a}}+2e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}}-\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{\ldots}\right.}{}$$

$$= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{5/3}}+\ldots$$

$$= -\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}\ldots\right.}{}$$

Mathematica [A] time = 0.128857, size = 174, normalized size = 0.92

$$\frac{\frac{\left(2\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}-\frac{2\sqrt{3}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{6b^{2/3}(c+x(d+ex))}{a+bx^3}}{18b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*(b^(1/3)*d + 2*a^(
1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(1/3)*
d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(b^(1/3)*d) + 2*a^(
1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(5/3
))
```

Maple [A] time = 0.009, size = 219, normalized size = 1.2

$$\frac{1}{bx^3+a}\left(-\frac{ex^2}{3b}-\frac{dx}{3b}-\frac{c}{3b}\right)+\frac{d}{9b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{18b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d\sqrt{3}}{9b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*e*x^2/b-1/3*d*x/b-1/3*c/b)/(b*x^3+a)+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*
a)^(1/3))*d-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+
1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-2
/9/b^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*e/(1/b*a)^(1/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^
(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.23643, size = 4988, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*e*x^2 + 2*(b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 +
8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e
*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a
^2*b^5))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a
^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3)
 - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1
/3)))^2*a^2*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^
2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3)
- 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/
3)))*a*b^2*d^2 + 8*a*d*e^2 + (b*d^3 + 8*a*e^3)*x) + 12*d*x - ((b^2*x^3 + a*
b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a
```

```
*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 +
 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) + 3*sqrt(1/3)*(b
^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b
^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1
)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))
)^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3
 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*
d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)
/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3
+ 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d
*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/
(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x + 3/2*sqrt
(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 -
 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d
^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e +
a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5)
 + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(
a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2
*a*b^3 + 32*d*e)/(a*b^3))) - ((b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)
^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8
*a*e^3)/(a^2*b^5))^(1/3))) - 3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3
)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5
))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2
*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*lo
g(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 -
 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d
^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e
+ 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 -
8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^
3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a*b^2*d^2 - 8
*a*d*e^2 + 2*(b*d^3 + 8*a*e^3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(
1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3
 - 8*a*e^3)/(a^2*b^5))^(1/3)))*a^2*b^3*e + a*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(
I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^
(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^
5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3))) + 12*
c)/(b^2*x^3 + a*b)
```

Sympy [A] time = 2.42221, size = 109, normalized size = 0.57

$$\operatorname{RootSum}\left(729t^3a^2b^5 + 54tab^2de + 8ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{162t^2a^2b^3e + 9tab^2d^2 + 8ade^2}{8ae^3 + bd^3}\right)\right)\right) - \frac{c + dx + ex^2}{3ab + 3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**3, Lambda(
_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**2 + 8*a*d*e**2)/(8*a
*e**3 + b*d**3)))) - (c + d*x + e*x**2)/(3*a*b + 3*b**2*x**3)
```

Giac [A] time = 1.0744, size = 258, normalized size = 1.36

$$-\frac{\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab}-\frac{x^2e+dx+c}{3\left(bx^3+a\right)b}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd-2\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) -
 1/3*(x^2*e + d*x + c)/((b*x^3 + a)*b) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b*d -
2*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a*b^3) + 1/18*((-a*b^2)^(1/3)*a*b^2*d + 2*(-a*b^2)^(2/3)*a*b*e)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.345 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} dx$

Optimal. Leaf size=200

$$\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}}-\frac{x\left(ae-\right.}{3ab\,(}$$

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^(2/3)*c + a^(2/3)*e)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(4
/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(4/3
)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(4/3)*b^(4/3))
```

Rubi [A] time = 0.15269, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}}-\frac{x\left(ae-\right.}{3ab\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```


```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(3*a*b*(a + b*x^3)) - ((b^(2/3)*c + a^(2/3)*e)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(4
/3)) - ((b^(2/3)*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(4/3
)) + ((b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(4/3)*b^(4/3))
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\int \frac{-ae-bcx}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}bc-2a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-\sqrt[3]{a}bc+a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,d}{18a^{4/3}b^{4/3}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{18a^{4/3}b^{4/3}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}}+$$

Mathematica [A] time = 0.173585, size = 186, normalized size = 0.93

$$\frac{-\left(a^{4/3}\sqrt[3]{b}e-a^{2/3}bc\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\left(a^{4/3}\sqrt[3]{b}e-a^{2/3}bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\left(a^{2/3}bc+a^{4/3}\sqrt[3]{b}e\right)\tan}{18a^2b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a*b^(2/3)*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) - 2*Sqrt[3]*(a^(2/3)
*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(
-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(2/3)*b
*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1
8*a^2*b^(5/3))
```

Maple [A] time = 0.007, size = 228, normalized size = 1.1

$$\frac{1}{bx^3+a}\left(\frac{cx^2}{3a}-\frac{ex}{3b}-\frac{d}{3b}\right)+\frac{e}{9b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{18b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{e\sqrt{3}}{9b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (1/3*c/a*x^2-1/3*e*x/b-1/3/b*d)/(b*x^3+a)+1/9/b^2*e/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))-1/18/b^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1
/9/b^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/
9/b/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/18/b/a/(1/b*a)^(1/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/9/b/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1
/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.77424, size = 5293, normalized size = 26.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*c*x^2 - 12*a*e*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
 - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
 - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c - 1/2*((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b
```

```
^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^
3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 + 2*a*b*c^2
*e + (b^2*c^3 + a^2*e^3)*x) - 12*a*d + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4
))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3))) + 3*sqrt(1/3)*(a*b^2*x^
3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^
4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) +
 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4)
)^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)
- 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4
) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(
I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^
4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3
)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*
e + 2*(b^2*c^3 + a^2*e^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1
/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b
^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)
^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)
/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 +
 a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16
*c*e)/(a^2*b^2))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2
*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^
(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c
^3 - a^2*e^3)/(a^4*b^4))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2
*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*
c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2
 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^
2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e
*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*
e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^
2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)
^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*
c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 - 2*a*b*c^2*e + 2*(b^2*c^3 + a^
2*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)
/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*s
qrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(
a^4*b^4))^(1/3)))*a^3*b^3*c + 2*a^3*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) -
 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4)
 - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2))))/
(a*b^2*x^3 + a^2*b)
```

Sympy [A] time = 1.66737, size = 124, normalized size = 0.62

$$\operatorname{RootSum}\left(729t^3a^4b^4 + 27ta^2b^2ce - a^2e^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^3b^3c + 9ta^3be^2 + 2abc^2e}{a^2e^3 + b^2c^3}\right)\right)\right) + \frac{-ad - aex + bcx}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b**2*c**3,
Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3*b*e**2 + 2*a*b*c**2
```

```
*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*x + b*c*x**2)/(3*a**2*b + 3*a*
b**2*x**3)
```

Giac [A] time = 1.10256, size = 273, normalized size = 1.36

$$-\frac{\left(bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae-\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{bcx^2-axe-ad}{3\left(bx^3+a\right)ab}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b*c*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*
b) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(b*c*x^2 - a*x*e - a*d)
/((b*x^3 + a)*a*b) + 1/18*((-a*b^2)^(1/3)*a*b^2*e + (-a*b^2)^(2/3)*b^2*c)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.346 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=199

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}-$$

```
[Out] -(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3))
 + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3))
 - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(2/3))
```

Rubi [A] time = 0.132274, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1854, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```


```
[Out] -(a*e - b*x*(c + d*x))/(3*a*b*(a + b*x^3)) - ((2*b^(1/3)*c + a^(1/3)*d)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(2/3))
 + ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(2/3))
 - ((2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(2/3))
```


Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^2}\,dx = -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-2c-dx}{a+bx^3}\,dx}{3a}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}c-\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{2/3}} + \frac{(2c+\ldots}{}$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} + \ldots$$

$$= -\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\ldots\right.}{}$$

Mathematica [A] time = 0.178538, size = 189, normalized size = 0.95

$$\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\left(4\sqrt[3]{a}b^{2/3}c-2a^{2/3}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{6a(bx(c+dx)-ae)}{a+bx^3}-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\ldots}{18a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

```
[Out] ((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(2*
b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (4*a^(
1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^
(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^2*b)
```

Maple [A] time = 0.002, size = 253, normalized size = 1.3

$$\frac{cx}{3a\left(bx^3+a\right)}+\frac{2c}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2c\sqrt{3}}{9ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*c*x/a/(b*x^3+a)+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9*c/a/b/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9*c/a/b/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*d*x^2/a/(b*x^3+a)-1/9
*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18*d/a/b/(1/b*a)^(1/3)*ln(x^2-(1
/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3/b/(b*x^3+a)*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.24559, size = 5011, normalized size = 25.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b*d*x^2 + 12*b*c*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b
*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d
*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) - 12*a*e + ((
a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2
```

```
) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/
(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))
+ 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b
*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3
)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^
3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*
sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1
/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2)
 + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) +
 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d
^3)*x + 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5
*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) -
 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3
)))*a^4*b*d + 8*a^2*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 +
a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(
I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5
*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b))) + ((a*b^2*x^3 + a^2*b)*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a
^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) - 3*sqrt(1/3)*(a*b^2*x^3 + a
^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8
*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b
*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*
b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d
^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^
2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3
) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(
1/3)))*a^2*b*c^2 - 4*a*c*d^2 + 2*(8*b*c^3 + a*d^3)*x - 3/4*sqrt(1/3)*(((1/2
)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a
^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3
)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*a^4*b*d + 8*a^2*b*c^2)*s
qrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
- a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*
c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*
c*d)/(a^3*b))))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] time = 1.28355, size = 116, normalized size = 0.58

$$\operatorname{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right)+\frac{-ae+bcx+bdx^2}{3a^2b+3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda(
_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d**3
 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(3*a**2*b + 3*a*b**2*x**3)
```

Giac [A] time = 1.07851, size = 266, normalized size = 1.34

$$-\frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{bdx^2+bcx-ae}{3\left(bx^3+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(d*(-a/b)^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1
/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(b*d*x^2 + b*c*x - a*e)/((b
*x^3 + a)*a*b) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*c + (-a*b^2)^(2/3)*a*b^2*d)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

3.347 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=222

$$-\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(a}{3}$$

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(
2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.312688, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}+\frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}-\frac{\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}}+\frac{x(a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```


```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) - ((2*b^(1/3)*d + a^(1/3)*e
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(
2/3)) + (c*Log[x])/a^2 + ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/(9*a^(5/3)*b^(2/3)) - ((2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^2)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-2bdx-bex^2}{x\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax} - \frac{b\left(2ad+aex-3bcx^2\right)}{a\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex-3bcx^2}{a+bx^3}\,dx}{3a^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\int \frac{2ad+aex}{a+bx^3}\,dx}{3a^2} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{c\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}d+a^{4/3}e\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}d+a^{4/3}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}\sqrt[3]{b}} + \frac{\left(2d - \cdots\right.}{\cdots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right.}{\cdots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots\right.}{18a^{5/3}b^{2/3}}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}}$$

Mathematica [A] time = 0.156545, size = 199, normalized size = 0.9

$$\frac{\frac{\left(a^{2/3}e-2\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{6a(c+x(d+ex))}{a+bx^3} - 6c\log\left(a\cdots\right.}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^
(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*c*Log[x]
 + (2*(2*a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
 ((-2*a^(1/3)*b^(1/3)*d + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/b^(2/3) - 6*c*Log[a + b*x^3])/(18*a^2)
```

Maple [A] time = 0.01, size = 274, normalized size = 1.2

$$\frac{ex^2}{3a\left(bx^3+a\right)} + \frac{dx}{3a\left(bx^3+a\right)} + \frac{c}{3a\left(bx^3+a\right)} + \frac{2d}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{d}{9ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2d\sqrt{3}}{9ab}\,\mathrm{a}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2/(b*x^3+a)*e+1/3/a*x/(b*x^3+a)*d+1/3/a/(b*x^3+a)*c+2/9/b/a*d/(1/b*
a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9/b/a*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))+2/9/b/a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x-1))-1/9/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/18/a/b/(1/b*a)
^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/9/a/b*3^(1/2)/(1/b*a)^(1/3
)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3*c*ln(b*x^3+a)/a^2+c*ln(x)
/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.16404, size = 12189, normalized size = 54.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(108*a*e*x^2 + 108*a*d*x - 2*(a^2*b*x^3 + a^3)*((-I*sqrt(3) + 1)*(9*c
^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a
*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 +
 a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(
-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e
^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^
6*b^2))^(1/3) + 54*c/a^2)*log(1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2
 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) +
 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d
^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1
/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1
/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 5
4*c/a^2)^2*a^4*b*e + 12*b*c*d^2 + 9*b*c^2*e + 4*a*d*e^2 - 1/9*(2*a^2*b*d^2
+ 3*a^2*b*c*e)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/
(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*
e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a
^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d
*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a
^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + (8*b*d^3 +
 a*e^3)*x) + 108*a*c - (162*b*c*x^3 - (a^2*b*x^3 + a^3)*((-I*sqrt(3) + 1)*(
9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 +
a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/
(a^6*b^2))^(1/3) + 54*c/a^2) + 162*a*c - 3*sqrt(1/3)*(a^2*b*x^3 + a^3)*sqrt
```

```
(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a
^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^
2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b
) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(
4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b - 108*((-I*sqrt(
3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9
*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(
27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sq
rt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8
*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d
*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c + 2916*b*c^2 + 2592*a*d*e)/(a
^4*b)))*log(-1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*
b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b
)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2
*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3
 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b*
e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2 + 3*a^2*b*c*e)*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1
/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 2*(8*b*d^3 + a*e^3)*x + 1/1
08*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(
-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e
^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^
6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*
e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^
2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^4*b*e + 72*
a^2*b*d^2 - 54*a^2*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 +
2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/
1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/16
2*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/14
58*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c
/a^2)^2*a^4*b - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4
*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*
b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c
 + 2916*b*c^2 + 2592*a*d*e)/(a^4*b))) - (162*b*c*x^3 - (a^2*b*x^3 + a^3)*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1
/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3
 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 162*a*c + 3*sqrt(1/3)*(a^2*
b*x^3 + a^3)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4
*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3
 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*
b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 +
2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^
3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)^2*a^4*b
 - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c
^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^
5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))
```

```
^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a
^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 -
 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2)*a^2*b*c + 2916*b*c^2
 + 2592*a*d*e)/(a^4*b)))*log(-1/324*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2
 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) +
 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d
^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1
/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1
/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 5
4*c/a^2)^2*a^4*b*e - 12*b*c*d^2 - 9*b*c^2*e - 4*a*d*e^2 + 1/9*(2*a^2*b*d^2
+ 3*a^2*b*c*e)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/
(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*
e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a
^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d
*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a
^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a^2) + 2*(8*b*d^3
 + a*e^3)*x - 1/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^2 + 2*
a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/14
58*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 -
9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*
(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458
*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 54*c/a
^2)*a^4*b*e + 72*a^2*b*d^2 - 54*a^2*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/
a^4 - (9*b*c^2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*
e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^
2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/
27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)
/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b
^2))^(1/3) + 54*c/a^2)^2*a^4*b - 108*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b*c^
2 + 2*a*d*e)/(a^4*b))/(-1/27*c^3/a^6 + 1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b)
+ 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) - 1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*
d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 +
1/162*(9*b*c^2 + 2*a*d*e)*c/(a^6*b) + 1/1458*(8*b*d^3 + a*e^3)/(a^5*b^2) -
1/1458*(27*b^2*c^3 + a^2*e^3 - 2*(4*d^3 - 9*c*d*e)*a*b)/(a^6*b^2))^(1/3) +
54*c/a^2)*a^2*b*c + 2916*b*c^2 + 2592*a*d*e)/(a^4*b))) + 324*(b*c*x^3 + a*c
)*log(x))/(a^2*b*x^3 + a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.09506, size = 311, normalized size = 1.4

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3a^2}+\frac{c\log\left(|x|\right)}{a^2}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2}+\frac{ax^2e+adx+ac}{3\left(bx^3+a\right)a^2}-\frac{\left(a^3b\left(-\frac{a}{b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/9*sqrt(3)*(2*(-a*b^2
)^(1/3)*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^2*b^2) + 1/3*(a*x^2*e + a*d*x + a*c)/((b*x^3 + a)*a^2) - 1/9*
(a^3*b*(-a/b)^(1/3)*e + 2*a^3*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a^5*b) + 1/18*(2*(-a*b^2)^(1/3)*a*b^3*d + (-a*b^2)^(2/3)*a*b^2*e)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4)
```

3.348 $\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=231

$$-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}$$

```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2
/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a
+ b*x^3])/(3*a^2)
```

Rubi [A] time = 0.343126, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(a^{2/3}e+2b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```


```
[Out] -(c/(a^2*x)) + (x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + (2*(2*b^(2
/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(7/3)*b^(1/3)) + (d*Log[x])/a^2 + (2*(2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(1/3)) - ((2*b^(2/3)*c + a^(2/3)*e)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3)*b^(1/3)) - (d*Log[a
+ b*x^3])/(3*a^2)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx = \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-2bex^2+\frac{b^2cx^3}{a}}{x^2\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^2}-\frac{3bd}{ax}-\frac{b\left(2ae-4bcx-3bdx^2\right)}{a\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx-3bdx^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\int \frac{2ae-4bcx}{a+bx^3}\,dx}{3a^2} - \frac{(bd)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^3\right)}{3a^2} + \frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{a}bc+4a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-4\sqrt[3]{a}bc-2a\sqrt[3]{b}e\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{9a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{d\log\left(a+bx^3\right)}{3a^2} -$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}} - \frac{\left(2b^{2/3}c+a^{2/3}e\right)\mathrm{l}}{}$$

$$= -\frac{c}{a^2x} + \frac{x\left(ae-bcx-bdx^2\right)}{3a^2\left(a+bx^3\right)} + \frac{2\left(2b^{2/3}c-a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^2} + \frac{2\left(2b^{2/3}c+a^{2/3}e\right.}{9a^{7}}$$

Mathematica [A] time = 0.261757, size = 213, normalized size = 0.92

$$-\frac{\frac{\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}} - \frac{2\left(2a^{2/3}b^{2/3}c+a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{2\sqrt{3}a^{2/3}\left(a^{2/3}e-2b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{3a\left(a(d+ex)-bcx^2\right)}{a+bx^3} + \cdots}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*Sqrt[3]*a^(
2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
)/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(1/3)
 + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/(9*a^3)
```

Maple [A] time = 0.012, size = 275, normalized size = 1.2

$$-\frac{bcx^2}{3a^2\left(bx^3+a\right)} + \frac{ex}{3a\left(bx^3+a\right)} + \frac{d}{3a\left(bx^3+a\right)} + \frac{2e}{9ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{e}{9ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2}{9}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3*b/a^2*x^2/(b*x^3+a)*c+1/3/a*x/(b*x^3+a)*e+1/3/a/(b*x^3+a)*d+2/9/a/b/(1
/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/9/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3
)*x+(1/b*a)^(2/3))*e+2/9/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/b*a)^(1/3)*x-1))*e+4/9/a^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9/a^2*c/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*c*3^(1/2)/(1/b*a
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^2+d*l
n(x)/a^2-1/a^2*c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.34174, size = 11634, normalized size = 50.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*c*x^3 - 108*a*e*x^2 - 108*a*d*x + 2*(a^2*b*x^4 + a^3*x)*((-I*
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*log(-1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c
*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3
+ 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3
)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c - 9*a*b
*c*d^2 + 16*a*b*c^2*e + 3*a^2*d*e^2 + 1/18*(6*a^3*b*c*d - a^4*e^2)*((-I*sqr
t(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 -
 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(
a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-
1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3
 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^
(1/3) + 54*d/a^2) - 2*(8*b^2*c^3 - a^2*e^3)*x) + 324*a*c + (162*b*d*x^4 + 1
62*a*d*x - (a^2*b*x^4 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*
e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)
*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
- 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) + 3*sqrt(1/3)*(a^2
*b*x^4 + a^3*x)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/
```

```
(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e
^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b)
)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 +
 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*
(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) +
 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e
)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b)
 - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d
^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(
3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
+ 54*d/a^2)*a^2*d + 2916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)
*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d
/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) -
4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/
a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d
^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 5
4*d/a^2)^2*a^5*b*c + 9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3
*b*c*d - a^4*e^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/2
7*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/
3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/14
58*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^
2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x + 1
/108*sqrt(1/3)*(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*
d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*
(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3)
 + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458
*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*
c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e
^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a
^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^
3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
 a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a
^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/
1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*
b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/1
62*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*
d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*
a^2*d + 2916*d^2 - 10368*c*e)/a^4)) + (162*b*d*x^4 + 162*a*d*x - (a^2*b*x^4
 + a^3*x)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^
6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3
 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*
(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b
^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 3*sqrt(1/3)*(a^2*b*x^4 + a^3*x)*sqrt(
-(((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/16
2*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d
*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(
3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 +
 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)
/(a^7*b))^(1/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d
^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
 a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2
 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)
/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^2*d + 2
916*d^2 - 10368*c*e)/a^4))*log(1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2
```

```
- 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2
*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^
2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 -
8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a
^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5*b*c +
9*a*b*c*d^2 - 16*a*b*c^2*e - 3*a^2*d*e^2 - 1/18*(6*a^3*b*c*d - a^4*e^2)*((-
I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*
d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a
*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) +
1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^
2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7
*b))^(1/3) + 54*d/a^2) - 4*(8*b^2*c^3 - a^2*e^3)*x - 1/108*sqrt(1/3)*(((-I*
sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^
2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b
)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)
*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*
e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b
))^(1/3) + 54*d/a^2)*a^5*b*c - 54*a^3*b*c*d - 18*a^4*e^2)*sqrt(-(((-I*sqrt(
3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8
*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^
7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/
27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 -
 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1
/3) + 54*d/a^2)^2*a^4 - 108*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/
a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*
a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a
^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/
a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4
/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)*a^2*d + 2916*d^2 - 10
368*c*e)/a^4)) - 324*(b*d*x^4 + a*d*x)*log(x))/(a^2*b*x^4 + a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10002, size = 328, normalized size = 1.42

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{d\log\left(\left|x\right|\right)}{a^2}-\frac{4\,bcx^3-ax^2e-adx+3\,ac}{3\left(bx^4+ax\right)a^2}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae-2\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 - 1/3*(4*b*c*x^3 - a*x^2
*e - a*d*x + 3*a*c)/((b*x^4 + a*x)*a^2) + 1/9*((-a*b^2)^(1/3)*a*e - 2*(-a*b
```

```
^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 2/9*sqrt(3)
*((-a*b^2)^(1/3)*a*b^2*e + 2*(-a*b^2)^(2/3)*b^2*c)*arctan(1/3*sqrt(3)*(2*x
+ (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 2/9*(2*a^2*b^2*c*(-a/b)^(1/3) - a
^3*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b)
```

3.349 $\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}\right.}{}$$

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^3)
) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/3)
*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/
3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
8/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.345395, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(4\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```



```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c + b*d*x + b*e*x^2))/(3*a^2*(a + b*x^3)
) + (b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (e*Log[x])/a^2 - (b^(1/3)*(5*b^(1/3)
*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)) + (b^(1/3)*(5*b^(1/
3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
8/3)) - (e*Log[a + b*x^3])/(3*a^2)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{2b^2cx^3}{a}+\frac{b^2dx^4}{a}}{x^3\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \left(-\frac{3bc}{ax^3}-\frac{3bd}{ax^2}-\frac{3be}{ax}+\frac{b^2\left(5c+4dx+3ex^2\right)}{a\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx+3ex^2}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{b\int \frac{5c+4dx}{a+bx^3}\,dx}{3a^2} - \frac{(be)\int \frac{x^2}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{e\log\left(a+bx^3\right)}{3a^2} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-5\sqrt[3]{b}\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{9a^{8/3}}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} - \frac{e\log\left(a+\right.}{3a^2}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}\right.}{}$$

$$= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} + \frac{e\log(x)}{a^2} - \frac{\sqrt[3]{b}\left(5\sqrt[3]{}\right.}{}$$

Mathematica [A] time = 0.169457, size = 221, normalized size = 0.91

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-4a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(4a^{2/3}d-5\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{6a(ae-bx(c+dx))}{a+bx^3}+2\sqrt{3}\sqrt[3]{a}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x^3) + 2*Sq
rt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)
/a^(1/3))/Sqrt[3]] + 18*a*e*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1/3)*c + 4*a^
(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 4*a^(2/3
)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 6*a*e*Log[a + b*x^3])
/(18*a^3)
```

Maple [A] time = 0.013, size = 276, normalized size = 1.1

$$-\frac{bdx^2}{3\,a^2\left(bx^3+a\right)} - \frac{bcx}{3\,a^2\left(bx^3+a\right)} + \frac{e}{3\,a\left(bx^3+a\right)} - \frac{5\,c}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5\,c}{18\,a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{5\,c}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*b*x^2/(b*x^3+a)*d-1/3*b/a^2*x/(b*x^3+a)*c+1/3/a/(b*x^3+a)*e-5/9/a^
2*c/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+5/18/a^2*c/(1/b*a)^(2/3)*ln(x^2-(1/b*
a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(1/b*a)^(1/3)*x-1))+4/9/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-2/9/a^
2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-4/9/a^2*3^(1/2)/(1/
b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3*e*ln(b*x^3+a)/a^
2-d/a^2/x-1/2*c/a^2/x^2+e*ln(x)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.80153, size = 12122, normalized size = 50.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/324*(432*b*d*x^4 + 270*b*c*x^3 - 108*a*e*x^2 + 324*a*d*x + 2*(a^2*b*x^5
+ a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*
e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*
b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^
(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^
7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3
- 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*log(1/81*((-I*sqrt(3) +
 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 6
4*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a
*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d + 160*a*b*c*d^2 - 75*a*b*c^2*e + 36*a^2*
d*e^2 + 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20
*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 +
1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*
(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1
/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1
458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*
e/a^2) + (125*b^2*c^3 + 64*a*b*d^3)*x) + 162*a*c + (162*b*e*x^5 + 162*a*e*x
^2 - (a^2*b*x^5 + a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e
^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*
c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c
*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d
 + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c
^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) - 3*sqr
```

```
t(1/3)*(a^2*b*x^5 + a^3*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*
d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/145
8*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d
^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*
(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(
125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2
)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/
27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^
3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^
8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e
/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e
^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d
 + 2916*a*e^2)/a^5))*log(-1/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*
a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125
*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 4
5*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*
c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^
2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^
6*d - 160*a*b*c*d^2 + 75*a*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24
*a^4*d*e)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7
 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 -
 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*
b*d^3)*x + 1/54*sqrt(1/3)*(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a
*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*
b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45
*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c
*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2
*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d
 - 225*a^3*b*c^2 - 108*a^4*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b
*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/
1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(1
6*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/1
62*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/145
8*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/
a^2)^2*a^5 - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(
-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a
*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)
/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2
)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^
2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c
*d + 2916*a*e^2)/a^5)) + (162*b*e*x^5 + 162*a*e*x^2 - (a^2*b*x^5 + a^3*x^2)
*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 +
1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/
1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458
*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^
3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b*x^5 + a^3*x^
2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e
^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b
/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(
1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7
 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 -
 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^5 - 108*((-I*sqrt(3)
 + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c
*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2
*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) +
```

```
1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 +
64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)
*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5))*log(-1
/81*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6
+ 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 -
1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1
458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16
*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d - 160*a*b*c*d^2 + 75*a
*b*c^2*e - 36*a^2*d*e^2 - 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1
)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d +
9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3
+ 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*
(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*
a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b
)/a^8)^(1/3) + 54*e/a^2) + 2*(125*b^2*c^3 + 64*a*b*d^3)*x - 1/54*sqrt(1/3)*
(2*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6
+ 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 -
1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/14
58*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*
d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^6*d - 225*a^3*b*c^2 - 108*a^4
*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/2
7*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3
)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8
)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/
a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^
3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^5 - 108*((-I*sqrt
(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*
b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*
b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3
) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^
3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d
*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*a^3*e + 25920*b*c*d + 2916*a*e^2)/a^5)) - 3
24*(b*e*x^5 + a*e*x^2)*log(x))/(a^2*b*x^5 + a^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09414, size = 344, normalized size = 1.42

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{e\log\left(|x|\right)}{a^2}-\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bc+4\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}+\frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/18*(5*(-a*b^2)^(1/3)
*b*c + 4*(-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
 + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) + 5*a^2*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^5*b) - 1/6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*x^2*e + 6*a*d*x + 3
*a*c)/((b*x^3 + a)*a^2*x^2) - 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*a*b^3*c - 4*(-a
*b^2)^(2/3)*a*b^2*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a^4*b^3)
```

3.350 $\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=262

$$-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\,(5\sqrt[3]{\ }}{}$$

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (2*b*c*Log[x])
/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(8/3)) + (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.403572, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\,(5\sqrt[3]{\ }}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(3*a^2*(a + b*x^3)) + (b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (2*b*c*Log[x])
/a^3 - (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(8/3)) + (b^(1/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(18*a^(8/3)) + (2*b*c*Log[a + b*x^3])/(3*a^3)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3bc-3bdx-3bex^2+\frac{3b^2cx^3}{a}+\frac{2b^2dx^4}{a}+\frac{b^2ex^5}{a}}{x^4(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)} - \frac{\int\left(-\frac{3bc}{ax^4}-\frac{3bd}{ax^3}-\frac{3be}{ax^2}+\frac{6b^2c}{a^2x}+\frac{b^2(5ad+4aex-6bcx^2)}{a^2(a+bx^3)}\right)dx}{3ab}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{b\int\frac{5ad+4aex-6bcx^2}{a+bx^3}dx}{3a^3}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{b\int\frac{5ad+4aex}{a+bx^3}dx}{3a^3}+\frac{\left(2b^2c\right)\int\frac{x^2}{a+bx^3}\,c}{a^3}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}+\frac{2bc\log\left(a+bx^3\right)}{3a^3}-\frac{b^{2/3}\int\frac{\sqrt[3]{a}(10a}{}}{}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{9a^{8/3}}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}-\frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{9a^{8/3}}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}-\frac{2bc\log}{a^3}$$

Mathematica [A] time = 0.174743, size = 225, normalized size = 0.86

$$\frac{\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}d-4a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(4a^{2/3}e-5\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\frac{6ab(c+x(d+ex))}{a+bx^3}+12bc\,l}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x)))/(a + b
*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (
2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 36*b*c*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1
/3)*d + 4*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*
d - 4*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 12*b*c*Lo
g[a + b*x^3])/(18*a^3)
```

Maple [A] time = 0.013, size = 289, normalized size = 1.1

$$-\frac{bex^2}{3\,a^2\left(bx^3+a\right)}-\frac{bdx}{3\,a^2\left(bx^3+a\right)}-\frac{bc}{3\,a^2\left(bx^3+a\right)}-\frac{5\,d}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,d}{18\,a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^2/(b*x^3+a)*b*e-1/3/a^2*b*x/(b*x^3+a)*d-1/3/a^2*b/(b*x^3+a)*c-5/
9/a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+5/18/a^2/(1/b*a)^(2/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))*d-5/9/a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(1/b*a)^(1/3)*x-1))*d+4/9/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/
9/a^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/a^2*e*3^(1/
2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/3*b*c*ln(b*x^3
+a)/a^3-1/2*d/a^2/x^2-e/a^2/x-1/3*c/x^3/a^2-2*b*c*ln(x)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.1793, size = 12670, normalized size = 48.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(48*a*b*e*x^5 + 30*a*b*d*x^4 + 24*a*b*c*x^3 + 36*a^2*e*x^2 + 18*a^2*d
*x + 12*a^2*c + 2*(a^3*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*
b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 +
64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^
2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*
b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)
/a^9)^(1/3) - 12*b*c/a^3)*log((8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^
6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*
b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 -
5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*
b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c
/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3
) - 12*b*c/a^3)^2*a^6*e + 150*b^2*c*d^2 + 144*b^2*c^2*e + 160*a*b*d*e^2 + 1
/2*(25*a^3*b*d^2 + 48*a^3*b*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2
/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^
3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3
 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(4
32*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*
b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(
1/3) - 12*b*c/a^3) + (125*b^2*d^3 + 64*a*b*e^3)*x) - (36*b^2*c*x^6 + 36*a*b
*c*x^3 + (a^3*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a
^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)
*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 -
 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432
*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*
```

```
c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/
3) - 12*b*c/a^3) + 3*sqrt(1/3)*(a^3*b*x^6 + a^4*x^3)*sqrt(-((8*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3
/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 +
(216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 -
72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^
3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^
9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21
6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*
d*e)/a^6))*log(-(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2
 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*
b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72
*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 +
(125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^
3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3
)^2*a^6*e - 150*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d
^2 + 48*a^3*b*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*
c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9
 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216
*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/
a^3) + 2*(125*b^2*d^3 + 64*a*b*e^3)*x + 3/2*sqrt(1/3)*(2*(8*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^
9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (21
6*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*
(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^6*e - 25*a^3*b*d^2 + 24*a^3*b*
c*e)*sqrt(-((8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5
*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*
c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d
*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125
*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^
3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*
a^6 + 24*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*
b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2
 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)
*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*
d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 +
 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b
*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6)) - (36*b^2*c*x^6 + 36*a*b*c*x^3 + (a^3
*b*x^6 + a^4*x^3)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c
^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(
9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 -
72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9
+ (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*
b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a
^3) - 3*sqrt(1/3)*(a^3*b*x^6 + a^4*x^3)*sqrt(-((8*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b
*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3
+ 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2
 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)
```

```
*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^
3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 6
4*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 +
5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*
b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*c^2 + 320*a*b*d*e)/a^6))*lo
g(-(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)
/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a
*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2
)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 +
64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^
2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6*e - 1
50*b^2*c*d^2 - 144*b^2*c^2*e - 160*a*b*d*e^2 - 1/2*(25*a^3*b*d^2 + 48*a^3*b
*c*e)*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d
*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 +
5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*
b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3
 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64
*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3) + 2*(125
*b^2*d^3 + 64*a*b*e^3)*x - 3/2*sqrt(1/3)*(2*(8*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(432*b^3*c^3/a^9 + (125*b*d^
3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 6
4*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 +
5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*
b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^6*e - 25*a^3*b*d^2 + 24*a^3*b*c*e)*sqrt(-((
8*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6
)/(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d
*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^
9)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a
*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*
e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)^2*a^6 + 24*(8*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*b^2*c^2/a^6 - (9*b^2*c^2 + 5*a*b*d*e)/a^6)/(
432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)
*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^
(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(432*b^3*c^3/a^9 + (125*b*d^3 + 64*a*e^
3)*b/a^8 - 72*(9*b^2*c^2 + 5*a*b*d*e)*b*c/a^9 + (216*b^3*c^3 + 64*a^2*b*e^3
 - 5*(25*d^3 - 72*c*d*e)*a*b^2)/a^9)^(1/3) - 12*b*c/a^3)*a^3*b*c + 144*b^2*
c^2 + 320*a*b*d*e)/a^6)) + 72*(b^2*c*x^6 + a*b*c*x^3)*log(x))/(a^3*b*x^6 +
a^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07759, size = 373, normalized size = 1.42

$$\frac{2\,bc\log\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{2\,bc\log\left(\left|x\right|\right)}{a^3}-\frac{\left(5\,\left(-ab^2\right)^{\frac{1}{3}}bd+4\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ab^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*c*log(abs(b*x^3 + a))/a^3 - 2*b*c*log(abs(x))/a^3 - 1/18*(5*(-a*b^2)^
(1/3)*b*d + 4*(-a*b^2)^(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^3*b) - 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*a*b^3*d - 4*(-a*b^2)^(2/3)*a*b^2*e)*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^3) + 1/9*(4*a^4
*b^2*(-a/b)^(1/3)*e + 5*a^4*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a^7*b) - 1/6*(8*a*b*x^5*e + 5*a*b*d*x^4 + 4*a*b*c*x^3 + 6*a^2*x^2*e + 3*a^
2*d*x + 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

3.351 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3} dx$

Optimal. Leaf size=215

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

```
[Out] -(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x^3
)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) +
b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3))
```

Rubi [A] time = 0.196885, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1823, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}}-\frac{\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```


```
[Out] -(c + d*x + e*x^2)/(6*b*(a + b*x^3)^2) + (x*(d + 2*e*x))/(18*a*b*(a + b*x^3
)) - ((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) +
b^(1/3)*x])/(27*a^(5/3)*b^(5/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3))
```


Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx = -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{d+2ex}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\int \frac{-2d-2ex}{a+bx^3}\,dx}{18ab}$$

$$= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\int \frac{\sqrt[3]{a}\left(-4\sqrt[3]{b}d-2\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(2\sqrt[3]{b}d-2\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{4/3}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\cdots}}{27a^{5/3}b}$$

$$= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}}+\frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}}{18a^{4/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\cdots\right.}{54a^{5/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}+\frac{x(d+2ex)}{18ab\left(a+bx^3\right)}-\frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{5/3}b^{4/3}}$$

Mathematica [A] time = 0.179103, size = 198, normalized size = 0.92

$$\frac{\frac{\left(\sqrt[3]{a}e-\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2\sqrt{3}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-\frac{9b^{2/3}(c+x(d+ex))}{\left(a+bx^3\right)^2}+\frac{3b^{2/3}x(d+2ex)}{a\left(a+bx^3\right)}}{54b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d + e*x)))/
(a + b*x^3)^2 - (2*Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(
1/3)*x])/a^(5/3) + ((-(b^(1/3)*d) + a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(5/3))
```

Maple [A] time = 0.01, size = 255, normalized size = 1.2

$$\frac{1}{\left(bx^3+a\right)^2}\left(\frac{ex^5}{9a}+\frac{dx^4}{18a}-\frac{ex^2}{18b}-\frac{dx}{9b}-\frac{c}{6b}\right)+\frac{d}{27b^2a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{54b^2a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d\sqrt{3}}{27b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9/a*e*x^5+1/18*d/a*x^4-1/18*e*x^2/b-1/9*d*x/b-1/6*c/b)/(b*x^3+a)^2+1/27/
b^2/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-1/54/b^2/a/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/27/b^2/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/27/a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))*e+1/54/a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/27
/a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.74419, size = 5122, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(12*b*e*x^5 + 6*b*d*x^4 - 6*a*e*x^2 - 12*a*d*x - 2*(a*b^3*x^6 + 2*a^2
*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) +
 (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3
*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*log(1/
4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)
/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a
*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^4*b^3*e - 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*
b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/
(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^2*b^2*d^2 + 2*a*d*e^2 + (b
*d^3 + a*e^3)*x) - 18*a*c + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^
(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3))) + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b
^2*x^3 + a^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b
^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)
/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2
*a^3*b^3 + 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^
3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e
*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*
e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*s
qrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5)
)^(1/3)))*a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x + 3/4*sqrt(1/3)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)
*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 -
a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d
^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d
*e)/(a^3*b^3))) + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3))) - 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a
^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d
^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*
((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 +
 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)
/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(
3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3)))^2*a^4*b^3*e + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*
b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1
)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*
a^2*b^2*d^2 - 2*a*d*e^2 + 2*(b*d^3 + a*e^3)*x - 3/4*sqrt(1/3)*(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1
/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5
) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^4*b^3*e + 2*a^2*b^2*d^2)*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3
)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^3*b^3 + 16*d*e)/(a^3*b
^3))))/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)
```

Sympy [A] time = 5.12363, size = 148, normalized size = 0.69

$$\operatorname{RootSum}\left(19683t^3a^5b^5 + 81ta^2b^2de + ae^3 - bd^3, \left(t \mapsto t\log\left(x + \frac{729t^2a^4b^3e + 27ta^2b^2d^2 + 2ade^2}{ae^3 + bd^3}\right)\right)\right) + \frac{-3ac - 2adx - a}{18a^3b + 36a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*d**3, Lamb
da(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b**2*d**2 + 2*a*d*e**
2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x - a*e*x**2 + b*d*x**4 + 2*b*e*x
**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6)
```

Giac [A] time = 1.11349, size = 288, normalized size = 1.34

$$-\frac{\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}} e + d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\, a^2 b} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}} bd - \left(-ab^2\right)^{\frac{2}{3}} e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\, a^2 b^3} + \frac{2\,bx^5 e + bdx^4 - ax^2 e -}{18\left(bx^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*((-a/b)^(1/3)*e + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b)
+ 1/27*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/18*(2*b*x^5*e + b*d*x^4 - a
*x^2*e - 2*a*d*x - 3*a*c)/((b*x^3 + a)^2*a*b) + 1/54*((-a*b^2)^(1/3)*a*b^2*
d + (-a*b^2)^(2/3)*a*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^4
)
```

3.352 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=239

$$\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}}-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}}-$$

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*b*
c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/3)
*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3
)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3
)*b^(4/3))
```

Rubi [A] time = 0.204025, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1828, 1854, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}}-\frac{\left(a^{2/3}e+2b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(6*a*b*(a + b*x^3)^2) - (3*a*d - x*(a*e + 4*b*
c*x))/(18*a^2*b*(a + b*x^3)) - ((2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(4/3)) - ((2*b^(2/3)
*c - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(4/3)) + ((2*b^(2/3
)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3
)*b^(4/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx = -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{\int \frac{-ae-4bcx-3bdx^2}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} + \frac{\int \frac{2ae+4bcx}{a+bx^3}\,dx}{18a^2b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{a}bc+4a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(4\sqrt[3]{a}bc-2a\sqrt[3]{b}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{4/3}} - \frac{\left(2b^{2/3}\right.}{}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2}\right.}{}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2}\right.}{}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b\left(a+bx^3\right)} - \frac{\left(2b^{2/3}c+a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{\left(2b^{2/3}c-a\right.}{}$$

Mathematica [A] time = 0.28205, size = 214, normalized size = 0.9

$$\frac{\frac{3ab^{2/3}\left(-a^2(3d+2ex)+abx^2\left(7c+ex^2\right)+4b^2cx^5\right)}{\left(a+bx^3\right)^2} + \left(2a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - 2\sqrt{3}a^{2/3}\sqrt[3]{b}\left(a^{2/3}e+2b^{2/3}c\right)\tan}{54a^3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*b^(2/3)*(4*b^2*c*x^5 - a^2*(3*d + 2*e*x) + a*b*x^2*(7*c + e*x^2)))/(a
 + b*x^3)^2 - 2*Sqrt[3]*a^(2/3)*b^(1/3)*(2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*a^(2/3)*b*c + a^(4/3)*b^(1/3)*e)
*Log[a^(1/3) + b^(1/3)*x] + (2*a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b^(5/3))
```

Maple [A] time = 0.01, size = 256, normalized size = 1.1

$$\frac{1}{\left(bx^3+a\right)^2}\left(\frac{2\,bcx^5}{9\,a^2}+\frac{ex^4}{18\,a}+\frac{7\,cx^2}{18\,a}-\frac{ex}{9\,b}-\frac{d}{6\,b}\right)+\frac{e}{27\,b^2a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{54\,b^2a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (2/9/a^2*c*b*x^5+1/18/a*e*x^4+7/18*c/a*x^2-1/9*e*x/b-1/6/b*d)/(b*x^3+a)^2+1
/27/a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/54/a/b^2/(1/b*a)^(2/3)*ln(x
^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/27/a/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan
```

```
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-2/27/b/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)
^(1/3))*c+1/27/b/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+
2/27/b/a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.65986, size = 5632, normalized size = 23.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*c*x^5 + 6*a*b*e*x^4 + 42*a*b*c*x^2 - 12*a^2*e*x - 18*a^2*d -
2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^
2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/
2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8
*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c - 1/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 +
8*a*b*c^2*e + (8*b^2*c^3 + a^2*e^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2
*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b
^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3
))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)
*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(
a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 3
2*c*e)/(a^4*b^2)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*
e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e
*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a
^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) +
4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4)
 - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*
b^2*c^3 + a^2*e^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*
c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)
^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b
^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(
1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^
3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^
```

```
3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2
 + 32*c*e)/(a^4*b^2))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 +
 a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3))) - 3*sqrt(1/3
)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)
*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)
+ 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^
4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*b^2)
))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) -
 (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)
/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4
))^(1/3)))^2*a^5*b^3*c + 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2
*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*
e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 -
a^2*e^3)/(a^7*b^4))^(1/3)))*a^4*b*e^2 - 8*a*b*c^2*e + 2*(8*b^2*c^3 + a^2*e^
3)*x - 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(
a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sq
rt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3
)/(a^7*b^4))^(1/3)))*a^5*b^3*c + a^4*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1
/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^
7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^4*b^2 + 32*c*e)/(a^4*
b^2))))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [A] time = 3.38543, size = 170, normalized size = 0.71

$$\operatorname{RootSum}\left(19683t^3a^7b^4+162ta^3b^2ce-a^2e^3+8b^2c^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^5b^3c+27ta^4be^2+8abc^2e}{a^2e^3+8b^2c^3}\right)\right)\right)+\frac{-3a^2d-}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3 + 8*b**2*c
**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27*_t*a**4*b*e**2 + 8*
a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-3*a**2*d - 2*a**2*e*x + 7*a*b*
c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a*
*2*b**3*x**6)
```

Giac [A] time = 1.10307, size = 306, normalized size = 1.28

$$-\frac{\left(2\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae-2\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2}+\frac{4\,b^2cx^5+abx}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b*c*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
^3*b) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*a*e - 2*(-a*b^2)^(2/3)*c)*arctan(1/3*s
```

```
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18*(4*b^2*c*x^5 + a
*b*x^4*e + 7*a*b*c*x^2 - 2*a^2*x*e - 3*a^2*d)/((b*x^3 + a)^2*a^2*b) + 1/54*
((-a*b^2)^(1/3)*a*b^2*e + 2*(-a*b^2)^(2/3)*b^2*c)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/(a^3*b^4)
```

3.353 $\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a +
b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)
*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)
*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rubi [A] time = 0.188495, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1854, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] (x*(5*c + 4*d*x))/(18*a^2*(a + b*x^3)) - (a*e - b*x*(c + d*x))/(6*a*b*(a +
b*x^3)^2) - ((5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + ((5*b^(1/3)*c - 2*a^(1/3)*d)
*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(2/3)) - ((5*b^(1/3)*c - 2*a^(1/3)
*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3))
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
```

```
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx &= -\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}-\frac{\int \frac{-5c-4dx}{\left(a+bx^3\right)^2}\,dx}{6a}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}+\frac{\int \frac{10c+4dx}{a+bx^3}\,dx}{18a^2}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}+\frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}c+4\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}c+4\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}}{27a^{8/3}}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x}}{54a^{8/3}b^{2/3}}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\right.}{54a^{8/3}b^{2/3}}\\
&= \frac{x(5c+4dx)}{18a^2\left(a+bx^3\right)}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}-\frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\right.}{27a^{8/3}b^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.286109, size = 213, normalized size = 0.95

$$\frac{\frac{3a(-3a^2e+abx(8c+7dx)+b^2x^4(5c+4dx))}{(a+bx^3)^2}+\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}d-5\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{\right.}{54a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(a + b*x^3)
^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)
*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-5*b^(1/3)*c + 2*a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b)
```

Maple [A] time = 0.003, size = 308, normalized size = 1.4

$$\frac{cx}{6a\left(bx^3+a\right)^2}+\frac{5cx}{18a^2\left(bx^3+a\right)}+\frac{5c}{27ba^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5c}{54ba^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5c\sqrt{3}}{27ba^2}\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*c/a*x/(b*x^3+a)^2+5/18*c/a^2*x/(b*x^3+a)+5/27*c/a^2/b/(1/b*a)^(2/3)*ln(
x+(1/b*a)^(1/3))-5/54*c/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))+5/27*c/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))+1/6*d/a*x^2/(b*x^3+a)^2+2/9*d/a^2*x^2/(b*x^3+a)-2/27*d/a^2/b/(1/b*
a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/27*d/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3
)*x+(1/b*a)^(2/3))+2/27*d/a^2*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))+1/6*e/a*x^3/(b*x^3+a)^2-1/6*e/a/b/(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.38672, size = 5612, normalized size = 24.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(24*b^2*d*x^5 + 30*b^2*c*x^4 + 42*a*b*d*x^2 + 48*a*b*c*x - 18*a^2*e -
 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125
*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(
1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (
125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1
)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
 - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/
(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 +
 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 +
40*a*c*d^2 + (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4
*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*
c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*
b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
)) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)
/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-
I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d
^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1
/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^
3 + 8*a*d^3)*x + 3/2*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 +
8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3
)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a
^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8
*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*
c*d)/(a^5*b))) + ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2
))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3
)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) - 3*sqrt(1/3)*(a^2*b
^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2
*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*(
(125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^
2*a^6*b*d + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(
3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^
8*b^2))^(1/3)))*a^3*b*c^2 - 40*a*c*d^2 + 2*(125*b*c^3 + 8*a*d^3)*x - 3/2*sq
rt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (1
25*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/
(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^
(1/3)))*a^6*b*d + 25*a^3*b*c^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/
2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (12
5*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b))))/(a^2*b^
3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [A] time = 2.11682, size = 163, normalized size = 0.72

$$\operatorname{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right)\right)\right) + \frac{-3a^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*d*
*2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e + 8*a*b*c*x + 7*a*b*d*x**2 + 5*
b**2*c*x**4 + 4*b**2*d*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b**3*
x**6)
```

Giac [A] time = 1.12697, size = 301, normalized size = 1.34

$$-\frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bc - 2\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2} + \frac{4\,b^2dx^5 + 5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3
+ 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18*(4*b^2*d*x^5 + 5*b^2
*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/((b*x^3 + a)^2*a^2*b) + 1/54*(5
*(-a*b^2)^(1/3)*a*b^3*c + 2*(-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^4*b^4)
```

3.354 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=257

$$-\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+$$

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9
*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log
[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/
3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.41382, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```


```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*d + 4*a*e*x - 9
*b*c*x^2))/(18*a^3*(a + b*x^3)) - ((5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(2/3)) + (c*Log
[x])/a^3 + ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/
3)*b^(2/3)) - ((5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(8/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^3)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-5bdx-4bex^2+\frac{3b^2cx^3}{a}}{x\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^2c+10b^2dx+4b^2ex^2}{x\left(a+bx^3\right)}\,dx}{18a^2b^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^2c}{ax} + \frac{2b^2\left(5ad+2aex-9bcx^2\right)}{a\left(a+bx^3\right)}\right)dx}{18a^2b^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex-9bcx^2}{a+bx^3}\,dx}{9a^3}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\int \frac{5ad+2aex}{a+bx^3}\,dx}{9a^3} - \frac{(bc)\int \frac{x^2}{a+bx^3}\,dx}{a^3}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} - \frac{c\log\left(a+bx^3\right)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}\left(10a\sqrt[3]{b}d+2a^{4/3}e\right)+\sqrt[3]{\ldots}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\ldots}}{27a^{11/3}\ldots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{c\log\ldots}{\ldots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{\ldots}\right.}{\ldots}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} + \frac{x\left(5ad+4aex-9bcx^2\right)}{18a^3\left(a+bx^3\right)} - \frac{\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{c\log(x)}{a^3} + \frac{\left(5\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.177574, size = 229, normalized size = 0.89

$$\frac{\frac{\left(2a^{2/3}e-5\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}d-2a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{9a^2(c+x(d+ex))}{\left(a+bx^3\right)^2} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{3a(6c+\ldots}{a\ldots}}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4*e*x)))/(a
 + b*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*c*Log[x] + (2*(5*a^(1/3)*b^(1/3)*
d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*d
 + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 1
8*c*Log[a + b*x^3])/(54*a^3)
```

Maple [A] time = 0.013, size = 331, normalized size = 1.3

$$\frac{2\,bex^{5}}{9\,a^{2}\left(bx^{3}+a\right)^{2}}+\frac{5\,bdx^{4}}{18\,a^{2}\left(bx^{3}+a\right)^{2}}+\frac{bcx^{3}}{3\,a^{2}\left(bx^{3}+a\right)^{2}}+\frac{7\,ex^{2}}{18\,a\left(bx^{3}+a\right)^{2}}+\frac{4\,dx}{9\,a\left(bx^{3}+a\right)^{2}}+\frac{c}{2\,a\left(bx^{3}+a\right)^{2}}+\frac{5\,d}{27\,ba^{2}}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 2/9/a^2/(b*x^3+a)^2*x^5*b*e+5/18/a^2/(b*x^3+a)^2*b*x^4*d+1/3/a^2/(b*x^3+a)^
2*b*c*x^3+7/18/a/(b*x^3+a)^2*x^2*e+4/9/a/(b*x^3+a)^2*x*d+1/2/a/(b*x^3+a)^2*
c+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-5/54/a^2/b/(1/b*a)^(2/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+5/27/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-2/27/a^2/b/(1/b*a)^(1/3)*ln(x+(1/b
*a)^(1/3))*e+1/27/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))
*e+2/27/a^2/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))*e-1/3*c*ln(b*x^3+a)/a^3+c*ln(x)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.6925, size = 13407, normalized size = 52.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2916*(648*a*b*e*x^5 + 810*a*b*d*x^4 + 972*a*b*c*x^3 + 1134*a^2*e*x^2 + 12
96*a^2*d*x + 1458*a^2*c - 2*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(3)
+ 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(
81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) -
1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1
/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(
a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8
*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*log(1/1
458*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c
^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*
e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a
*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^
2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366
*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 4
86*c/a^3)^2*a^6*b*e + 225*b*c*d^2 + 162*b*c^2*e + 40*a*d*e^2 - 1/54*(25*a^3
*b*d^2 + 36*a^3*b*c*e)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e
)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/3936
```

```
6*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(2
5*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^
9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/
(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(
a^9*b^2))^(1/3) + 486*c/a^3) + (125*b*d^3 + 8*a*e^3)*x) - (1458*b^2*c*x^6 +
 2916*a*b*c*x^3 + 1458*a^2*c - (a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(
3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/145
8*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2)
 - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))
^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*
c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3
+ 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) - 3*
sqrt(1/3)*(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^
2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 +
10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(72
9*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(
I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/
39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 -
5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I
*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 +
 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^
8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9
*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a
*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^
2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3
)*a^3*b*c + 236196*b*c^2 + 116640*a*d*e)/(a^6*b)))*log(-1/1458*((-I*sqrt(3)
 + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*
(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) -
 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(
1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/
(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 +
8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*
b*e - 225*b*c*d^2 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3*
b*c*e)*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/2
7*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8
*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e
)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b
*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39
366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3)
+ 486*c/a^3) + 2*(125*b*d^3 + 8*a*e^3)*x + 1/486*sqrt(1/3)*(((-I*sqrt(3) +
1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81
*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/
39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3
) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^
9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a
^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^6*b*e +
 675*a^3*b*d^2 - 486*a^3*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*
b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/
(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 +
8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) +
1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b
*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 -
54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1
)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*
b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/3
9366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3)
 + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9
*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^
2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c +
```

```
236196*b*c^2 + 116640*a*d*e)/(a^6*b))) - (1458*b^2*c*x^6 + 2916*a*b*c*x^3 +
 1458*a^2*c - (a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)*((-I*sqrt(3) + 1)*(81*c^2/a
^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*
a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b
^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*s
qrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/393
66*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(
25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) + 3*sqrt(1/3)*(a^3*b^
2*x^6 + 2*a^4*b*x^3 + a^5)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2
 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*
b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2
*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3
+ 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*
d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1)*(81
*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2
 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*
(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 72
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) +
 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3
 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c + 23619
6*b*c^2 + 116640*a*d*e)/(a^6*b)))*log(-1/1458*((-I*sqrt(3) + 1)*(81*c^2/a^6
 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*
d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2
*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqr
t(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366
*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25
*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b*e - 225*b*c*d^2
 - 162*b*c^2*e - 40*a*d*e^2 + 1/54*(25*a^3*b*d^2 + 36*a^3*b*c*e)*((-I*sqrt(
3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/145
8*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2)
 - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))
^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*
c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3
+ 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3) + 2*
(125*b*d^3 + 8*a*e^3)*x - 1/486*sqrt(1/3)*(((-I*sqrt(3) + 1)*(81*c^2/a^6 -
(81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e
)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^
3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(1
25*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^
3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^6*b*e + 675*a^3*b*d^2 -
486*a^3*b*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*c^2/a^6 - (81*b*c^2 + 10*a*d*e)
/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366
*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25
*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*c^3/a^9
 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(
a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a
^9*b^2))^(1/3) + 486*c/a^3)^2*a^6*b - 972*((-I*sqrt(3) + 1)*(81*c^2/a^6 - (
81*b*c^2 + 10*a*d*e)/(a^6*b))/(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)
*c/(a^9*b) + 1/39366*(125*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3
 + 8*a^2*e^3 - 5*(25*d^3 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^9 + 1/1458*(81*b*c^2 + 10*a*d*e)*c/(a^9*b) + 1/39366*(12
5*b*d^3 + 8*a*e^3)/(a^8*b^2) - 1/39366*(729*b^2*c^3 + 8*a^2*e^3 - 5*(25*d^3
 - 54*c*d*e)*a*b)/(a^9*b^2))^(1/3) + 486*c/a^3)*a^3*b*c + 236196*b*c^2 + 11
6640*a*d*e)/(a^6*b))) + 2916*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*log(x))/(a^3
*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11628, size = 359, normalized size = 1.4

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{c\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd-2\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2}+\frac{4\,abx^5e+5\,abdx^4+6\,abcx^3\cdots}{18\left(bx^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b^
2)^(1/3)*b*d - 2*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/(a^3*b^2) + 1/18*(4*a*b*x^5*e + 5*a*b*d*x^4 + 6*a*b*c*x^3 + 7
*a^2*x^2*e + 8*a^2*d*x + 9*a^2*c)/((b*x^3 + a)^2*a^3) - 1/27*(2*a^4*b*(-a/b
)^(1/3)*e + 5*a^4*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) + 1/
54*(5*(-a*b^2)^(1/3)*a*b^3*d + 2*(-a*b^2)^(2/3)*a*b^2*e)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^4*b^4)
```

3.355 $\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=267

$$-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}$$

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*
e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)
*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x
^3])/(3*a^3)
```

Rubi [A] time = 0.461799, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}+\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}}+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{9\sqrt{3}a^{10/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -(c/(a^3*x)) + (x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + (x*(5*a*
e - 10*b*c*x - 9*b*d*x^2))/(18*a^3*(a + b*x^3)) + ((14*b^(2/3)*c - 5*a^(2/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)
*b^(1/3)) + (d*Log[x])/a^3 + ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(1/3) + b^
(1/3)*x])/(27*a^(10/3)*b^(1/3)) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(1/3)) - (d*Log[a + b*x
^3])/(3*a^3)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} - \frac{\int \frac{-6bc-6bdx-5bex^2+\frac{4b^2cx^3}{a}+\frac{3b^2dx^4}{a}}{x^2(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{\int \frac{18b^3c+18b^3dx+10b^3ex^2-\frac{10b^4cx^3}{a}}{x^2(a+bx^3)}\,dx}{18a^2b^3}$$

$$= \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{\int \left(\frac{18b^3c}{ax^2}+\frac{18b^3d}{ax}+\frac{2b^3(5ae-14bcx-9bdx^2)}{a(a+bx^3)}\right)dx}{18a^2b^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx-9bdx^2}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{\int \frac{5ae-14bcx}{a+bx^3}\,dx}{9a^3} - \frac{(bd)\int \frac{x}{a+}}{a^3}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} - \frac{d\log(a+bx^3)}{3a^3} + \frac{\int \frac{\sqrt[3]{a}(-1}{}}{}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{(14b^{2/3}c+5a^{2/3}e)\log(\sqrt[3]{a}}{27a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{d\log(x)}{a^3} + \frac{(14b^{2/3}c+5a^{2/3}e)\log(\sqrt[3]{a}}{27a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{c}{a^3x} + \frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{(14b^{2/3}c-5a^{2/3}e)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}} +$$

Mathematica [A] time = 0.221198, size = 248, normalized size = 0.93

$$\frac{-\frac{(14a^{2/3}b^{2/3}c+5a^{4/3}e)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{\sqrt[3]{b}} + \frac{2(14a^{2/3}b^{2/3}c+5a^{4/3}e)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{b}} - \frac{2\sqrt{3}a^{2/3}(5a^{2/3}e-14b^{2/3}c)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{9a^2(a(d+ex)-b}{(a+bx^3)^2}}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) + (9*a^2*(-
(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*a^(2/3)*(-14*b^(2/3)*c
 + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 54*a
*d*Log[x] + (2*(14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - ((14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/b^(1/3) - 18*a*d*Log[a + b*x^3])/(54*a^4)
```

Maple [A] time = 0.017, size = 334, normalized size = 1.3

$$-\frac{5\,b^2cx^5}{9\,a^3\left(bx^3+a\right)^2}+\frac{5\,bex^4}{18\,a^2\left(bx^3+a\right)^2}+\frac{bdx^3}{3\,a^2\left(bx^3+a\right)^2}-\frac{13\,bcx^2}{18\,a^2\left(bx^3+a\right)^2}+\frac{4\,ex}{9\,a\left(bx^3+a\right)^2}+\frac{d}{2\,a\left(bx^3+a\right)^2}+\frac{5\,e}{27\,ba^2}\ln\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -5/9/a^3/(b*x^3+a)^2*b^2*x^5*c+5/18/a^2/(b*x^3+a)^2*x^4*b*e+1/3/a^2/(b*x^3+
a)^2*b*d*x^3-13/18/a^2/(b*x^3+a)^2*b*x^2*c+4/9/a/(b*x^3+a)^2*x*e+1/2/a/(b*x
^3+a)^2*d+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-5/54/a^2/b/(1/b*a)
^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+5/27/a^2/b/(1/b*a)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+14/27/a^3/(1/b*a)^(1/3)*ln
(x+(1/b*a)^(1/3))*c-7/27/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*c-14/27/a^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)
*x-1))*c-1/3*d*ln(b*x^3+a)/a^3+d*ln(x)/a^3-c/a^3/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.0854, size = 13300, normalized size = 49.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2916*(4536*b^2*c*x^6 - 810*a*b*e*x^5 - 972*a*b*d*x^4 + 7938*a*b*c*x^3 -
1296*a^2*e*x^2 - 1458*a^2*d*x + 2916*a^2*c + 2*(a^3*b^2*x^7 + 2*a^4*b*x^4 +
 a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a
^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3
)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 7
0*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)
*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486
*d/a^3)*log(-7/1458*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458
*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3
 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))
^(1/3) + 486*d/a^3)^2*a^7*b*c - 1134*a*b*c*d^2 + 1960*a*b*c^2*e + 225*a^2*d
*e^2 + 1/54*(252*a^4*b*c*d - 25*a^5*e^2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (8
1*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39
```

366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/
39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125
*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 1
25*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3) - (2744*b^2*c^3 - 125*a^2*e^3)*x)
+ (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2*a^4*b*
x^4 + a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*
d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*
e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^
2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^
2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c
*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
+ 486*d/a^3) + 3*sqrt(1/3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*sqrt(-(((-I*
sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(
81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 -
70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(
1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9
+ 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b
) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6
- 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^
9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)
/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70
*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*
a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*
d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*log(7/1458*((-I*sqrt(3) + 1)*
(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c
*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*
b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I
sqrt(3) + 1)(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(27
44*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(
2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^7*b*c + 1134*a
*b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25*a^5*e^
2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 +
1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(
27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e
)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)
/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^
3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x + 1/486*sqrt(1/3)*(7*((-I*sqrt(3) + 1
)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70
*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*
a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*
(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(
2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366
*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^7*b*c - 3402*a
^4*b*c*d - 675*a^5*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70
*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*
b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(274
4*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a
^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 -
27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3
)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 -
(81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/
39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) -
1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)

```
/a^6)) + (1458*b^2*d*x^7 + 2916*a*b*d*x^4 + 1458*a^2*d*x - (a^3*b^2*x^7 + 2
*a^4*b*x^4 + a^5*x)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/
(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 1
25*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 -
 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458
*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3
 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))
^(1/3) + 486*d/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*sqrt(
-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1
/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(2
7*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^1
0*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)
*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/
(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3
)^2*a^6 - 972*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27
*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2
*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a
^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d
^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*
c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
 + 486*d/a^3)*a^3*d + 236196*d^2 - 816480*c*e)/a^6))*log(7/1458*((-I*sqrt(3
) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2
 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*
d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) +
 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39
366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/
39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^7*b*c +
 1134*a*b*c*d^2 - 1960*a*b*c^2*e - 225*a^2*d*e^2 - 1/54*(252*a^4*b*c*d - 25
*a^5*e^2)*((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3
/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3
 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e
^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 -
 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*
e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 4
86*d/a^3) - 2*(2744*b^2*c^3 - 125*a^2*e^3)*x - 1/486*sqrt(1/3)*(7*((-I*sqrt
(3) + 1)*(81*d^2/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d
^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*
c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3)
 + 729*(I*sqrt(3) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/
39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) -
1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^7*b*c -
 3402*a^4*b*c*d - 675*a^5*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*d^2/a^6 - (81*d
^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366
*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/393
66*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/2
7*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*c^3 + 125*a^
2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^2*c^3 - 125*
a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)^2*a^6 - 972*((-I*sqrt(3) + 1)*(81*d^2
/a^6 - (81*d^2 - 70*c*e)/a^6)/(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a
^9 + 1/39366*(2744*b^2*c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^1
0*b) - 1/39366*(2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 729*(I*sqrt(3
) + 1)*(-1/27*d^3/a^9 + 1/1458*(81*d^2 - 70*c*e)*d/a^9 + 1/39366*(2744*b^2*
c^3 + 125*a^2*e^3 - 27*(27*d^3 - 70*c*d*e)*a*b)/(a^10*b) - 1/39366*(2744*b^
2*c^3 - 125*a^2*e^3)/(a^10*b))^(1/3) + 486*d/a^3)*a^3*d + 236196*d^2 - 8164
80*c*e)/a^6)) - 2916*(b^2*d*x^7 + 2*a*b*d*x^4 + a^2*d*x)*log(x))/(a^3*b^2*x
^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11718, size = 377, normalized size = 1.41

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{d\log\left(|x|\right)}{a^3}+\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}ae-14\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}ab^2e+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 + 1/54*(5*(-a*b^2)^(1/3)
*a*e - 14*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b
) + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*a*b^2*e + 14*(-a*b^2)^(2/3)*b^2*c)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^3) - 1/18*(28*b^2*c
*x^6 - 5*a*b*x^5*e - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*x^2*e - 9*a^2*d*x +
 18*a^2*c)/((b*x^3 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^(1/3) - 5*a^4*
b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.356 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3} dx$

Optimal. Leaf size=276

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{\ldots}\right.}{27\ldots}$$

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^3)
^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)
*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*
a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c
 - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3
)) - (e*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.499915, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}-\frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{\ldots}\right.}{27\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```


```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c + b*d*x + b*e*x^2))/(6*a^2*(a + b*x^3)
^2) - (x*(11*b*c + 10*b*d*x + 9*b*e*x^2))/(18*a^3*(a + b*x^3)) + (2*b^(1/3)
*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(9*Sqrt[3]*a^(11/3)) + (e*Log[x])/a^3 - (2*b^(1/3)*(10*b^(1/3)*c - 7*
a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)) + (b^(1/3)*(10*b^(1/3)*c
 - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3
)) - (e*Log[a + b*x^3])/(3*a^3)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-6bex^2+\frac{5b^2cx^3}{a}+\frac{4b^2dx^4}{a}+\frac{3b^2ex^5}{a}}{x^3\left(a+bx^3\right)^2}dx}{6ab}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{22b^4cx^3}{a}-\frac{10b^4dx^4}{a}}{x^3\left(a+bx^3\right)}dx}{18a^2b^3}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \left(\frac{18b^3c}{ax^3}+\frac{18b^3d}{ax^2}+\frac{18b^3e}{ax}-\frac{2b^4\left(20c+14dx+9ex^2\right)}{a\left(a+bx^3\right)}\right) d}{18a^2b^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c+14dx+9ex^2}{a+bx^3}dx}{9a^3}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{b\int \frac{20c+14dx}{a+bx^3}dx}{9a^3} - (b$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{e\log\left(a+bx^3\right)}{3a^3} - \frac{b^2}{}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}\right.}{27a}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{e\log(x)}{a^3} - \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}\right.}{27a}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc+10bdx+9bex^2\right)}{18a^3\left(a+bx^3\right)} + \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\right.}{9\sqrt{3}a^{11/3}}$$

Mathematica [A] time = 0.209617, size = 253, normalized size = 0.92

$$\frac{2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{9a^2(ae-bx(c+dx))}{\left(a+bx^3\right)^2}+4\sqrt[3]{b}\left(7a^{2/3}d-10\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\frac{3a(6}{}}{54a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a + b*x^3)^2 +
 (3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*Sqrt[3]*a^(1/3)*b^(1/3
)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
+ 54*a*e*Log[x] + 4*b^(1/3)*(-10*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/
3) + b^(1/3)*x] + 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 18*a*e*Log[a + b*x^3])/(54*a^4)
```

Maple [A] time = 0.016, size = 337, normalized size = 1.2

$$-\frac{5\,b^2dx^5}{9\,a^3\left(bx^3+a\right)^2}-\frac{11\,b^2cx^4}{18\,a^3\left(bx^3+a\right)^2}+\frac{bex^3}{3\,a^2\left(bx^3+a\right)^2}-\frac{13\,bdx^2}{18\,a^2\left(bx^3+a\right)^2}-\frac{7\,bcx}{9\,a^2\left(bx^3+a\right)^2}+\frac{e}{2\,a\left(bx^3+a\right)^2}-\frac{20\,c}{27\,a^3}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -5/9/a^3/(b*x^3+a)^2*x^5*b^2*d-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c+1/3*b/a^2/(b
*x^3+a)^2*e*x^3-13/18/a^2/(b*x^3+a)^2*b*x^2*d-7/9/a^2/(b*x^3+a)^2*b*x*c+1/2
/a/(b*x^3+a)^2*e-20/27/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c+10/27/a^3/(1
/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-20/27/a^3/(1/b*a)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+14/27/a^3/(1/b*a)^(1/3
)*ln(x+(1/b*a)^(1/3))*d-7/27/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))*d-14/27/a^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))*d-1/3*e*ln(b*x^3+a)/a^3-d/a^3/x-1/2*c/a^3/x^2+e*ln(x)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 9.17148, size = 13346, normalized size = 48.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2916*(4536*b^2*d*x^7 + 3240*b^2*c*x^6 - 972*a*b*e*x^5 + 7938*a*b*d*x^4 +
 5184*a*b*c*x^3 - 1458*a^2*e*x^2 + 2916*a^2*d*x + 1458*a^2*c + 2*(a^3*b^2*x
^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81
*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/1968
3*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 5
6*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^
9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)
*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b
)/a^11)^(1/3) + 486*e/a^3)*log(7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*
b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^1
0 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a
^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1
/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*
c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^8*d + 3920*a*b*c*d^2 - 1800*a*b*c^
2*e + 567*a^2*d*e^2 + 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(
81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
```

+ 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(
I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 4*(1000*b^2*c^3 +
343*a*b*d^3)*x) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2*e*x^2 - (a^3*
b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d
+ 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4
/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^
3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 486*e/a^3) - 3*sqrt(1/3)*(a^3*b^2*x^8 + 2*a^4*b*x^5 +
a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)
/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^
3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 -
135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*
a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683
*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56
*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9
+ 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7))*log(-
7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e
^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a
*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e
)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d
+ 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(80
00*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e
/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e^2 - 1/27*(100
*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*
e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 +
1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/
a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a
^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x + 1/972*sqrt(1/3
)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3
/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d
^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*
a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d +
81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000
*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a
^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^
2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81
*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^
2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqr
t(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(
49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3)
+ 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280
*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/393
66*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) +
729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10
+ 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2
e^3 - 56(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*a^4*e + 326592

```
0*b*c*d + 236196*a*e^2)/a^7)) + (1458*b^2*e*x^8 + 2916*a*b*e*x^5 + 1458*a^2
*e*x^2 - (a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^
6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e
^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^
3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3)
 + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000
*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d
^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 3*sqrt(1/3)*(a^3*b^2*x^8 +
2*a^4*b*x^5 + a^5*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d +
81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19
683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 -
 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/
a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^
3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a
*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (2
80*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/
a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 72
9*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3
 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 1
35*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*a^4*e + 3265920*b*c*d + 236196*a*e^
2)/a^7))*log(-7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)
/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000
*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d
^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1
458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11
 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)
^(1/3) + 486*e/a^3)^2*a^8*d - 3920*a*b*c*d^2 + 1800*a*b*c^2*e - 567*a^2*d*e
^2 - 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280
*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^
10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*
a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 +
 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135
*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3) + 8*(1000*b^2*c^3 + 343*a*b*d^3)*x -
1/972*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a
^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b
*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3
 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/145
8*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 -
 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(
1/3) + 486*e/a^3)*a^8*d - 10800*a^4*b*c^2 - 3402*a^5*d*e)*sqrt(-(((-I*sqrt(
3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(
280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/
39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3
) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^
10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*
a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^7 - 972
*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9
 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*
b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)
/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*
a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2
*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*
a^4*e + 3265920*b*c*d + 236196*a*e^2)/a^7)) - 2916*(b^2*e*x^8 + 2*a*b*e*x^5
 + a^2*e*x^2)*log(x))/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09321, size = 390, normalized size = 1.41

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{e\log\left(|x|\right)}{a^3}-\frac{\left(10\,\left(-ab^2\right)^{\frac{1}{3}}bc+7\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b}-\frac{28\,b^2dx^7+20\,b^2cx^6-6\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 1/27*(10*(-a*b^2)^(1/3
)*b*c + 7*(-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b
) - 1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*x^5*e + 49*a*b*d*x^4 + 32*a*b
*c*x^3 - 9*a^2*x^2*e + 18*a^2*d*x + 9*a^2*c)/((b*x^4 + a*x)^2*a^3) - 2/27*s
qrt(3)*(10*(-a*b^2)^(1/3)*a*b^3*c - 7*(-a*b^2)^(2/3)*a*b^2*d)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b^3) + 2/27*(7*a^3*b^2*d*(-a/
b)^(1/3) + 10*a^3*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.357 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}+\frac{bc\log\left(a\right.}{a^4}$$

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(18*
a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^
4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```

Rubi [A] time = 0.589456, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{x\left(-\frac{15b^2cx^2}{a}+11bd+10bex\right)}{18a^3\left(a+bx^3\right)}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}+\frac{bc\log\left(a\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```


```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d + 10*b*e*x - (15*b^2*c*x^2)/a))/(18*
a^3*(a + b*x^3)) + (2*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)) - (3*b*c*Log[x])/a^
4 - (2*b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(11/3)) + (b^(1/3)*(10*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(27*a^(11/3)) + (b*c*Log[a + b*x^3])/a^4
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6bc-6bdx-6bex^2+\frac{6b^2cx^3}{a}+\frac{5b^2dx^4}{a}+\frac{4b^2ex^5}{a}-\frac{3b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^2}dx}{6ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} + \frac{\int \frac{18b^3c+18b^3dx+18b^3ex^2-\frac{36b^4cx^3}{a}-\frac{22b^4dx^4}{a}-\frac{10b^4ex^5}{a}}{x^4\left(a+bx^3\right)}}{18a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} + \frac{\int\left(\frac{18b^3c}{ax^4}+\frac{18b^3d}{ax^3}+\frac{18b^3e}{ax^2}-\frac{54b^4c}{a^2x}-\frac{2b^4(20ad+}{a^2(}\right.}{18a^2b^3}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \frac{b\int}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \frac{b\int}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} + \frac{bc\,l}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \frac{2\sqrt[3]{b}}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} - \frac{3bc\log(x)}{a^4} - \frac{2\sqrt[3]{b}}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3\left(a+bx^3\right)} + \frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d+7\right.}{9}$$

Mathematica [A] time = 0.370094, size = 255, normalized size = 0.86

$$-\frac{-2\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{9a^2b(c+x(d+ex))}{\left(a+bx^3\right)^2}+4\sqrt[3]{b}\left(10\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -((18*a*c)/x^3 + (27*a*d)/x^2 + (54*a*e)/x + (9*a^2*b*(c + x*(d + e*x)))/(a
 + b*x^3)^2 + (3*a*b*(12*c + x*(11*d + 10*e*x)))/(a + b*x^3) - 4*Sqrt[3]*a^
(1/3)*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]] + 162*b*c*Log[x] + 4*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*
e)*Log[a^(1/3) + b^(1/3)*x] - 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 54*b*c*Log[a + b*x^3])/(
54*a^4)
```

Maple [A] time = 0.017, size = 351, normalized size = 1.2

$$-\frac{5\,x^{5}e b^{2}}{9\,a^{3}\left(b x^{3}+a\right)^{2}}-\frac{11\,x^{4}b^{2}d}{18\,a^{3}\left(b x^{3}+a\right)^{2}}-\frac{2\,b^{2}c x^{3}}{3\,a^{3}\left(b x^{3}+a\right)^{2}}-\frac{13\,b e x^{2}}{18\,a^{2}\left(b x^{3}+a\right)^{2}}-\frac{7\,b d x}{9\,a^{2}\left(b x^{3}+a\right)^{2}}-\frac{5\,b c}{6\,a^{2}\left(b x^{3}+a\right)^{2}}-\frac{20\,d}{27\,a^{3}}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -5/9/a^3/(b*x^3+a)^2*x^5*e*b^2-11/18/a^3/(b*x^3+a)^2*x^4*b^2*d-2/3/a^3*b^2/
(b*x^3+a)^2*c*x^3-13/18/a^2/(b*x^3+a)^2*x^2*b*e-7/9/a^2/(b*x^3+a)^2*b*x*d-5
/6/a^2*b/(b*x^3+a)^2*c-20/27/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+10/27/
a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-20/27/a^3/(1/b*a)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+14/27/a^3*e/(1/b
*a)^(1/3)*ln(x+(1/b*a)^(1/3))-7/27/a^3*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))-14/27/a^3*e*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))+b*c*ln(b*x^3+a)/a^4-1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-
3*b*c*ln(x)/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 12.2192, size = 14256, normalized size = 47.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(168*a*b^2*e*x^8 + 120*a*b^2*d*x^7 + 108*a*b^2*c*x^6 + 294*a^2*b*e*x
^5 + 192*a^2*b*d*x^4 + 162*a^2*b*c*x^3 + 108*a^3*e*x^2 + 54*a^3*d*x + 36*a^
3*c + 2*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12
+ 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^
12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12
)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 +
 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c
^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/
a^4)*log(7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^
2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^
11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*
e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*
c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3
 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e + 5400*b^2*c*d^2 + 5
```

```
103*b^2*c^2*e + 3920*a*b*d*e^2 + (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(
39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 +
280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567
*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^1
2 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/
a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^
12)^(1/3) - 54*b*c/a^4) + 4*(1000*b^2*d^3 + 343*a*b*e^3)*x) - (162*b^3*c*x^
9 + 324*a*b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^
3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*
b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(7
29*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(
200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(3936
6*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*
a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d
*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4) + 3*sqrt(1/3)*(a^4*b^2*x^9 + 2*a^5*b*x
^6 + a^6*x^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (72
9*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e
^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 274
4*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(
729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*
(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)
/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2
+ 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 5
67*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a
^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*
c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/
a^12)^(1/3) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*log(
-7/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*
a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*
(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40
*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39
366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 28
0*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c
*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2*
c^2*e - 3920*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^
3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*
d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*
a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1
000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (
19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3
) - 54*b*c/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x + 3/4*sqrt(1/3)*(7*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a
^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c
^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3
- 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^
3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)
*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^
2)/a^12)^(1/3) - 54*b*c/a^4)*a^8*e - 400*a^4*b*d^2 + 378*a^4*b*c*e)*sqrt(-(
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d
*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*
b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200
*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b
^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b
*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)
*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^(2/3)*(-I*sqrt(3) +
```

```
1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12
+ 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^
12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12
)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 +
 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c
^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/
a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8)) - (162*b^3*c*x^9 + 324*a*
b^2*c*x^6 + 162*a^2*b*c*x^3 + (a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)*(2*(1/2
)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8
)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2
 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 -
567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/
a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b
*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)
/a^12)^(1/3) - 54*b*c/a^4) - 3*sqrt(1/3)*(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x
^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2
+ 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11
 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^
3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^
2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 -
 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^
3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*
d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*
a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1
000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (
19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3
) - 54*b*c/a^4)*a^4*b*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8))*log(-7/4*(2*(1
/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a
^8)/(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c
^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3
- 567*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^
3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)
*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^
2)/a^12)^(1/3) - 54*b*c/a^4)^2*a^8*e - 5400*b^2*c*d^2 - 5103*b^2*c^2*e - 39
20*a*b*d*e^2 - (100*a^4*b*d^2 + 189*a^4*b*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12
 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a
^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^1
2)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3
+ 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*
c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c
/a^4) + 8*(1000*b^2*d^3 + 343*a*b*e^3)*x - 3/4*sqrt(1/3)*(7*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366
*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a
*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*
e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8
*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12
+ (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(
1/3) - 54*b*c/a^4)*a^8*e - 400*a^4*b*d^2 + 378*a^4*b*c*e)*sqrt(-((2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(729*b^2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(
39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 +
280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567
*c*d*e)*a*b^2)/a^12)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^1
2 + 8*(1000*b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/
a^12 + (19683*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^
12)^(1/3) - 54*b*c/a^4)^2*a^8 + 108*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(729*b^
2*c^2/a^8 - (729*b^2*c^2 + 280*a*b*d*e)/a^8)/(39366*b^3*c^3/a^12 + 8*(1000*
```

```
b*d^3 + 343*a*e^3)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (1968
3*b^3*c^3 + 2744*a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*(39366*b^3*c^3/a^12 + 8*(1000*b*d^3 + 343*a*e^3
)*b/a^11 - 81*(729*b^2*c^2 + 280*a*b*d*e)*b*c/a^12 + (19683*b^3*c^3 + 2744*
a^2*b*e^3 - 40*(200*d^3 - 567*c*d*e)*a*b^2)/a^12)^(1/3) - 54*b*c/a^4)*a^4*b
*c + 2916*b^2*c^2 + 4480*a*b*d*e)/a^8)) + 324*(b^3*c*x^9 + 2*a*b^2*c*x^6 +
a^2*b*c*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08487, size = 421, normalized size = 1.41

$$\frac{bc\log\left(\left|bx^3+a\right|\right)}{a^4}-\frac{3\,bc\log\left(|x|\right)}{a^4}-\frac{\left(10\,\left(-ab^2\right)^{\frac{1}{3}}bd+7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b}-\frac{2\,\sqrt{3}\left(10\,\left(-ab^2\right)^{\frac{1}{3}}ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] b*c*log(abs(b*x^3 + a))/a^4 - 3*b*c*log(abs(x))/a^4 - 1/27*(10*(-a*b^2)^(1/
3)*b*d + 7*(-a*b^2)^(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*
b) - 2/27*sqrt(3)*(10*(-a*b^2)^(1/3)*a*b^3*d - 7*(-a*b^2)^(2/3)*a*b^2*e)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b^3) + 2/27*(7*a^5
*b^2*(-a/b)^(1/3)*e + 10*a^5*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))
/(a^9*b) - 1/18*(28*a*b^2*x^8*e + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*
b*x^5*e + 32*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*x^2*e + 9*a^3*d*x + 6*a^
3*c)/((b*x^3 + a)^2*a^4*x^3)
```

3.358 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4} dx$

Optimal. Leaf size=248

$$-\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}}-\frac{\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}}+$$

```
[Out] -(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x^3
)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^(1/3)*d + 4*a^(1/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(8/3)
*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(
8/3)*b^(5/3)) - ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(486*a^(8/3)*b^(5/3))
```

Rubi [A] time = 0.242314, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1823, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}+\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}}-\frac{\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```


```
[Out] -(c + d*x + e*x^2)/(9*b*(a + b*x^3)^3) + (x*(d + 2*e*x))/(54*a*b*(a + b*x^3
)^2) + (x*(5*d + 8*e*x))/(162*a^2*b*(a + b*x^3)) - ((5*b^(1/3)*d + 4*a^(1/3
)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(8/3)
*b^(5/3)) + ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(
8/3)*b^(5/3)) - ((5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(486*a^(8/3)*b^(5/3))
```


Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
```

```
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx = -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{\int \frac{d+2ex}{\left(a+bx^3\right)^3}\,dx}{9b}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}-\frac{\int \frac{-5d-8ex}{\left(a+bx^3\right)^2}\,dx}{54ab}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\int \frac{10d+8ex}{a+bx^3}\,dx}{162a^2b}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(20\sqrt[3]{b}d+8\sqrt[3]{a}e\right)+\sqrt[3]{b}\left(-10\sqrt[3]{b}d+8\sqrt[3]{a}e\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{486a^{8/3}b^{4/3}}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}}-\frac{(5}{}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}+\frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}}-\frac{(5}{}$$

$$= -\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}+\frac{x(d+2ex)}{54ab\left(a+bx^3\right)^2}+\frac{x(5d+8ex)}{162a^2b\left(a+bx^3\right)}-\frac{\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}}+\frac{(}{}$$

Mathematica [A] time = 0.232971, size = 230, normalized size = 0.93

$$\frac{\frac{3b^{2/3}x(5d+8ex)}{a^2\left(a+bx^3\right)}+\frac{\left(4\sqrt[3]{a}e-5\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{2\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}-\frac{2\sqrt{3}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}-\frac{54b^{2/3}(c+x(d+ex))}{\left(a+bx^3\right)^3}}{486b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d + 8*e*x))/
(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^3 - (2*Sqrt[
3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3)
 + ((-5*b^(1/3)*d + 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/a^(8/3))/(486*b^(5/3))
```

Maple [A] time = 0.01, size = 275, normalized size = 1.1

$$\frac{1}{\left(bx^3+a\right)^3}\left(\frac{4\,bex^8}{81\,a^2}+\frac{5\,bdx^7}{162\,a^2}+\frac{11\,ex^5}{81\,a}+\frac{13\,dx^4}{162\,a}-\frac{2\,ex^2}{81\,b}-\frac{5\,dx}{81\,b}-\frac{c}{9\,b}\right)+\frac{5\,d}{243\,b^2a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,d}{486\,b^2a^2}\ln\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] (4/81/a^2*b*e*x^8+5/162/a^2*d*b*x^7+11/81/a*e*x^5+13/162*d/a*x^4-2/81*e*x^2
/b-5/81*d*x/b-1/9*c/b)/(b*x^3+a)^3+5/243/b^2/a^2*d/(1/b*a)^(2/3)*ln(x+(1/b*
a)^(1/3))-5/486/b^2/a^2*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))+5/243/b^2/a^2*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3
)*x-1))-4/243/b^2/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+2/243/b^2/a^2*e/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+4/243/b^2/a^2*e*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.78528, size = 5952, normalized size = 24.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(48*b^2*e*x^8 + 30*b^2*d*x^7 + 132*a*b*e*x^5 + 78*a*b*d*x^4 - 24*a^2*
e*x^2 - 60*a^2*d*x - 108*a^2*c - 2*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2
*x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)
/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(
-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))*a^3*b^2*d^2 + 160*a*d*e
^2 + (125*b*d^3 + 64*a*e^3)*x) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*
x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3)
 + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^
3 + a^5*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8
*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sq
rt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e
^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(
I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(
a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3
+ 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3
*e + 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) +
(125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) +
```

```
1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8
*b^5))^(1/3)))*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x + 3/2
*sqrt(1/3)*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5
) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3
) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/
(a^8*b^5))^(1/3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(
3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5
))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*
e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*
d*e)/(a^5*b^3))) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
- 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
))) - 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b
*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^
5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))
^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)))*log(-((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3
) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a
^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e + 25/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 6
4*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((
125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))
*a^3*b^2*d^2 - 160*a*d*e^2 + 2*(125*b*d^3 + 64*a*e^3)*x - 3/2*sqrt(1/3)*(2*
((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3
 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^
3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/
3)))*a^6*b^3*e + 25*a^3*b^2*d^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*
b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*
(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5)
 + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 320*d*e)/(a^5*b^3)
)))/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 + a^5*b)
```

Sympy [A] time = 13.5946, size = 201, normalized size = 0.81

$$\operatorname{RootSum}\left(14348907t^3a^8b^5 + 14580ta^3b^2de + 64ae^3 - 125bd^3, \left(t \mapsto t\log\left(x + \frac{236196t^2a^6b^3e + 6075ta^3b^2d^2 + 160ade^2}{64ae^3 + 125bd^3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*e**3 - 125
*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*e + 6075*_t*a**3*b**
2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**3)))) + (-18*a**2*c - 10*a**2*
d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8*b**
2*e*x**8)/(162*a**5*b + 486*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*a**2*
b**4*x**9)
```

Giac [A] time = 1.07474, size = 333, normalized size = 1.34

$$-\frac{\left(4\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+5d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^3b}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}bd-4\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b^3}+\frac{8b^2x^8e+5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*(4*(-a/b)^(1/3)*e + 5*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^
3*b) + 1/243*sqrt(3)*(5*(-a*b^2)^(1/3)*b*d - 4*(-a*b^2)^(2/3)*e)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/162*(8*b^2*x^8*e
+ 5*b^2*d*x^7 + 22*a*b*x^5*e + 13*a*b*d*x^4 - 4*a^2*x^2*e - 10*a^2*d*x - 18
*a^2*c)/((b*x^3 + a)^3*a^2*b) + 1/486*(5*(-a*b^2)^(1/3)*a*b^2*d + 4*(-a*b^2
)^(2/3)*a*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^4)
```

3.359 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=270

$$\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}}-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}$$

```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x))
/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x^3
)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*
Log[a^(1/3) + b^(1/3)*x])/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/
3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/3)
)
```

Rubi [A] time = 0.25355, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1828, 1854, 1855, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}}-\frac{\left(5a^{2/3}e+14b^{2/3}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```


```
[Out] -(x*(a*e - b*c*x - b*d*x^2))/(9*a*b*(a + b*x^3)^3) + (x*(5*a*e + 28*b*c*x))
/(162*a^3*b*(a + b*x^3)) - (6*a*d - x*(a*e + 7*b*c*x))/(54*a^2*b*(a + b*x^3
)^2) - ((14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(81*Sqrt[3]*a^(10/3)*b^(4/3)) - ((14*b^(2/3)*c - 5*a^(2/3)*e)*
Log[a^(1/3) + b^(1/3)*x])/(243*a^(10/3)*b^(4/3)) + ((14*b^(2/3)*c - 5*a^(2/
3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(486*a^(10/3)*b^(4/3)
)
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
```

```
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}dx = -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} - \frac{\int \frac{-ae-7bcx-6bdx^2}{\left(a+bx^3\right)^3}dx}{9ab}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} + \frac{\int \frac{5ae+28bcx}{\left(a+bx^3\right)^2}dx}{54a^2b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-10ae-28bcx}{a+bx^3}dx}{162a^3b}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{\sqrt[3]{a}\left(-28\sqrt[3]{a}bc-20a\sqrt[3]{b}e\right)+\sqrt[3]{b}\left(-28\sqrt[3]{}\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{486a^{11/3}b^{4/3}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{243a^{10/3}b^{4/3}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{243a^{10/3}b^{4/3}}$$

$$= -\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3} + \frac{x(5ae+28bcx)}{162a^3b\left(a+bx^3\right)} - \frac{6ad-x(ae+7bcx)}{54a^2b\left(a+bx^3\right)^2} - \frac{\left(14b^{2/3}c+5a^{2/3}e\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{81\sqrt{3}a^{10/3}b^{4/3}}$$

Mathematica [A] time = 0.345652, size = 241, normalized size = 0.89

$$\frac{\frac{3ab^{2/3}\left(a^2bx^2\left(67c+13ex^2\right)-2a^3(9d+5ex)+ab^2x^5\left(77c+5ex^2\right)+28b^3cx^8\right)}{\left(a+bx^3\right)^3} + a^{2/3}\sqrt[3]{b}\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-2\sqrt{3}a^{2/3}\sqrt[3]{}}{486a^4b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

```
[Out] ((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77*c + 5*e*x
^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*Sqrt[3]*a^(2/3)*b^(1/
3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
 + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] + a^(
2/3)*b^(1/3)*(14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(486*a^4*b^(5/3))
```

Maple [A] time = 0.012, size = 278, normalized size = 1.

$$\frac{1}{\left(bx^3+a\right)^3}\left(\frac{14\,b^2cx^8}{81\,a^3}+\frac{5\,bex^7}{162\,a^2}+\frac{77\,bcx^5}{162\,a^2}+\frac{13\,ex^4}{162\,a}+\frac{67\,cx^2}{162\,a}-\frac{5\,ex}{81\,b}-\frac{d}{9\,b}\right)+\frac{5\,e}{243\,b^2a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,e}{486\,b^2a^2}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] (14/81*c/a^3*b^2*x^8+5/162/a^2*b*e*x^7+77/162/a^2*c*b*x^5+13/162/a*e*x^4+67
/162*c/a*x^2-5/81*e*x/b-1/9/b*d)/(b*x^3+a)^3+5/243/b^2/a^2*e/(1/b*a)^(2/3)*
ln(x+(1/b*a)^(1/3))-5/486/b^2/a^2*e/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))+5/243/b^2/a^2*e/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/b*a)^(1/3)*x-1))-14/243/b/a^3*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/243/b/
a^3*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+14/243/b/a^3*c*3^
(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 8.83698, size = 6597, normalized size = 24.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*c*x^8 + 30*a*b^2*e*x^7 + 462*a*b^2*c*x^5 + 78*a^2*b*e*x^4 +
402*a^2*b*c*x^2 - 60*a^3*e*x - 108*a^3*d - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 +
 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a
^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(
1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^1
0*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*log(7/2*((1/2)^(1
/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^
3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(
a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*
e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c - 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10
*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3
 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)
))*a^5*b*e^2 + 1960*a*b*c^2*e + (2744*b^2*c^3 + 125*a^2*e^3)*x) + ((a^3*b^4
*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sq
rt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
- (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*
sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*
c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a^6*b^2)))*lo
g(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4
) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-
I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^
2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c + 25/2*((1/2)^(1/3)*(I
*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 12
5*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^
```

```
2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(
a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(2744*b^2*c^3 + 125*a^2*e
^3)*x + 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*
a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*
(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^
10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^7*b^3*c + 25*a
^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)
/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2
/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
- (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 1120*c*e)/(a
^6*b^2))) + ((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(
1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c
^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/
(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2
*e^3)/(a^10*b^4))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^
5*b^2*x^3 + a^6*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140
*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a
^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^6*b^2 + 112
0*c*e)/(a^6*b^2)))*log(-7/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 1
25*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 1
40*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/
(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c +
 25/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4)
 - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I
*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2
*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*a^5*b*e^2 - 1960*a*b*c^2*e + 2*(274
4*b^2*c^3 + 125*a^2*e^3)*x - 3/2*sqrt(1/3)*(7*((1/2)^(1/3)*(I*sqrt(3) + 1)*
((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^
10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c
^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/
3)))*a^7*b^3*c + 25*a^5*b*e^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b
^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))
^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125
*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a
^6*b^2 + 1120*c*e)/(a^6*b^2))))/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^
3 + a^6*b)
```

Sympy [A] time = 7.04865, size = 214, normalized size = 0.79

$$\operatorname{RootSum}\left(14348907t^3a^{10}b^4 + 51030ta^4b^2ce - 125a^2e^3 + 2744b^2c^3, \left(t \mapsto t\log\left(x + \frac{826686t^2a^7b^3c + 6075ta^5be^2 + 1960}{125a^2e^3 + 2744b^2c^3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*a**2*e**3
+ 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a**7*b**3*c + 6075*_t
*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**3 + 2744*b**2*c**3)))) + (-18*
a**3*d - 10*a**3*e*x + 67*a**2*b*c*x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*c*x*
*5 + 5*a*b**2*e*x**7 + 28*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 4
86*a**4*b**3*x**6 + 162*a**3*b**4*x**9)
```

Giac [A] time = 1.08782, size = 346, normalized size = 1.28

$$-\frac{\left(14\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4b}+\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}ae-14\,\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2}+\frac{28\,b^3cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/243*(14*b*c*(-a/b)^(1/3) + 5*a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a^4*b) + 1/243*sqrt(3)*(5*(-a*b^2)^(1/3)*a*e - 14*(-a*b^2)^(2/3)*c)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) + 1/162*(28*b^3
*c*x^8 + 5*a*b^2*x^7*e + 77*a*b^2*c*x^5 + 13*a^2*b*x^4*e + 67*a^2*b*c*x^2 -
 10*a^3*x*e - 18*a^3*d)/((b*x^3 + a)^3*a^3*b) + 1/486*(5*(-a*b^2)^(1/3)*a*b
^2*e + 14*(-a*b^2)^(2/3)*b^2*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^4*b^4)
```

3.360 $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

Optimal. Leaf size=250

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{81\sqrt{3}a^{11/3}b^{2/3}}$$

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a
+ b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c
+ 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[
3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x])/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rubi [A] time = 0.222237, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1854, 1855, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{81\sqrt{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] (x*(8*c + 7*d*x))/(54*a^2*(a + b*x^3)^2) + (2*x*(10*c + 7*d*x))/(81*a^3*(a
+ b*x^3)) - (a*e - b*x*(c + d*x))/(9*a*b*(a + b*x^3)^3) - (2*(20*b^(1/3)*c
+ 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[
3]*a^(11/3)*b^(2/3)) + (2*(20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x])/(243*a^(11/3)*b^(2/3)) - ((20*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3))
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
```

```
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^4}\,dx = -\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} - \frac{\int \frac{-8c-7dx}{\left(a+bx^3\right)^3}\,dx}{9a}$$

$$= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} - \frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} + \frac{\int \frac{40c+28dx}{\left(a+bx^3\right)^2}\,dx}{54a^2}$$

$$= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} - \frac{\int \frac{-80c-28dx}{a+bx^3}\,dx}{162a^3}$$

$$= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} - \frac{\int \frac{\sqrt[3]{a}\left(-160\sqrt[3]{b}c-28\sqrt[3]{a}d\right)+\sqrt[3]{b}\left(80\sqrt[3]{b}c-28\sqrt[3]{a}d\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{486a^{11/3}\sqrt[3]{b}} + \cdots$$

$$= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}\cdots\right.}{\cdots}$$

$$= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} + \frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} - \frac{\left(20\sqrt[3]{b}\cdots\right.}{\cdots}$$

$$= \frac{x(8c+7dx)}{54a^2\left(a+bx^3\right)^2} + \frac{2x(10c+7dx)}{81a^3\left(a+bx^3\right)} - \frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3} - \frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} + \frac{2\left(20\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.231645, size = 239, normalized size = 0.96

$$\frac{\frac{2\left(7a^{2/3}d-20\sqrt[3]{a}\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{54a^3(ae-bx(c+dx))}{b\left(a+bx^3\right)^3} + \frac{9a^2x(8c+7dx)}{\left(a+bx^3\right)^2} - \frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\tan\cdots}{b^{2/3}}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

```
[Out] ((9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3
) - (54*a^3*(a*e - b*x*(c + d*x)))/(b*(a + b*x^3)^3) - (4*Sqrt[3]*a^(1/3)*(
20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + (4*(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b
^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

Maple [A] time = 0.004, size = 360, normalized size = 1.4

$$\frac{cx}{9a\left(bx^3+a\right)^3} + \frac{4cx}{27a^2\left(bx^3+a\right)^2} + \frac{20cx}{81a^3\left(bx^3+a\right)} + \frac{40c}{243a^3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{20c}{243a^3b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^4,x)
```

```
[Out] 1/9*c/a*x/(b*x^3+a)^3+4/27*c/a^2*x/(b*x^3+a)^2+20/81*c/a^3*x/(b*x^3+a)+40/2
43*c/a^3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-20/243*c/a^3/b/(1/b*a)^(2/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+40/243*c/a^3/b/(1/b*a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/9*d/a*x^2/(b*x^3+a)^3+7/54*d/a^2
*x^2/(b*x^3+a)^2+14/81*d/a^3*x^2/(b*x^3+a)-14/243*d/a^3/b/(1/b*a)^(1/3)*ln(
x+(1/b*a)^(1/3))+7/243*d/a^3/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))+14/243*d/a^3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^
(1/3)*x-1))+1/9*e/a*x^3/(b*x^3+a)^3+1/9*e/a^2*x^3/(b*x^3+a)^2-1/9*e/a^2/b/(
b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 6.22442, size = 6020, normalized size = 24.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/972*(168*b^3*d*x^8 + 240*b^3*c*x^7 + 462*a*b^2*d*x^5 + 624*a*b^2*c*x^4 +
402*a^2*b*d*x^2 + 492*a^2*b*c*x - 108*a^3*e - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^
6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^
3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*
d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)
*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a
*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*
a^4*b*c^2 + 7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^4*x^9 +
3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^
3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 14
0*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2)
+ (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 +
 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8
000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1
/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^
11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/
(a^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sq
rt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*
```

```
a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*
b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)
- 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2
 + 8*(8000*b*c^3 + 343*a*d^3)*x + 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)
*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2)
)^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a
^4*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))) + ((a^3*b^4*x^9 + 3*
a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3
+ 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*
4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +
(8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) - 3*sqrt(1/3)*(a^3*b^4*x^9 + 3
*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((800
0*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3
) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11
*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a
^7*b)))*log(-7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b
^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d + 400*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*
c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) -
140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 - 7840*a*c*d^2 +
 8*(8000*b*c^3 + 343*a*d^3)*x - 3/4*sqrt(1/3)*(7*(4^(1/3)*(I*sqrt(3) + 1)*(
(8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^
(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(
a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^8*b*d + 1600*a^4
*b*c^2)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2
) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3
) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^
3)/(a^11*b^2))^(1/3)))^2*a^7*b + 8960*c*d)/(a^7*b))))/(a^3*b^4*x^9 + 3*a^4*
b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

Sympy [A] time = 4.04936, size = 202, normalized size = 0.81

$$\operatorname{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 194400ta^4bc^2 + 7\ldots}{1372ad^3 + 32000bc^3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

```
[Out] RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 64
000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4*b
*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (-18*a**3*e + 82*a
**2*b*c*x + 67*a**2*b*d*x**2 + 104*a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 40*b*
*3*c*x**7 + 28*b**3*d*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**4*b**
3*x**6 + 162*a**3*b**4*x**9)
```

Giac [A] time = 1.08396, size = 333, normalized size = 1.33

$$-\frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{4}}+\frac{2\,\sqrt{3}\left(20\,\left(-ab^{2}\right)^{\frac{1}{3}}\,bc-7\,\left(-ab^{2}\right)^{\frac{2}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^{4}b^{2}}+\frac{28\,b^{3}d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^
4 + 2/243*sqrt(3)*(20*(-a*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2) + 1/162*(28*b^3*d*x^8 +
 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67*a^2*b*d*x^2 + 82*a^2*
b*c*x - 18*a^3*e)/((b*x^3 + a)^3*a^3*b) + 1/243*(20*(-a*b^2)^(1/3)*a*b^3*c
+ 7*(-a*b^2)^(2/3)*a*b^2*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b
^4)
```

3.361 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{81\sqrt{3}a^{11/3}b^{2/3}}$$

```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 1
5*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(
162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a
^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3
)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 0.516882, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}}-\frac{2\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{81\sqrt{3}a^{11/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```


```
[Out] (x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*d + 7*a*e*x - 1
5*b*c*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*d + 28*a*e*x - 99*b*c*x^2))/(
162*a^4*(a + b*x^3)) - (2*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(11/3)*b^(2/3)) + (c*Log[x])/a
^4 + (2*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(11/3
)*b^(2/3)) - ((20*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(243*a^(11/3)*b^(2/3)) - (c*Log[a + b*x^3])/(3*a^4)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-8bdx-7bex^2+\frac{6b^2cx^3}{a}}{x\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^2c+40b^2dx+28b^2ex^2-\frac{45b^3cx^3}{a}}{x\left(a+bx^3\right)^2}\,dx}{54a^2b^2}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-162b^3c-80b^3dx-28b}{x\left(a+bx^3\right)}}{162a^3b^3}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{162b^3c}{ax} - \frac{2b^3(40}{}\right.}{162}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \frac{\int 40ad+1}{}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \frac{\int \frac{40ad+1}{a+b}}{81}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} - \frac{c\log\left(a\right.}{3a}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \frac{2\left(20\sqrt[3]{b}\right.}{}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{c\log(x)}{a^4} + \frac{2\left(20\sqrt[3]{b}\right.}{}$$

$$= \frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ad+7aex-15bcx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ad+28aex-99bcx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{2\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)}{81\sqrt{3}a^1}$$

Mathematica [A] time = 0.227502, size = 259, normalized size = 0.89

$$\frac{\frac{2\left(7a^{2/3}e-20\sqrt[3]{a}\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{54a^3(c+x(d+ex))}{\left(a+bx^3\right)^3} + \frac{9a^2(9c+x(8d+7ex))}{\left(a+bx^3\right)^2} - \frac{4\sqrt{3}\sqrt[3]{a}\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)}{b^{2/3}}}{486a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

```
[Out] ((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d + 7*e*x)))
/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a + b*x^3) - (4*Sqrt[3]
*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 486*c*Log[x] + (4*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Lo
g[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3)*e)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a + b*x
```

```
^3])/(486*a^4)
```

Maple [A] time = 0.016, size = 394, normalized size = 1.4

$$\frac{14\,b^2ex^8}{81\,a^3\left(bx^3+a\right)^3}+\frac{20\,b^2dx^7}{81\,a^3\left(bx^3+a\right)^3}+\frac{b^2cx^6}{3\,a^3\left(bx^3+a\right)^3}+\frac{77\,bex^5}{162\,a^2\left(bx^3+a\right)^3}+\frac{52\,bdx^4}{81\,a^2\left(bx^3+a\right)^3}+\frac{5\,bcx^3}{6\,a^2\left(bx^3+a\right)^3}+\frac{}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^4,x)
```

```
[Out] 14/81/a^3/(b*x^3+a)^3*b^2*e*x^8+20/81/a^3/(b*x^3+a)^3*b^2*d*x^7+1/3/a^3/(b*
x^3+a)^3*c*b^2*x^6+77/162/a^2/(b*x^3+a)^3*b*e*x^5+52/81/a^2/(b*x^3+a)^3*b*d
*x^4+5/6/a^2/(b*x^3+a)^3*b*c*x^3+67/162/a/(b*x^3+a)^3*e*x^2+41/81/a/(b*x^3+
a)^3*d*x+11/18/a/(b*x^3+a)^3*c+40/243/a^3*d/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1
/3))-20/243/a^3*d/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+40/
243/a^3*d/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
-14/243/a^3*e/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+7/243/a^3*e/b/(1/b*a)^(1/
3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+14/243/a^3*e*3^(1/2)/b/(1/b*a)^(1/
3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*c*ln(b*x^3+a)/a^4+c*ln(x)/
a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.40697, size = 15555, normalized size = 53.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] 1/236196*(40824*a*b^2*e*x^8 + 58320*a*b^2*d*x^7 + 78732*a*b^2*c*x^6 + 11226
6*a^2*b*e*x^5 + 151632*a^2*b*d*x^4 + 196830*a^2*b*c*x^3 + 97686*a^3*e*x^2 +
 119556*a^3*d*x + 144342*a^3*c - 2*(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x
^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b
))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348
907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744
*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt
(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4
/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3
+ 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c
/a^4)*log(7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*
```

```
e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
+ 39366*c/a^4)^2*a^8*b*e + 64800*b*c*d^2 + 45927*b*c^2*e + 7840*a*d*e^2 - 1
/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (656
1*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*
a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/2869
7814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*
b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
+ 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/
(a^12*b^2))^(1/3) + 39366*c/a^4) + 4*(8000*b*d^3 + 343*a*e^3)*x) - (118098*
b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c - (a^4*b
^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c^2/a^8
 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)
/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561
*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*
b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e
)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) - 3*sqrt(1/3)*(a^4*b^3*x^9 + 3*a^5*
b^2*x^6 + 3*a^6*b*x^3 + a^7)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561
*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a
*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697
814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b
^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 +
 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1
/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(
a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2
/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b
*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^
2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*
a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(
6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a
^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c
*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 52
9079040*a*d*e)/(a^8*b)))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6
561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 56
0*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28
697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^1
2*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^
2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2)
- 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b
)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927*b*c^2*
e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3) + 1)*
(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/11809
8*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)
/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 170
1*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1
/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*
a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3
 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 8*(8000*b*d^3 + 343*
a*e^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c
^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e
)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*
(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))
```

```
^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560
*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/286
97814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12
*b^2))^(1/3) + 39366*c/a^4)*a^8*b*e + 388800*a^4*b*d^2 - 275562*a^4*b*c*e)*
sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/
(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907
*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^
2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3)
 + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14
348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2
744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^
4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*
e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 5904
9*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
+ 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 529079040*a*d*e)/(a^8*b))) - (1
18098*b^3*c*x^9 + 354294*a*b^2*c*x^6 + 354294*a^2*b*c*x^3 + 118098*a^3*c -
(a^4*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*((-I*sqrt(3) + 1)*(6561*c
^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561
*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*
b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e
)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098
*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/
(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701
*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^9 +
3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 -
 (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 +
 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1
/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(
a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b
*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^
2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*
a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(65
61*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(
6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a
^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c
*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/11
8098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e
^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 -
1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^
2 + 529079040*a*d*e)/(a^8*b)))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*c^2/a^
8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^
2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2)
- 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b
)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(656
1*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11
*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*
e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4)^2*a^8*b*e - 64800*b*c*d^2 - 45927*
b*c^2*e - 7840*a*d*e^2 + 1/243*(400*a^4*b*d^2 + 567*a^4*b*c*e)*((-I*sqrt(3)
 + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1
/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*
a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3
 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^
12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3
+ 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(8
00*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 39366*c/a^4) + 8*(8000*b*d^3
```

```
+ 343*a*e^3)*x - 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (65
61*b*c^2 + 560*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560
*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/286
97814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12
*b^2))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2
 + 560*a*d*e)*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) -
 1/28697814*(531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)
/(a^12*b^2))^(1/3) + 39366*c/a^4)*a^8*b*e + 388800*a^4*b*d^2 - 275562*a^4*b
*c*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 560*a*d*e)/(a^
8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b) + 4/14
348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c^3 + 2
744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 59049*(I*s
qrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^12*b)
+ 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441*b^2*c
^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3) + 3936
6*c/a^4)^2*a^8*b - 78732*((-I*sqrt(3) + 1)*(6561*c^2/a^8 - (6561*b*c^2 + 56
0*a*d*e)/(a^8*b))/(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)*c/(a^
12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(531441
*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^(1/3)
+ 59049*(I*sqrt(3) + 1)*(-1/27*c^3/a^12 + 1/118098*(6561*b*c^2 + 560*a*d*e)
*c/(a^12*b) + 4/14348907*(8000*b*d^3 + 343*a*e^3)/(a^11*b^2) - 1/28697814*(
531441*b^2*c^3 + 2744*a^2*e^3 - 80*(800*d^3 - 1701*c*d*e)*a*b)/(a^12*b^2))^
(1/3) + 39366*c/a^4)*a^4*b*c + 1549681956*b*c^2 + 529079040*a*d*e)/(a^8*b))
) + 236196*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*log(x))/(a^4
*b^3*x^9 + 3*a^5*b^2*x^6 + 3*a^6*b*x^3 + a^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09416, size = 409, normalized size = 1.41

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{c\log\left(|x|\right)}{a^4}+\frac{2\,\sqrt{3}\left(20\,\left(-ab^2\right)^{\frac{1}{3}}bd-7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^4b^2}+\frac{28\,ab^2x^8e+40\,ab^2dx^7+5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^4 + c*log(abs(x))/a^4 + 2/243*sqrt(3)*(20*(-a*
b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^4*b^2) + 1/162*(28*a*b^2*x^8*e + 40*a*b^2*d*x^7 + 54*a*b
^2*c*x^6 + 77*a^2*b*x^5*e + 104*a^2*b*d*x^4 + 135*a^2*b*c*x^3 + 67*a^3*x^2*
e + 82*a^3*d*x + 99*a^3*c)/((b*x^3 + a)^3*a^4) + 1/243*(20*(-a*b^2)^(1/3)*a
*b^3*d + 7*(-a*b^2)^(2/3)*a*b^2*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
```

```
/(a^5*b^4) - 2/243*(7*a^5*b*(-a/b)^(1/3)*e + 20*a^5*b*d)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^9*b)
```

3.362 $\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4} dx$

Optimal. Leaf size=301

$$-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\tan^{-1}\left(\sqrt[3]{}\right.}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}$$

```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*
e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x
 - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3
)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^
3])/(3*a^4)
```

Rubi [A] time = 0.601175, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{10\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(2a^{2/3}e+7b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}+\frac{20\left(7b^{2/3}c-2a^{2/3}e\right)\tan^{-1}\left(\sqrt[3]{}\right.}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] -(c/(a^4*x)) + (x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + (x*(8*a*
e - 16*b*c*x - 15*b*d*x^2))/(54*a^3*(a + b*x^3)^2) + (x*(40*a*e - 118*b*c*x
 - 99*b*d*x^2))/(162*a^4*(a + b*x^3)) + (20*(7*b^(2/3)*c - 2*a^(2/3)*e)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(13/3)*b^(1/3
)) + (d*Log[x])/a^4 + (20*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)
*x])/(243*a^(13/3)*b^(1/3)) - (10*(7*b^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(13/3)*b^(1/3)) - (d*Log[a + b*x^
3])/(3*a^4)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx = \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-8bex^2+\frac{7b^2cx^3}{a}+\frac{6b^2dx^4}{a}}{x^2\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+40b^3ex^2-\frac{64b^4cx^3}{a}-\frac{45b^4dx^4}{a}}{x^2\left(a+bx^3\right)^2}\,dx}{54a^2b^3}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-162b^5c-162b^5dx-}{x^2(a+}}{162}$$

$$= \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{162b^5c}{ax^2}-\frac{162b}{ax}\right.}{}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{d\log(x)}{a^4}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{d\log(x)}{a^4}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{d\log(x)}{a^4}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{d\log(x)}{a^4}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{d\log(x)}{a^4}$$

$$= -\frac{c}{a^4x} + \frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3} + \frac{x\left(8ae-16bcx-15bdx^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{x\left(40ae-118bcx-99bdx^2\right)}{162a^4\left(a+bx^3\right)} + \frac{20\left(7b^{2/3}\right.}{}$$

Mathematica [A] time = 0.29171, size = 279, normalized size = 0.93

$$\frac{-\frac{20\left(7a^{2/3}b^{2/3}c+2a^{4/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{40\left(7a^{2/3}b^{2/3}c+2a^{4/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{40\sqrt{3}a^{2/3}\left(2a^{2/3}e-7b^{2/3}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{54a^3\left(a(d+ex)-bcx\right.}{\left(a+bx^3\right)^3}}{486a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

```
[Out] ((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3)^2 + (6*a
*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^3*(-(b*c*x^2) + a*(d
 + e*x)))/(a + b*x^3)^3 - (40*Sqrt[3]*a^(2/3)*(-7*b^(2/3)*c + 2*a^(2/3)*e)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 486*a*d*Log[x] + (40
*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (2
```

```
0*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3])/(486*a^5)
```

Maple [A] time = 0.018, size = 397, normalized size = 1.3

$$-\frac{59\,b^3cx^8}{81\,a^4\left(bx^3+a\right)^3}+\frac{20\,b^2ex^7}{81\,a^3\left(bx^3+a\right)^3}+\frac{b^2dx^6}{3\,a^3\left(bx^3+a\right)^3}-\frac{142\,b^2cx^5}{81\,a^3\left(bx^3+a\right)^3}+\frac{52\,bex^4}{81\,a^2\left(bx^3+a\right)^3}+\frac{5\,bdx^3}{6\,a^2\left(bx^3+a\right)^3}-\frac{}{81\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x)
```

```
[Out] -59/81/a^4/(b*x^3+a)^3*b^3*c*x^8+20/81/a^3/(b*x^3+a)^3*x^7*b^2*e+1/3/a^3/(b
*x^3+a)^3*b^2*d*x^6-142/81/a^3/(b*x^3+a)^3*c*b^2*x^5+52/81/a^2/(b*x^3+a)^3*
b*e*x^4+5/6/a^2/(b*x^3+a)^3*b*d*x^3-92/81/a^2/(b*x^3+a)^3*b*c*x^2+41/81/a/(
b*x^3+a)^3*e*x+11/18/a/(b*x^3+a)^3*d+40/243/a^3*e/b/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))-20/243/a^3*e/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))+40/243/a^3*e/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)
*x-1))+140/243/a^4*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-70/243/a^4*c/(1/b*a)
^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-140/243/a^4*c*3^(1/2)/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^4-c/a^4
/x+d*ln(x)/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.53068, size = 15244, normalized size = 50.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/236196*(408240*b^3*c*x^9 - 58320*a*b^2*e*x^8 - 78732*a*b^2*d*x^7 + 11226
60*a*b^2*c*x^6 - 151632*a^2*b*e*x^5 - 196830*a^2*b*d*x^4 + 976860*a^2*b*c*x
^3 - 119556*a^3*e*x^2 - 144342*a^3*d*x + 236196*a^3*c + 2*(a^4*b^3*x^10 + 3
*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561
*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
```

```
/3) + 39366*d/a^4)*log(-7/236196*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^
2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12
+ 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)
*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) +
59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 39366*d/a^4)^2*a^9*b*c - 45927*a*b*c*d^2 + 78400*a*b*c^2*e + 6480*a^2*d*
e^2 + 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 -
(6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)
*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 560
0*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^
(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c
*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 -
5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b
))^(1/3) + 39366*d/a^4) - 400*(343*b^2*c^3 - 8*a^2*e^3)*x) + (118098*b^3*d*
x^10 + 354294*a*b^2*d*x^7 + 354294*a^2*b*d*x^4 + 118098*a^3*d*x - (a^4*b^3*
x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8
 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c
*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 -
5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b
))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 560
0*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3
 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^1
3*b))^(1/3) + 39366*d/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*
a^6*b*x^4 + a^7*x)*sqrt(-(((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 560
0*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/286
97814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(
a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(
I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/
28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b
)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 3936
6*d/a^4)^2*a^8 - 78732*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c
*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/286978
14*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^1
3*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*s
qrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/286
97814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(
a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d
/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e)/a^8))*log(7/236196*((-I*sqrt
(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118
098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*
e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^
3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/
118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a
^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2
*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^9*b*c + 45927*a*b*c*d^
2 - 78400*a*b*c^2*e - 6480*a^2*d*e^2 - 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*(
(-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12
 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64
000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(34
3*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a
^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 +
 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*
(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) - 800*(343*b^2*c^3
 - 8*a^2*e^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6
561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
```

/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e
)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 56
00*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))
^(1/3) + 39366*d/a^4)*a^9*b*c - 275562*a^5*b*c*d - 38880*a^6*e^2)*sqrt(-(((
-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12
+ 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 640
00*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343
*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^
12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 +
64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(
343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*
sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1
/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*
a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^
2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12
+ 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 640
00*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343
*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2
 - 5290790400*c*e)/a^8)) + (118098*b^3*d*x^10 + 354294*a*b^2*d*x^7 + 354294
*a^2*b*d*x^4 + 118098*a^3*d*x - (a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4
 + a^7*x)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/
27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b
^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/1
4348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(
-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(274400
0*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 400
0/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4) - 3*sqr
t(1/3)*(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)*sqrt(-(((-I*sqr
t(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/11
8098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2
e^3 - 243(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c
^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1
/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*
a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^
2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*sqrt(3
) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/11809
8*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^
3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3
- 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/11
8098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2
e^3 - 243(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c
^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 529
0790400*c*e)/a^8))*log(7/236196*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2
 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 +
 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*
a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 5
9049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^1
2 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*
e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
+ 39366*d/a^4)^2*a^9*b*c + 45927*a*b*c*d^2 - 78400*a*b*c^2*e - 6480*a^2*d*e
^2 - 1/243*(567*a^5*b*c*d - 40*a^6*e^2)*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (
6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*
d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600
*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(
1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*
e)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5
600*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b)
)^(1/3) + 39366*d/a^4) - 800*(343*b^2*c^3 - 8*a^2*e^3)*x - 1/78732*sqrt(1/3
)*(7*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561*d^2 - 5600*c*e)/a^8)/(-1/27*d^

```
3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2*c^
3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/143489
07*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27
*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^12 + 1/28697814*(2744000*b^2
*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d*e)*a*b)/(a^13*b) - 4000/143
48907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3) + 39366*d/a^4)*a^9*b*c - 27
5562*a^5*b*c*d - 38880*a^6*e^2)*sqrt(-(((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6
561*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e
)*d/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 56
00*c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))
^(1/3) + 39366*d/a^4)^2*a^8 - 78732*((-I*sqrt(3) + 1)*(6561*d^2/a^8 - (6561
*d^2 - 5600*c*e)/a^8)/(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d/a^
12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*c*d
*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1/3)
 + 59049*(I*sqrt(3) + 1)*(-1/27*d^3/a^12 + 1/118098*(6561*d^2 - 5600*c*e)*d
/a^12 + 1/28697814*(2744000*b^2*c^3 + 64000*a^2*e^3 - 243*(2187*d^3 - 5600*
c*d*e)*a*b)/(a^13*b) - 4000/14348907*(343*b^2*c^3 - 8*a^2*e^3)/(a^13*b))^(1
/3) + 39366*d/a^4)*a^4*d + 1549681956*d^2 - 5290790400*c*e)/a^8)) - 236196*
(b^3*d*x^10 + 3*a*b^2*d*x^7 + 3*a^2*b*d*x^4 + a^3*d*x)*log(x))/(a^4*b^3*x^1
0 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07262, size = 427, normalized size = 1.42

$$-\frac{d\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{4}}+\frac{d\log\left(|x|\right)}{a^{4}}+\frac{10\left(2\left(-ab^{2}\right)^{\frac{1}{3}}ae-7\left(-ab^{2}\right)^{\frac{2}{3}}c\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^{5}b}+\frac{20\sqrt{3}\left(2\left(-ab^{2}\right)^{\frac{1}{3}}ab^{2}e\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^4 + d*log(abs(x))/a^4 + 10/243*(2*(-a*b^2)^(1/
3)*a*e - 7*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*
b) + 20/243*sqrt(3)*(2*(-a*b^2)^(1/3)*a*b^2*e + 7*(-a*b^2)^(2/3)*b^2*c)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b^3) - 1/162*(280*b
^3*c*x^9 - 40*a*b^2*x^8*e - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 - 104*a^2*b*x^
5*e - 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*x^2*e - 99*a^3*d*x + 162*a
^3*c)/((b*x^3 + a)^3*a^4*x) + 20/243*(7*a^4*b^2*c*(-a/b)^(1/3) - 2*a^5*b*e)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b)
```

3.363 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx$

Optimal. Leaf size=310

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)}-\frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x}{}$$

```
[Out] -c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^3)
^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(139
*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(
1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(8
1*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3
)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c -
 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)
) - (e*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 0.65622, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}-\frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)}-\frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```


```
[Out] -c/(2*a^4*x^2) - d/(a^4*x) - (x*(b*c + b*d*x + b*e*x^2))/(9*a^2*(a + b*x^3)
^3) - (x*(17*b*c + 16*b*d*x + 15*b*e*x^2))/(54*a^3*(a + b*x^3)^2) - (x*(139
*b*c + 118*b*d*x + 99*b*e*x^2))/(162*a^4*(a + b*x^3)) + (20*b^(1/3)*(11*b^(
1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(8
1*Sqrt[3]*a^(14/3)) + (e*Log[x])/a^4 - (20*b^(1/3)*(11*b^(1/3)*c - 7*a^(1/3
)*d)*Log[a^(1/3) + b^(1/3)*x])/(243*a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*c -
 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)
) - (e*Log[a + b*x^3])/(3*a^4)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{8b^2cx^3}{a}+\frac{7b^2dx^4}{a}+\frac{6b^2ex^5}{a}}{x^3\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{85b^4cx^3}{a}-\frac{64b^4dx^4}{a}-\frac{45b^4ex^5}{a}}{x^3\left(a+bx^3\right)^2}}{54a^2b^3}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-162b^5c-}{}}{}$$

$$= -\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99bex^2\right)}{162a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{162b^5}{ax^3}\right.}{}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99be\right.}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99be\right.}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99be\right.}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99be\right.}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99be\right.}{162a^4\left(a+bx^3\right)}$$

$$= -\frac{c}{2a^4x^2} - \frac{d}{a^4x} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bc+16bdx+15bex^2\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bc+118bdx+99be\right.}{162a^4\left(a+bx^3\right)}$$

Mathematica [A] time = 0.286433, size = 284, normalized size = 0.92

$$20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{54a^3(ae-bx(c+dx))}{\left(a+bx^3\right)^3}+\frac{9a^2(9ae-bx(17c+16dx))}{\left(a+bx^3\right)^2}+40\sqrt[3]{b}\left(7a^{2/3}d-11\sqrt[3]{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

```
[Out] ((-243*a*c)/x^2 - (486*a*d)/x + (54*a^3*(a*e - b*x*(c + d*x)))/(a + b*x^3)^
3 + (9*a^2*(9*a*e - b*x*(17*c + 16*d*x)))/(a + b*x^3)^2 + (3*a*(54*a*e - b*
x*(139*c + 118*d*x)))/(a + b*x^3) + 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/3)*
c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 486*a*e*Log[
x] + 40*b^(1/3)*(-11*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)
*x] + 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)
```

```
*b^(1/3)*x + b^(2/3)*x^2] - 162*a*e*Log[a + b*x^3])/(486*a^5)
```

Maple [A] time = 0.019, size = 400, normalized size = 1.3

$$-\frac{59\,b^3dx^8}{81\,a^4\left(bx^3+a\right)^3}-\frac{139\,b^3cx^7}{162\,a^4\left(bx^3+a\right)^3}+\frac{b^2ex^6}{3\,a^3\left(bx^3+a\right)^3}-\frac{142\,b^2dx^5}{81\,a^3\left(bx^3+a\right)^3}-\frac{329\,b^2cx^4}{162\,a^3\left(bx^3+a\right)^3}+\frac{5\,bex^3}{6\,a^2\left(bx^3+a\right)^3}-\frac{9}{81\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x)
```

```
[Out] -59/81*b^3/a^4/(b*x^3+a)^3*d*x^8-139/162*b^3/a^4/(b*x^3+a)^3*c*x^7+1/3*b^2/
a^3/(b*x^3+a)^3*e*x^6-142/81*b^2/a^3/(b*x^3+a)^3*d*x^5-329/162*b^2/a^3/(b*x
^3+a)^3*c*x^4+5/6*b/a^2/(b*x^3+a)^3*e*x^3-92/81*b/a^2/(b*x^3+a)^3*d*x^2-104
/81*b/a^2/(b*x^3+a)^3*c*x+11/18/a/(b*x^3+a)^3*e-220/243/a^4*c/(1/b*a)^(2/3)
*ln(x+(1/b*a)^(1/3))+110/243/a^4*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))-220/243/a^4*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x-1))+140/243/a^4*d/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-70/243/a^4*d
/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-140/243/a^4*d*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*e*ln(b*x^3+a)/a
^4-1/2*c/a^4/x^2-d/a^4/x+e*ln(x)/a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 7.40401, size = 15485, normalized size = 49.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/236196*(408240*b^3*d*x^10 + 320760*b^3*c*x^9 - 78732*a*b^2*e*x^8 + 11226
60*a*b^2*d*x^7 + 833976*a*b^2*c*x^6 - 196830*a^2*b*e*x^5 + 976860*a^2*b*d*x
^4 + 657558*a^2*b*c*x^3 - 144342*a^3*e*x^2 + 236196*a^3*d*x + 118098*a^3*c
+ 2*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1
)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11809
8*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3
)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 -
2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/11
8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*
d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3
 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*log(7/236196*((-I*sqrt(3) +
```

```
1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/1180
98*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^
3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 -
 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/1
18098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a
*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^
3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^10*d + 431200*a*b*c*d^2
 - 196020*a*b*c^2*e + 45927*a^2*d*e^2 + 1/243*(1210*a^5*b*c^2 - 567*a^6*d*e
)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*
e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331
*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3
- 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/
27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1
331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e
^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 400*(133
1*b^2*c^3 + 343*a*b*d^3)*x) + (118098*b^3*e*x^11 + 354294*a*b^2*e*x^8 + 354
294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*
b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^
2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000
/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 +
531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sq
rt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4
000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3
 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/
a^4) - 3*sqrt(1/3)*(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*s
qrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1
/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(
1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*
e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*
(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/1434890
7*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a
^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^9
- 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(
-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907
*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^
2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1
)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348
907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441
*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^5*
e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9))*log(-7/236196*((-I*sqrt(3)
+ 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/11
8098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*
d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3
 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1
/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343
*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*
d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^10*d - 431200*a*b*c*d
^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/243*(1210*a^5*b*c^2 - 567*a^6*d
*e)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/2
7*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(13
31*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^
3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-
1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*
(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2
*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 800*(1
331*b^2*c^3 + 343*a*b*d^3)*x + 1/78732*sqrt(1/3)*(7*((-I*sqrt(3) + 1)*(6561
*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(3080
0*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^1
4 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*
```

```
d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(3
0800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/
a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673
*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^10*d - 1176120*a^5*b*c^2 - 275562
*a^6*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^
2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000
/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 +
531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sq
rt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4
000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3
 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/
a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*
e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 40
00/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3
+ 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*
sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 +
 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c
^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*
e/a^4)*a^5*e + 29099347200*b*c*d + 1549681956*a*e^2)/a^9)) + (118098*b^3*e*
x^11 + 354294*a*b^2*e*x^8 + 354294*a^2*b*e*x^5 + 118098*a^3*e*x^2 - (a^4*b^
3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)*((-I*sqrt(3) + 1)*(6561*e^2
/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*
c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 -
1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)
*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800
*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14
 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d
*e)*a*b)/a^14)^(1/3) + 39366*e/a^4) + 3*sqrt(1/3)*(a^4*b^3*x^11 + 3*a^5*b^2
*x^8 + 3*a^6*b*x^5 + a^7*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (308
00*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*
a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*
(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)
^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 65
61*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/286978
14*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^
14)^(1/3) + 39366*e/a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (3
0800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 656
1*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/2869781
4*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^1
4)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d +
6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/2869
7814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/
a^14)^(1/3) + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 1549681956*a*e^2)/a^
9))*log(-7/236196*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e
^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 400
0/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 +
 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*s
qrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 +
4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^
3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e
/a^4)^2*a^10*d - 431200*a*b*c*d^2 + 196020*a*b*c^2*e - 45927*a^2*d*e^2 - 1/
243*(1210*a^5*b*c^2 - 567*a^6*d*e)*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800
*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*
e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(1
0648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(
1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561
*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814
*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14
)^(1/3) + 39366*e/a^4) + 800*(1331*b^2*c^3 + 343*a*b*d^3)*x - 1/78732*sqrt(
```

```
1/3)*(7*((-I*sqrt(3) + 1)*(6561*e^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(
-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907
*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^
2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1
)*(-1/27*e^3/a^12 + 1/118098*(30800*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348
907*(1331*b*c^3 + 343*a*d^3)*b/a^14 - 1/28697814*(10648000*b^2*c^3 + 531441
*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^10
*d - 1176120*a^5*b*c^2 - 275562*a^6*d*e)*sqrt(-(((-I*sqrt(3) + 1)*(6561*e^2
/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*b*
c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14 -
1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*e)
*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(30800
*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14
 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d
*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)^2*a^9 - 78732*((-I*sqrt(3) + 1)*(6561*e
^2/a^8 - (30800*b*c*d + 6561*a*e^2)/a^9)/(-1/27*e^3/a^12 + 1/118098*(30800*
b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^14
- 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c*d*
e)*a*b)/a^14)^(1/3) + 59049*(I*sqrt(3) + 1)*(-1/27*e^3/a^12 + 1/118098*(308
00*b*c*d + 6561*a*e^2)*e/a^13 + 4000/14348907*(1331*b*c^3 + 343*a*d^3)*b/a^
14 - 1/28697814*(10648000*b^2*c^3 + 531441*a^2*e^3 - 2800*(980*d^3 - 2673*c
*d*e)*a*b)/a^14)^(1/3) + 39366*e/a^4)*a^5*e + 29099347200*b*c*d + 154968195
6*a*e^2)/a^9)) - 236196*(b^3*e*x^11 + 3*a*b^2*e*x^8 + 3*a^2*b*e*x^5 + a^3*e
*x^2)*log(x))/(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a^6*b*x^5 + a^7*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07115, size = 441, normalized size = 1.42

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^4}+\frac{e\log\left(|x|\right)}{a^4}-\frac{10\left(11\,\left(-ab^2\right)^{\frac{1}{3}}bc+7\,\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{20\,\sqrt{3}\left(11\,\left(-ab^2\right)^{\frac{1}{3}}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^4 + e*log(abs(x))/a^4 - 10/243*(11*(-a*b^2)^(1
/3)*b*c + 7*(-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5
*b) - 20/243*sqrt(3)*(11*(-a*b^2)^(1/3)*a*b^3*c - 7*(-a*b^2)^(2/3)*a*b^2*d)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b^3) + 20/243*(
7*a^4*b^2*d*(-a/b)^(1/3) + 11*a^4*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/(a^9*b) - 1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*x^8*e + 77
0*a*b^2*d*x^7 + 572*a*b^2*c*x^6 - 135*a^2*b*x^5*e + 670*a^2*b*d*x^4 + 451*a
^2*b*c*x^3 - 99*a^3*x^2*e + 162*a^3*d*x + 81*a^3*c)/((b*x^3 + a)^3*a^4*x^2)
```

3.364 $\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx$

Optimal. Leaf size=340

$$-\frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4\left(a+bx^3\right)}-\frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^2\right.}{243a^{14/3}}$$

```
[Out] -c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(54*
a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a^4
*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5
 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*
a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)
```

Rubi [A] time = 0.7727, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{x\left(-\frac{234b^2cx^2}{a}+139bd+118bex\right)}{162a^4\left(a+bx^3\right)}-\frac{x\left(-\frac{24b^2cx^2}{a}+17bd+16bex\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^2\right.}{243a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -c/(3*a^4*x^3) - d/(2*a^4*x^2) - e/(a^4*x) - (x*(b*d + b*e*x - (b^2*c*x^2)/
a))/(9*a^2*(a + b*x^3)^3) - (x*(17*b*d + 16*b*e*x - (24*b^2*c*x^2)/a))/(54*
a^3*(a + b*x^3)^2) - (x*(139*b*d + 118*b*e*x - (234*b^2*c*x^2)/a))/(162*a^4
*(a + b*x^3)) + (20*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(81*Sqrt[3]*a^(14/3)) - (4*b*c*Log[x])/a^5
 - (20*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(243*
a^(14/3)) + (10*b^(1/3)*(11*b^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(243*a^(14/3)) + (4*b*c*Log[a + b*x^3])/(3*a^5)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{\int \frac{-9bc-9bdx-9bex^2+\frac{9b^2cx^3}{a}+\frac{8b^2dx^4}{a}+\frac{7b^2ex^5}{a}-\frac{6b^3cx^6}{a^2}}{x^4\left(a+bx^3\right)^3}\,dx}{9ab}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} + \frac{\int \frac{54b^3c+54b^3dx+54b^3ex^2-\frac{108b^4cx^3}{a}-\frac{85b^4dx^4}{a}-\frac{64b^4ex^5}{a}+72\cdots}{x^4\left(a+bx^3\right)^2}}{54a^2b^3}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \frac{\int \frac{-162b^5c-162\cdots}{\cdots}}{\cdots}$$

$$= -\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} - \frac{\int\left(-\frac{162b^5c}{ax^4}-\cdots\right.}{\cdots}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\cdots\right.}{162a^4\left(a+bx\cdots\right.}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\cdots\right.}{162a^4\left(a+bx\cdots\right.}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\cdots\right.}{162a^4\left(a+bx\cdots\right.}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\cdots\right.}{162a^4\left(a+bx\cdots\right.}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\cdots\right.}{162a^4\left(a+bx\cdots\right.}$$

$$= -\frac{c}{3a^4x^3} - \frac{d}{2a^4x^2} - \frac{e}{a^4x} - \frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2} - \frac{x\left(139bd+118bex-\cdots\right.}{162a^4\left(a+bx\cdots\right.}$$

Mathematica [A] time = 0.47782, size = 284, normalized size = 0.84

$$-\frac{-20\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}d-7a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{54a^3b(c+x(d+ex))}{\left(a+bx^3\right)^3}+\frac{9a^2b(18c+x(17d+16ex))}{\left(a+bx^3\right)^2}+40\sqrt[3]{b}\left(11\sqrt[3]{a}\sqrt[3]{b}d-7a^{2}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

```
[Out] -((162*a*c)/x^3 + (243*a*d)/x^2 + (486*a*e)/x + (54*a^3*b*(c + x*(d + e*x))
)/(a + b*x^3)^3 + (9*a^2*b*(18*c + x*(17*d + 16*e*x)))/(a + b*x^3)^2 + (3*a
*b*(162*c + x*(139*d + 118*e*x)))/(a + b*x^3) - 40*Sqrt[3]*a^(1/3)*b^(1/3)*
(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] +
1944*b*c*Log[x] + 40*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(1/
3) + b^(1/3)*x] - 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e)*Log[a^(2/
```

```
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 648*b*c*Log[a + b*x^3])/(486*a^5)
```

Maple [A] time = 0.019, size = 415, normalized size = 1.2

$$-\frac{59\,b^3ex^8}{81\,a^4\left(bx^3+a\right)^3}-\frac{139\,b^3dx^7}{162\,a^4\left(bx^3+a\right)^3}-\frac{b^3cx^6}{a^4\left(bx^3+a\right)^3}-\frac{142\,b^2ex^5}{81\,a^3\left(bx^3+a\right)^3}-\frac{329\,b^2dx^4}{162\,a^3\left(bx^3+a\right)^3}-\frac{7\,b^2cx^3}{3\,a^3\left(bx^3+a\right)^3}-\frac{}{81\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x)
```

```
[Out] -59/81/a^4*b^3/(b*x^3+a)^3*e*x^8-139/162/a^4*b^3/(b*x^3+a)^3*d*x^7-1/a^4*b^
3/(b*x^3+a)^3*c*x^6-142/81/a^3*b^2/(b*x^3+a)^3*e*x^5-329/162/a^3*b^2/(b*x^3
+a)^3*d*x^4-7/3/a^3*b^2/(b*x^3+a)^3*c*x^3-92/81/a^2*b/(b*x^3+a)^3*e*x^2-104
/81/a^2*b/(b*x^3+a)^3*d*x-13/9/a^2*b/(b*x^3+a)^3*c-220/243/a^4*d/(1/b*a)^(2
/3)*ln(x+(1/b*a)^(1/3))+110/243/a^4*d/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+
(1/b*a)^(2/3))-220/243/a^4*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/b*a)^(1/3)*x-1))+140/243/a^4*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-70/243/a^
4*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-140/243/a^4*e*3^(1/
2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+4/3*b*c*ln(b*x^3
+a)/a^5-1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-4*b*c*ln(x)/a^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 12.2973, size = 15115, normalized size = 44.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="fricas")
```

```
[Out] -1/486*(840*a*b^3*e*x^11 + 660*a*b^3*d*x^10 + 648*a*b^3*c*x^9 + 2310*a^2*b^
2*e*x^8 + 1716*a^2*b^2*d*x^7 + 1620*a^2*b^2*c*x^6 + 2010*a^3*b*e*x^5 + 1353
*a^3*b*d*x^4 + 1188*a^3*b*c*x^3 + 486*a^4*e*x^2 + 243*a^4*d*x + 162*a^4*c +
 2*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt
(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*
b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 19
25*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1
701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^
15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e
)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)
*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*log(7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^
```

```
2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 12
5*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a
^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)
/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*
d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531
441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/
3) - 324*b*c/a^5)^2*a^10*e + 784080*b^2*c*d^2 + 734832*b^2*c^2*e + 431200*a
*b*d*e^2 + 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1)*(65
61*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15
 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*
b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a
*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(13
31*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
 (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15
)^(1/3) - 324*b*c/a^5) + 400*(1331*b^2*d^3 + 343*a*b*e^3)*x) - (972*b^4*c*x
^12 + 2916*a*b^3*c*x^9 + 2916*a^2*b^2*c*x^6 + 972*a^3*b*c*x^3 + (a^5*b^3*x^
12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561
*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 +
 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*
c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b
^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331
*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (
531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^
(1/3) - 324*b*c/a^5) + 3*sqrt(1/3)*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*
x^6 + a^8*x^3)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*
b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343
*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c
^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1
/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2
*a^10 + 648*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 +
1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqr
t(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243
*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 1
04976*b^2*c^2 + 123200*a*b*d*e)/a^10))*log(-7*(4^(2/3)*(-I*sqrt(3) + 1)*(65
61*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15
 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*
b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a
*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(13
31*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
 (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15
)^(1/3) - 324*b*c/a^5)^2*a^10*e - 784080*b^2*c*d^2 - 734832*b^2*c^2*e - 431
200*a*b*d*e^2 - 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1
)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3
/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*
d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d
*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 12
5*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a
^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)
/a^15)^(1/3) - 324*b*c/a^5) + 800*(1331*b^2*d^3 + 343*a*b*e^3)*x + 3*sqrt(1
/3)*(7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*
a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14
 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*
b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3)
+ 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(656
```

1*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 27
5*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^10*e - 2420*a^
5*b*d^2 + 2268*a^5*b*c*e)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^
10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*
b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (5
31441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(
1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 34
3*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*
c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324
*b*c/a^5)^2*a^10 + 648*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561
*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 34
3*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*
c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(
1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b
/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 4287
5*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*
a^5*b*c + 104976*b^2*c^2 + 123200*a*b*d*e)/a^10)) - (972*b^4*c*x^12 + 2916*
a*b^3*c*x^9 + 2916*a^2*b^2*c*x^6 + 972*a^3*b*c*x^3 + (a^5*b^3*x^12 + 3*a^6*
b^2*x^9 + 3*a^7*b*x^6 + a^8*x^3)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^
10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*
b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (5
31441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(
1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 34
3*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*
c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324
*b*c/a^5) - 3*sqrt(1/3)*(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a^7*b*x^6 + a^8*x
^3)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1
925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a
^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*
a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt
(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*
(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
- 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2*a^10 + 648
(4^(2/3)(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*
e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*
(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
- 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(
1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*
c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605
*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 104976*b^2*c
^2 + 123200*a*b*d*e)/a^10))*log(-7*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/
a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(133
1*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 +
(531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)
^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 +
343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^
3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 3
24*b*c/a^5)^2*a^10*e - 784080*b^2*c*d^2 - 734832*b^2*c^2*e - 431200*a*b*d*e
^2 - 4*(605*a^5*b*d^2 + 1134*a^5*b*c*e)*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2
*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125
*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^
15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/
a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d
^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (5314
41*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3
) - 324*b*c/a^5) + 800*(1331*b^2*d^3 + 343*a*b*e^3)*x - 3*sqrt(1/3)*(7*(4^(
2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 + 1925*a*b*d*e)/a^
10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561
*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275

```
*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqrt(3) + 1)*(10628
82*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 +
 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3 - 275*(605*d^3
- 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^10*e - 2420*a^5*b*d^2 + 2
268*a^5*b*c*e)*sqrt(-((4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*
b^2*c^2 + 1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343
*a*e^3)*b/a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c
^3 + 42875*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1
/3)*(I*sqrt(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)^2
*a^10 + 648*(4^(2/3)*(-I*sqrt(3) + 1)*(6561*b^2*c^2/a^10 - (6561*b^2*c^2 +
1925*a*b*d*e)/a^10)/(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/
a^14 - 243*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875
*a^2*b*e^3 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) + 4^(1/3)*(I*sqr
t(3) + 1)*(1062882*b^3*c^3/a^15 + 125*(1331*b*d^3 + 343*a*e^3)*b/a^14 - 243
*(6561*b^2*c^2 + 1925*a*b*d*e)*b*c/a^15 + (531441*b^3*c^3 + 42875*a^2*b*e^3
 - 275*(605*d^3 - 1701*c*d*e)*a*b^2)/a^15)^(1/3) - 324*b*c/a^5)*a^5*b*c + 1
04976*b^2*c^2 + 123200*a*b*d*e)/a^10)) + 1944*(b^4*c*x^12 + 3*a*b^3*c*x^9 +
 3*a^2*b^2*c*x^6 + a^3*b*c*x^3)*log(x))/(a^5*b^3*x^12 + 3*a^6*b^2*x^9 + 3*a
^7*b*x^6 + a^8*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06862, size = 459, normalized size = 1.35

$$\frac{4\,bc\log\left(\left|bx^3+a\right|\right)}{3\,a^5}-\frac{4\,bc\log\left(|x|\right)}{a^5}-\frac{10\left(11\,\left(-ab^2\right)^{\frac{1}{3}}bd+7\,\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b}-\frac{280\,b^3x^{11}e+220\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="giac")
```

```
[Out] 4/3*b*c*log(abs(b*x^3 + a))/a^5 - 4*b*c*log(abs(x))/a^5 - 10/243*(11*(-a*b^
2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^5*b) - 1/162*(280*b^3*x^11*e + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b
^2*x^8*e + 572*a*b^2*d*x^7 + 540*a*b^2*c*x^6 + 670*a^2*b*x^5*e + 451*a^2*b*
d*x^4 + 396*a^2*b*c*x^3 + 162*a^3*x^2*e + 81*a^3*d*x + 54*a^3*c)/((b*x^4 +
a*x)^3*a^4) - 20/243*sqrt(3)*(11*(-a*b^2)^(1/3)*a*b^3*d - 7*(-a*b^2)^(2/3)*
a*b^2*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b^3) +
20/243*(7*a^6*b^2*(-a/b)^(1/3)*e + 11*a^6*b^2*d)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a^11*b)
```

3.365 $\int \frac{2ax-x^2}{a^3+x^3} dx$

Optimal. Leaf size=29

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rubi [A] time = 0.0578282, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1593, 1868, 31, 617, 204}

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2ax-x^2}{a^3+x^3}\,dx &= \int \frac{(2a-x)x}{a^3+x^3}\,dx\\
&= a\int \frac{1}{a^2-ax+x^2}\,dx - \int \frac{1}{a+x}\,dx\\
&= -\log(a+x) + 2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{a}\right)\\
&= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)
\end{aligned}$$

Mathematica [A] time = 0.0138302, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(\log\left(a^2-ax+x^2\right)-\log\left(a^3+x^3\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x - x^2)/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x + x^2] - Log[a^3 + x^3])/3
```

Maple [A] time = 0.007, size = 29, normalized size = 1.

$$-\ln(a+x)+\frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-x^2)/(a^3+x^3),x)
```

```
[Out] -ln(a+x)+2/3*3^(1/2)*arctan(1/3*(2*x-a)/a*3^(1/2))
```

Maxima [A] time = 1.41899, size = 35, normalized size = 1.21

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Fricas [A] time = 0.991842, size = 80, normalized size = 2.76

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Sympy [C] time = 0.287255, size = 54, normalized size = 1.86

$$-\log\left(a+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x**2)/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

Giac [A] time = 1.06164, size = 36, normalized size = 1.24

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log\left(|a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

3.366 $\int \frac{(2a-x)x}{a^3+x^3}\,dx$

Optimal. Leaf size=29

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rubi [A] time = 0.0347537, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1868, 31, 617, 204}

$$-\log(a+x)-\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (-2*ArcTan[(a - 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a + x]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = a\int \frac{1}{a^2-ax+x^2}\,dx - \int \frac{1}{a+x}\,dx$$
$$= -\log(a+x) + 2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{a}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

Mathematica [A] time = 0.0052049, size = 57, normalized size = 1.97

$$\frac{1}{3}\left(\log\left(a^2-ax+x^2\right) - \log\left(a^3+x^3\right) - 2\log(a+x) + 2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2*a - x)*x)/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2] - Log[a^3 + x^3])/3
```

Maple [A] time = 0.006, size = 29, normalized size = 1.

$$-\ln(a+x) + \frac{2\sqrt{3}}{3}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-x)*x/(a^3+x^3),x)
```

```
[Out] -ln(a+x)+2/3*3^(1/2)*arctan(1/3*(2*x-a)/a*3^(1/2))
```

Maxima [A] time = 1.42322, size = 35, normalized size = 1.21

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - \log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Fricas [A] time = 1.00505, size = 80, normalized size = 2.76

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - \log(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

Sympy [C] time = 0.286357, size = 54, normalized size = 1.86

$$-\log{(a+x)}-\frac{\sqrt{3}i\log{\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}}{3}+\frac{\sqrt{3}i\log{\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a**3+x**3),x)
```

```
[Out] -log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a/
2 + sqrt(3)*I*a/2 + x)/3
```

Giac [A] time = 1.07708, size = 36, normalized size = 1.24

$$\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-\log\left(|a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-x)*x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

3.367 $\int \frac{2ax+x^2}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rubi [A] time = 0.0564489, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1593, 1868, 31, 617, 204}

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2ax+x^2}{a^3-x^3}\,dx &= \int \frac{x(2a+x)}{a^3-x^3}\,dx\\ &= -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right)-\int \frac{1}{-a+x}\,dx\\ &= -\log(a-x)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{a}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a-x)\end{aligned}$$

Mathematica [A] time = 0.0130022, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(\log\left(a^2+ax+x^2\right)-\log\left(x^3-a^3\right)-2\log(x-a)-2\sqrt{3}\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a*x + x^2)/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

Maple [A] time = 0.006, size = 29, normalized size = 0.9

$$-\frac{2\sqrt{3}}{3}\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)-\ln(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x+x^2)/(a^3-x^3),x)
```

```
[Out] -2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)-ln(-a+x)
```

Maxima [A] time = 1.43221, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Fricas [A] time = 1.02411, size = 81, normalized size = 2.61

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Sympy [C] time = 0.287166, size = 54, normalized size = 1.74

$$-\log(-a+x)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x**2)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

Giac [A] time = 1.05652, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log(|-a+x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

3.368 $\int \frac{x(2a+x)}{a^3-x^3}\,dx$

Optimal. Leaf size=31

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rubi [A] time = 0.0367465, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1868, 31, 617, 204}

$$-\log(a-x)-\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*ArcTan[(a + 2*x)/(Sqrt[3]*a)])/Sqrt[3] - Log[a - x]
```

Rule 1868

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Dist[C/b, I
nt[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /
; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && PolyQ
[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\left(a\int \frac{1}{a^2+ax+x^2}\,dx\right) - \int \frac{1}{-a+x}\,dx$$
$$= -\log(a-x) + 2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{a}\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a-x)$$

Mathematica [A] time = 0.0057692, size = 58, normalized size = 1.87

$$\frac{1}{3}\left(\log\left(a^2+ax+x^2\right) - \log\left(x^3-a^3\right) - 2\log(x-a) - 2\sqrt{3}\tan^{-1}\left(\frac{a+2x}{\sqrt{3}a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*a + x))/(a^3 - x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x +
 x^2] - Log[-a^3 + x^3])/3
```

Maple [A] time = 0.005, size = 29, normalized size = 0.9

$$-\frac{2\sqrt{3}}{3}\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right) - \ln(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*a+x)/(a^3-x^3),x)
```

```
[Out] -2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)-ln(-a+x)
```

Maxima [A] time = 1.41686, size = 38, normalized size = 1.23

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Fricas [A] time = 0.941276, size = 81, normalized size = 2.61

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(-a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

Sympy [C] time = 0.288601, size = 54, normalized size = 1.74

$$-\log(-a+x)+\frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a**3-x**3),x)
```

```
[Out] -log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/2
 + sqrt(3)*I*a/2 + x)/3
```

Giac [A] time = 1.05676, size = 39, normalized size = 1.26

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right)-\log\left(|-a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

3.369 $\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=50

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1
/3) + x])/b
```

Rubi [A] time = 0.086742, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1867, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(1
/3) + x])/b
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx = \frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}+x}\,dx}{b} - \frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}-\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$
$$= \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b} - \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$
$$= \frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

Mathematica [B] time = 0.0410268, size = 146, normalized size = 2.92

$$\frac{C\left(-\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)+2\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))/(3
*a^(1/3)*b)
```

Maple [A] time = 0.004, size = 87, normalized size = 1.7

$$\frac{2C}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)-\frac{C}{3b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)+\frac{C\ln\left(bx^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*(1/b*a)^(1/3)*C+C*x)/(b*x^3+a),x)
```

```
[Out] 2/3*C*ln(x+(1/b*a)^(1/3))/b-1/3*C/b*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2
/3*C/b*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C/b*ln(b*x^3+a
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08227, size = 139, normalized size = 2.78

$$-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-3C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) - 3
*C*log(x + (a/b)^(1/3)))/b
```

Sympy [C] time = 0.413596, size = 100, normalized size = 2.

$$\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqrt
(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [B] time = 1.11064, size = 235, normalized size = 4.7

$$-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-2\left(ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}-\frac{\sqrt{3}\left(ab^2-\sqrt{3}\sqrt{a^2b^4}i\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}+\frac{\left(3ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(-a/b)^(2/3) - 2*(a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a*b) - 1/3*sqrt(3)*(a*b^2 - sqrt(3)*sqrt(a^2*b^4)*i)
*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*(3*a
*b^2 - sqrt(3)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a*b^3)
```

3.370 $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a
/b))^(1/3) + x])/b
```

Rubi [A] time = 0.0931049, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1867, 31, 617, 204}

$$-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a
/b))^(1/3) + x])/b
```

Rule 1867

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[C/b,
Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]]
/; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Poly
Q[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx &= -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}+x}\,dx}{b}+\frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}-\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}\\ &= -\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}+\frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}\\ &= -\frac{2C\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.0692506, size = 149, normalized size = 2.81

$$-\frac{C\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)-2\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+\dots}{\sqrt{3}}\right.\right.}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(1/3
)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a -
b*x^3]))/(3*a^(1/3)*b)
```

Maple [B] time = 0.006, size = 135, normalized size = 2.6

$$\frac{2C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2C\sqrt{3}}{3b}\sqrt[3]{-\frac{a}{b}}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*(-1/b*a)^(1/3)*C+C*x)/(-b*x^3+a),x)
```

```
[Out] 2/3*C*(-1/b*a)^(1/3)/b/(1/b*a)^(1/3)*ln(x-(1/b*a)^(1/3))-1/3*C*(-1/b*a)^(1/
3)/b/(1/b*a)^(1/3)*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*(-1/b*a)^(1/
3)*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))-1/3*C/
b*ln(b*x^3-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.10121, size = 142, normalized size = 2.68

$$-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+3C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) +
3*C*log(x + (-a/b)^(1/3)))/b
```

Sympy [C] time = 0.435654, size = 110, normalized size = 2.08

$$-\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [B] time = 1.09418, size = 223, normalized size = 4.21

$$-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\left(-ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}+\frac{\sqrt{3}\left(ab^2+\sqrt{3}\sqrt{a^2b^4}i\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3}-\frac{\left(3ab^2+\sqrt{3}\sqrt{a^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(C*b*(a/b)^(2/3) - 2*(-a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs
(x - (a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(a*b^2 + sqrt(3)*sqrt(a^2*b^4)*i)*C*
arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/6*(3*a*b^2
+ sqrt(3)*sqrt(a^2*b^4)*i)*C*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3)
```

3.371 $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

Optimal. Leaf size=54

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a/
b))^(1/3) - x])/b
```

Rubi [A] time = 0.0772552, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1869, 31, 617, 204}

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}+\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a/
b))^(1/3) - x])/b
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx = -\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}\,dx}{b}+\frac{\left(\sqrt[3]{-\frac{a}{b}}C\right)\int \frac{1}{\left(-\frac{a}{b}\right)^{2/3}+\sqrt[3]{-\frac{a}{b}}x+x^2}\,dx}{b}$$
$$=\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{(2C)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}$$
$$=\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.0415237, size = 148, normalized size = 2.74

$$\frac{C\left(\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a+bx^3\right)-2\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

```
[Out] (C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] + (-(a/b))^(1/3)
*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b
*x^3]))/(3*a^(1/3)*b)
```

Maple [B] time = 0.003, size = 132, normalized size = 2.4

$$-\frac{2C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C}{3b}\sqrt[3]{-\frac{a}{b}}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{2C\sqrt{3}}{3b}\sqrt[3]{-\frac{a}{b}}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{C}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*(-1/b*a)^(1/3)*C+C*x)/(b*x^3+a),x)
```

```
[Out] -2/3*C*(-1/b*a)^(1/3)/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/3*C*(-1/b*a)^(1
/3)/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3*C*(-1/b*a)^(1
/3)*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*C
/b*ln(b*x^3+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.1221, size = 142, normalized size = 2.63

$$-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-3C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) -
3*C*log(x - (-a/b)^(1/3)))/b
```

Sympy [C] time = 0.421174, size = 109, normalized size = 2.02

$$\frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

```
[Out] C*(log(a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3))
 + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

Giac [B] time = 1.06755, size = 131, normalized size = 2.43

$$\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^{2}\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/3
*(C*b*(-a/b)^(2/3) + 2*(-a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b)
```

3.372 $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

Optimal. Leaf size=53

$$-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(
1/3) - x])/b
```

Rubi [A] time = 0.0755327, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1869, 31, 617, 204}

$$-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}-\frac{2C\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] (-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(
1/3) - x])/b
```

Rule 1869

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-(a/b))^(1/3)}, -Dist[C
/b, Int[1/(q - x), x], x] + Dist[(B - C*q)/b, Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-(a/b))^(1/3)*B - 2*(-(a/b))^(2/3)*C, 0]] /; FreeQ[{a, b},
x] && PolyQ[P2, x, 2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}\,dx}{b}-\frac{\left(\sqrt[3]{\frac{a}{b}}C\right)\int \frac{1}{\left(\frac{a}{b}\right)^{2/3}+\sqrt[3]{\frac{a}{b}}x+x^2}\,dx}{b}$$

$$=-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}+\frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$=-\frac{2C\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

Mathematica [B] time = 0.0533153, size = 147, normalized size = 2.77

$$-\frac{C\left(-\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\sqrt[3]{a}\log\left(a-bx^3\right)+2\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{b}\sqrt[3]{\frac{a}{b}}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{3\sqrt[3]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

```
[Out] -(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)*
Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a - b*x^3]))/(
3*a^(1/3)*b)
```

Maple [A] time = 0.005, size = 90, normalized size = 1.7

$$-\frac{2C}{3b}\ln\left(x-\sqrt[3]{\frac{a}{b}}\right)+\frac{C}{3b}\ln\left(x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\frac{2C\sqrt{3}}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(1+2x\frac{1}{\sqrt[3]{\frac{a}{b}}}\right)\right)-\frac{C\ln\left(bx^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*(1/b*a)^(1/3)*C+C*x)/(-b*x^3+a),x)
```

```
[Out] -2/3*C/b*ln(x-(1/b*a)^(1/3))+1/3*C/b*ln(x^2+(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-
2/3*C*arctan(1/3*(1+2/(1/b*a)^(1/3)*x)*3^(1/2))/b*3^(1/2)-1/3*C/b*ln(b*x^3-
a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.03239, size = 139, normalized size = 2.62

$$-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+3C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) + 3
*C*log(x - (a/b)^(1/3)))/b
```

Sympy [C] time = 0.430672, size = 102, normalized size = 1.92

$$-\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

```
[Out] -C*(log(-a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) + s
qrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

Giac [A] time = 1.09216, size = 122, normalized size = 2.3

$$-\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^{2}\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*
(C*b*(a/b)^(2/3) + 2*(a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs(x - (
a/b)^(1/3)))/(a*b)
```

3.373 $\int x^4 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=97

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Rubi [A] time = 0.121369, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^4 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(acx^4+adx^5+aex^6+(bc+af)x^7+(bd+ag)x^8+(be+ah$$
$$=\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+ag)x^9+\frac{1}{10}($$

Mathematica [A] time = 0.0280747, size = 97, normalized size = 1.

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*g
)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x^1
3)/13
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^5}{5} + \frac{adx^6}{6} + \frac{aex^7}{7} + \frac{(af + bc)x^8}{8} + \frac{(ag + bd)x^9}{9} + \frac{(ah + be)x^{10}}{10} + \frac{bfx^{11}}{11} + \frac{bgx^{12}}{12} + \frac{bhx^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/1
0*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

Maxima [A] time = 0.954434, size = 107, normalized size = 1.1

$$\frac{1}{13}\,bhx^{13} + \frac{1}{12}\,bgx^{12} + \frac{1}{11}\,bfx^{11} + \frac{1}{10}\,(be + ah)x^{10} + \frac{1}{9}\,(bd + ag)x^9 + \frac{1}{7}\,aex^7 + \frac{1}{8}\,(bc + af)x^8 + \frac{1}{6}\,adx^6 + \frac{1}{5}\,acx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/9
*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*a*
c*x^5
```

Fricas [A] time = 0.844004, size = 239, normalized size = 2.46

$$\frac{1}{13}x^{13}hb + \frac{1}{12}x^{12}gb + \frac{1}{11}x^{11}fb + \frac{1}{10}x^{10}eb + \frac{1}{10}x^{10}ha + \frac{1}{9}x^9db + \frac{1}{9}x^9ga + \frac{1}{8}x^8cb + \frac{1}{8}x^8fa + \frac{1}{7}x^7ea + \frac{1}{6}x^6da + \frac{1}{5}x^5ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*x^13*h*b + 1/12*x^12*g*b + 1/11*x^11*f*b + 1/10*x^10*e*b + 1/10*x^10*h
*a + 1/9*x^9*d*b + 1/9*x^9*g*a + 1/8*x^8*c*b + 1/8*x^8*f*a + 1/7*x^7*e*a +
1/6*x^6*d*a + 1/5*x^5*c*a
```

Sympy [A] time = 0.070904, size = 90, normalized size = 0.93

$$\frac{acx^5}{5} + \frac{adx^6}{6} + \frac{aex^7}{7} + \frac{bfx^{11}}{11} + \frac{bgx^{12}}{12} + \frac{bhx^{13}}{13} + x^{10}\left(\frac{ah}{10} + \frac{be}{10}\right) + x^9\left(\frac{ag}{9} + \frac{bd}{9}\right) + x^8\left(\frac{af}{8} + \frac{bc}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/12 + b*h*x*
*13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9) + x**8*(a*f/8 + b*c
```

```
/8)
```

Giac [A] time = 1.09469, size = 117, normalized size = 1.21

$$\frac{1}{13}bhx^{13} + \frac{1}{12}bgx^{12} + \frac{1}{11}bfx^{11} + \frac{1}{10}ahx^{10} + \frac{1}{10}bx^{10}e + \frac{1}{9}bdx^9 + \frac{1}{9}agx^9 + \frac{1}{8}bcx^8 + \frac{1}{8}afx^8 + \frac{1}{7}ax^7e + \frac{1}{6}adx^6 + \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*a*h*x^10 + 1/10*b*x^10
*e + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*x^8 + 1/7*a*x^7*e +
1/6*a*d*x^6 + 1/5*a*c*x^5
```

3.374 $\int x^3 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=97

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Rubi [A] time = 0.100829, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx = \int \left(acx^3+adx^4+aex^5+(bc+af)x^6+(bd+ag)x^7+(be+ah)x^8\right.$$
$$=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)x^8+\frac{1}{9}(be+$$

Mathematica [A] time = 0.027894, size = 97, normalized size = 1.

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*g
)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^12)
/12
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{(af+bc)\,x^7}{7} + \frac{(ag+bd)\,x^8}{8} + \frac{(ah+be)\,x^9}{9} + \frac{bfx^{10}}{10} + \frac{bgx^{11}}{11} + \frac{bhx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/9
*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

Maxima [A] time = 0.936407, size = 107, normalized size = 1.1

$$\frac{1}{12}\,bhx^{12} + \frac{1}{11}\,bgx^{11} + \frac{1}{10}\,bfx^{10} + \frac{1}{9}\,(be+ah)x^9 + \frac{1}{8}\,(bd+ag)x^8 + \frac{1}{6}\,aex^6 + \frac{1}{7}\,(bc+af)x^7 + \frac{1}{5}\,adx^5 + \frac{1}{4}\,acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*(
b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*c*
x^4
```

Fricas [A] time = 0.875431, size = 234, normalized size = 2.41

$$\frac{1}{12}x^{12}hb + \frac{1}{11}x^{11}gb + \frac{1}{10}x^{10}fb + \frac{1}{9}x^9eb + \frac{1}{9}x^9ha + \frac{1}{8}x^8db + \frac{1}{8}x^8ga + \frac{1}{7}x^7cb + \frac{1}{7}x^7fa + \frac{1}{6}x^6ea + \frac{1}{5}x^5da + \frac{1}{4}x^4ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/12*x^12*h*b + 1/11*x^11*g*b + 1/10*x^10*f*b + 1/9*x^9*e*b + 1/9*x^9*h*a +
 1/8*x^8*d*b + 1/8*x^8*g*a + 1/7*x^7*c*b + 1/7*x^7*f*a + 1/6*x^6*e*a + 1/5*
x^5*d*a + 1/4*x^4*c*a
```

Sympy [A] time = 0.072214, size = 90, normalized size = 0.93

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{bfx^{10}}{10} + \frac{bgx^{11}}{11} + \frac{bhx^{12}}{12} + x^9\left(\frac{ah}{9} + \frac{be}{9}\right) + x^8\left(\frac{ag}{8} + \frac{bd}{8}\right) + x^7\left(\frac{af}{7} + \frac{bc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/11 + b*h*x*
*12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) + x**7*(a*f/7 + b*c/7)
```

Giac [A] time = 1.06642, size = 117, normalized size = 1.21

$$\frac{1}{12}bhx^{12} + \frac{1}{11}bgx^{11} + \frac{1}{10}bfx^{10} + \frac{1}{9}ahx^9 + \frac{1}{9}bx^9e + \frac{1}{8}bdx^8 + \frac{1}{8}agx^8 + \frac{1}{7}bcx^7 + \frac{1}{7}afx^7 + \frac{1}{6}ax^6e + \frac{1}{5}adx^5 + \frac{1}{4}acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*a*h*x^9 + 1/9*b*x^9*e +
 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7 + 1/6*a*x^6*e + 1/5*
a*d*x^5 + 1/4*a*c*x^4
```

3.375 $\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=97

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Rubi [A] time = 0.0914639, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(acx^2+adx^3+aex^4+(bc+af)x^5+(bd+ag)x^6+(be+ah$$
$$= \frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7+\frac{1}{8}(b$$

Mathematica [A] time = 0.0242234, size = 97, normalized size = 1.

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*g
)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)/1
1
```

Maple [A] time = 0.001, size = 80, normalized size = 0.8

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{(af + bc)x^6}{6} + \frac{(ag + bd)x^7}{7} + \frac{(ah + be)x^8}{8} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/8
*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

Maxima [A] time = 0.941309, size = 107, normalized size = 1.1

$$\frac{1}{11}\,bhx^{11} + \frac{1}{10}\,bgx^{10} + \frac{1}{9}\,bfx^9 + \frac{1}{8}\,(be + ah)x^8 + \frac{1}{7}\,(bd + ag)x^7 + \frac{1}{5}\,aex^5 + \frac{1}{6}\,(bc + af)x^6 + \frac{1}{4}\,adx^4 + \frac{1}{3}\,acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b*
d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*x^
3
```

Fricas [A] time = 0.838869, size = 231, normalized size = 2.38

$$\frac{1}{11}x^{11}hb + \frac{1}{10}x^{10}gb + \frac{1}{9}x^9fb + \frac{1}{8}x^8eb + \frac{1}{8}x^8ha + \frac{1}{7}x^7db + \frac{1}{7}x^7ga + \frac{1}{6}x^6cb + \frac{1}{6}x^6fa + \frac{1}{5}x^5ea + \frac{1}{4}x^4da + \frac{1}{3}x^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/11*x^11*h*b + 1/10*x^10*g*b + 1/9*x^9*f*b + 1/8*x^8*e*b + 1/8*x^8*h*a + 1
/7*x^7*d*b + 1/7*x^7*g*a + 1/6*x^6*c*b + 1/6*x^6*f*a + 1/5*x^5*e*a + 1/4*x^
4*d*a + 1/3*x^3*c*a
```

Sympy [A] time = 0.071216, size = 90, normalized size = 0.93

$$\frac{acx^3}{3} + \frac{adx^4}{4} + \frac{aex^5}{5} + \frac{bfx^9}{9} + \frac{bgx^{10}}{10} + \frac{bhx^{11}}{11} + x^8\left(\frac{ah}{8} + \frac{be}{8}\right) + x^7\left(\frac{ag}{7} + \frac{bd}{7}\right) + x^6\left(\frac{af}{6} + \frac{bc}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10 + b*h*x**1
1/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x**6*(a*f/6 + b*c/6)
```

Giac [A] time = 1.32095, size = 117, normalized size = 1.21

$$\frac{1}{11}bhx^{11} + \frac{1}{10}bgx^{10} + \frac{1}{9}bfx^9 + \frac{1}{8}ahx^8 + \frac{1}{8}bx^8e + \frac{1}{7}bdx^7 + \frac{1}{7}agx^7 + \frac{1}{6}bcx^6 + \frac{1}{6}afx^6 + \frac{1}{5}ax^5e + \frac{1}{4}adx^4 + \frac{1}{3}acx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*a*h*x^8 + 1/8*b*x^8*e + 1
/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 + 1/5*a*x^5*e + 1/4*a*
d*x^4 + 1/3*a*c*x^3
```

3.376 $\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=97

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Rubi [A] time = 0.0802172, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1820}

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx=\int\left(acx+adx^2+aex^3+(bc+af)x^4+(bd+ag)x^5+(be+ah)x^6+\right.$$
$$=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6+\frac{1}{7}(be+$$

Mathematica [A] time = 0.0173656, size = 97, normalized size = 1.

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*g
)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/10
```

Maple [A] time = 0.002, size = 80, normalized size = 0.8

$$\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{\left(af+bc\right)x^5}{5}+\frac{\left(ag+bd\right)x^6}{6}+\frac{(ah+be)\,x^7}{7}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+\frac{bhx^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/7
*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

Maxima [A] time = 0.951365, size = 107, normalized size = 1.1

$$\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^{9}+\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,(be+ah)x^{7}+\frac{1}{6}\,\big(bd+ag\big)x^{6}+\frac{1}{4}\,aex^{4}+\frac{1}{5}\,\big(bc+af\big)x^{5}+\frac{1}{3}\,adx^{3}+\frac{1}{2}\,acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
+ a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^2
```

Fricas [A] time = 0.893111, size = 228, normalized size = 2.35

$$\frac{1}{10}x^{10}hb+\frac{1}{9}x^{9}gb+\frac{1}{8}x^{8}fb+\frac{1}{7}x^{7}eb+\frac{1}{7}x^{7}ha+\frac{1}{6}x^{6}db+\frac{1}{6}x^{6}ga+\frac{1}{5}x^{5}cb+\frac{1}{5}x^{5}fa+\frac{1}{4}x^{4}ea+\frac{1}{3}x^{3}da+\frac{1}{2}x^{2}ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/10*x^10*h*b + 1/9*x^9*g*b + 1/8*x^8*f*b + 1/7*x^7*e*b + 1/7*x^7*h*a + 1/6
*x^6*d*b + 1/6*x^6*g*a + 1/5*x^5*c*b + 1/5*x^5*f*a + 1/4*x^4*e*a + 1/3*x^3*
d*a + 1/2*x^2*c*a
```

Sympy [A] time = 0.070509, size = 90, normalized size = 0.93

$$\frac{acx^{2}}{2}+\frac{adx^{3}}{3}+\frac{aex^{4}}{4}+\frac{bfx^{8}}{8}+\frac{bgx^{9}}{9}+\frac{bhx^{10}}{10}+x^{7}\left(\frac{ah}{7}+\frac{be}{7}\right)+x^{6}\left(\frac{ag}{6}+\frac{bd}{6}\right)+x^{5}\left(\frac{af}{5}+\frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 + b*h*x**10/
10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5*(a*f/5 + b*c/5)
```

Giac [A] time = 1.05363, size = 117, normalized size = 1.21

$$\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^{9}+\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,ahx^{7}+\frac{1}{7}\,bx^{7}e+\frac{1}{6}\,bdx^{6}+\frac{1}{6}\,agx^{6}+\frac{1}{5}\,bcx^{5}+\frac{1}{5}\,afx^{5}+\frac{1}{4}\,ax^{4}e+\frac{1}{3}\,adx^{3}+\frac{1}{2}\,acx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*a*h*x^7 + 1/7*b*x^7*e + 1/6
*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/4*a*x^4*e + 1/3*a*d*
x^3 + 1/2*a*c*x^2
```

3.377 $\int \left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=92

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Rubi [A] time = 0.0734568, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {1850}

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \int \left(ac+adx+aex^2+(bc+af)x^3+(bd+ag)x^4+(be+ah)x^5+\right.\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5+\frac{1}{6}(be+a\end{aligned}$$

Mathematica [A] time = 0.0132528, size = 92, normalized size = 1.

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5)
/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

Maple [A] time = 0., size = 77, normalized size = 0.8

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{(af+bc)\,x^4}{4}+\frac{(ag+bd)\,x^5}{5}+\frac{(ah+be)\,x^6}{6}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h+
b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

Maxima [A] time = 0.933405, size = 103, normalized size = 1.12

$$\frac{1}{9}bhx^9 + \frac{1}{8}bgx^8 + \frac{1}{7}bfx^7 + \frac{1}{6}(be + ah)x^6 + \frac{1}{5}\left(bd + ag\right)x^5 + \frac{1}{3}aex^3 + \frac{1}{4}\left(bc + af\right)x^4 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A] time = 0.846603, size = 217, normalized size = 2.36

$$\frac{1}{9}x^9hb + \frac{1}{8}x^8gb + \frac{1}{7}x^7fb + \frac{1}{6}x^6eb + \frac{1}{6}x^6ha + \frac{1}{5}x^5db + \frac{1}{5}x^5ga + \frac{1}{4}x^4cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*h*b + 1/8*x^8*g*b + 1/7*x^7*f*b + 1/6*x^6*e*b + 1/6*x^6*h*a + 1/5*x
^5*d*b + 1/5*x^5*g*a + 1/4*x^4*c*b + 1/4*x^4*f*a + 1/3*x^3*e*a + 1/2*x^2*d*
a + x*c*a
```

Sympy [A] time = 0.069543, size = 87, normalized size = 0.95

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{bfx^7}{7} + \frac{bgx^8}{8} + \frac{bhx^9}{9} + x^6\left(\frac{ah}{6} + \frac{be}{6}\right) + x^5\left(\frac{ag}{5} + \frac{bd}{5}\right) + x^4\left(\frac{af}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x**9/9 + x*
*6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4 + b*c/4)
```

Giac [A] time = 1.06718, size = 113, normalized size = 1.23

$$\frac{1}{9}bhx^9 + \frac{1}{8}bgx^8 + \frac{1}{7}bfx^7 + \frac{1}{6}ahx^6 + \frac{1}{6}bx^6e + \frac{1}{5}bdx^5 + \frac{1}{5}agx^5 + \frac{1}{4}bcx^4 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*a*h*x^6 + 1/6*b*x^6*e + 1/5*b
*d*x^5 + 1/5*a*g*x^5 + 1/4*b*c*x^4 + 1/4*a*f*x^4 + 1/3*a*x^3*e + 1/2*a*d*x^
2 + a*c*x
```

3.378 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} \, dx$

Optimal. Leaf size=88

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Rubi [A] time = 0.0578865, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} \, dx = \int \left(ad+\frac{ac}{x}+aex+(bc+af)x^2+(bd+ag)x^3+(be+ah)x^4+bfx^5\right.$$
$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bf$$

Mathematica [A] time = 0.0287393, size = 88, normalized size = 1.

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e + a
*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

Maple [A] time = 0.003, size = 81, normalized size = 0.9

$$\frac{bhx^8}{8}+\frac{bgx^7}{7}+\frac{bfx^6}{6}+\frac{x^5ah}{5}+\frac{bex^5}{5}+\frac{x^4ag}{4}+\frac{bdx^4}{4}+\frac{x^3af}{3}+\frac{bcx^3}{3}+\frac{aex^2}{2}+adx+ac\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/8*b*h*x^8+1/7*b*g*x^7+1/6*b*f*x^6+1/5*x^5*a*h+1/5*b*e*x^5+1/4*x^4*a*g+1/4
*b*d*x^4+1/3*x^3*a*f+1/3*b*c*x^3+1/2*a*e*x^2+a*d*x+a*c*ln(x)
```

Maxima [A] time = 0.943872, size = 100, normalized size = 1.14

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}(be + ah)x^5 + \frac{1}{4}\left(bd + ag\right)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}\left(bc + af\right)x^3 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

Fricas [A] time = 0.992809, size = 192, normalized size = 2.18

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}(be + ah)x^5 + \frac{1}{4}\left(bd + ag\right)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}\left(bc + af\right)x^3 + adx + ac\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
)
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

Sympy [A] time = 0.339668, size = 85, normalized size = 0.97

$$ac\log(x) + adx + \frac{aex^2}{2} + \frac{bfx^6}{6} + \frac{bgx^7}{7} + \frac{bhx^8}{8} + x^5\left(\frac{ah}{5} + \frac{be}{5}\right) + x^4\left(\frac{ag}{4} + \frac{bd}{4}\right) + x^3\left(\frac{af}{3} + \frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a*c*log(x) + a*d*x + a*e*x**2/2 + b*f*x**6/6 + b*g*x**7/7 + b*h*x**8/8 + x*
*5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b*d/4) + x**3*(a*f/3 + b*c/3)
```

Giac [A] time = 1.05718, size = 112, normalized size = 1.27

$$\frac{1}{8}bhx^8 + \frac{1}{7}bgx^7 + \frac{1}{6}bfx^6 + \frac{1}{5}ahx^5 + \frac{1}{5}bx^5e + \frac{1}{4}bdx^4 + \frac{1}{4}agx^4 + \frac{1}{3}bcx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx + ac\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*a*h*x^5 + 1/5*b*x^5*e + 1/4*b
*d*x^4 + 1/4*a*g*x^4 + 1/3*b*c*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x + a*
c*log(abs(x))
```

3.379 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx$

Optimal. Leaf size=86

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Rubi [A] time = 0.0662083, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2} \, dx = \int \left(ae+\frac{ac}{x^2}+\frac{ad}{x}+(bc+af)x+(bd+ag)x^2+(be+ah)x^3+bfx$$
$$= -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3+\frac{1}{4}(be+ah)x^4+\frac{1}{5}bf$$

Mathematica [A] time = 0.0412215, size = 86, normalized size = 1.

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a*
h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

Maple [A] time = 0.004, size = 81, normalized size = 0.9

$$\frac{bhx^7}{7}+\frac{bgx^6}{6}+\frac{bfx^5}{5}+\frac{x^4ah}{4}+\frac{bex^4}{4}+\frac{x^3ag}{3}+\frac{bdx^3}{3}+\frac{afx^2}{2}+\frac{bcx^2}{2}+aex+ad\ln(x)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*x^4*a*h+1/4*b*e*x^4+1/3*x^3*a*g+1/3
*b*d*x^3+1/2*a*f*x^2+1/2*b*c*x^2+a*e*x+a*d*ln(x)-a*c/x
```

Maxima [A] time = 0.9478, size = 100, normalized size = 1.16

$$\frac{1}{7}\,bhx^7+\frac{1}{6}\,bgx^6+\frac{1}{5}\,bfx^5+\frac{1}{4}\,(be+ah)x^4+\frac{1}{3}\,\left(bd+ag\right)x^3+aex+\frac{1}{2}\,\left(bc+af\right)x^2+ad\log\left(x\right)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxim
a")
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(b*e + a*h)*x^4 + 1/3*(b*d +
a*g)*x^3 + a*e*x + 1/2*(b*c + a*f)*x^2 + a*d*log(x) - a*c/x
```

Fricas [A] time = 1.02038, size = 212, normalized size = 2.47

$$\frac{60\,bhx^8+70\,bgx^7+84\,bfx^6+105\,(be+ah)x^5+140\,\left(bd+ag\right)x^4+420\,aex^2+210\,\left(bc+af\right)x^3+420\,adx\log\left(x\right)-420\,a}{420\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="frica
s")
```

```
[Out] 1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5 + 140*(b*
d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*a*d*x*log(x) - 420*a
*c)/x
```

Sympy [A] time = 0.347942, size = 82, normalized size = 0.95

$$-\frac{ac}{x}+ad\log\left(x\right)+aex+\frac{bfx^5}{5}+\frac{bgx^6}{6}+\frac{bhx^7}{7}+x^4\left(\frac{ah}{4}+\frac{be}{4}\right)+x^3\left(\frac{ag}{3}+\frac{bd}{3}\right)+x^2\left(\frac{af}{2}+\frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + x**4*(
a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b*c/2)
```

Giac [A] time = 1.07097, size = 112, normalized size = 1.3

$$\frac{1}{7}\,bhx^7+\frac{1}{6}\,bgx^6+\frac{1}{5}\,bfx^5+\frac{1}{4}\,ahx^4+\frac{1}{4}\,bx^4e+\frac{1}{3}\,bdx^3+\frac{1}{3}\,agx^3+\frac{1}{2}\,bcx^2+\frac{1}{2}\,afx^2+axe+ad\log\left(|x|\right)-\frac{ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
```

```
[Out] 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*a*h*x^4 + 1/4*b*x^4*e + 1/3*b
*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*x*e + a*d*log(abs(x))
- a*c/x
```

3.380 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=86

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*
h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*Log[x]
```

Rubi [A] time = 0.0724331, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a*c)/(2*x^2) - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a*
h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = \int \left(bc\left(1+\frac{af}{bc}\right)+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+(bd+ag)x+(be+ah)x^2+bfx^3\right.$$
$$=-\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah)x^3+\frac{1}{4}bfx^4+$$

Mathematica [A] time = 0.0597417, size = 78, normalized size = 0.91

$$\frac{a(-3c-6dx+6fx^3+3gx^4+2hx^5)}{6x^2}+ae\log(x)+bcx+\frac{1}{60}bx^2\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2) + (b*x^2*(
30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a*e*Log[x]
```

Maple [A] time = 0.005, size = 78, normalized size = 0.9

$$\frac{bhx^6}{6}+\frac{bgx^5}{5}+\frac{bfx^4}{4}+\frac{x^3ah}{3}+\frac{bex^3}{3}+\frac{x^2ag}{2}+\frac{bdx^2}{2}+afx+bcx+ae\ln(x)-\frac{ac}{2x^2}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*x^3*a*h+1/3*b*e*x^3+1/2*x^2*a*g+1/2
*b*d*x^2+a*f*x+b*c*x+a*e*ln(x)-1/2*a*c/x^2-a*d/x
```

Maxima [A] time = 0.940463, size = 100, normalized size = 1.16

$$\frac{1}{6}bhx^6+\frac{1}{5}bgx^5+\frac{1}{4}bfx^4+\frac{1}{3}(be+ah)x^3+\frac{1}{2}\left(bd+ag\right)x^2+ae\log(x)+\left(bc+af\right)x-\frac{2adx+ac}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(b*e + a*h)*x^3 + 1/2*(b*d +
a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x + a*c)/x^2
```

Fricas [A] time = 1.11308, size = 205, normalized size = 2.38

$$\frac{10\,bhx^8+12\,bgx^7+15\,bfx^6+20\,(be+ah)x^5+30\left(bd+ag\right)x^4+60\,aex^2\log(x)+60\left(bc+af\right)x^3-60\,adx-30\,ac}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] 1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 + 30*(b*d +
 a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60*a*d*x - 30*a*c)/x^2
```

Sympy [A] time = 0.456412, size = 82, normalized size = 0.95

$$ae\log(x)+\frac{bfx^4}{4}+\frac{bgx^5}{5}+\frac{bhx^6}{6}+x^3\left(\frac{ah}{3}+\frac{be}{3}\right)+x^2\left(\frac{ag}{2}+\frac{bd}{2}\right)+x\left(af+bc\right)-\frac{ac+2adx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 + b*e/3) +
x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) - (a*c + 2*a*d*x)/(2*x**2)
```

Giac [A] time = 1.06629, size = 108, normalized size = 1.26

$$\frac{1}{6}bhx^{6}+\frac{1}{5}bgx^{5}+\frac{1}{4}bfx^{4}+\frac{1}{3}ahx^{3}+\frac{1}{3}bx^{3}e+\frac{1}{2}bdx^{2}+\frac{1}{2}agx^{2}+bcx+afx+ae\log\left(|x|\right)-\frac{2\,adx+ac}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
```

```
[Out] 1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*a*h*x^3 + 1/3*b*x^3*e + 1/2*b
*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c
)/x^2
```

3.381 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=86

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

```
[Out] -(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2
)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*Log[x]
```

Rubi [A] time = 0.070949, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a*c)/(3*x^3) - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^2
)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^4}\,dx=\int \left(bd\left(1+\frac{ag}{bd}\right)+\frac{ac}{x^4}+\frac{ad}{x^3}+\frac{ae}{x^2}+\frac{bc+af}{x}+(be+ah)x+bfx^2\right.$$

$$=-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4$$

Mathematica [A] time = 0.0605947, size = 76, normalized size = 0.88

$$\log(x)(af+bc)-\frac{a\left(2c+3x\left(d+2ex+x^3(-(2g+hx))\right)\right)}{6x^3}+\frac{1}{60}bx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (b*x*(60*d + x*(30
*e + x*(20*f + 15*g*x + 12*h*x^2))))/60 + (b*c + a*f)*Log[x]
```

Maple [A] time = 0.006, size = 76, normalized size = 0.9

$$\frac{bhx^5}{5} + \frac{bgx^4}{4} + \frac{bfx^3}{3} + \frac{x^2ah}{2} + \frac{bex^2}{2} + agx + bdx + \ln(x)\,af + \ln(x)\,bc - \frac{ac}{3x^3} - \frac{ad}{2x^2} - \frac{ae}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/5*b*h*x^5+1/4*b*g*x^4+1/3*b*f*x^3+1/2*x^2*a*h+1/2*b*e*x^2+a*g*x+b*d*x+ln(
x)*a*f+ln(x)*b*c-1/3*a*c/x^3-1/2*a*d/x^2-a*e/x
```

Maxima [A] time = 0.942655, size = 101, normalized size = 1.17

$$\frac{1}{5}\,bhx^5 + \frac{1}{4}\,bgx^4 + \frac{1}{3}\,bfx^3 + \frac{1}{2}\,(be + ah)x^2 + \left(bd + ag\right)x + \left(bc + af\right)\log(x) - \frac{6\,aex^2 + 3\,adx + 2\,ac}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(b*e + a*h)*x^2 + (b*d + a*g)
*x + (b*c + a*f)*log(x) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^3
```

Fricas [A] time = 1.24613, size = 205, normalized size = 2.38

$$\frac{12\,bhx^8 + 15\,bgx^7 + 20\,bfx^6 + 30\,(be + ah)x^5 + 60\left(bd + ag\right)x^4 + 60\left(bc + af\right)x^3\log(x) - 60\,aex^2 - 30\,adx - 20\,ac}{60\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] 1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 + 60*(b*d +
 a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30*a*d*x - 20*a*c)/x^3
```

Sympy [A] time = 0.732298, size = 82, normalized size = 0.95

$$\frac{bfx^3}{3} + \frac{bgx^4}{4} + \frac{bhx^5}{5} + x^2\left(\frac{ah}{2} + \frac{be}{2}\right) + x\left(ag + bd\right) + \left(af + bc\right)\log(x) - \frac{2ac + 3adx + 6aex^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(a*g + b*d)
 + (a*f + b*c)*log(x) - (2*a*c + 3*a*d*x + 6*a*e*x**2)/(6*x**3)
```

Giac [A] time = 1.05191, size = 107, normalized size = 1.24

$$\frac{1}{5}\,bhx^5 + \frac{1}{4}\,bgx^4 + \frac{1}{3}\,bfx^3 + \frac{1}{2}\,ahx^2 + \frac{1}{2}\,bx^2e + bdx + agx + \left(bc + af\right)\log\left(|x|\right) - \frac{6\,ax^2e + 3\,adx + 2\,ac}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
```

```
[Out] 1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*a*h*x^2 + 1/2*b*x^2*e + b*d*x
 + a*g*x + (b*c + a*f)*log(abs(x)) - 1/6*(6*a*x^2*e + 3*a*d*x + 2*a*c)/x^3
```

3.382 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal. Leaf size=86

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

```
[Out] -(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h
)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*Log[x]
```

Rubi [A] time = 0.0729936, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1820}

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a*c)/(4*x^4) - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*h
)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx &= \int \left(be\left(1+\frac{ah}{be}\right)+\frac{ac}{x^5}+\frac{ad}{x^4}+\frac{ae}{x^3}+\frac{bc+af}{x^2}+\frac{bd+ag}{x}+bfx+bgx^2\right.\\ &= -\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bh\end{aligned}$$

Mathematica [A] time = 0.0607921, size = 77, normalized size = 0.9

$$\log(x)(ag+bd)-\frac{a\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)}{12x^4}+b\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4*d*x + 6*x
^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*Log[x]
```

Maple [A] time = 0.005, size = 76, normalized size = 0.9

$$\frac{bhx^4}{4}+\frac{bgx^3}{3}+\frac{bfx^2}{2}+ahx+bxe+\ln(x)\,ag+\ln(x)\,bd-\frac{ad}{3x^3}-\frac{ac}{4x^4}-\frac{ae}{2x^2}-\frac{af}{x}-\frac{bc}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/4*b*h*x^4+1/3*b*g*x^3+1/2*b*f*x^2+a*h*x+b*x*e+ln(x)*a*g+ln(x)*b*d-1/3*a*d
/x^3-1/4*a*c/x^4-1/2*a*e/x^2-1/x*a*f-1/x*b*c
```

Maxima [A] time = 1.20498, size = 101, normalized size = 1.17

$$\frac{1}{4}\,bhx^4+\frac{1}{3}\,bgx^3+\frac{1}{2}\,bfx^2+(be+ah)x+\left(bd+ag\right)\log(x)-\frac{6\,aex^2+12\left(bc+af\right)x^3+4\,adx+3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxim
a")
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (b*e + a*h)*x + (b*d + a*g)*log(x
) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4
```

Fricas [A] time = 1.23532, size = 197, normalized size = 2.29

$$\frac{3\,bhx^8+4\,bgx^7+6\,bfx^6+12\,(be+ah)x^5+12\left(bd+ag\right)x^4\log(x)-6\,aex^2-12\left(bc+af\right)x^3-4\,adx-3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="frica
s")
```

```
[Out] 1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12*(b*d + a*
g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*x - 3*a*c)/x^4
```

Sympy [A] time = 2.88505, size = 82, normalized size = 0.95

$$\frac{bfx^2}{2}+\frac{bgx^3}{3}+\frac{bhx^4}{4}+x\,(ah+be)+\left(ag+bd\right)\log(x)-\frac{3ac+4adx+6aex^2+x^3\left(12af+12bc\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d)*log(x) -
 (3*a*c + 4*a*d*x + 6*a*e*x**2 + x**3*(12*a*f + 12*b*c))/(12*x**4)
```

Giac [A] time = 1.07568, size = 104, normalized size = 1.21

$$\frac{1}{4}\,bhx^4+\frac{1}{3}\,bgx^3+\frac{1}{2}\,bfx^2+ahx+bxe+\left(bd+ag\right)\log\left(|x|\right)-\frac{12\left(bc+af\right)x^3+6\,ax^2e+4\,adx+3\,ac}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] 1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + a*h*x + b*x*e + (b*d + a*g)*log(a
bs(x)) - 1/12*(12*(b*c + a*f)*x^3 + 6*a*x^2*e + 4*a*d*x + 3*a*c)/x^4
```

3.383 $\int x^4 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=163

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2$$

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Rubi [A] time = 0.210111, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^4 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(a^2cx^4+a^2dx^5+a^2ex^6+a(2bc+af)x^7+a(2bd+ag)x^8+\right.$$

$$=\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2bd+ag$$

Mathematica [A] time = 0.0395111, size = 163, normalized size = 1.

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 + (
a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^11)
/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^14)/
14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

Maple [A] time = 0.001, size = 152, normalized size = 0.9

$$\frac{b^2hx^{16}}{16}+\frac{b^2gx^{15}}{15}+\frac{b^2fx^{14}}{14}+\frac{(2\,abh+b^2e)\,x^{13}}{13}+\frac{(2\,abg+b^2d)\,x^{12}}{12}+\frac{(2\,abf+b^2c)\,x^{11}}{11}+\frac{(a^2h+2\,aeb)\,x^{10}}{10}+\frac{(a^2g+2\,b}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2*e)*x^13+1
/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10*(a^2*h+2*a*b*e)*x^1
0+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)*x^8+1/7*a^2*e*x^7+1/6*a^2*d*x
^6+1/5*a^2*c*x^5
```

Maxima [A] time = 0.956301, size = 204, normalized size = 1.25

$$\frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\,(b^2e+2\,abh)x^{13}+\frac{1}{12}\,(b^2d+2\,abg)x^{12}+\frac{1}{11}\,(b^2c+2\,abf)x^{11}+\frac{1}{10}\,(2\,abe+a^2h$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*h
)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10*(
2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*a^2
*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

Fricas [A] time = 1.11735, size = 414, normalized size = 2.54

$$\frac{1}{16}x^{16}hb^2+\frac{1}{15}x^{15}gb^2+\frac{1}{14}x^{14}fb^2+\frac{1}{13}x^{13}eb^2+\frac{2}{13}x^{13}hba+\frac{1}{12}x^{12}db^2+\frac{1}{6}x^{12}gba+\frac{1}{11}x^{11}cb^2+\frac{2}{11}x^{11}fba+\frac{1}{5}x^{10}eba+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/16*x^16*h*b^2 + 1/15*x^15*g*b^2 + 1/14*x^14*f*b^2 + 1/13*x^13*e*b^2 + 2/1
3*x^13*h*b*a + 1/12*x^12*d*b^2 + 1/6*x^12*g*b*a + 1/11*x^11*c*b^2 + 2/11*x^
11*f*b*a + 1/5*x^10*e*b*a + 1/10*x^10*h*a^2 + 2/9*x^9*d*b*a + 1/9*x^9*g*a^2
 + 1/4*x^8*c*b*a + 1/8*x^8*f*a^2 + 1/7*x^7*e*a^2 + 1/6*x^6*d*a^2 + 1/5*x^5*
c*a^2
```

Sympy [A] time = 0.095411, size = 167, normalized size = 1.02

$$\frac{a^2cx^5}{5}+\frac{a^2dx^6}{6}+\frac{a^2ex^7}{7}+\frac{b^2fx^{14}}{14}+\frac{b^2gx^{15}}{15}+\frac{b^2hx^{16}}{16}+x^{13}\left(\frac{2abh}{13}+\frac{b^2e}{13}\right)+x^{12}\left(\frac{abg}{6}+\frac{b^2d}{12}\right)+x^{11}\left(\frac{2abf}{11}+\frac{b^2c}{11}\right)+x^{10}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 + b**2*g*x*
*15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/13) + x**12*(a*b*g/6
+ b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11) + x**10*(a**2*h/10 + a*b*e/5)
 + x**9*(a**2*g/9 + 2*a*b*d/9) + x**8*(a**2*f/8 + a*b*c/4)
```

Giac [A] time = 1.07411, size = 216, normalized size = 1.33

$$\frac{1}{16}\,b^2hx^{16} + \frac{1}{15}\,b^2gx^{15} + \frac{1}{14}\,b^2fx^{14} + \frac{2}{13}\,abhx^{13} + \frac{1}{13}\,b^2x^{13}e + \frac{1}{12}\,b^2dx^{12} + \frac{1}{6}\,abgx^{12} + \frac{1}{11}\,b^2cx^{11} + \frac{2}{11}\,abfx^{11} + \frac{1}{10}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 2/13*a*b*h*x^13 + 1/1
3*b^2*x^13*e + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*b^2*c*x^11 + 2/11*a*
b*f*x^11 + 1/10*a^2*h*x^10 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/9*a^2*g*x^9
 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 + 1/7*a^2*x^7*e + 1/6*a^2*d*x^6 + 1/5*a^2*
c*x^5
```

3.384 $\int x^3 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=163

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah$$

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Rubi [A] time = 0.15901, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(a^2cx^3+a^2dx^4+a^2ex^5+a(2bc+af)x^6+a(2bd+ag)x^7+a(\right.$$
$$=\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8$$

Mathematica [A] time = 0.027851, size = 163, normalized size = 1.

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 + (
a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)/1
0 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)/13
 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

Maple [A] time = 0.001, size = 152, normalized size = 0.9

$$\frac{b^2hx^{15}}{15}+\frac{b^2gx^{14}}{14}+\frac{b^2fx^{13}}{13}+\frac{(2\,abh+b^2e)\,x^{12}}{12}+\frac{(2\,abg+b^2d)\,x^{11}}{11}+\frac{(2\,abf+b^2c)\,x^{10}}{10}+\frac{(a^2h+2\,aeb)\,x^9}{9}+\frac{(a^2g+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2*e)*x^12+1
/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*(a^2*h+2*a*b*e)*x^9+
1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x^7+1/6*a^2*e*x^6+1/5*a^2*d*x^5
+1/4*a^2*c*x^4
```

Maxima [A] time = 0.935679, size = 204, normalized size = 1.25

$$\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,(b^2e+2\,abh)x^{12}+\frac{1}{11}\,(b^2d+2\,abg)x^{11}+\frac{1}{10}\,(b^2c+2\,abf)x^{10}+\frac{1}{9}\,(2\,abe+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*h
)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*(2
*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^2*d
*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

Fricas [A] time = 1.0839, size = 409, normalized size = 2.51

$$\frac{1}{15}x^{15}hb^2+\frac{1}{14}x^{14}gb^2+\frac{1}{13}x^{13}fb^2+\frac{1}{12}x^{12}eb^2+\frac{1}{6}x^{12}hba+\frac{1}{11}x^{11}db^2+\frac{2}{11}x^{11}gba+\frac{1}{10}x^{10}cb^2+\frac{1}{5}x^{10}fba+\frac{2}{9}x^9eba+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/15*x^15*h*b^2 + 1/14*x^14*g*b^2 + 1/13*x^13*f*b^2 + 1/12*x^12*e*b^2 + 1/6
*x^12*h*b*a + 1/11*x^11*d*b^2 + 2/11*x^11*g*b*a + 1/10*x^10*c*b^2 + 1/5*x^1
0*f*b*a + 2/9*x^9*e*b*a + 1/9*x^9*h*a^2 + 1/4*x^8*d*b*a + 1/8*x^8*g*a^2 + 2
/7*x^7*c*b*a + 1/7*x^7*f*a^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^
2
```

Sympy [A] time = 0.13967, size = 167, normalized size = 1.02

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}+x^{12}\left(\frac{abh}{6}+\frac{b^2e}{12}\right)+x^{11}\left(\frac{2abg}{11}+\frac{b^2d}{11}\right)+x^{10}\left(\frac{abf}{5}+\frac{b^2c}{10}\right)+x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 + b**2*g*x*
*14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12) + x**11*(2*a*b*g/11
+ b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) + x**9*(a**2*h/9 + 2*a*b*e/9) +
x**8*(a**2*g/8 + a*b*d/4) + x**7*(a**2*f/7 + 2*a*b*c/7)
```

Giac [A] time = 1.06638, size = 216, normalized size = 1.33

$$\frac{1}{15}b^2hx^{15} + \frac{1}{14}b^2gx^{14} + \frac{1}{13}b^2fx^{13} + \frac{1}{6}abhx^{12} + \frac{1}{12}b^2x^{12}e + \frac{1}{11}b^2dx^{11} + \frac{2}{11}abgx^{11} + \frac{1}{10}b^2cx^{10} + \frac{1}{5}abfx^{10} + \frac{1}{9}a^2hx^9 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/6*a*b*h*x^12 + 1/12
*b^2*x^12*e + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*b^2*c*x^10 + 1/5*a*b
*f*x^10 + 1/9*a^2*h*x^9 + 2/9*a*b*x^9*e + 1/4*a*b*d*x^8 + 1/8*a^2*g*x^8 + 2
/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^
4
```

3.385 $\int x^2 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=158

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2$$

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (
a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(
b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
 + (c*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.125777, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1582, 1850}

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 + (
a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b*(
b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
 + (c*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx&=\frac{c\left(a+bx^3\right)^3}{9b}+\int\left(a+bx^3\right)^2\left(-cx^2+x^2\left(c+dx+ex^2+fx^3+\right.\right.\\&=\frac{c\left(a+bx^3\right)^3}{9b}+\int\left(a^2dx^3+a^2ex^4+a^2fx^5+a(2bd+ag)x^6+\right.\\&=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{7}a(2bd+ag)x^7+\frac{1}{8}a(2be+ah\end{aligned}$$

Mathematica [A] time = 0.0759694, size = 150, normalized size = 0.95

$$a^2\left(\frac{cx^3}{3}+\frac{dx^4}{4}+\frac{ex^5}{5}+\frac{fx^6}{6}+\frac{gx^7}{7}+\frac{hx^8}{8}\right)+ab\left(\frac{cx^6}{3}+\frac{2dx^7}{7}+\frac{ex^8}{4}+\frac{2fx^9}{9}+\frac{gx^{10}}{5}+\frac{2hx^{11}}{11}\right)+\frac{b^2x^9\left(20020c+3x\left(6006\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 + (h*x^8)/8)
 + a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)/9 + (g*x^10)/5 + (2
*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*d + 5460*e*x + 55*x^2*(91*f +
84*g*x + 78*h*x^2))))/180180
```

Maple [A] time = 0.001, size = 152, normalized size = 1.

$$\frac{b^2hx^{14}}{14}+\frac{b^2gx^{13}}{13}+\frac{b^2fx^{12}}{12}+\frac{\left(2abh+b^2e\right)x^{11}}{11}+\frac{\left(2abg+b^2d\right)x^{10}}{10}+\frac{\left(2abf+b^2c\right)x^9}{9}+\frac{\left(a^2h+2aeb\right)x^8}{8}+\frac{\left(a^2g+2bd\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2*e)*x^11+1
/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a^2*h+2*a*b*e)*x^8+1/
7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1
/3*a^2*c*x^3
```

Maxima [A] time = 0.959115, size = 204, normalized size = 1.29

$$\frac{1}{14}b^2hx^{14}+\frac{1}{13}b^2gx^{13}+\frac{1}{12}b^2fx^{12}+\frac{1}{11}\left(b^2e+2abh\right)x^{11}+\frac{1}{10}\left(b^2d+2abg\right)x^{10}+\frac{1}{9}\left(b^2c+2abf\right)x^9+\frac{1}{8}\left(2abe+a^2h\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*h
)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2*a
*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*d*x
^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

Fricas [A] time = 0.861869, size = 405, normalized size = 2.56

$$\frac{1}{14}x^{14}hb^2+\frac{1}{13}x^{13}gb^2+\frac{1}{12}x^{12}fb^2+\frac{1}{11}x^{11}eb^2+\frac{2}{11}x^{11}hba+\frac{1}{10}x^{10}db^2+\frac{1}{5}x^{10}gba+\frac{1}{9}x^9cb^2+\frac{2}{9}x^9fba+\frac{1}{4}x^8eba+\frac{1}{8}x^8h$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/14*x^14*h*b^2 + 1/13*x^13*g*b^2 + 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 2/1
1*x^11*h*b*a + 1/10*x^10*d*b^2 + 1/5*x^10*g*b*a + 1/9*x^9*c*b^2 + 2/9*x^9*f
*b*a + 1/4*x^8*e*b*a + 1/8*x^8*h*a^2 + 2/7*x^7*d*b*a + 1/7*x^7*g*a^2 + 1/3*
x^6*c*b*a + 1/6*x^6*f*a^2 + 1/5*x^5*e*a^2 + 1/4*x^4*d*a^2 + 1/3*x^3*c*a^2
```

Sympy [A] time = 0.100332, size = 167, normalized size = 1.06

$$\frac{a^2cx^3}{3} + \frac{a^2dx^4}{4} + \frac{a^2ex^5}{5} + \frac{b^2fx^{12}}{12} + \frac{b^2gx^{13}}{13} + \frac{b^2hx^{14}}{14} + x^{11}\left(\frac{2abh}{11} + \frac{b^2e}{11}\right) + x^{10}\left(\frac{abg}{5} + \frac{b^2d}{10}\right) + x^9\left(\frac{2abf}{9} + \frac{b^2c}{9}\right) + x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 + b**2*g*x*
*13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/11) + x**10*(a*b*g/5
+ b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) + x**8*(a**2*h/8 + a*b*e/4) + x*
*7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a**2*f/6 + a*b*c/3)
```

Giac [A] time = 1.07267, size = 216, normalized size = 1.37

$$\frac{1}{14}\,b^2hx^{14} + \frac{1}{13}\,b^2gx^{13} + \frac{1}{12}\,b^2fx^{12} + \frac{2}{11}\,abhx^{11} + \frac{1}{11}\,b^2x^{11}e + \frac{1}{10}\,b^2dx^{10} + \frac{1}{5}\,abgx^{10} + \frac{1}{9}\,b^2cx^9 + \frac{2}{9}\,abfx^9 + \frac{1}{8}\,a^2hx^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 2/11*a*b*h*x^11 + 1/1
1*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b^2*c*x^9 + 2/9*a*b*f
*x^9 + 1/8*a^2*h*x^8 + 1/4*a*b*x^8*e + 2/7*a*b*d*x^7 + 1/7*a^2*g*x^7 + 1/3*
a*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2*x^5*e + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.386 $\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=158

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+$$

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (
a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*
e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 +
 (d*(a + b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.129048, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1582, 1850}

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 + (
a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(b*
e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13 +
 (d*(a + b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{d\left(a+bx^3\right)^3}{9b}+\int \left(a+bx^3\right)^2\left(-dx^2+x\left(c+dx+ex^2+fx^3+gx^4\right.\right.\\ &= \frac{d\left(a+bx^3\right)^3}{9b}+\int \left(a^2cx+a^2ex^3+a(2bc+af)x^4+a^2gx^5+a(2be\right.\\ &= \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a^2gx^6+\frac{1}{7}a(2be+ah)x^7+\end{aligned}$$

Mathematica [A] time = 0.0238715, size = 163, normalized size = 1.03

$$\frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{1}{9}bx^9(2ag+bd)+\frac{1}{6}ax^6(ag+2bd)+\frac{1}{10}bx^{10}(2ah+$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (
a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8
+ (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (
b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

Maple [A] time = 0.001, size = 152, normalized size = 1.

$$\frac{b^2hx^{13}}{13}+\frac{b^2gx^{12}}{12}+\frac{b^2fx^{11}}{11}+\frac{\left(2\,abh+b^2e\right)x^{10}}{10}+\frac{\left(2\,abg+b^2d\right)x^9}{9}+\frac{\left(2\,abf+b^2c\right)x^8}{8}+\frac{\left(a^2h+2\,aeb\right)x^7}{7}+\frac{\left(a^2g+2\,b\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2*e)*x^10+1
/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2*h+2*a*b*e)*x^7+1/6*
(a^2*g+2*a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2
*a^2*c*x^2
```

Maxima [A] time = 0.947235, size = 204, normalized size = 1.29

$$\frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\left(b^2e+2\,abh\right)x^{10}+\frac{1}{9}\left(b^2d+2\,abg\right)x^9+\frac{1}{8}\left(b^2c+2\,abf\right)x^8+\frac{1}{7}\left(2\,abe+a^2h\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*h
)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a*b
*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*x^3
 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

Fricas [A] time = 0.862469, size = 400, normalized size = 2.53

$$\frac{1}{13}x^{13}hb^2+\frac{1}{12}x^{12}gb^2+\frac{1}{11}x^{11}fb^2+\frac{1}{10}x^{10}eb^2+\frac{1}{5}x^{10}hba+\frac{1}{9}x^9db^2+\frac{2}{9}x^9gba+\frac{1}{8}x^8cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba+\frac{1}{7}x^7ha$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/13*x^13*h*b^2 + 1/12*x^12*g*b^2 + 1/11*x^11*f*b^2 + 1/10*x^10*e*b^2 + 1/5
*x^10*h*b*a + 1/9*x^9*d*b^2 + 2/9*x^9*g*b*a + 1/8*x^8*c*b^2 + 1/4*x^8*f*b*a
 + 2/7*x^7*e*b*a + 1/7*x^7*h*a^2 + 1/3*x^6*d*b*a + 1/6*x^6*g*a^2 + 2/5*x^5*
c*b*a + 1/5*x^5*f*a^2 + 1/4*x^4*e*a^2 + 1/3*x^3*d*a^2 + 1/2*x^2*c*a^2
```

Sympy [A] time = 0.10222, size = 167, normalized size = 1.06

$$\frac{a^2cx^2}{2} + \frac{a^2dx^3}{3} + \frac{a^2ex^4}{4} + \frac{b^2fx^{11}}{11} + \frac{b^2gx^{12}}{12} + \frac{b^2hx^{13}}{13} + x^{10}\left(\frac{abh}{5} + \frac{b^2e}{10}\right) + x^9\left(\frac{2abg}{9} + \frac{b^2d}{9}\right) + x^8\left(\frac{abf}{4} + \frac{b^2c}{8}\right) + x^7\left(\frac{a^2h}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 + b**2*g*x*
*12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10) + x**9*(2*a*b*g/9 +
b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*(a**2*h/7 + 2*a*b*e/7) + x**6*
(a**2*g/6 + a*b*d/3) + x**5*(a**2*f/5 + 2*a*b*c/5)
```

Giac [A] time = 1.05239, size = 216, normalized size = 1.37

$$\frac{1}{13}b^2hx^{13} + \frac{1}{12}b^2gx^{12} + \frac{1}{11}b^2fx^{11} + \frac{1}{5}abhx^{10} + \frac{1}{10}b^2x^{10}e + \frac{1}{9}b^2dx^9 + \frac{2}{9}abgx^9 + \frac{1}{8}b^2cx^8 + \frac{1}{4}abfx^8 + \frac{1}{7}a^2hx^7 + \frac{2}{7}ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/5*a*b*h*x^10 + 1/10
*b^2*x^10*e + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c*x^8 + 1/4*a*b*f*x^8
 + 1/7*a^2*h*x^7 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 1/6*a^2*g*x^6 + 2/5*a*b*
c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*x^4*e + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.387 $\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=153

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e(a+bx^3)^3}{9b}+$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5
 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a
*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a +
 b*x^3)^3)/(9*b)
```

Rubi [A] time = 0.12658, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1582, 1850}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e(a+bx^3)^3}{9b}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/5
 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2*a
*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(a +
 b*x^3)^3)/(9*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{e(a+bx^3)^3}{9b}+\int (a+bx^3)^2(c+dx+fx^3+gx^4+hx^5)\,dx$$

$$= \frac{e(a+bx^3)^3}{9b}+\int (a^2c+a^2dx+a(2bc+af)x^3+a(2bd+ag)x^4$$

$$= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{4}a(2bc+af)x^4+\frac{1}{5}a(2bd+ag)x^5+\frac{1}{6}a^2hx^6+$$

Mathematica [A] time = 0.0742271, size = 125, normalized size = 0.82

$$\frac{462a^2x\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+22abx^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx))))}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))
) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))) +
 22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))
)/27720
```

Maple [A] time = 0.002, size = 149, normalized size = 1.

$$\frac{b^2hx^{12}}{12}+\frac{b^2gx^{11}}{11}+\frac{b^2fx^{10}}{10}+\frac{\left(2abh+b^2e\right)x^9}{9}+\frac{\left(2abg+b^2d\right)x^8}{8}+\frac{\left(2abf+b^2c\right)x^7}{7}+\frac{\left(a^2h+2aeb\right)x^6}{6}+\frac{\left(a^2g+2bda\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*e)*x^9+1/8
*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h+2*a*b*e)*x^6+1/5*(a
^2*g+2*a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c
*x
```

Maxima [A] time = 0.947005, size = 200, normalized size = 1.31

$$\frac{1}{12}b^2hx^{12}+\frac{1}{11}b^2gx^{11}+\frac{1}{10}b^2fx^{10}+\frac{1}{9}\left(b^2e+2abh\right)x^9+\frac{1}{8}\left(b^2d+2abg\right)x^8+\frac{1}{7}\left(b^2c+2abf\right)x^7+\frac{1}{6}\left(2abe+a^2h\right)x^6+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h)
*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b*e
 + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^2 +
 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

Fricas [A] time = 0.900123, size = 387, normalized size = 2.53

$$\frac{1}{12}x^{12}hb^2+\frac{1}{11}x^{11}gb^2+\frac{1}{10}x^{10}fb^2+\frac{1}{9}x^9eb^2+\frac{2}{9}x^9hba+\frac{1}{8}x^8db^2+\frac{1}{4}x^8gba+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7fba+\frac{1}{3}x^6eba+\frac{1}{6}x^6ha^2+\frac{2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/12*x^12*h*b^2 + 1/11*x^11*g*b^2 + 1/10*x^10*f*b^2 + 1/9*x^9*e*b^2 + 2/9*x
^9*h*b*a + 1/8*x^8*d*b^2 + 1/4*x^8*g*b*a + 1/7*x^7*c*b^2 + 2/7*x^7*f*b*a +
1/3*x^6*e*b*a + 1/6*x^6*h*a^2 + 2/5*x^5*d*b*a + 1/5*x^5*g*a^2 + 1/2*x^4*c*b
*a + 1/4*x^4*f*a^2 + 1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.087431, size = 163, normalized size = 1.07

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{b^2fx^{10}}{10} + \frac{b^2gx^{11}}{11} + \frac{b^2hx^{12}}{12} + x^9\left(\frac{2abh}{9} + \frac{b^2e}{9}\right) + x^8\left(\frac{abg}{4} + \frac{b^2d}{8}\right) + x^7\left(\frac{2abf}{7} + \frac{b^2c}{7}\right) + x^6\left(\frac{a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2*g*x**11/1
1 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**8*(a*b*g/4 + b**2*d/
8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2*h/6 + a*b*e/3) + x**5*(a**2*g
/5 + 2*a*b*d/5) + x**4*(a**2*f/4 + a*b*c/2)
```

Giac [A] time = 1.05691, size = 212, normalized size = 1.39

$$\frac{1}{12}b^2hx^{12} + \frac{1}{11}b^2gx^{11} + \frac{1}{10}b^2fx^{10} + \frac{2}{9}abhx^9 + \frac{1}{9}b^2x^9e + \frac{1}{8}b^2dx^8 + \frac{1}{4}abgx^8 + \frac{1}{7}b^2cx^7 + \frac{2}{7}abfx^7 + \frac{1}{6}a^2hx^6 + \frac{1}{3}ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 2/9*a*b*h*x^9 + 1/9*b
^2*x^9*e + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^7 + 2/7*a*b*f*x^7 +
1/6*a^2*h*x^6 + 1/3*a*b*x^6*e + 2/5*a*b*d*x^5 + 1/5*a^2*g*x^5 + 1/2*a*b*c*x
^4 + 1/4*a^2*f*x^4 + 1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

3.388 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=149

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+}{9b}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2
*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*
a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) +
 a^2*c*Log[x]
```

Rubi [A] time = 0.105703, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f(a+}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(2
*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e + 2*
a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b) +
 a^2*c*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= \frac{f(a+bx^3)^3}{9b}+\int \frac{(a+bx^3)^2(c+dx+ex^2+gx^4+hx^5)}{x}\,dx\\ &= \frac{f(a+bx^3)^3}{9b}+\int \left(a^2d+\frac{a^2c}{x}+a^2ex+2abcx^2+a(2bd+ag)x^3+a\right.\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{4}a(2bd+ag)x^4+\frac{1}{5}a(2be+ah)x^5+\frac{1}{6}b\end{aligned}$$

Mathematica [A] time = 0.0438508, size = 154, normalized size = 1.03

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{6}bx^6(2af+bc)+\frac{1}{3}ax^3(af+2bc)+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+b$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4
 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 + (b*(b*d + 2*a*g)*x^7
)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9 + (b^2*g*x^10)/10 + (b^2*h*x^
11)/11 + a^2*c*Log[x]
```

Maple [A] time = 0.003, size = 153, normalized size = 1.

$$\frac{b^2hx^{11}}{11}+\frac{b^2gx^{10}}{10}+\frac{x^9b^2f}{9}+\frac{x^8abh}{4}+\frac{b^2ex^8}{8}+\frac{2x^7abg}{7}+\frac{b^2dx^7}{7}+\frac{x^6abf}{3}+\frac{b^2cx^6}{6}+\frac{x^5a^2h}{5}+\frac{2abex^5}{5}+\frac{x^4a^2g}{4}+\frac{abdx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/11*b^2*h*x^11+1/10*b^2*g*x^10+1/9*x^9*b^2*f+1/4*x^8*a*b*h+1/8*b^2*e*x^8+2
/7*x^7*a*b*g+1/7*b^2*d*x^7+1/3*x^6*a*b*f+1/6*b^2*c*x^6+1/5*x^5*a^2*h+2/5*a*
b*e*x^5+1/4*x^4*a^2*g+1/2*a*b*d*x^4+1/3*a^2*f*x^3+2/3*a*b*c*x^3+1/2*a^2*e*x
^2+a^2*d*x+a^2*c*ln(x)
```

Maxima [A] time = 0.934238, size = 197, normalized size = 1.32

$$\frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}\left(b^2e+2abh\right)x^8+\frac{1}{7}\left(b^2d+2abg\right)x^7+\frac{1}{6}\left(b^2c+2abf\right)x^6+\frac{1}{5}\left(2abe+a^2h\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Fricas [A] time = 0.937602, size = 351, normalized size = 2.36

$$\frac{1}{11}b^2hx^{11}+\frac{1}{10}b^2gx^{10}+\frac{1}{9}b^2fx^9+\frac{1}{8}\left(b^2e+2abh\right)x^8+\frac{1}{7}\left(b^2d+2abg\right)x^7+\frac{1}{6}\left(b^2c+2abf\right)x^6+\frac{1}{5}\left(2abe+a^2h\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*x
^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e +
 a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*(2*
a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

Sympy [A] time = 0.47338, size = 162, normalized size = 1.09

$$a^2c\log(x) + a^2dx + \frac{a^2ex^2}{2} + \frac{b^2fx^9}{9} + \frac{b^2gx^{10}}{10} + \frac{b^2hx^{11}}{11} + x^8\left(\frac{abh}{4} + \frac{b^2e}{8}\right) + x^7\left(\frac{2abg}{7} + \frac{b^2d}{7}\right) + x^6\left(\frac{abf}{3} + \frac{b^2c}{6}\right) + x^5\left(\frac{a^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g*x**10/10
+ b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2*a*b*g/7 + b**2*d/7)
 + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5 + 2*a*b*e/5) + x**4*(a**2*g/4
 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*b*c/3)
```

Giac [A] time = 1.06798, size = 211, normalized size = 1.42

$$\frac{1}{11}b^2hx^{11} + \frac{1}{10}b^2gx^{10} + \frac{1}{9}b^2fx^9 + \frac{1}{4}abhx^8 + \frac{1}{8}b^2x^8e + \frac{1}{7}b^2dx^7 + \frac{2}{7}abgx^7 + \frac{1}{6}b^2cx^6 + \frac{1}{3}abfx^6 + \frac{1}{5}a^2hx^5 + \frac{2}{5}abx^5e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/4*a*b*h*x^8 + 1/8*b^2
*x^8*e + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6 + 1/3*a*b*f*x^6 + 1/
5*a^2*h*x^5 + 2/5*a*b*x^5*e + 1/2*a*b*d*x^4 + 1/4*a^2*g*x^4 + 2/3*a*b*c*x^3
 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x + a^2*c*log(abs(x))
```

3.389 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g(a+$$

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/3 + (a*(2*
b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^6)/6 + (b*(b*e + 2*a
*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10 + (g*(a + b*x^3)^3)/(9*b) + a^
2*d*Log[x]
```

Rubi [A] time = 0.127544, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+\frac{1}{4}ax^4(ah+2be)+\frac{g(a+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/3 + (a*(2*
b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^6)/6 + (b*(b*e + 2*a
*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10 + (g*(a + b*x^3)^3)/(9*b) + a^
2*d*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx=\frac{g(a+bx^3)^3}{9b}+\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+hx^5)}{x^2}\,dx$$

$$=\frac{g(a+bx^3)^3}{9b}+\int\left(a^2e+\frac{a^2c}{x^2}+\frac{a^2d}{x}+a(2bc+af)x+2abdx^2+\right.$$

$$=-\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{2}{3}abdx^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}$$

Mathematica [A] time = 0.0605502, size = 152, normalized size = 1.03

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{1}{6}bx^6(2ag+bd)+\frac{1}{3}ax^3(ag+2bd)+\frac{1}{7}bx^7(2ah+be)+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*g)*x^3)/3
+ (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b*(b*d + 2*a*g)*x^6)
/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 + (b^2*h*x^10)
/10 + a^2*d*Log[x]
```

Maple [A] time = 0.006, size = 152, normalized size = 1.

$$\frac{b^2hx^{10}}{10}+\frac{b^2gx^9}{9}+\frac{b^2fx^8}{8}+\frac{2x^7abh}{7}+\frac{b^2ex^7}{7}+\frac{x^6abg}{3}+\frac{b^2dx^6}{6}+\frac{2x^5abf}{5}+\frac{b^2cx^5}{5}+\frac{x^4a^2h}{4}+\frac{abex^4}{2}+\frac{x^3a^2g}{3}+\frac{2abdx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/10*b^2*h*x^10+1/9*b^2*g*x^9+1/8*b^2*f*x^8+2/7*x^7*a*b*h+1/7*b^2*e*x^7+1/3
*x^6*a*b*g+1/6*b^2*d*x^6+2/5*x^5*a*b*f+1/5*b^2*c*x^5+1/4*x^4*a^2*h+1/2*a*b*
e*x^4+1/3*x^3*a^2*g+2/3*a*b*d*x^3+1/2*a^2*f*x^2+a*b*c*x^2+a^2*e*x+a^2*d*ln(
x)-a^2*c/x
```

Maxima [A] time = 0.958382, size = 197, normalized size = 1.34

$$\frac{1}{10}b^2hx^{10}+\frac{1}{9}b^2gx^9+\frac{1}{8}b^2fx^8+\frac{1}{7}\left(b^2e+2abh\right)x^7+\frac{1}{6}\left(b^2d+2abg\right)x^6+\frac{1}{5}\left(b^2c+2abf\right)x^5+\frac{1}{4}\left(2abe+a^2h\right)x^4+a^2ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(b^2*e + 2*a*b*h)*x^7
 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^5 + 1/4*(2*a*b*e + a
^2*h)*x^4 + a^2*e*x + 1/3*(2*a*b*d + a^2*g)*x^3 + a^2*d*log(x) + 1/2*(2*a*b
*c + a^2*f)*x^2 - a^2*c/x
```

Fricas [A] time = 0.992344, size = 379, normalized size = 2.58

$$\frac{252\,b^2hx^{11}+280\,b^2gx^{10}+315\,b^2fx^9+360\left(b^2e+2\,abh\right)x^8+420\left(b^2d+2\,abg\right)x^7+504\left(b^2c+2\,abf\right)x^6+630\left(2\,abe+\right.}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^2*e + 2*a*
b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a*b*f)*x^6 + 630*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 2520*a^2*d
*x*log(x) + 1260*(2*a*b*c + a^2*f)*x^3 - 2520*a^2*c)/x
```

Sympy [A] time = 0.499578, size = 156, normalized size = 1.06

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{b^2fx^8}{8}+\frac{b^2gx^9}{9}+\frac{b^2hx^{10}}{10}+x^7\left(\frac{2abh}{7}+\frac{b^2e}{7}\right)+x^6\left(\frac{abg}{3}+\frac{b^2d}{6}\right)+x^5\left(\frac{2abf}{5}+\frac{b^2c}{5}\right)+x^4\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**9/9 + b**2
*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g/3 + b**2*d/6) + x**
5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a*b*e/2) + x**3*(a**2*g/3 + 2*a
*b*d/3) + x**2*(a**2*f/2 + a*b*c)
```

Giac [A] time = 1.05546, size = 209, normalized size = 1.42

$$\frac{1}{10}b^2hx^{10}+\frac{1}{9}b^2gx^9+\frac{1}{8}b^2fx^8+\frac{2}{7}abhx^7+\frac{1}{7}b^2x^7e+\frac{1}{6}b^2dx^6+\frac{1}{3}abgx^6+\frac{1}{5}b^2cx^5+\frac{2}{5}abfx^5+\frac{1}{4}a^2hx^4+\frac{1}{2}abx^4e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 2/7*a*b*h*x^7 + 1/7*b^2*x
^7*e + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 + 2/5*a*b*f*x^5 + 1/4*
a^2*h*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*g*x^3 + a*b*c*x^2 + 1/2
*a^2*f*x^2 + a^2*x*e + a^2*d*log(abs(x)) - a^2*c/x
```

3.390 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}$$

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*g)*x^2)/2
+ (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d + 2*a*g)*x^5)/5 + (b^
2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 + (h*(a + b*x^3)^3)/(9*b) + a^2*
e*Log[x]
```

Rubi [A] time = 0.128372, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*g)*x^2)/2
+ (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d + 2*a*g)*x^5)/5 + (b^
2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 + (h*(a + b*x^3)^3)/(9*b) + a^2*
e*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^3}\,dx &= \frac{h\left(a+bx^3\right)^3}{9b}+\int \frac{\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx\\ &= \frac{h\left(a+bx^3\right)^3}{9b}+\int \left(a(2bc+af)+\frac{a^2c}{x^3}+\frac{a^2d}{x^2}+\frac{a^2e}{x}+a(2bd+ag)x\right.\\ &= -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a(2bc+af)x+\frac{1}{2}a(2bd+ag)x^2+\frac{2}{3}abex^3+\frac{1}{4}b(bc+\end{aligned}$$

Mathematica [A] time = 0.0706433, size = 127, normalized size = 0.86

$$\frac{a^2\left(-3c-6dx+x^3\left(6f+3gx+2hx^2\right)\right)}{6x^2}+a^2e\log(x)+\frac{1}{30}abx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b*x*(60*c +
 x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/30 + (b^2*x^4*(630*c
+ x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/2520 + a^2*e*Log[
x]
```

Maple [A] time = 0.006, size = 150, normalized size = 1.

$$\frac{b^2hx^9}{9}+\frac{b^2gx^8}{8}+\frac{b^2fx^7}{7}+\frac{x^6abh}{3}+\frac{b^2ex^6}{6}+\frac{2x^5abg}{5}+\frac{b^2dx^5}{5}+\frac{x^4abf}{2}+\frac{b^2cx^4}{4}+\frac{x^3a^2h}{3}+\frac{2abex^3}{3}+\frac{x^2a^2g}{2}+abdx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/9*b^2*h*x^9+1/8*b^2*g*x^8+1/7*b^2*f*x^7+1/3*x^6*a*b*h+1/6*b^2*e*x^6+2/5*x
^5*a*b*g+1/5*b^2*d*x^5+1/2*x^4*a*b*f+1/4*b^2*c*x^4+1/3*x^3*a^2*h+2/3*a*b*e*
x^3+1/2*x^2*a^2*g+a*b*d*x^2+a^2*f*x+2*a*b*c*x+a^2*e*ln(x)-1/2*a^2*c/x^2-a^2
*d/x
```

Maxima [A] time = 0.939303, size = 197, normalized size = 1.34

$$\frac{1}{9}b^2hx^9+\frac{1}{8}b^2gx^8+\frac{1}{7}b^2fx^7+\frac{1}{6}\left(b^2e+2abh\right)x^6+\frac{1}{5}\left(b^2d+2abg\right)x^5+\frac{1}{4}\left(b^2c+2abf\right)x^4+\frac{1}{3}\left(2abe+a^2h\right)x^3+a^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(b^2*e + 2*a*b*h)*x^6 +
 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4 + 1/3*(2*a*b*e + a^2
*h)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g)*x^2 + (2*a*b*c + a^2*f)*x -
1/2*(2*a^2*d*x + a^2*c)/x^2
```

Fricas [A] time = 1.00032, size = 383, normalized size = 2.61

$$\frac{280\,b^2hx^{11}+315\,b^2gx^{10}+360\,b^2fx^9+420\left(b^2e+2\,abh\right)x^8+504\left(b^2d+2\,abg\right)x^7+630\left(b^2c+2\,abf\right)x^6+840\left(2\,abe\right.}{2520\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^2*e + 2*a*
b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a*b*f)*x^6 + 840*(2*a
*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2*log(x) + 1260*(2*a*b*d + a^2*g)*x^4 - 25
20*a^2*d*x + 2520*(2*a*b*c + a^2*f)*x^3 - 1260*a^2*c)/x^2
```

Sympy [A] time = 0.550674, size = 156, normalized size = 1.06

$$a^2e\log(x) + \frac{b^2fx^7}{7} + \frac{b^2gx^8}{8} + \frac{b^2hx^9}{9} + x^6\left(\frac{abh}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{2abg}{5} + \frac{b^2d}{5}\right) + x^4\left(\frac{abf}{2} + \frac{b^2c}{4}\right) + x^3\left(\frac{a^2h}{3} + \frac{2abe}{3}\right) + x^2\left(\frac{a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x**6*(a*b*h
/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*b*f/2 + b**2*c/4) +
x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2 + a*b*d) + x*(a**2*f + 2*a*b*c
) - (a**2*c + 2*a**2*d*x)/(2*x**2)
```

Giac [A] time = 1.07399, size = 207, normalized size = 1.41

$$\frac{1}{9}b^2hx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{7}b^2fx^7 + \frac{1}{3}abhx^6 + \frac{1}{6}b^2x^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}abgx^5 + \frac{1}{4}b^2cx^4 + \frac{1}{2}abfx^4 + \frac{1}{3}a^2hx^3 + \frac{2}{3}abx^3e + abd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/3*a*b*h*x^6 + 1/6*b^2*x^6
*e + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/2*a*b*f*x^4 + 1/3*a^
2*h*x^3 + 2/3*a*b*x^3*e + a*b*d*x^2 + 1/2*a^2*g*x^2 + 2*a*b*c*x + a^2*f*x +
 a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

3.391 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax$$

```
[Out] -(a^2*c)/(3*x^3) - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*x + (a*(2*
b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d + 2*a*g)*x^4)/4 + (b*
(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8 + a*(2
*b*c + a*f)*Log[x]
```

Rubi [A] time = 0.118, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*x + (a*(2*
b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d + 2*a*g)*x^4)/4 + (b*
(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8 + a*(2
*b*c + a*f)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \int \left(a(2bd+ag)+\frac{a^2c}{x^4}+\frac{a^2d}{x^3}+\frac{a^2e}{x^2}+\frac{a(2bc+af)}{x}+a(2be+ah\right.$$

$$=-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2+\frac{1}{3}b(bc+$$

Mathematica [A] time = 0.0806839, size = 123, normalized size = 0.81

$$-\frac{a^2\left(2c+3x\left(d+2ex+x^3(-(2g+hx))\right)\right)}{6x^3}+a\log(x)(af+2bc)+\frac{1}{30}abx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (a*b*x*(60*d + x
*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/30 + (b^2*x^3*(280*c + x*(210*d +
```

```
x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/840 + a*(2*b*c + a*f)*Log[x]
```

Maple [A] time = 0.005, size = 149, normalized size = 1.

$$\frac{b^2hx^8}{8}+\frac{b^2gx^7}{7}+\frac{b^2fx^6}{6}+\frac{2x^5abh}{5}+\frac{x^5b^2e}{5}+\frac{x^4abg}{2}+\frac{b^2dx^4}{4}+\frac{2x^3abf}{3}+\frac{b^2cx^3}{3}+\frac{x^2a^2h}{2}+aebx^2+a^2gx+2bdax+\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*x^5*a*b*h+1/5*x^5*b^2*e+1/2*x
^4*a*b*g+1/4*b^2*d*x^4+2/3*x^3*a*b*f+1/3*b^2*c*x^3+1/2*x^2*a^2*h+a*e*b*x^2+
a^2*g*x+2*b*d*a*x+ln(x)*a^2*f+2*ln(x)*a*b*c-1/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2
*e/x
```

Maxima [A] time = 0.9255, size = 198, normalized size = 1.3

$$\frac{1}{8}b^2hx^8+\frac{1}{7}b^2gx^7+\frac{1}{6}b^2fx^6+\frac{1}{5}\left(b^2e+2abh\right)x^5+\frac{1}{4}\left(b^2d+2abg\right)x^4+\frac{1}{3}\left(b^2c+2abf\right)x^3+\frac{1}{2}\left(2abe+a^2h\right)x^2+\left(2abd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(b^2*e + 2*a*b*h)*x^5 +
 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3 + 1/2*(2*a*b*e + a^2
*h)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^2*f)*log(x) - 1/6*(6*a^2*e*x^2
 + 3*a^2*d*x + 2*a^2*c)/x^3
```

Fricas [A] time = 0.991457, size = 375, normalized size = 2.47

$$\frac{105\,b^2hx^{11}+120\,b^2gx^{10}+140\,b^2fx^9+168\left(b^2e+2\,abh\right)x^8+210\left(b^2d+2\,abg\right)x^7+280\left(b^2c+2\,abf\right)x^6+420\left(2\,abe+a\right.}{840\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2*e + 2*a*b
*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*
b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 840*(2*a*b*c
 + a^2*f)*x^3*log(x) - 420*a^2*d*x - 280*a^2*c)/x^3
```

Sympy [A] time = 0.872923, size = 156, normalized size = 1.03

$$a\left(af+2bc\right)\log\left(x\right)+\frac{b^2fx^6}{6}+\frac{b^2gx^7}{7}+\frac{b^2hx^8}{8}+x^5\left(\frac{2abh}{5}+\frac{b^2e}{5}\right)+x^4\left(\frac{abg}{2}+\frac{b^2d}{4}\right)+x^3\left(\frac{2abf}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2h}{2}+abe\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + x*
*5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) + x**3*(2*a*b*f/3 + b
**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*g + 2*a*b*d) - (2*a**2*c + 3*a
**2*d*x + 6*a**2*e*x**2)/(6*x**3)
```

Giac [A] time = 1.07006, size = 207, normalized size = 1.36

$$\frac{1}{8}b^2hx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{2}{5}abhx^5 + \frac{1}{5}b^2x^5e + \frac{1}{4}b^2dx^4 + \frac{1}{2}abgx^4 + \frac{1}{3}b^2cx^3 + \frac{2}{3}abfx^3 + \frac{1}{2}a^2hx^2 + abx^2e + 2\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 2/5*a*b*h*x^5 + 1/5*b^2*x^5
*e + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*f*x^3 + 1/2*a^
2*h*x^2 + a*b*x^2*e + 2*a*b*d*x + a^2*g*x + (2*a*b*c + a^2*f)*log(abs(x)) -
 1/6*(6*a^2*x^2*e + 3*a^2*d*x + 2*a^2*c)/x^3
```

3.392 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+$$

```
[Out] -(a^2*c)/(4*x^4) - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c + a*f))/x
+ a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d + 2*a*g)*x^3)/3 + (
b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7 + a*
(2*b*d + a*g)*Log[x]
```

Rubi [A] time = 0.117274, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a^2*c)/(4*x^4) - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c + a*f))/x
+ a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d + 2*a*g)*x^3)/3 + (
b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7 + a*
(2*b*d + a*g)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx=\int \left(a(2be+ah)+\frac{a^2c}{x^5}+\frac{a^2d}{x^4}+\frac{a^2e}{x^3}+\frac{a(2bc+af)}{x^2}+\frac{a(2bd+ag)}{x}+\right.$$

$$=-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+\frac{1}{2}b(bc+2af)x^2$$

Mathematica [A] time = 0.0725245, size = 125, normalized size = 0.82

$$-\frac{a^2\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)}{12x^4}-\frac{2abc}{x}+a\log(x)(ag+2bd)+\frac{1}{6}abx(12e+x(6f+x(4g+3hx)))+\frac{1}{420}b^2x^2\left(210\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) +
 (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x))))/6 + (b^2*x^2*(210*c + x*(140*d
```

```
+ x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/420 + a*(2*b*d + a*g)*Log[x]
```

Maple [A] time = 0.006, size = 149, normalized size = 1.

$$\frac{b^2hx^7}{7}+\frac{b^2gx^6}{6}+\frac{b^2fx^5}{5}+\frac{x^4abh}{2}+\frac{x^4b^2e}{4}+\frac{2x^3abg}{3}+\frac{b^2dx^3}{3}+x^2abf+\frac{x^2b^2c}{2}+a^2hx+2aebx+\ln(x)a^2g+2\ln(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*b^2*f*x^5+1/2*x^4*a*b*h+1/4*x^4*b^2*e+2/3*x
^3*a*b*g+1/3*b^2*d*x^3+x^2*a*b*f+1/2*x^2*b^2*c+a^2*h*x+2*a*e*b*x+ln(x)*a^2*
g+2*ln(x)*a*b*d-1/3*a^2*d/x^3-1/4*a^2*c/x^4-1/2*a^2*e/x^2-a^2/x*f-2*a/x*b*c
```

Maxima [A] time = 0.947725, size = 198, normalized size = 1.3

$$\frac{1}{7}b^2hx^7+\frac{1}{6}b^2gx^6+\frac{1}{5}b^2fx^5+\frac{1}{4}\left(b^2e+2abh\right)x^4+\frac{1}{3}\left(b^2d+2abg\right)x^3+\frac{1}{2}\left(b^2c+2abf\right)x^2+\left(2abe+a^2h\right)x+\left(2abd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(b^2*e + 2*a*b*h)*x^4 +
 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2 + (2*a*b*e + a^2*h)*
x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*x + 12*(2*a*b*c
+ a^2*f)*x^3 + 3*a^2*c)/x^4
```

Fricas [A] time = 0.990554, size = 371, normalized size = 2.44

$$\frac{60b^2hx^{11}+70b^2gx^{10}+84b^2fx^9+105\left(b^2e+2abh\right)x^8+140\left(b^2d+2abg\right)x^7+210\left(b^2c+2abf\right)x^6+420\left(2abe+a\right.}{420x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e + 2*a*b*h)
*x^8 + 140*(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b*e
 + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log(x) - 210*a^2*e*x^2 - 140*a^2*
d*x - 420*(2*a*b*c + a^2*f)*x^3 - 105*a^2*c)/x^4
```

Sympy [A] time = 2.7749, size = 155, normalized size = 1.02

$$a\left(ag+2bd\right)\log(x)+\frac{b^2fx^5}{5}+\frac{b^2gx^6}{6}+\frac{b^2hx^7}{7}+x^4\left(\frac{abh}{2}+\frac{b^2e}{4}\right)+x^3\left(\frac{2abg}{3}+\frac{b^2d}{3}\right)+x^2\left(abf+\frac{b^2c}{2}\right)+x\left(a^2h+2ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + x*
*4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) + x**2*(a*b*f + b**2*
c/2) + x*(a**2*h + 2*a*b*e) - (3*a**2*c + 4*a**2*d*x + 6*a**2*e*x**2 + x**3
*(12*a**2*f + 24*a*b*c))/(12*x**4)
```

Giac [A] time = 1.06372, size = 205, normalized size = 1.35

$$\frac{1}{7}\,b^2hx^7+\frac{1}{6}\,b^2gx^6+\frac{1}{5}\,b^2fx^5+\frac{1}{2}\,abhx^4+\frac{1}{4}\,b^2x^4e+\frac{1}{3}\,b^2dx^3+\frac{2}{3}\,abgx^3+\frac{1}{2}\,b^2cx^2+abfx^2+a^2hx+2\,abxe+\left(2\,abd+a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/2*a*b*h*x^4 + 1/4*b^2*x^4
*e + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*b*f*x^2 + a^2*h*x +
2*a*b*x*e + (2*a*b*d + a^2*g)*log(abs(x)) - 1/12*(6*a^2*x^2*e + 4*a^2*d*x +
 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

3.393 $\int x^4 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=223

$$\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x$$

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rubi [A] time = 0.292612, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^4 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \int \left(a^3cx^4+a^3dx^5+a^3ex^6+a^2(3bc+af)x^7+a^2(3bd+ag)x^8\right.$$

$$=\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+a$$

Mathematica [A] time = 0.0509596, size = 223, normalized size = 1.

$$\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8 +
 (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c + a*
```

```
f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (b^2
*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a*h)*
x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

Maple [A] time = 0.001, size = 224, normalized size = 1.

$$\frac{b^3hx^{19}}{19}+\frac{b^3gx^{18}}{18}+\frac{b^3fx^{17}}{17}+\frac{\left(3\,b^2ah+b^3e\right)x^{16}}{16}+\frac{\left(3\,b^2ag+b^3d\right)x^{15}}{15}+\frac{\left(3\,b^2af+b^3c\right)x^{14}}{14}+\frac{\left(3\,ba^2h+3\,aeb^2\right)x^{13}}{13}+\frac{(3\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/19*b^3*h*x^19+1/18*b^3*g*x^18+1/17*b^3*f*x^17+1/16*(3*a*b^2*h+b^3*e)*x^16
+1/15*(3*a*b^2*g+b^3*d)*x^15+1/14*(3*a*b^2*f+b^3*c)*x^14+1/13*(3*a^2*b*h+3*
a*b^2*e)*x^13+1/12*(3*a^2*b*g+3*a*b^2*d)*x^12+1/11*(3*a^2*b*f+3*a*b^2*c)*x^
11+1/10*(a^3*h+3*a^2*b*e)*x^10+1/9*(a^3*g+3*a^2*b*d)*x^9+1/8*(a^3*f+3*a^2*b
*c)*x^8+1/7*a^3*e*x^7+1/6*a^3*d*x^6+1/5*a^3*c*x^5
```

Maxima [A] time = 0.957263, size = 293, normalized size = 1.31

$$\frac{1}{19}\,b^3hx^{19}+\frac{1}{18}\,b^3gx^{18}+\frac{1}{17}\,b^3fx^{17}+\frac{1}{16}\left(b^3e+3\,ab^2h\right)x^{16}+\frac{1}{15}\left(b^3d+3\,ab^2g\right)x^{15}+\frac{1}{14}\left(b^3c+3\,ab^2f\right)x^{14}+\frac{3}{13}\left(ab^2e+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^2
*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14 +
3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b^2*
c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6*a^3
*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c + a^3
*f)*x^8
```

Fricas [A] time = 0.873177, size = 591, normalized size = 2.65

$$\frac{1}{19}x^{19}hb^3+\frac{1}{18}x^{18}gb^3+\frac{1}{17}x^{17}fb^3+\frac{1}{16}x^{16}eb^3+\frac{3}{16}x^{16}hb^2a+\frac{1}{15}x^{15}db^3+\frac{1}{5}x^{15}gb^2a+\frac{1}{14}x^{14}cb^3+\frac{3}{14}x^{14}fb^2a+\frac{3}{13}x^{13}eb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/19*x^19*h*b^3 + 1/18*x^18*g*b^3 + 1/17*x^17*f*b^3 + 1/16*x^16*e*b^3 + 3/1
6*x^16*h*b^2*a + 1/15*x^15*d*b^3 + 1/5*x^15*g*b^2*a + 1/14*x^14*c*b^3 + 3/1
4*x^14*f*b^2*a + 3/13*x^13*e*b^2*a + 3/13*x^13*h*b*a^2 + 1/4*x^12*d*b^2*a +
 1/4*x^12*g*b*a^2 + 3/11*x^11*c*b^2*a + 3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a
^2 + 1/10*x^10*h*a^3 + 1/3*x^9*d*b*a^2 + 1/9*x^9*g*a^3 + 3/8*x^8*c*b*a^2 +
1/8*x^8*f*a^3 + 1/7*x^7*e*a^3 + 1/6*x^6*d*a^3 + 1/5*x^5*c*a^3
```

Sympy [A] time = 0.099741, size = 246, normalized size = 1.1

$$\frac{a^3cx^5}{5}+\frac{a^3dx^6}{6}+\frac{a^3ex^7}{7}+\frac{b^3fx^{17}}{17}+\frac{b^3gx^{18}}{18}+\frac{b^3hx^{19}}{19}+x^{16}\left(\frac{3ab^2h}{16}+\frac{b^3e}{16}\right)+x^{15}\left(\frac{ab^2g}{5}+\frac{b^3d}{15}\right)+x^{14}\left(\frac{3ab^2f}{14}+\frac{b^3c}{14}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 + b**3*g*x*
*18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*e/16) + x**15*(a*b**
2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b**3*c/14) + x**13*(3*a**2*b*h/
13 + 3*a*b**2*e/13) + x**12*(a**2*b*g/4 + a*b**2*d/4) + x**11*(3*a**2*b*f/1
1 + 3*a*b**2*c/11) + x**10*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/9 + a
**2*b*d/3) + x**8*(a**3*f/8 + 3*a**2*b*c/8)
```

Giac [A] time = 1.06827, size = 315, normalized size = 1.41

$$\frac{1}{19}b^3hx^{19}+\frac{1}{18}b^3gx^{18}+\frac{1}{17}b^3fx^{17}+\frac{3}{16}ab^2hx^{16}+\frac{1}{16}b^3x^{16}e+\frac{1}{15}b^3dx^{15}+\frac{1}{5}ab^2gx^{15}+\frac{1}{14}b^3cx^{14}+\frac{3}{14}ab^2fx^{14}+\frac{3}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 3/16*a*b^2*h*x^16 + 1
/16*b^3*x^16*e + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1/14*b^3*c*x^14 + 3/1
4*a*b^2*f*x^14 + 3/13*a^2*b*h*x^13 + 3/13*a*b^2*x^13*e + 1/4*a*b^2*d*x^12 +
 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^11 + 3/11*a^2*b*f*x^11 + 1/10*a^3*h*x^10
 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x^8 +
1/8*a^3*f*x^8 + 1/7*a^3*x^7*e + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5
```

3.394 $\int x^3 (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

Optimal. Leaf size=223

$$\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3a$$

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rubi [A] time = 0.227574, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3a$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3 (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \int \left(a^3cx^3+a^3dx^4+a^3ex^5+a^2(3bc+af)x^6+a^2(3bd+ag)x^7+a\right.\\ &= \frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x\end{aligned}$$

Mathematica [A] time = 0.0468082, size = 223, normalized size = 1.

$$\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7 +
 (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*f)
```

```
*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^2*(
b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h)*x^
15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

Maple [A] time = 0.002, size = 224, normalized size = 1.

$$\frac{b^3hx^{18}}{18}+\frac{b^3gx^{17}}{17}+\frac{b^3fx^{16}}{16}+\frac{\left(3\,b^2ah+b^3e\right)x^{15}}{15}+\frac{\left(3\,b^2ag+b^3d\right)x^{14}}{14}+\frac{\left(3\,b^2af+b^3c\right)x^{13}}{13}+\frac{\left(3\,ba^2h+3\,aeb^2\right)x^{12}}{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/18*b^3*h*x^18+1/17*b^3*g*x^17+1/16*b^3*f*x^16+1/15*(3*a*b^2*h+b^3*e)*x^15
+1/14*(3*a*b^2*g+b^3*d)*x^14+1/13*(3*a*b^2*f+b^3*c)*x^13+1/12*(3*a^2*b*h+3*
a*b^2*e)*x^12+1/11*(3*a^2*b*g+3*a*b^2*d)*x^11+1/10*(3*a^2*b*f+3*a*b^2*c)*x^
10+1/9*(a^3*h+3*a^2*b*e)*x^9+1/8*(a^3*g+3*a^2*b*d)*x^8+1/7*(a^3*f+3*a^2*b*c
)*x^7+1/6*a^3*e*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4
```

Maxima [A] time = 0.931833, size = 293, normalized size = 1.31

$$\frac{1}{18}\,b^3hx^{18}+\frac{1}{17}\,b^3gx^{17}+\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\left(b^3e+3\,ab^2h\right)x^{15}+\frac{1}{14}\left(b^3d+3\,ab^2g\right)x^{14}+\frac{1}{13}\left(b^3c+3\,ab^2f\right)x^{13}+\frac{1}{4}\left(ab^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^2
*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13 +
1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b^2*
c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a^3*d
*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c + a^3*f
)*x^7
```

Fricas [A] time = 0.869605, size = 586, normalized size = 2.63

$$\frac{1}{18}x^{18}hb^3+\frac{1}{17}x^{17}gb^3+\frac{1}{16}x^{16}fb^3+\frac{1}{15}x^{15}eb^3+\frac{1}{5}x^{15}hb^2a+\frac{1}{14}x^{14}db^3+\frac{3}{14}x^{14}gb^2a+\frac{1}{13}x^{13}cb^3+\frac{3}{13}x^{13}fb^2a+\frac{1}{4}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/18*x^18*h*b^3 + 1/17*x^17*g*b^3 + 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/5
*x^15*h*b^2*a + 1/14*x^14*d*b^3 + 3/14*x^14*g*b^2*a + 1/13*x^13*c*b^3 + 3/1
3*x^13*f*b^2*a + 1/4*x^12*e*b^2*a + 1/4*x^12*h*b*a^2 + 3/11*x^11*d*b^2*a +
3/11*x^11*g*b*a^2 + 3/10*x^10*c*b^2*a + 3/10*x^10*f*b*a^2 + 1/3*x^9*e*b*a^2
 + 1/9*x^9*h*a^3 + 3/8*x^8*d*b*a^2 + 1/8*x^8*g*a^3 + 3/7*x^7*c*b*a^2 + 1/7*
x^7*f*a^3 + 1/6*x^6*e*a^3 + 1/5*x^5*d*a^3 + 1/4*x^4*c*a^3
```

Sympy [A] time = 0.100897, size = 246, normalized size = 1.1

$$\frac{a^3cx^4}{4} + \frac{a^3dx^5}{5} + \frac{a^3ex^6}{6} + \frac{b^3fx^{16}}{16} + \frac{b^3gx^{17}}{17} + \frac{b^3hx^{18}}{18} + x^{15}\left(\frac{ab^2h}{5} + \frac{b^3e}{15}\right) + x^{14}\left(\frac{3ab^2g}{14} + \frac{b^3d}{14}\right) + x^{13}\left(\frac{3ab^2f}{13} + \frac{b^3c}{13}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 + b**3*g*x*
*17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/15) + x**14*(3*a*b**2
*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b**3*c/13) + x**12*(a**2*b*h/4
+ a*b**2*e/4) + x**11*(3*a**2*b*g/11 + 3*a*b**2*d/11) + x**10*(3*a**2*b*f/1
0 + 3*a*b**2*c/10) + x**9*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3*a**2
*b*d/8) + x**7*(a**3*f/7 + 3*a**2*b*c/7)
```

Giac [A] time = 1.07661, size = 315, normalized size = 1.41

$$\frac{1}{18}\,b^3hx^{18} + \frac{1}{17}\,b^3gx^{17} + \frac{1}{16}\,b^3fx^{16} + \frac{1}{5}\,ab^2hx^{15} + \frac{1}{15}\,b^3x^{15}e + \frac{1}{14}\,b^3dx^{14} + \frac{3}{14}\,ab^2gx^{14} + \frac{1}{13}\,b^3cx^{13} + \frac{3}{13}\,ab^2fx^{13} + \frac{1}{4}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/5*a*b^2*h*x^15 + 1/
15*b^3*x^15*e + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1/13*b^3*c*x^13 + 3/1
3*a*b^2*f*x^13 + 1/4*a^2*b*h*x^12 + 1/4*a*b^2*x^12*e + 3/11*a*b^2*d*x^11 +
3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^10 + 3/10*a^2*b*f*x^10 + 1/9*a^3*h*x^9 +
 1/3*a^2*b*x^9*e + 3/8*a^2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 + 1/7*
a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.395 $\int x^2 \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$

Optimal. Leaf size=212

$$\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+$$

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 +
 (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10
+ (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12)/4 + (b^2*(b*d + 3*a*g)*x^13)
/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^
3*h*x^17)/17 + (c*(a + b*x^3)^4)/(12*b)
```

Rubi [A] time = 0.179245, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1582, 1850}

$$\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7 +
 (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10
+ (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12)/4 + (b^2*(b*d + 3*a*g)*x^13)
/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^
3*h*x^17)/17 + (c*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a+bx^3\right)^3\left(-cx^2+x^2\left(c+dx+ex^2+fx^3+\right.\right.\\ &= \frac{c\left(a+bx^3\right)^4}{12b}+\int \left(a^3dx^3+a^3ex^4+a^3fx^5+a^2(3bd+ag)x^6+\right.\\ &= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+\end{aligned}$$

Mathematica [A] time = 0.0480482, size = 223, normalized size = 1.05

$$\frac{1}{6}a^2x^6(af+3bc)+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{12}b^2x^{12}(3af+bc)+\frac{1}{13}b^2x^{13}(3a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f)*x^6)/6 +
 (a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a*b*(b*c + a*f)*x
^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (b^2*(b
*c + 3*a*f)*x^12)/12 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)*x^1
4)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

Maple [A] time = 0.001, size = 224, normalized size = 1.1

$$\frac{b^3hx^{17}}{17}+\frac{b^3gx^{16}}{16}+\frac{b^3fx^{15}}{15}+\frac{\left(3\,b^2ah+b^3e\right)x^{14}}{14}+\frac{\left(3\,b^2ag+b^3d\right)x^{13}}{13}+\frac{\left(3\,b^2af+b^3c\right)x^{12}}{12}+\frac{\left(3\,ba^2h+3\,aeb^2\right)x^{11}}{11}+\frac{(3\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/17*b^3*h*x^17+1/16*b^3*g*x^16+1/15*b^3*f*x^15+1/14*(3*a*b^2*h+b^3*e)*x^14
+1/13*(3*a*b^2*g+b^3*d)*x^13+1/12*(3*a*b^2*f+b^3*c)*x^12+1/11*(3*a^2*b*h+3*
a*b^2*e)*x^11+1/10*(3*a^2*b*g+3*a*b^2*d)*x^10+1/9*(3*a^2*b*f+3*a*b^2*c)*x^9
+1/8*(a^3*h+3*a^2*b*e)*x^8+1/7*(a^3*g+3*a^2*b*d)*x^7+1/6*(a^3*f+3*a^2*b*c)*
x^6+1/5*a^3*e*x^5+1/4*a^3*d*x^4+1/3*a^3*c*x^3
```

Maxima [A] time = 0.942734, size = 293, normalized size = 1.38

$$\frac{1}{17}\,b^3hx^{17}+\frac{1}{16}\,b^3gx^{16}+\frac{1}{15}\,b^3fx^{15}+\frac{1}{14}\,\left(b^3e+3\,ab^2h\right)x^{14}+\frac{1}{13}\,\left(b^3d+3\,ab^2g\right)x^{13}+\frac{1}{12}\,\left(b^3c+3\,ab^2f\right)x^{12}+\frac{3}{11}\,\left(ab^2e+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="max
ima")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^2
*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12 +
3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b^2*
c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^3*d*
x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a^3*f)
*x^6
```

Fricas [A] time = 0.828252, size = 583, normalized size = 2.75

$$\frac{1}{17}x^{17}hb^3+\frac{1}{16}x^{16}gb^3+\frac{1}{15}x^{15}fb^3+\frac{1}{14}x^{14}eb^3+\frac{3}{14}x^{14}hb^2a+\frac{1}{13}x^{13}db^3+\frac{3}{13}x^{13}gb^2a+\frac{1}{12}x^{12}cb^3+\frac{1}{4}x^{12}fb^2a+\frac{3}{11}x^{11}eb^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fri
cas")
```

```
[Out] 1/17*x^17*h*b^3 + 1/16*x^16*g*b^3 + 1/15*x^15*f*b^3 + 1/14*x^14*e*b^3 + 3/1
4*x^14*h*b^2*a + 1/13*x^13*d*b^3 + 3/13*x^13*g*b^2*a + 1/12*x^12*c*b^3 + 1/
4*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/11*x^11*h*b*a^2 + 3/10*x^10*d*b^2*a
+ 3/10*x^10*g*b*a^2 + 1/3*x^9*c*b^2*a + 1/3*x^9*f*b*a^2 + 3/8*x^8*e*b*a^2 +
 1/8*x^8*h*a^3 + 3/7*x^7*d*b*a^2 + 1/7*x^7*g*a^3 + 1/2*x^6*c*b*a^2 + 1/6*x^
6*f*a^3 + 1/5*x^5*e*a^3 + 1/4*x^4*d*a^3 + 1/3*x^3*c*a^3
```

Sympy [A] time = 0.098678, size = 246, normalized size = 1.16

$$\frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{b^3fx^{15}}{15} + \frac{b^3gx^{16}}{16} + \frac{b^3hx^{17}}{17} + x^{14}\left(\frac{3ab^2h}{14} + \frac{b^3e}{14}\right) + x^{13}\left(\frac{3ab^2g}{13} + \frac{b^3d}{13}\right) + x^{12}\left(\frac{ab^2f}{4} + \frac{b^3c}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 + b**3*g*x*
*16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*e/14) + x**13*(3*a*b
**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b**3*c/12) + x**11*(3*a**2*b*h/
11 + 3*a*b**2*e/11) + x**10*(3*a**2*b*g/10 + 3*a*b**2*d/10) + x**9*(a**2*b*
f/3 + a*b**2*c/3) + x**8*(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 + 3*a**
2*b*d/7) + x**6*(a**3*f/6 + a**2*b*c/2)
```

Giac [A] time = 1.07915, size = 315, normalized size = 1.49

$$\frac{1}{17}b^3hx^{17} + \frac{1}{16}b^3gx^{16} + \frac{1}{15}b^3fx^{15} + \frac{3}{14}ab^2hx^{14} + \frac{1}{14}b^3x^{14}e + \frac{1}{13}b^3dx^{13} + \frac{3}{13}ab^2gx^{13} + \frac{1}{12}b^3cx^{12} + \frac{1}{4}ab^2fx^{12} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gia
c")
```

```
[Out] 1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 3/14*a*b^2*h*x^14 + 1
/14*b^3*x^14*e + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 + 1/12*b^3*c*x^12 + 1/
4*a*b^2*f*x^12 + 3/11*a^2*b*h*x^11 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10
+ 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*f*x^9 + 1/8*a^3*h*x^8 + 3
/8*a^2*b*x^8*e + 3/7*a^2*b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1/6*a^
3*f*x^6 + 1/5*a^3*x^5*e + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.396 $\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

Optimal. Leaf size=212

$$\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)$$

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*g*x^6)/6 +
 (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8 + (a^2*b*g*x^9)/3 +
(3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (a*b^2*g*x^12)/
4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*
h*x^16)/16 + (d*(a + b*x^3)^4)/(12*b)
```

Rubi [A] time = 0.177018, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1582, 1850}

$$\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*g*x^6)/6 +
 (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8 + (a^2*b*g*x^9)/3 +
(3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (a*b^2*g*x^12)/
4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*
h*x^16)/16 + (d*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a+bx^3\right)^3\left(-dx^2+x\left(c+dx+ex^2+fx^3+gx^4\right.\right.\\ &= \frac{d\left(a+bx^3\right)^4}{12b}+\int\left(a^3cx+a^3ex^3+a^2(3bc+af)x^4+a^3gx^5+a^2(3b\right.\\ &= \frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^3gx^6+\frac{1}{7}a^2(3be+ah)x^7\end{aligned}$$

Mathematica [A] time = 0.0334001, size = 223, normalized size = 1.05

$$\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{6}a^2x^6(ag+3bd)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{12}b^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 +
 (a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)
*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c
 + 3*a*f)*x^11)/11 + (b^2*(b*d + 3*a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x^13)
/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

Maple [A] time = 0., size = 224, normalized size = 1.1

$$\frac{b^3hx^{16}}{16}+\frac{b^3gx^{15}}{15}+\frac{b^3fx^{14}}{14}+\frac{\left(3\,b^2ah+b^3e\right)x^{13}}{13}+\frac{\left(3\,b^2ag+b^3d\right)x^{12}}{12}+\frac{\left(3\,b^2af+b^3c\right)x^{11}}{11}+\frac{\left(3\,ba^2h+3\,aeb^2\right)x^{10}}{10}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/16*b^3*h*x^16+1/15*b^3*g*x^15+1/14*b^3*f*x^14+1/13*(3*a*b^2*h+b^3*e)*x^13
+1/12*(3*a*b^2*g+b^3*d)*x^12+1/11*(3*a*b^2*f+b^3*c)*x^11+1/10*(3*a^2*b*h+3*
a*b^2*e)*x^10+1/9*(3*a^2*b*g+3*a*b^2*d)*x^9+1/8*(3*a^2*b*f+3*a*b^2*c)*x^8+1
/7*(a^3*h+3*a^2*b*e)*x^7+1/6*(a^3*g+3*a^2*b*d)*x^6+1/5*(a^3*f+3*a^2*b*c)*x^
5+1/4*a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

Maxima [A] time = 0.954929, size = 293, normalized size = 1.38

$$\frac{1}{16}\,b^3hx^{16}+\frac{1}{15}\,b^3gx^{15}+\frac{1}{14}\,b^3fx^{14}+\frac{1}{13}\left(b^3e+3\,ab^2h\right)x^{13}+\frac{1}{12}\left(b^3d+3\,ab^2g\right)x^{12}+\frac{1}{11}\left(b^3c+3\,ab^2f\right)x^{11}+\frac{3}{10}\left(ab^2e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^2
*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11 +
3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2*c
+ a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*d*x^
3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3*f)*x
^5
```

Fricas [A] time = 0.876503, size = 578, normalized size = 2.73

$$\frac{1}{16}x^{16}hb^3+\frac{1}{15}x^{15}gb^3+\frac{1}{14}x^{14}fb^3+\frac{1}{13}x^{13}eb^3+\frac{3}{13}x^{13}hb^2a+\frac{1}{12}x^{12}db^3+\frac{1}{4}x^{12}gb^2a+\frac{1}{11}x^{11}cb^3+\frac{3}{11}x^{11}fb^2a+\frac{3}{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] 1/16*x^16*h*b^3 + 1/15*x^15*g*b^3 + 1/14*x^14*f*b^3 + 1/13*x^13*e*b^3 + 3/1
3*x^13*h*b^2*a + 1/12*x^12*d*b^3 + 1/4*x^12*g*b^2*a + 1/11*x^11*c*b^3 + 3/1
1*x^11*f*b^2*a + 3/10*x^10*e*b^2*a + 3/10*x^10*h*b*a^2 + 1/3*x^9*d*b^2*a +
1/3*x^9*g*b*a^2 + 3/8*x^8*c*b^2*a + 3/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/7
*x^7*h*a^3 + 1/2*x^6*d*b*a^2 + 1/6*x^6*g*a^3 + 3/5*x^5*c*b*a^2 + 1/5*x^5*f*
a^3 + 1/4*x^4*e*a^3 + 1/3*x^3*d*a^3 + 1/2*x^2*c*a^3
```

Sympy [A] time = 0.099559, size = 246, normalized size = 1.16

$$\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{b^3fx^{14}}{14}+\frac{b^3gx^{15}}{15}+\frac{b^3hx^{16}}{16}+x^{13}\left(\frac{3ab^2h}{13}+\frac{b^3e}{13}\right)+x^{12}\left(\frac{ab^2g}{4}+\frac{b^3d}{12}\right)+x^{11}\left(\frac{3ab^2f}{11}+\frac{b^3c}{11}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 + b**3*g*x*
*15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*e/13) + x**12*(a*b**
2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b**3*c/11) + x**10*(3*a**2*b*h/
10 + 3*a*b**2*e/10) + x**9*(a**2*b*g/3 + a*b**2*d/3) + x**8*(3*a**2*b*f/8 +
 3*a*b**2*c/8) + x**7*(a**3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**2*b*d
/2) + x**5*(a**3*f/5 + 3*a**2*b*c/5)
```

Giac [A] time = 1.0781, size = 315, normalized size = 1.49

$$\frac{1}{16}b^3hx^{16}+\frac{1}{15}b^3gx^{15}+\frac{1}{14}b^3fx^{14}+\frac{3}{13}ab^2hx^{13}+\frac{1}{13}b^3x^{13}e+\frac{1}{12}b^3dx^{12}+\frac{1}{4}ab^2gx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{11}ab^2fx^{11}+\frac{3}{10}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] 1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 3/13*a*b^2*h*x^13 + 1
/13*b^3*x^13*e + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1/11*b^3*c*x^11 + 3/1
1*a*b^2*f*x^11 + 3/10*a^2*b*h*x^10 + 3/10*a*b^2*x^10*e + 1/3*a*b^2*d*x^9 +
1/3*a^2*b*g*x^9 + 3/8*a*b^2*c*x^8 + 3/8*a^2*b*f*x^8 + 1/7*a^3*h*x^7 + 3/7*a
^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*f*
x^5 + 1/4*a^3*x^4*e + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.397 $\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

Optimal. Leaf size=207

$$\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^
5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/
8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10)/10 + (b^2*(b*d + 3*a*g)*x^11
)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/
15 + (e*(a + b*x^3)^4)/(12*b)
```

Rubi [A] time = 0.177319, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1582, 1850}

$$\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x^
5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8)/
8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10)/10 + (b^2*(b*d + 3*a*g)*x^11
)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/
15 + (e*(a + b*x^3)^4)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx &= \frac{e\left(a+bx^3\right)^4}{12b}+\int \left(a+bx^3\right)^3 \left(c+dx+fx^3+gx^4+hx^5\right) dx\\ &= \frac{e\left(a+bx^3\right)^4}{12b}+\int \left(a^3c+a^3dx+a^2(3bc+af)x^3+a^2(3bd+ag)x\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)x^5+\frac{1}{6}a^3hx^6\end{aligned}$$

Mathematica [A] time = 0.101428, size = 170, normalized size = 0.82

$$x\left(143a^2bx^3(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))+2002a^3\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10h\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

```
[Out] (x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55
*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x
^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*(455*e + 420*f*x + 390*g*x^2 +
 364*h*x^3))) + 143*a^2*b*x^3*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x
*(9*g + 8*h*x)))))))/120120
```

Maple [A] time = 0.001, size = 221, normalized size = 1.1

$$\frac{b^3hx^{15}}{15}+\frac{b^3gx^{14}}{14}+\frac{b^3fx^{13}}{13}+\frac{\left(3b^2ah+b^3e\right)x^{12}}{12}+\frac{\left(3b^2ag+b^3d\right)x^{11}}{11}+\frac{\left(3b^2af+b^3c\right)x^{10}}{10}+\frac{\left(3ba^2h+3aeb^2\right)x^9}{9}+\frac{(3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] 1/15*b^3*h*x^15+1/14*b^3*g*x^14+1/13*b^3*f*x^13+1/12*(3*a*b^2*h+b^3*e)*x^12
+1/11*(3*a*b^2*g+b^3*d)*x^11+1/10*(3*a*b^2*f+b^3*c)*x^10+1/9*(3*a^2*b*h+3*a
*b^2*e)*x^9+1/8*(3*a^2*b*g+3*a*b^2*d)*x^8+1/7*(3*a^2*b*f+3*a*b^2*c)*x^7+1/6
*(a^3*h+3*a^2*b*e)*x^6+1/5*(a^3*g+3*a^2*b*d)*x^5+1/4*(a^3*f+3*a^2*b*c)*x^4+
1/3*a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 0.948258, size = 289, normalized size = 1.4

$$\frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{12}\left(b^3e+3ab^2h\right)x^{12}+\frac{1}{11}\left(b^3d+3ab^2g\right)x^{11}+\frac{1}{10}\left(b^3c+3ab^2f\right)x^{10}+\frac{1}{3}\left(ab^2e+a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^2
*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10 +
1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c +
a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*x^2
+ 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

Fricas [A] time = 0.832962, size = 564, normalized size = 2.72

$$\frac{1}{15}x^{15}hb^3+\frac{1}{14}x^{14}gb^3+\frac{1}{13}x^{13}fb^3+\frac{1}{12}x^{12}eb^3+\frac{1}{4}x^{12}hb^2a+\frac{1}{11}x^{11}db^3+\frac{3}{11}x^{11}gb^2a+\frac{1}{10}x^{10}cb^3+\frac{3}{10}x^{10}fb^2a+\frac{1}{3}x^9eb^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] 1/15*x^15*h*b^3 + 1/14*x^14*g*b^3 + 1/13*x^13*f*b^3 + 1/12*x^12*e*b^3 + 1/4
*x^12*h*b^2*a + 1/11*x^11*d*b^3 + 3/11*x^11*g*b^2*a + 1/10*x^10*c*b^3 + 3/1
0*x^10*f*b^2*a + 1/3*x^9*e*b^2*a + 1/3*x^9*h*b*a^2 + 3/8*x^8*d*b^2*a + 3/8*
x^8*g*b*a^2 + 3/7*x^7*c*b^2*a + 3/7*x^7*f*b*a^2 + 1/2*x^6*e*b*a^2 + 1/6*x^6
*h*a^3 + 3/5*x^5*d*b*a^2 + 1/5*x^5*g*a^3 + 3/4*x^4*c*b*a^2 + 1/4*x^4*f*a^3
+ 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.095226, size = 243, normalized size = 1.17

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{b^3fx^{13}}{13} + \frac{b^3gx^{14}}{14} + \frac{b^3hx^{15}}{15} + x^{12}\left(\frac{ab^2h}{4} + \frac{b^3e}{12}\right) + x^{11}\left(\frac{3ab^2g}{11} + \frac{b^3d}{11}\right) + x^{10}\left(\frac{3ab^2f}{10} + \frac{b^3c}{10}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3*g*x**14/1
4 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) + x**11*(3*a*b**2*g/11
 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c/10) + x**9*(a**2*b*h/3 + a*b*
*2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a*b**
2*c/7) + x**6*(a**3*h/6 + a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5) + x*
*4*(a**3*f/4 + 3*a**2*b*c/4)
```

Giac [A] time = 1.06531, size = 311, normalized size = 1.5

$$\frac{1}{15}b^3hx^{15} + \frac{1}{14}b^3gx^{14} + \frac{1}{13}b^3fx^{13} + \frac{1}{4}ab^2hx^{12} + \frac{1}{12}b^3x^{12}e + \frac{1}{11}b^3dx^{11} + \frac{3}{11}ab^2gx^{11} + \frac{1}{10}b^3cx^{10} + \frac{3}{10}ab^2fx^{10} + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] 1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/4*a*b^2*h*x^12 + 1/
12*b^3*x^12*e + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1/10*b^3*c*x^10 + 3/1
0*a*b^2*f*x^10 + 1/3*a^2*b*h*x^9 + 1/3*a*b^2*x^9*e + 3/8*a*b^2*d*x^8 + 3/8*
a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7*a^2*b*f*x^7 + 1/6*a^3*h*x^6 + 1/2*a^2*b
*x^6*e + 3/5*a^2*b*d*x^5 + 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f*x^4
+ 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

3.398 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

Optimal. Leaf size=200

$$a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3$$

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3
*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(
b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e
 + 3*a*h)*x^11)/11 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/
(12*b) + a^3*c*Log[x]
```

Rubi [A] time = 0.145609, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```


```
[Out] a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(3
*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b*(
b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e
 + 3*a*h)*x^11)/11 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + (f*(a + b*x^3)^4)/
(12*b) + a^3*c*Log[x]
```


Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x}\,dx &= \frac{f\left(a+bx^3\right)^4}{12b}+\int \frac{\left(a+bx^3\right)^3\left(c+dx+ex^2+gx^4+hx^5\right)}{x}\,dx\\ &= \frac{f\left(a+bx^3\right)^4}{12b}+\int \left(a^3d+\frac{a^3c}{x}+a^3ex+3a^2bcx^2+a^2(3bd+ag)x^3+\right.\\ &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5+\frac{1}{2}\end{aligned}$$

Mathematica [A] time = 0.0777477, size = 214, normalized size = 1.07

$$\frac{1}{3}a^2x^3(af+3bc)+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{9}b^2x^9(3af+bc)+\frac{1}{10}b^2x^{10}(3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d + a*g)*x^
4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6)/2 + (3*a*b*(b*d +
a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^2*(b*c + 3*a*f)*x^9)/9 + (b^2*
(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*f*x^12)/12 + (b
^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*Log[x]
```

Maple [A] time = 0.004, size = 224, normalized size = 1.1

$$\frac{b^3hx^{14}}{14}+\frac{b^3gx^{13}}{13}+\frac{fx^{12}b^3}{12}+\frac{3\,x^{11}ab^2h}{11}+\frac{b^3ex^{11}}{11}+\frac{3\,x^{10}ab^2g}{10}+\frac{b^3dx^{10}}{10}+\frac{x^9ab^2f}{3}+\frac{b^3cx^9}{9}+\frac{3\,x^8a^2bh}{8}+\frac{3\,ab^2ex^8}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] 1/14*b^3*h*x^14+1/13*b^3*g*x^13+1/12*f*x^12*b^3+3/11*x^11*a*b^2*h+1/11*b^3*
e*x^11+3/10*x^10*a*b^2*g+1/10*b^3*d*x^10+1/3*x^9*a*b^2*f+1/9*b^3*c*x^9+3/8*
x^8*a^2*b*h+3/8*a*b^2*e*x^8+3/7*x^7*a^2*b*g+3/7*a*b^2*d*x^7+1/2*x^6*a^2*b*f
+1/2*a*b^2*c*x^6+1/5*x^5*a^3*h+3/5*a^2*b*e*x^5+1/4*x^4*a^3*g+3/4*a^2*b*d*x^
4+1/3*a^3*f*x^3+a^2*b*c*x^3+1/2*a^3*e*x^2+a^3*d*x+a^3*c*ln(x)
```

Maxima [A] time = 0.960838, size = 286, normalized size = 1.43

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\left(b^3e+3\,ab^2h\right)x^{11}+\frac{1}{10}\left(b^3d+3\,ab^2g\right)x^{10}+\frac{1}{9}\left(b^3c+3\,ab^2f\right)x^9+\frac{3}{8}\left(ab^2e+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

Fricas [A] time = 0.997019, size = 497, normalized size = 2.48

$$\frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\left(b^3e+3\,ab^2h\right)x^{11}+\frac{1}{10}\left(b^3d+3\,ab^2g\right)x^{10}+\frac{1}{9}\left(b^3c+3\,ab^2f\right)x^9+\frac{3}{8}\left(ab^2e+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="frica
s")
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^2
*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 + 3/
8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c + a^
2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/4*(3
*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

Sympy [A] time = 0.604787, size = 240, normalized size = 1.2

$$a^3c\log(x) + a^3dx + \frac{a^3ex^2}{2} + \frac{b^3fx^{12}}{12} + \frac{b^3gx^{13}}{13} + \frac{b^3hx^{14}}{14} + x^{11}\left(\frac{3ab^2h}{11} + \frac{b^3e}{11}\right) + x^{10}\left(\frac{3ab^2g}{10} + \frac{b^3d}{10}\right) + x^9\left(\frac{ab^2f}{3} + \frac{b^3c}{9}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3*g*x**13/1
3 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11) + x**10*(3*a*b**2*g
/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/9) + x**8*(3*a**2*b*h/8 + 3*a*
b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b**2*
c/2) + x**5*(a**3*h/5 + 3*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4) + x*
*3*(a**3*f/3 + a**2*b*c)
```

Giac [A] time = 1.06867, size = 308, normalized size = 1.54

$$\frac{1}{14}b^3hx^{14} + \frac{1}{13}b^3gx^{13} + \frac{1}{12}b^3fx^{12} + \frac{3}{11}ab^2hx^{11} + \frac{1}{11}b^3x^{11}e + \frac{1}{10}b^3dx^{10} + \frac{3}{10}ab^2gx^{10} + \frac{1}{9}b^3cx^9 + \frac{1}{3}ab^2fx^9 + \frac{3}{8}a^2bhx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac"
)
```

```
[Out] 1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 3/11*a*b^2*h*x^11 + 1
/11*b^3*x^11*e + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 + 1/9*b^3*c*x^9 + 1/3*
a*b^2*f*x^9 + 3/8*a^2*b*h*x^8 + 3/8*a*b^2*x^8*e + 3/7*a*b^2*d*x^7 + 3/7*a^2
*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^2*b*f*x^6 + 1/5*a^3*h*x^5 + 3/5*a^2*b*x^
5*e + 3/4*a^2*b*d*x^4 + 1/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 + 1/2*a
^3*x^2*e + a^3*d*x + a^3*c*log(abs(x))
```

3.399 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

Optimal. Leaf size=198

$$\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3a$$

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3 + (a^2*(3*
b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b*(b
*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8 + (b^3*d*x^9)/9 + (b^2*(b*e +
3*a*h)*x^10)/10 + (b^3*f*x^11)/11 + (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12
*b) + a^3*d*Log[x]
```

Rubi [A] time = 0.182106, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] -((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3 + (a^2*(3*
b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b*(b
*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8 + (b^3*d*x^9)/9 + (b^2*(b*e +
3*a*h)*x^10)/10 + (b^3*f*x^11)/11 + (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)/(12
*b) + a^3*d*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx=\frac{g(a+bx^3)^4}{12b}+\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+hx^5)}{x^2}\,dx$$

$$=\frac{g(a+bx^3)^4}{12b}+\int \left(a^3e+\frac{a^3c}{x^2}+\frac{a^3d}{x}+a^2(3bc+af)x+3a^2bdx^2\right.$$

$$=-\frac{a^3c}{x}+a^3ex+\frac{1}{2}a^2(3bc+af)x^2+a^2bdx^3+\frac{1}{4}a^2(3be+ah)x^4+$$

Mathematica [A] time = 0.136937, size = 172, normalized size = 0.87

$$\frac{1}{140}a^2bx^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)+a^3\left(-\frac{c}{x}+ex+\frac{1}{12}x^2\left(6f+4gx+3hx^2\right)\right)+a^3d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

```
[Out] a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*(6435*c +
5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x^3)))/51480 + (a^2*b
*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/140 +
(a*b^2*x^5*(504*c + x*(420*d + x*(360*e + 315*f*x + 280*g*x^2 + 252*h*x^3))
))/840 + a^3*d*Log[x]
```

Maple [A] time = 0.007, size = 224, normalized size = 1.1

$$\frac{b^3hx^{13}}{13}+\frac{b^3gx^{12}}{12}+\frac{b^3fx^{11}}{11}+\frac{3x^{10}ab^2h}{10}+\frac{b^3ex^{10}}{10}+\frac{x^9ab^2g}{3}+\frac{b^3dx^9}{9}+\frac{3x^8ab^2f}{8}+\frac{b^3cx^8}{8}+\frac{3x^7a^2bh}{7}+\frac{3ab^2ex^7}{7}+\frac{x^6a^2b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 1/13*b^3*h*x^13+1/12*b^3*g*x^12+1/11*b^3*f*x^11+3/10*x^10*a*b^2*h+1/10*b^3*
e*x^10+1/3*x^9*a*b^2*g+1/9*b^3*d*x^9+3/8*x^8*a*b^2*f+1/8*b^3*c*x^8+3/7*x^7*
a^2*b*h+3/7*a*b^2*e*x^7+1/2*x^6*a^2*b*g+1/2*a*b^2*d*x^6+3/5*x^5*a^2*b*f+3/5
*a*b^2*c*x^5+1/4*x^4*a^3*h+3/4*a^2*b*e*x^4+1/3*x^3*a^3*g+a^2*b*d*x^3+1/2*a^
3*f*x^2+3/2*a^2*b*c*x^2+a^3*e*x+a^3*d*ln(x)-a^3*c/x
```

Maxima [A] time = 0.945867, size = 286, normalized size = 1.44

$$\frac{1}{13}b^3hx^{13}+\frac{1}{12}b^3gx^{12}+\frac{1}{11}b^3fx^{11}+\frac{1}{10}\left(b^3e+3ab^2h\right)x^{10}+\frac{1}{9}\left(b^3d+3ab^2g\right)x^9+\frac{1}{8}\left(b^3c+3ab^2f\right)x^8+\frac{3}{7}\left(ab^2e+a^2bh\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="max
ima")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*(b^3*e + 3*a*b^2
*h)*x^10 + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3*a*b^2*f)*x^8 + 3/7*
(a*b^2*e + a^2*b*h)*x^7 + 1/2*(a*b^2*d + a^2*b*g)*x^6 + 3/5*(a*b^2*c + a^2*
b*f)*x^5 + a^3*e*x + 1/4*(3*a^2*b*e + a^3*h)*x^4 + a^3*d*log(x) + 1/3*(3*a^
2*b*d + a^3*g)*x^3 - a^3*c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2
```

Fricas [A] time = 1.02106, size = 570, normalized size = 2.88

$$\frac{27720\,b^3hx^{14}+30030\,b^3gx^{13}+32760\,b^3fx^{12}+36036\left(b^3e+3\,ab^2h\right)x^{11}+40040\left(b^3d+3\,ab^2g\right)x^{10}+45045\left(b^3c+3\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/360360*(27720*b^3*h*x^14 + 30030*b^3*g*x^13 + 32760*b^3*f*x^12 + 36036*(b
^3*e + 3*a*b^2*h)*x^11 + 40040*(b^3*d + 3*a*b^2*g)*x^10 + 45045*(b^3*c + 3*
a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8 + 180180*(a*b^2*d + a^2*b*g)*
x^7 + 216216*(a*b^2*c + a^2*b*f)*x^6 + 360360*a^3*e*x^2 + 90090*(3*a^2*b*e
+ a^3*h)*x^5 + 360360*a^3*d*x*log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4 - 360
360*a^3*c + 180180*(3*a^2*b*c + a^3*f)*x^3)/x
```

Sympy [A] time = 0.633058, size = 236, normalized size = 1.19

$$-\frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{b^3fx^{11}}{11} + \frac{b^3gx^{12}}{12} + \frac{b^3hx^{13}}{13} + x^{10}\left(\frac{3ab^2h}{10} + \frac{b^3e}{10}\right) + x^9\left(\frac{ab^2g}{3} + \frac{b^3d}{9}\right) + x^8\left(\frac{3ab^2f}{8} + \frac{b^3c}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] -a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x**12/12 +
b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x**9*(a*b**2*g/3 + b*
*3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x**7*(3*a**2*b*h/7 + 3*a*b**2*e/
7) + x**6*(a**2*b*g/2 + a*b**2*d/2) + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5) +
x**4*(a**3*h/4 + 3*a**2*b*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a**3*f/
2 + 3*a**2*b*c/2)
```

Giac [A] time = 1.06084, size = 308, normalized size = 1.56

$$\frac{1}{13}b^3hx^{13} + \frac{1}{12}b^3gx^{12} + \frac{1}{11}b^3fx^{11} + \frac{3}{10}ab^2hx^{10} + \frac{1}{10}b^3x^{10}e + \frac{1}{9}b^3dx^9 + \frac{1}{3}ab^2gx^9 + \frac{1}{8}b^3cx^8 + \frac{3}{8}ab^2fx^8 + \frac{3}{7}a^2bhx^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 3/10*a*b^2*h*x^10 + 1
/10*b^3*x^10*e + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*b^3*c*x^8 + 3/8*a*b^
2*f*x^8 + 3/7*a^2*b*h*x^7 + 3/7*a*b^2*x^7*e + 1/2*a*b^2*d*x^6 + 1/2*a^2*b*g
*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*f*x^5 + 1/4*a^3*h*x^4 + 3/4*a^2*b*x^4*e
+ a^2*b*d*x^3 + 1/3*a^3*g*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*x*e +
 a^3*d*log(abs(x)) - a^3*c/x
```

3.400 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

Optimal. Leaf size=198

$$a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+$$

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g)*x^2
)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*(b*d + a*g)*x^5)/5 +
 (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7 + (b^2*(b*d + 3*a*g)*x^8)/8 +
(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b
) + a^3*e*Log[x]
```

Rubi [A] time = 0.196774, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1583, 1820}

$$a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] -(a^3*c)/(2*x^2) - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g)*x^2
)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*(b*d + a*g)*x^5)/5 +
 (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7 + (b^2*(b*d + 3*a*g)*x^8)/8 +
(b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(12*b
) + a^3*e*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx &= \frac{h(a+bx^3)^4}{12b}+\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx\\ &= \frac{h(a+bx^3)^4}{12b}+\int \left(a^2(3bc+af)+\frac{a^3c}{x^3}+\frac{a^3d}{x^2}+\frac{a^3e}{x}+a^2(3bd+ag)\right.\\ &= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^2(3bc+af)x+\frac{1}{2}a^2(3bd+ag)x^2+a^2bex^3+\frac{3}{4}ab(bc\end{aligned}$$

Mathematica [A] time = 0.117362, size = 174, normalized size = 0.88

$$\frac{1}{20}a^2bx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right)+\frac{a^3\left(-3c-6dx+x^3\left(6f+3gx+2hx^2\right)\right)}{6x^2}+a^3e\log(x)+\frac{}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

```
[Out] (a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3*x^7*(3960
*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))))/27720 + (a^
2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/20 + (a*
b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/84
0 + a^3*e*Log[x]
```

Maple [A] time = 0.008, size = 222, normalized size = 1.1

$$\frac{b^3hx^{12}}{12}+\frac{b^3gx^{11}}{11}+\frac{b^3fx^{10}}{10}+\frac{x^9ab^2h}{3}+\frac{b^3ex^9}{9}+\frac{3x^8ab^2g}{8}+\frac{b^3dx^8}{8}+\frac{3x^7ab^2f}{7}+\frac{b^3cx^7}{7}+\frac{x^6a^2bh}{2}+\frac{ab^2ex^6}{2}+\frac{3x^5a^2b}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] 1/12*b^3*h*x^12+1/11*b^3*g*x^11+1/10*b^3*f*x^10+1/3*x^9*a*b^2*h+1/9*b^3*e*x
^9+3/8*x^8*a*b^2*g+1/8*b^3*d*x^8+3/7*x^7*a*b^2*f+1/7*b^3*c*x^7+1/2*x^6*a^2*
b*h+1/2*a*b^2*e*x^6+3/5*x^5*a^2*b*g+3/5*a*b^2*d*x^5+3/4*x^4*a^2*b*f+3/4*a*b
^2*c*x^4+1/3*x^3*a^3*h+a^2*b*e*x^3+1/2*x^2*a^3*g+3/2*a^2*b*d*x^2+a^3*f*x+3*
a^2*b*c*x+a^3*e*ln(x)-1/2*a^3*c/x^2-a^3*d/x
```

Maxima [A] time = 0.951091, size = 286, normalized size = 1.44

$$\frac{1}{12}b^3hx^{12}+\frac{1}{11}b^3gx^{11}+\frac{1}{10}b^3fx^{10}+\frac{1}{9}\left(b^3e+3ab^2h\right)x^9+\frac{1}{8}\left(b^3d+3ab^2g\right)x^8+\frac{1}{7}\left(b^3c+3ab^2f\right)x^7+\frac{1}{2}\left(ab^2e+a^2bh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(b^3*e + 3*a*b^2*
h)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a*b^2*f)*x^7 + 1/2*(a
*b^2*e + a^2*b*h)*x^6 + 3/5*(a*b^2*d + a^2*b*g)*x^5 + 3/4*(a*b^2*c + a^2*b*
f)*x^4 + a^3*e*log(x) + 1/3*(3*a^2*b*e + a^3*h)*x^3 + 1/2*(3*a^2*b*d + a^3*
g)*x^2 + (3*a^2*b*c + a^3*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

Fricas [A] time = 1.01098, size = 551, normalized size = 2.78

$$\frac{2310b^3hx^{14}+2520b^3gx^{13}+2772b^3fx^{12}+3080\left(b^3e+3ab^2h\right)x^{11}+3465\left(b^3d+3ab^2g\right)x^{10}+3960\left(b^3c+3ab^2f\right)x^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 3080*(b^3*e
+ 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 3960*(b^3*c + 3*a*b^2*f
)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632*(a*b^2*d + a^2*b*g)*x^7 + 207
90*(a*b^2*c + a^2*b*f)*x^6 + 27720*a^3*e*x^2*log(x) + 9240*(3*a^2*b*e + a^3
*h)*x^5 - 27720*a^3*d*x + 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c + 277
20*(3*a^2*b*c + a^3*f)*x^3)/x^2
```

Sympy [A] time = 0.716463, size = 236, normalized size = 1.19

$$a^3e\log(x) + \frac{b^3fx^{10}}{10} + \frac{b^3gx^{11}}{11} + \frac{b^3hx^{12}}{12} + x^9\left(\frac{ab^2h}{3} + \frac{b^3e}{9}\right) + x^8\left(\frac{3ab^2g}{8} + \frac{b^3d}{8}\right) + x^7\left(\frac{3ab^2f}{7} + \frac{b^3c}{7}\right) + x^6\left(\frac{a^2bh}{2} + \frac{ab^2e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/12 + x**9*
(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8) + x**7*(3*a*b**2*f
/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/2) + x**5*(3*a**2*b*g/5 + 3*a*
b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*a*b**2*c/4) + x**3*(a**3*h/3 + a**2*b*e)
 + x**2*(a**3*g/2 + 3*a**2*b*d/2) + x*(a**3*f + 3*a**2*b*c) - (a**3*c + 2*a
**3*d*x)/(2*x**2)
```

Giac [A] time = 1.06294, size = 305, normalized size = 1.54

$$\frac{1}{12}b^3hx^{12} + \frac{1}{11}b^3gx^{11} + \frac{1}{10}b^3fx^{10} + \frac{1}{3}ab^2hx^9 + \frac{1}{9}b^3x^9e + \frac{1}{8}b^3dx^8 + \frac{3}{8}ab^2gx^8 + \frac{1}{7}b^3cx^7 + \frac{3}{7}ab^2fx^7 + \frac{1}{2}a^2bhx^6 + \frac{1}{2}al$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/3*a*b^2*h*x^9 + 1/9
*b^3*x^9*e + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*c*x^7 + 3/7*a*b^2*f*
x^7 + 1/2*a^2*b*h*x^6 + 1/2*a*b^2*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*b*g*x^5
 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^4 + 1/3*a^3*h*x^3 + a^2*b*x^3*e + 3/2*a^
2*b*d*x^2 + 1/2*a^3*g*x^2 + 3*a^2*b*c*x + a^3*f*x + a^3*e*log(abs(x)) - 1/2
*(2*a^3*d*x + a^3*c)/x^2
```

3.401 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

Optimal. Leaf size=209

$$a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}$$

```
[Out] -(a^3*c)/(3*x^3) - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g)*x + (a^2
*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b*d + a*g)*x^4)/4 + (
3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)*x^6)/6 + (b^2*(b*d + 3*a*g)*x
^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*
h*x^11)/11 + a^2*(3*b*c + a*f)*Log[x]
```

Rubi [A] time = 0.179548, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^3*c)/(3*x^3) - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g)*x + (a^2
*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b*d + a*g)*x^4)/4 + (
3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)*x^6)/6 + (b^2*(b*d + 3*a*g)*x
^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*
h*x^11)/11 + a^2*(3*b*c + a*f)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx &= \int \left(a^2(3bd+ag)+\frac{a^3c}{x^4}+\frac{a^3d}{x^3}+\frac{a^3e}{x^2}+\frac{a^2(3bc+af)}{x}+a^2(3be+\right. \\ &= -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x^2+ab(bc+\end{aligned}$$

Mathematica [A] time = 0.126123, size = 172, normalized size = 0.82

$$a^2\log(x)(af+3bc)+\frac{1}{20}a^2bx\left(60d+x\left(30e+x\left(20f+15gx+12hx^2\right)\right)\right)-\frac{a^3\left(2c+3x\left(d+2ex+x^3(-(2g+hx))\right)\right)}{6x^3}+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

```
[Out] -(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/(6*x^3) + (a^2*b*x*(60*d +
 x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/20 + (a*b^2*x^3*(280*c + x*(210*
d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/280 + (b^3*x^6*(4620*c +
 x*(3960*d + 7*x*(495*e + 4*x*(110*f + 99*g*x + 90*h*x^2)))))/27720 + a^2*(
3*b*c + a*f)*Log[x]
```

Maple [A] time = 0.006, size = 220, normalized size = 1.1

$$\frac{b^3hx^{11}}{11}+\frac{b^3gx^{10}}{10}+\frac{b^3fx^9}{9}+\frac{3x^8ab^2h}{8}+\frac{x^8b^3e}{8}+\frac{3x^7ab^2g}{7}+\frac{x^7b^3d}{7}+\frac{x^6ab^2f}{2}+\frac{x^6b^3c}{6}+\frac{3x^5a^2bh}{5}+\frac{3x^5ab^2e}{5}+\frac{3x^4a^2bg}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] 1/11*b^3*h*x^11+1/10*b^3*g*x^10+1/9*b^3*f*x^9+3/8*x^8*a*b^2*h+1/8*x^8*b^3*e
+3/7*x^7*a*b^2*g+1/7*x^7*b^3*d+1/2*x^6*a*b^2*f+1/6*x^6*b^3*c+3/5*x^5*a^2*b*
h+3/5*x^5*a*b^2*e+3/4*x^4*a^2*b*g+3/4*a*b^2*d*x^4+x^3*a^2*b*f+x^3*a*b^2*c+1
/2*x^2*a^3*h+3/2*a^2*b*e*x^2+a^3*g*x+3*a^2*b*d*x+ln(x)*a^3*f+3*ln(x)*a^2*b*
c-1/3*a^3*c/x^3-1/2*a^3*d/x^2-a^3*e/x
```

Maxima [A] time = 0.946772, size = 286, normalized size = 1.37

$$\frac{1}{11}b^3hx^{11}+\frac{1}{10}b^3gx^{10}+\frac{1}{9}b^3fx^9+\frac{1}{8}\left(b^3e+3ab^2h\right)x^8+\frac{1}{7}\left(b^3d+3ab^2g\right)x^7+\frac{1}{6}\left(b^3c+3ab^2f\right)x^6+\frac{3}{5}\left(ab^2e+a^2bh\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(b^3*e + 3*a*b^2*h)
*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b^2*f)*x^6 + 3/5*(a*b
^2*e + a^2*b*h)*x^5 + 3/4*(a*b^2*d + a^2*b*g)*x^4 + (a*b^2*c + a^2*b*f)*x^3
 + 1/2*(3*a^2*b*e + a^3*h)*x^2 + (3*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a^3*f
)*log(x) - 1/6*(6*a^3*e*x^2 + 3*a^3*d*x + 2*a^3*c)/x^3
```

Fricas [A] time = 1.00583, size = 551, normalized size = 2.64

$$\frac{2520\,b^3hx^{14}+2772\,b^3gx^{13}+3080\,b^3fx^{12}+3465\left(b^3e+3\,ab^2h\right)x^{11}+3960\left(b^3d+3\,ab^2g\right)x^{10}+4620\left(b^3c+3\,ab^2f\right)x^9+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 3465*(b^3*e
+ 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 4620*(b^3*c + 3*a*b^2*f
)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790*(a*b^2*d + a^2*b*g)*x^7 + 277
20*(a*b^2*c + a^2*b*f)*x^6 - 27720*a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h)*x^
5 - 13860*a^3*d*x + 27720*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c + a^3*
```

```
f)*x^3*log(x) - 9240*a^3*c)/x^3
```

Sympy [A] time = 1.06122, size = 235, normalized size = 1.12

$$a^2\left(af+3bc\right)\log\left(x\right)+\frac{b^3fx^9}{9}+\frac{b^3gx^{10}}{10}+\frac{b^3hx^{11}}{11}+x^8\left(\frac{3ab^2h}{8}+\frac{b^3e}{8}\right)+x^7\left(\frac{3ab^2g}{7}+\frac{b^3d}{7}\right)+x^6\left(\frac{ab^2f}{2}+\frac{b^3c}{6}\right)+x^5\left(\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**3*h*x**11/
11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/7 + b**3*d/7) + x**6
*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5 + 3*a*b**2*e/5) + x**4*(3*a**
2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**2*b*f + a*b**2*c) + x**2*(a**3*h/2 + 3*a
**2*b*e/2) + x*(a**3*g + 3*a**2*b*d) - (2*a**3*c + 3*a**3*d*x + 6*a**3*e*x*
*2)/(6*x**3)
```

Giac [A] time = 1.06246, size = 304, normalized size = 1.45

$$\frac{1}{11}b^3hx^{11}+\frac{1}{10}b^3gx^{10}+\frac{1}{9}b^3fx^9+\frac{3}{8}ab^2hx^8+\frac{1}{8}b^3x^8e+\frac{1}{7}b^3dx^7+\frac{3}{7}ab^2gx^7+\frac{1}{6}b^3cx^6+\frac{1}{2}ab^2fx^6+\frac{3}{5}a^2bhx^5+\frac{3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 3/8*a*b^2*h*x^8 + 1/8*b
^3*x^8*e + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*x^6 + 1/2*a*b^2*f*x^
6 + 3/5*a^2*b*h*x^5 + 3/5*a*b^2*x^5*e + 3/4*a*b^2*d*x^4 + 3/4*a^2*b*g*x^4 +
 a*b^2*c*x^3 + a^2*b*f*x^3 + 1/2*a^3*h*x^2 + 3/2*a^2*b*x^2*e + 3*a^2*b*d*x
+ a^3*g*x + (3*a^2*b*c + a^3*f)*log(abs(x)) - 1/6*(6*a^3*x^2*e + 3*a^3*d*x
+ 2*a^3*c)/x^3
```

3.402 $\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

Optimal. Leaf size=209

$$-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7$$

```
[Out] -(a^3*c)/(4*x^4) - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*c + a*f))/
x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a*b*(b*d + a*g)*x^3 +
 (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*f)*x^5)/5 + (b^2*(b*d + 3*a*g)
*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*
h*x^10)/10 + a^2*(3*b*d + a*g)*Log[x]
```

Rubi [A] time = 0.176742, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1820}

$$-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] -(a^3*c)/(4*x^4) - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*c + a*f))/
x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a*b*(b*d + a*g)*x^3 +
 (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*f)*x^5)/5 + (b^2*(b*d + 3*a*g)
*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*
h*x^10)/10 + a^2*(3*b*d + a*g)*Log[x]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = \int \left(a^2(3be+ah)+\frac{a^3c}{x^5}+\frac{a^3d}{x^4}+\frac{a^3e}{x^3}+\frac{a^2(3bc+af)}{x^2}+\frac{a^2(3bd+ag}{x}\right.$$
$$= -\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\frac{3}{2}ab(bc+af)$$

Mathematica [A] time = 0.120628, size = 170, normalized size = 0.81

$$\frac{630a^2bx^3\left(x^2\left(12e+6fx+4gx^2+3hx^3\right)-12c\right)-210a^3\left(3c+4dx+6x^2\left(e+2fx-2hx^3\right)\right)+18ab^2x^6\left(210c+x\left(140d+\right.\right.}{2520x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

```
[Out] (-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b*x^3*(-12*
c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*x^6*(210*c + x*(140*
d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)) + b^3*x^9*(504*c + x*(420*d
+ 360*e*x + 315*f*x^2 + 280*g*x^3 + 252*h*x^4)))/(2520*x^4) + a^2*(3*b*d +
a*g)*Log[x]
```

Maple [A] time = 0.007, size = 220, normalized size = 1.1

$$\frac{b^3hx^{10}}{10}+\frac{b^3gx^9}{9}+\frac{b^3fx^8}{8}+\frac{3x^7ab^2h}{7}+\frac{x^7b^3e}{7}+\frac{x^6ab^2g}{2}+\frac{x^6b^3d}{6}+\frac{3x^5ab^2f}{5}+\frac{x^5b^3c}{5}+\frac{3x^4a^2bh}{4}+\frac{3x^4ab^2e}{4}+x^3a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] 1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*x^7*a*b^2*h+1/7*x^7*b^3*e+1
/2*x^6*a*b^2*g+1/6*x^6*b^3*d+3/5*x^5*a*b^2*f+1/5*x^5*b^3*c+3/4*x^4*a^2*b*h+
3/4*x^4*a*b^2*e+x^3*a^2*b*g+a*b^2*d*x^3+3/2*x^2*a^2*b*f+3/2*x^2*a*b^2*c+a^3
*h*x+3*a^2*b*e*x+ln(x)*a^3*g+3*ln(x)*a^2*b*d-1/3*a^3*d/x^3-1/4*a^3*c/x^4-1/
2*a^3*e/x^2-a^3/x*f-3*a^2/x*b*c
```

Maxima [A] time = 0.947768, size = 286, normalized size = 1.37

$$\frac{1}{10}b^3hx^{10}+\frac{1}{9}b^3gx^9+\frac{1}{8}b^3fx^8+\frac{1}{7}\left(b^3e+3ab^2h\right)x^7+\frac{1}{6}\left(b^3d+3ab^2g\right)x^6+\frac{1}{5}\left(b^3c+3ab^2f\right)x^5+\frac{3}{4}\left(ab^2e+a^2bh\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(b^3*e + 3*a*b^2*h)*x
^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2*f)*x^5 + 3/4*(a*b^2
*e + a^2*b*h)*x^4 + (a*b^2*d + a^2*b*g)*x^3 + 3/2*(a*b^2*c + a^2*b*f)*x^2 +
 (3*a^2*b*e + a^3*h)*x + (3*a^2*b*d + a^3*g)*log(x) - 1/12*(6*a^3*e*x^2 + 4
*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

Fricas [A] time = 0.97709, size = 528, normalized size = 2.53

$$\frac{252b^3hx^{14}+280b^3gx^{13}+315b^3fx^{12}+360\left(b^3e+3ab^2h\right)x^{11}+420\left(b^3d+3ab^2g\right)x^{10}+504\left(b^3c+3ab^2f\right)x^9+1890}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b^3*e + 3*a
*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*c + 3*a*b^2*f)*x^9 +
 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d + a^2*b*g)*x^7 + 3780*(a*b^2*
c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 + 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520*(3*
a^2*b*d + a^3*g)*x^4*log(x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*c + a
```

```
^3*f)*x^3)/x^4
```

Sympy [A] time = 2.97899, size = 233, normalized size = 1.11

$$a^2\left(ag+3bd\right)\log\left(x\right)+\frac{b^3fx^8}{8}+\frac{b^3gx^9}{9}+\frac{b^3hx^{10}}{10}+x^7\left(\frac{3ab^2h}{7}+\frac{b^3e}{7}\right)+x^6\left(\frac{ab^2g}{2}+\frac{b^3d}{6}\right)+x^5\left(\frac{3ab^2f}{5}+\frac{b^3c}{5}\right)+x^4\left(\frac{3a^2b}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*h*x**10/10
 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 + b**3*d/6) + x**5*(3*
a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 + 3*a*b**2*e/4) + x**3*(a**2*b*
g + a*b**2*d) + x**2*(3*a**2*b*f/2 + 3*a*b**2*c/2) + x*(a**3*h + 3*a**2*b*e
) - (3*a**3*c + 4*a**3*d*x + 6*a**3*e*x**2 + x**3*(12*a**3*f + 36*a**2*b*c)
)/(12*x**4)
```

Giac [A] time = 1.07367, size = 302, normalized size = 1.44

$$\frac{1}{10}b^3hx^{10}+\frac{1}{9}b^3gx^9+\frac{1}{8}b^3fx^8+\frac{3}{7}ab^2hx^7+\frac{1}{7}b^3x^7e+\frac{1}{6}b^3dx^6+\frac{1}{2}ab^2gx^6+\frac{1}{5}b^3cx^5+\frac{3}{5}ab^2fx^5+\frac{3}{4}a^2bhx^4+\frac{3}{4}ab^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 3/7*a*b^2*h*x^7 + 1/7*b^3
*x^7*e + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^5 + 3/5*a*b^2*f*x^5
+ 3/4*a^2*b*h*x^4 + 3/4*a*b^2*x^4*e + a*b^2*d*x^3 + a^2*b*g*x^3 + 3/2*a*b^2
*c*x^2 + 3/2*a^2*b*f*x^2 + a^3*h*x + 3*a^2*b*x*e + (3*a^2*b*d + a^3*g)*log(
abs(x)) - 1/12*(6*a^3*x^2*e + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*
x^3)/x^4
```

3.403 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}$$

```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)
*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b
^(10/3)) - (a^(2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log[a +
b*x^3])/(3*b^3)
```

Rubi [A] time = 1.06967, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] -((a*(b*e - a*h)*x)/b^3) + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(10/3)) + (a^(2/3)
*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b
^(10/3)) - (a^(2/3)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) - (a*(b*d - a*g)*Log[a +
b*x^3])/(3*b^3)
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^7}{7b}+\frac{\int \frac{x^4(7bc+7bdx+7(be-ah)x^2+7bfx^3+7bgx^4)}{a+bx^3}dx}{7b}$$

$$= \frac{gx^6}{6b}+\frac{hx^7}{7b}+\frac{\int \frac{x^4(42b^2c+42b(bd-ag)x+42b(be-ah)x^2+42b^2fx^3)}{a+bx^3}dx}{42b^2}$$

$$= \frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}+\frac{\int \frac{x^4(210b^2(bc-af)+210b^2(bd-ag)x+210b^2(be-ah)x^2)}{a+bx^3}dx}{210b^3}$$

$$= \frac{fx^5}{5b}+\frac{gx^6}{6b}+\frac{hx^7}{7b}+\frac{\int \Big(-210a(be-ah)+210b(bc-af)x+210b(bd-a}{}$$

$$= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+$$

$$= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+$$

$$= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+$$

$$= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+$$

$$= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+$$

$$= -\frac{a(be-ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}+$$

Mathematica [A] time = 0.438062, size = 334, normalized size = 1.01

$$\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{6b^{10/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+\right.}{3b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(3
*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^7)
/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(b^(5
/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/(3
*b^(10/3)) + (a^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) + (a*(-(b*d)
 + a*g)*Log[a + b*x^3])/(3*b^3)
```

Maple [B] time = 0.006, size = 533, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3/b*d*x^3+1/6*g*x^6/b+1/7*h*x^7/b-1/3*a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/4/b^2*x^4*a*h+1/3*a^2/b^3*ln(b*x^3+a
)*g+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^2*e-1/6/b^3/(1/b*a)^(2/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^2*e-1/3*a^3/b^4/(1/b*a)^(2/3)*ln(x+(
1/b*a)^(1/3))*h+1/6*a^3/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2
/3))*h-1/3/b^3*a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+1/2/b*x^2*c+1/6/b^3*
a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/4/b*x^4*e-1/6*a
/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*a/b^2/(1/b*a
)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/3/b^2*ln(b*x^3+a)*a*d-1/2/b^2*x^2*a*f-1/3/b
^2*x^3*a*g+1/b^3*a^2*h*x+1/5*f*x^5/b-1/b^2*a*e*x-1/3*a^3/b^4/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/3/b^3/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*e+1/3/b^3*a^2*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [B] time = 29.1659, size = 874, normalized size = 2.64

$$\mathrm{RootSum}\Big(27t^3b^{10} + t^2\left(-27a^2b^7g + 27ab^8d\right) + t\left(-9a^4b^4fh + 9a^4b^4g^2 + 9a^3b^5ch - 18a^3b^5dg + 9a^3b^5ef - 9a^2b^6ce + 9a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**10 + _t**2*(-27*a**2*b**7*g + 27*a*b**8*d) + _t*(-9*a**
4*b**4*f*h + 9*a**4*b**4*g**2 + 9*a**3*b**5*c*h - 18*a**3*b**5*d*g + 9*a**3
```

```
*b**5*e*f - 9*a**2*b**6*c*e + 9*a**2*b**6*d**2) + a**7*h**3 - 3*a**6*b*e*h*
*2 + 3*a**6*b*f*g*h - a**6*b*g**3 - 3*a**5*b**2*c*g*h - 3*a**5*b**2*d*f*h +
 3*a**5*b**2*d*g**2 + 3*a**5*b**2*e**2*h - 3*a**5*b**2*e*f*g + a**5*b**2*f*
*3 + 3*a**4*b**3*c*d*h + 3*a**4*b**3*c*e*g - 3*a**4*b**3*c*f**2 - 3*a**4*b*
*3*d**2*g + 3*a**4*b**3*d*e*f - a**4*b**3*e**3 + 3*a**3*b**4*c**2*f - 3*a**
3*b**4*c*d*e + a**3*b**4*d**3 - a**2*b**5*c**3, Lambda(_t, _t*log(x + (-9*_
t**2*a*b**7*f + 9*_t**2*b**8*c - 3*_t*a**4*b**3*h**2 + 6*_t*a**3*b**4*e*h +
 6*_t*a**3*b**4*f*g - 6*_t*a**2*b**5*c*g - 6*_t*a**2*b**5*d*f - 3*_t*a**2*b
**5*e**2 + 6*_t*a*b**6*c*d + a**6*g*h**2 - a**5*b*d*h**2 - 2*a**5*b*e*g*h +
 2*a**5*b*f**2*h - a**5*b*f*g**2 - 4*a**4*b**2*c*f*h + a**4*b**2*c*g**2 + 2
*a**4*b**2*d*e*h + 2*a**4*b**2*d*f*g + a**4*b**2*e**2*g - 2*a**4*b**2*e*f**
2 + 2*a**3*b**3*c**2*h - 2*a**3*b**3*c*d*g + 4*a**3*b**3*c*e*f - a**3*b**3*
d**2*f - a**3*b**3*d*e**2 - 2*a**2*b**4*c**2*e + a**2*b**4*c*d**2)/(a**6*h*
*3 - 3*a**5*b*e*h**2 + 3*a**4*b**2*e**2*h - a**4*b**2*f**3 + 3*a**3*b**3*c*
f**2 - a**3*b**3*e**3 - 3*a**2*b**4*c**2*f + a*b**5*c**3)))) + f*x**5/(5*b)
 + g*x**6/(6*b) + h*x**7/(7*b) - x**4*(a*h - b*e)/(4*b**2) - x**3*(a*g - b*
d)/(3*b**2) - x**2*(a*f - b*c)/(2*b**2) + x*(a**2*h - a*b*e)/b**3
```

Giac [A] time = 1.07807, size = 513, normalized size = 1.55

$$-\frac{\left(abd-a^2g\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^3}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe-\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*b*d - a^2*g)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*
a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3
)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70*b^6*
g*x^6 + 84*b^6*f*x^5 - 105*a*b^5*h*x^4 + 105*b^6*x^4*e + 140*b^6*d*x^3 - 14
0*a*b^5*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 + 420*a^2*b^4*h*x - 420*a*b
^5*x*e)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(1/3) + a^3*b^
12*h - a^2*b^13*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^15)
```

3.404 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

Optimal. Leaf size=313

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}b^{8/3}}$$

```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2)
 + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^
(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c -
 a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.988245, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```


```
[Out] ((b*c - a*f)*x)/b^2 + ((b*d - a*g)*x^2)/(2*b^2) + ((b*e - a*h)*x^3)/(3*b^2)
 + (f*x^4)/(4*b) + (g*x^5)/(5*b) + (h*x^6)/(6*b) + (a^(1/3)*(b^(4/3)*c + a^
(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b
*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*c -
 a*f) - a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(8/3)) - (a*(b*e - a*h)*Log[a + b*x^3])/(3*b^3)
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^6}{6b} + \frac{\int \frac{x^3(6bc+6bdx+6(be-ah)x^2+6bfx^3+6bgx^4)}{a+bx^3}dx}{6b}$$

$$= \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \frac{x^3(30b^2c+30b(bd-ag)x+30b(be-ah)x^2+30b^2fx^3)}{a+bx^3}dx}{30b^2}$$

$$= \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \frac{x^3(120b^2(bc-af)+120b^2(bd-ag)x+120b^2(be-ah)x^2)}{a+bx^3}dx}{120b^3}$$

$$= \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\int \left(120b(bc-af)+120b(bd-ag)x+120b(be-ah)x^2\right.}{120b^3}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \int \frac{ab(bc-af)+a}{}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\int \frac{ab(bc-af)+a}{a+b}}{b^3}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{a(be-ah)\log}{3b}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-\right.}{}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-\right.}{}$$

$$= \frac{(bc-af)x}{b^2} + \frac{(bd-ag)x^2}{2b^2} + \frac{(be-ah)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} + \frac{\sqrt[3]{a}\left(b^{4/3}c+\right.}{}$$

Mathematica [A] time = 0.238224, size = 299, normalized size = 0.96

$$\frac{10\sqrt[3]{a}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)-20\sqrt[3]{a}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3 + 15*b^2*
f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3)*b^(1/3)*(-(b^(4/3)
*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f +
 a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1
/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

Maple [B] time = 0.005, size = 505, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/6*h*x^6/b+1/5*g*x^5/b+1/4*f*x^4/b-1/3/b^2*x^3*a*h+1/3*e*x^3/b-1/2/b^2*x^2
*a*g+1/2*d*x^2/b-1/b^2*a*f*x+c*x/b+1/3/b^3*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(
1/3))*f-1/3*a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b^3*a^2/(1/b*a)^(
2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6*a/b^2/(1/b*a)^(2/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b^3*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3*a/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3/b^3*a^2/(1/b*a)^(1/3)*ln(x+(1/b*a
)^(1/3))*g+1/3/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a*d+1/6/b^3*a^2/(1/b*a
)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/6/b^2/(1/b*a)^(1/3)*ln(x^
2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a*d+1/3/b^3*a^2*3^(1/2)/(1/b*a)^(1/3)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g-1/3/b^2*3^(1/2)/(1/b*a)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*d+1/3/b^3*a^2*ln(b*x^3+a)*h-1/3*a*e*
ln(b*x^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [B] time = 29.5259, size = 842, normalized size = 2.69

$$\text{RootSum}\left(27t^3b^9 + t^2\left(-27a^2b^6h + 27ab^7e\right) + t\left(9a^4b^3h^2 - 18a^3b^4eh + 9a^3b^4fg - 9a^2b^5cg - 9a^2b^5df + 9a^2b^5e^2 + 9a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**9 + _t**2*(-27*a**2*b**6*h + 27*a*b**7*e) + _t*(9*a**4*
b**3*h**2 - 18*a**3*b**4*e*h + 9*a**3*b**4*f*g - 9*a**2*b**5*c*g - 9*a**2*b
**5*d*f + 9*a**2*b**5*e**2 + 9*a*b**6*c*d) - a**6*h**3 + 3*a**5*b*e*h**2 -
3*a**5*b*f*g*h + a**5*b*g**3 + 3*a**4*b**2*c*g*h + 3*a**4*b**2*d*f*h - 3*a*
```

```
*4*b**2*d*g**2 - 3*a**4*b**2*e**2*h + 3*a**4*b**2*e*f*g - a**4*b**2*f**3 -
3*a**3*b**3*c*d*h - 3*a**3*b**3*c*e*g + 3*a**3*b**3*c*f**2 + 3*a**3*b**3*d*
*2*g - 3*a**3*b**3*d*e*f + a**3*b**3*e**3 - 3*a**2*b**4*c**2*f + 3*a**2*b**
4*c*d*e - a**2*b**4*d**3 + a*b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a*b*
*6*g - 9*_t**2*b**7*d - 6*_t*a**3*b**3*g*h + 6*_t*a**2*b**4*d*h + 6*_t*a**2
*b**4*e*g + 3*_t*a**2*b**4*f**2 - 6*_t*a*b**5*c*f - 6*_t*a*b**5*d*e + 3*_t*
b**6*c**2 + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**4*b*f**2*h +
2*a**4*b*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a**3*b**2*d*e*
h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a**2*b**3*c**
2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2*f - a**2*b**
3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)/(a**4*b*g**3 - 3*a**3*b**2*d*g*
*2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g + 3*a*b**4*c*
*2*f - a*b**4*d**3 - b**5*c**3)))) + f*x**4/(4*b) + g*x**5/(5*b) + h*x**6/(
6*b) - x**3*(a*h - b*e)/(3*b**2) - x**2*(a*g - b*d)/(2*b**2) - x*(a*f - b*c
)/b**2
```

Giac [A] time = 1.09472, size = 477, normalized size = 1.52

$$\frac{\left(a^2h-abe\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^3}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{\left(\left(-ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(a^2*h - a*b*e)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b
^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)
*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*b^5*g*
x^5 + 15*b^5*f*x^4 - 20*a*b^4*h*x^3 + 20*b^5*x^3*e + 30*b^5*d*x^2 - 30*a*b^
4*g*x^2 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/3) - a^2
*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b^13)
```

3.405 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

Optimal. Leaf size=294

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}c\right.}{\sqrt{3}b^{8/3}}$$

```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(
4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3
)) - (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)
*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.976263, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6b^{8/3}}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}c\right.}{\sqrt{3}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```


```
[Out] ((b*d - a*g)*x)/b^2 + ((b*e - a*h)*x^2)/(2*b^2) + (f*x^3)/(3*b) + (g*x^4)/(
4*b) + (h*x^5)/(5*b) + (a^(1/3)*(b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3
)) - (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(8/3)) + (a^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3)) + ((b*c - a*f)
*Log[a + b*x^3])/(3*b^2)
```


Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx = \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(5bc+5bdx+5(be-ah)x^2+5bfx^3+5bgx^4\right)}{a+bx^3}dx}{5b}$$

$$= \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(20b^2c+20b(bd-ag)x+20b(be-ah)x^2+20b^2fx^3\right)}{a+bx^3}dx}{20b^2}$$

$$= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \frac{x^2\left(60b^2(bc-af)+60b^2(bd-ag)x+60b^2(be-ah)x^2\right)}{a+bx^3}dx}{60b^3}$$

$$= \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\int \left(60b(bd-ag)+60b(be-ah)x - \frac{60\left(ab(bd-ag)+ab(be-\right.}{a+b}\right.}{60b^3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag)+ab(be-ah)x-b^2(b}{a+bx^3}}{b^3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\int \frac{ab(bd-ag)+ab(be-ah)x}{a+bx^3}dx}{b^3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{(bc-af)\log\left(a+bx^3\right)}{3b^2}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(\right.}{3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(\right.}{3}$$

$$= \frac{(bd-ag)x}{b^2} + \frac{(be-ah)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} + \frac{\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{\right.}}{}$$

Mathematica [A] time = 0.26219, size = 290, normalized size = 0.99

$$10\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+20\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be+a\sqrt[3]{b}g-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/3)*f*x^3 +
 15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a
^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) + a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*
Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*
g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(2/3)*
(b*c - a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

Maple [B] time = 0.003, size = 483, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/5*h*x^5/b+1/4*g*x^4/b+1/3*f*x^3/b-1/2/b^2*x^2*a*h+1/2*e*x^2/b-1/b^2*a*g*x
+d*x/b+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^2*g-1/3/b^2/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))*a*d-1/6/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))*a^2*g+1/6/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3)
)*a*d+1/3/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1
))*a^2*g-1/3/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*a*d-1/3/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a^2*h+1/3/b^2*a/(1/b*a)
^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(
1/b*a)^(2/3))*a^2*h-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))*e+1/3/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*a^2*h-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))*e-1/3*a/b^2*ln(b*x^3+a)*f+1/3*c*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [B] time = 63.1816, size = 789, normalized size = 2.68

$$\mathrm{RootSum}\left(27t^3b^8 + t^2\left(27ab^6f - 27b^7c\right) + t\left(9a^3b^3gh - 9a^2b^4dh - 9a^2b^4eg + 9a^2b^4f^2 - 18ab^5cf + 9ab^5de + 9b^6c^2\right) + a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**8 + _t**2*(27*a*b**6*f - 27*b**7*c) + _t*(9*a**3*b**3*g
*h - 9*a**2*b**4*d*h - 9*a**2*b**4*e*g + 9*a**2*b**4*f**2 - 18*a*b**5*c*f +
 9*a*b**5*d*e + 9*b**6*c**2) + a**5*h**3 - 3*a**4*b*e*h**2 + 3*a**4*b*f*g*h
 - a**4*b*g**3 - 3*a**3*b**2*c*g*h - 3*a**3*b**2*d*f*h + 3*a**3*b**2*d*g**2
 + 3*a**3*b**2*e**2*h - 3*a**3*b**2*e*f*g + a**3*b**2*f**3 + 3*a**2*b**3*c*
```

```
d*h + 3*a**2*b**3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*b**3*d**2*g + 3*a**2*
b**3*d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3*a*b**4*c*d*e + a*b**4*d**
3 - b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a*b**5*h - 9*_t**2*b**6*e + 6
*_t*a**2*b**3*f*h + 3*_t*a**2*b**3*g**2 - 6*_t*a*b**4*c*h - 6*_t*a*b**4*d*g
 - 6*_t*a*b**4*e*f + 6*_t*b**5*c*e + 3*_t*b**5*d**2 + 2*a**4*g*h**2 - 2*a**
3*b*d*h**2 - 4*a**3*b*e*g*h + a**3*b*f**2*h + a**3*b*f*g**2 - 2*a**2*b**2*c
*f*h - a**2*b**2*c*g**2 + 4*a**2*b**2*d*e*h - 2*a**2*b**2*d*f*g + 2*a**2*b*
*2*e**2*g - a**2*b**2*e*f**2 + a*b**3*c**2*h + 2*a*b**3*c*d*g + 2*a*b**3*c*
e*f + a*b**3*d**2*f - 2*a*b**3*d*e**2 - b**4*c**2*e - b**4*c*d**2)/(a**4*h*
*3 - 3*a**3*b*e*h**2 + a**3*b*g**3 - 3*a**2*b**2*d*g**2 + 3*a**2*b**2*e**2*
h + 3*a*b**3*d**2*g - a*b**3*e**3 - b**4*d**3)))) + f*x**3/(3*b) + g*x**4/(
4*b) + h*x**5/(5*b) - x**2*(a*h - b*e)/(2*b**2) - x*(a*g - b*d)/b**2
```

Giac [A] time = 1.08309, size = 450, normalized size = 1.53

$$\frac{\left(bc-af\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d - \left(-ab^2\right)^{\frac{1}{3}}abg + \left(-ab^2\right)^{\frac{2}{3}}ah - \left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - ((-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d
 - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*b^2
*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g*x^4
+ 20*b^4*f*x^3 - 30*a*b^3*h*x^2 + 30*b^4*x^2*e + 60*b^4*d*x - 60*a*b^3*g*x)
/b^5 - 1/3*(a^2*b^9*h*(-a/b)^(1/3) - a*b^10*(-a/b)^(1/3)*e - a*b^10*d + a^2
*b^9*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

3.406 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

Optimal. Leaf size=275

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}b^{7/3}}-\tan^{-1}\Big(\ldots$$

```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(
5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) +
 a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(
2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.921478, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1836, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3\sqrt[3]{a}b^{7/3}}-\tan^{-1}\Big(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] ((b*e - a*h)*x)/b^2 + (f*x^2)/(2*b) + (g*x^3)/(3*b) + (h*x^4)/(4*b) - ((b^(
5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(7/3)) - ((b^(2/3)*(b*c - a*f) +
 a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(7/3)) + ((b^(
2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(1/3)*b^(7/3)) + ((b*d - a*g)*Log[a + b*x^3])/(3*b^2)
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{hx^4}{4b} + \frac{\int \frac{x(4bc+4bdx+4(be-ah)x^2+4bfx^3+4bgx^4)}{a+bx^3}dx}{4b}$$
$$= \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(12b^2c+12b(bd-ag)x+12b(be-ah)x^2+12b^2fx^3)}{a+bx^3}dx}{12b^2}$$
$$= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \frac{x(24b^2(bc-af)+24b^2(bd-ag)x+24b^2(be-ah)x^2)}{a+bx^3}dx}{24b^3}$$
$$= \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{\int \left(24b(be-ah) - \frac{24(ab(be-ah)-b^2(bc-af)x-b^2(bd-ag)x^2)}{a+bx^3}\right)dx}{24b^3}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x-b^2(bd-ag)x^2}{a+bx^3}dx}{b^3}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\int \frac{ab(be-ah)-b^2(bc-af)x}{a+bx^3}dx}{b^3} + \frac{(bd-ag)\int \frac{x^2}{a+bx^3}}{b}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} + \frac{(bd-ag)\log(a+bx^3)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}(-\sqrt[3]{a}b^2(bc-af}{}}{}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{2/3}(bc-af)+a^{2/3}(be-ah))\log(\sqrt[3]{a}+\sqrt[3]{}}{3\sqrt[3]{a}b^{7/3}}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{2/3}(bc-af)+a^{2/3}(be-ah))\log(\sqrt[3]{a}+\sqrt[3]{}}{3\sqrt[3]{a}b^{7/3}}$$
$$= \frac{(be-ah)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\sqrt{}}\right.}{\sqrt{3}\sqrt[3]{a}b^{7/3}}$$

Mathematica [A] time = 0.372748, size = 272, normalized size = 0.99

$$\frac{\frac{2\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c)}{\sqrt[3]{a}} + \frac{4\log(\sqrt[3]{a}+\sqrt[3]{b}x)(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c)}{\sqrt[3]{a}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+}{\sqrt[3]{a}}}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3*b^(4/3)*h
*x^4 - (4*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-(b^(5/3)*c) - a^(2/3
)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2*(b^
(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3])/(12*b
^(7/3))
```

Maple [B] time = 0.003, size = 455, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/4*h*x^4/b+1/3*g*x^3/b+1/2*f*x^2/b-1/b^2*a*h*x+e*x/b+1/3/b^3/(1/b*a)^(2/3)
*ln(x+(1/b*a)^(1/3))*a^2*h-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/
6/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^2*h+1/6/b^2*a/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/3/b^3/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*h-1/3/b^2*a/(1/b*a)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/b^2*a/(1/b*a)^
(1/3)*ln(x+(1/b*a)^(1/3))*f-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-1/6/b
^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6/b/(1/b*a)^(1
/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/b^2*a*3^(1/2)/(1/b*a)^(1/3)
*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/3/b*3^(1/2)/(1/b*a)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3/b^2*ln(b*x^3+a)*a*g+1/3*d*ln(
b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [B] time = 33.0156, size = 811, normalized size = 2.95

$$\operatorname{RootSum}\left(27t^3ab^7 + t^2\left(27a^2b^5g - 27ab^6d\right) + t\left(-9a^3b^3fh + 9a^3b^3g^2 + 9a^2b^4ch - 18a^2b^4dg + 9a^2b^4ef - 9ab^5ce + 9a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**7 + _t**2*(27*a**2*b**5*g - 27*a*b**6*d) + _t*(-9*a**
3*b**3*f*h + 9*a**3*b**3*g**2 + 9*a**2*b**4*c*h - 18*a**2*b**4*d*g + 9*a**2
*b**4*e*f - 9*a*b**5*c*e + 9*a*b**5*d**2) - a**5*h**3 + 3*a**4*b*e*h**2 - 3
```

```
*a**4*b*f*g*h + a**4*b*g**3 + 3*a**3*b**2*c*g*h + 3*a**3*b**2*d*f*h - 3*a**
3*b**2*d*g**2 - 3*a**3*b**2*e**2*h + 3*a**3*b**2*e*f*g - a**3*b**2*f**3 - 3
*a**2*b**3*c*d*h - 3*a**2*b**3*c*e*g + 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**
2*g - 3*a**2*b**3*d*e*f + a**2*b**3*e**3 - 3*a*b**4*c**2*f + 3*a*b**4*c*d*e
 - a*b**4*d**3 + b**5*c**3, Lambda(_t, _t*log(x + (-9*_t**2*a**2*b**5*f + 9
*_t**2*a*b**6*c + 3*_t*a**4*b**2*h**2 - 6*_t*a**3*b**3*e*h - 6*_t*a**3*b**3
*f*g + 6*_t*a**2*b**4*c*g + 6*_t*a**2*b**4*d*f + 3*_t*a**2*b**4*e**2 - 6*_t
*a*b**5*c*d + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h + 2*a**4*b*f**2*
h - a**4*b*f*g**2 - 4*a**3*b**2*c*f*h + a**3*b**2*c*g**2 + 2*a**3*b**2*d*e*
h + 2*a**3*b**2*d*f*g + a**3*b**2*e**2*g - 2*a**3*b**2*e*f**2 + 2*a**2*b**3
*c**2*h - 2*a**2*b**3*c*d*g + 4*a**2*b**3*c*e*f - a**2*b**3*d**2*f - a**2*b
**3*d*e**2 - 2*a*b**4*c**2*e + a*b**4*c*d**2)/(a**5*h**3 - 3*a**4*b*e*h**2
+ 3*a**3*b**2*e**2*h - a**3*b**2*f**3 + 3*a**2*b**3*c*f**2 - a**2*b**3*e**3
 - 3*a*b**4*c**2*f + b**5*c**3)))) + f*x**2/(2*b) + g*x**3/(3*b) + h*x**4/(
4*b) - x*(a*h - b*e)/b**2
```

Giac [A] time = 1.08277, size = 428, normalized size = 1.56

$$\frac{(bd-ag)\log\left(|bx^3+a|\right)}{3\,b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h-\left(-ab^2\right)^{\frac{1}{3}}abe-\left(-ab^2\right)^{\frac{2}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{\left(\left(-ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*d - a*g)*log(abs(b*x^3 + a))/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*h
 - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*((-a*b^2)^(1/3)
*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3) + 1/12*(3*b^3*h*x^4 + 4*b^3*
g*x^3 + 6*b^3*f*x^2 - 12*a*b^2*h*x + 12*b^3*x*e)/b^4 - 1/3*(b^9*c*(-a/b)^(1
/3) - a*b^8*f*(-a/b)^(1/3) + a^2*b^7*h - a*b^8*e)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a*b^9)
```

3.407 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$

Optimal. Leaf size=259

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6a^{2/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}}{}$$

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(
1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(
b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(5
/3)) + ((b*e - a*h)*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.373397, antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}\sqrt[3]{}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^(
1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*c - a*f - (a^(1/3)*(b*d - a*g
))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4
/3)) + ((b*e - a*h)*Log[a + b*x^3])/(3*b^2)
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \int \left(\frac{f}{b}+\frac{gx}{b}+\frac{hx^2}{b}+\frac{bc-af+(bd-ag)x+(be-ah)x^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x+(be-ah)x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\int \frac{bc-af+(bd-ag)x}{a+bx^3}\,dx}{b}+\frac{(be-ah)\int \frac{x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bc-af)+\sqrt[3]{a}(bd-ag)\right)+\sqrt[3]{b}\left(-\sqrt[3]{b}(b\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{3a^{2/3}b^{4/3}}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}+\frac{(be-ah)\log\left(a+b\right.}{3b^2}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}+\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right.}{}$$

$$= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}-\frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{\left(bc-af\right.}{}$$

Mathematica [A] time = 0.293717, size = 254, normalized size = 0.98

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f-b^{4/3}\right.}{a^{2/3}}}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

```
[Out] (6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*Sqrt[3]*(-(b^(4/3)*
c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)
*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*c - a^(1/3)*b*d - a*b^(1/
3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) +
 (2*(b*e - a*h)*Log[a + b*x^3])/b^(1/3))/(6*b^(5/3))
```

Maple [B] time = 0.003, size = 429, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*h*x^3/b+1/2*g*x^2/b+f*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f
+1/3*c/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/6*c/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))*f+1/3*c/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x-1))+1/3/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*a*g-1/3*d/b/(1/b*a
)^(1/3)*ln(x+(1/b*a)^(1/3))-1/6/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))*a*g+1/6*d/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3)
)-1/3/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a
*g+1/3*d*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-
1/3/b^2*ln(b*x^3+a)*a*h+1/3*e*ln(b*x^3+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 28.6818, size = 804, normalized size = 3.1

$$\operatorname{RootSum}\left(27t^3a^2b^6 + t^2\left(27a^3b^4h - 27a^2b^5e\right) + t\left(9a^4b^2h^2 - 18a^3b^3eh + 9a^3b^3fg - 9a^2b^4cg - 9a^2b^4df + 9a^2b^4e^2 + 9ab^5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**6 + _t**2*(27*a**3*b**4*h - 27*a**2*b**5*e) + _t*(
9*a**4*b**2*h**2 - 18*a**3*b**3*e*h + 9*a**3*b**3*f*g - 9*a**2*b**4*c*g - 9
*a**2*b**4*d*f + 9*a**2*b**4*e**2 + 9*a*b**5*c*d) + a**5*h**3 - 3*a**4*b*e*
h**2 + 3*a**4*b*f*g*h - a**4*b*g**3 - 3*a**3*b**2*c*g*h - 3*a**3*b**2*d*f*h
 + 3*a**3*b**2*d*g**2 + 3*a**3*b**2*e**2*h - 3*a**3*b**2*e*f*g + a**3*b**2*
f**3 + 3*a**2*b**3*c*d*h + 3*a**2*b**3*c*e*g - 3*a**2*b**3*c*f**2 - 3*a**2*
b**3*d**2*g + 3*a**2*b**3*d*e*f - a**2*b**3*e**3 + 3*a*b**4*c**2*f - 3*a*b*
*4*c*d*e + a*b**4*d**3 - b**5*c**3, Lambda(_t, _t*log(x + (9*_t**2*a**3*b**
4*g - 9*_t**2*a**2*b**5*d + 6*_t*a**4*b**2*g*h - 6*_t*a**3*b**3*d*h - 6*_t*
a**3*b**3*e*g - 3*_t*a**3*b**3*f**2 + 6*_t*a**2*b**4*c*f + 6*_t*a**2*b**4*d
*e - 3*_t*a*b**5*c**2 + a**5*g*h**2 - a**4*b*d*h**2 - 2*a**4*b*e*g*h - a**4
*b*f**2*h + 2*a**4*b*f*g**2 + 2*a**3*b**2*c*f*h - 2*a**3*b**2*c*g**2 + 2*a*
*3*b**2*d*e*h - 4*a**3*b**2*d*f*g + a**3*b**2*e**2*g + a**3*b**2*e*f**2 - a
**2*b**3*c**2*h + 4*a**2*b**3*c*d*g - 2*a**2*b**3*c*e*f + 2*a**2*b**3*d**2*
f - a**2*b**3*d*e**2 + a*b**4*c**2*e - 2*a*b**4*c*d**2)/(a**4*b*g**3 - 3*a*
*3*b**2*d*g**2 + a**3*b**2*f**3 - 3*a**2*b**3*c*f**2 + 3*a**2*b**3*d**2*g +
 3*a*b**4*c**2*f - a*b**4*d**3 - b**5*c**3)))) + f*x/b + g*x**2/(2*b) + h*x
**3/(3*b)
```

Giac [A] time = 1.07956, size = 397, normalized size = 1.53

$$-\frac{(ah-be)\log\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{2\,b^2hx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(a*h - b*e)*log(abs(b*x^3 + a))/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*
c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*(2*b^2*h*x^3 +
 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 + 1/6*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*
a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a*b^3) - 1/3*(b^7*d*(-a/b)^(1/3) - a*b^6*g*(-a/b)^(1/3) + b
^7*c - a*b^6*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7)
```

3.408 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=258

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6a^{2/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b
*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(5/
3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.470611, antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\cdots}{\sqrt{3}\sqrt[3]{\cdots}}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - a*g - (a^(1/3)*(b*e - a*h)
)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/
3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)}dx = \int\left(\frac{g}{b}+\frac{c}{ax}+\frac{hx}{b}+\frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{ab\left(a+bx^3\right)}\right)dx$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int\frac{a(bd-ag)+a(be-ah)x-b(bc-af)x^2}{a+bx^3}dx}{ab}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\int\frac{a(bd-ag)+a(be-ah)x}{a+bx^3}dx}{ab}-\frac{(bc-af)\int\frac{x^2}{a+bx^3}dx}{a}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{\int\frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}(bd-ag)+a^{4/3}(be-ah)\right)+}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}}{3a^{5/}}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-af)\log}{3ab}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{c\log(x)}{a}+\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{\left(bd-ag-\right.}{}$$

$$= \frac{gx}{b}+\frac{hx^2}{2b}-\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{c\log(x)}{a}+$$

Mathematica [A] time = 0.212392, size = 258, normalized size = 1.

$$-\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)+2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

```
[Out] (6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a^
(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)
*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e
 - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
- 2*b^(2/3)*(b*c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

Maple [B] time = 0.006, size = 426, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x)
```

```
[Out] 1/2*h*x^2/b+g*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*g+1/3/b/(1/b*
a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*g-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))*d-1/3/b^2*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-
1))*g+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*d+1/3/b^2*a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*h-1/3/b/(1/b*a)^(1/3)*ln(x+(
```

```
1/b*a)^(1/3))*e-1/6/b^2*a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))*h+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3/b^2*a*
3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/3/b*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/b*ln(b*x
^3+a)*f-1/3*c*ln(b*x^3+a)/a+c*ln(x)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06272, size = 409, normalized size = 1.59

$$\frac{c\log\left(|x|\right)}{a} - \frac{\left(bc - af\right)\log\left(\left|bx^3 + a\right|\right)}{3\,ab} + \frac{bhx^2 + 2\,bgx}{2\,b^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}} b^2 d - \left(-ab^2\right)^{\frac{1}{3}} abg + \left(-ab^2\right)^{\frac{2}{3}} ah - \left(-ab^2\right)^{\frac{2}{3}} be\right) \mathrm{arcta}}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))/a - 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/(a*b) + 1/2*(b*h*x^2
+ 2*b*g*x)/b^2 + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g +
 (-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/6*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a
*b*g - (-a*b^2)^(2/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a*b^3) + 1/3*(a^3*b^2*h*(-a/b)^(1/3) - a^2*b^3*(-a/b)^(1/3)*
e - a^2*b^3*d + a^3*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3
)
```

3.409 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=253

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(4/3
)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e
- a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3))
 - ((b*d - a*g)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.453954, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```


```
[Out] -(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(4/3
)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e
- a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(4/3))
 - ((b*d - a*g)*Log[a + b*x^3])/(3*a*b)
```


Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}dx = \int \left(\frac{h}{b}+\frac{c}{ax^2}+\frac{d}{ax}+\frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{ab(a+bx^3)}\right)dx$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x-b(bd-ag)x^2}{a+bx^3}dx}{ab}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\int \frac{a(be-ah)-b(bc-af)x}{a+bx^3}dx}{ab}-\frac{(bd-ag)\int \frac{x^2}{a+bx^3}dx}{a}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}-\frac{(bd-ag)\log(a+bx^3)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(-\sqrt[3]{a}b(bc-af)+2a\sqrt[3]{b}(be-ah)\right)+}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{3a^{5/3}}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}}-\frac{(bd-}{}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}}-\frac{(b^{2/3}(}{}$$

$$= -\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{4/3}}+\frac{d\log(x)}{a}+\frac{(b^{2/}}{}$$

Mathematica [A] time = 0.280359, size = 257, normalized size = 1.02

$$\frac{1}{6}\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be+a^{5/3}h+ab^{2/3}f-b^{5/3}c\right)}{a^{4/3}b^{4/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+b^{5/3}c\right)}{a^{4/3}b^{4/3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

```
[Out] ((-6*c)/(a*x) + (6*h*x)/b + (2*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)
*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(4/3)*b^(4/
3)) + (6*d*Log[x])/a + (2*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*
h)*Log[a^(1/3) + b^(1/3)*x])/(a^(4/3)*b^(4/3)) + ((-(b^(5/3)*c) - a^(2/3)*b
*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(4/3)*b^(4/3)) + (2*(-(b*d) + a*g)*Log[a + b*x^3])/(a*b))/6
```

Maple [B] time = 0.007, size = 423, normalized size = 1.7

$$\frac{hx}{b}-\frac{ah}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{e}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ah}{6b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{e}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] h*x/b-1/3/b^2*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*h+1/3/b/(1/b*a)^(2/3)*ln(
x+(1/b*a)^(1/3))*e+1/6/b^2*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
```

```
2/3))*h-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-1/3/b^2
*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/3/b/
(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/3/b/(1/
b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+
1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1/6/a/(1/b*a)^(
1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3*3^(1/2)/b/(1/b*a)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3/a*3^(1/2)/(1/b*a)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3/b*ln(b*x^3+a)*g-1/3*d*ln(b*x^3+a
)/a+d*ln(x)/a-c/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09191, size = 404, normalized size = 1.6

$$\frac{hx}{b} + \frac{d\log\left(|x|\right)}{a} - \frac{\left(bd - ag\right)\log\left(\left|bx^3 + a\right|\right)}{3\,ab} - \frac{c}{ax} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h - \left(-ab^2\right)^{\frac{1}{3}}abe - \left(-ab^2\right)^{\frac{2}{3}}bc + \left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan}{3\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] h*x/b + d*log(abs(x))/a - 1/3*(b*d - a*g)*log(abs(b*x^3 + a))/(a*b) - c/(a*
x) - 1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2
/3)*b*c + (-a*b^2)^(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^2*b^2) - 1/6*((-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e + (-a
*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(a^2*b^2) + 1/3*(a*b^4*c*(-a/b)^(1/3) - a^2*b^3*f*(-a/b)^(1/3) + a^3*b^
2*h - a^2*b^3*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

3.410 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=260

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{6a^{5/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}+\tan^{-1}\Big(\cdots$$

```
[Out] -c/(2*a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*
g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(2
/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1
/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*
d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(2/3
)) - ((b*e - a*h)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.380025, antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}+\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cdots}{\sqrt{3}\sqrt[3]{a}\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] -c/(2*a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/3)*
g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(2
/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log[a^(1
/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*c - a*f - (a^(1/3)*(b*d - a*g))
/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(1/3
)) - ((b*e - a*h)*Log[a + b*x^3])/(3*a*b)
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}dx = \int \left(\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}+\frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}+\frac{\int \frac{-bc+af-(bd-ag)x-(be-ah)x^2}{a+bx^3}dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}+\frac{\int \frac{-bc+af+(-bd+ag)x}{a+bx^3}dx}{a}+\frac{(-be+ah)\int \frac{x^2}{a+bx^3}dx}{a}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}+\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(-bc+af)+\sqrt[3]{a}(-bd+a\ldots\right.}{a^{2/3}-\ldots}}{\ldots}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{(be-ah)\,\mathrm{l}\ldots}{3\ldots}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}-\frac{\left(bc-af-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{\left(bc-af-\ldots\right.}{\ldots}$$

$$= -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{e\log(x)}{a}-\ldots$$

Mathematica [A] time = 0.32134, size = 257, normalized size = 0.99

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4\ldots}\right.}{b^{2/3}}\ldots}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a^(1/3)*b*d
 - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^
(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d - a*
b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2
/3) + (2*a^(2/3)*(-(b*e) + a*h)*Log[a + b*x^3])/b)/(6*a^(5/3))
```

Maple [B] time = 0.007, size = 423, normalized size = 1.6

$$\frac{f}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{f}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{6a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^
(1/3))*c-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/6/a/
```

```
(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/3/b/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-1/3/a/(1/b*a)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c-1/3/b/(1/b*a)^(1/3)*ln(x+(1
/b*a)^(1/3))*g+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d+1/6/b/(1/b*a)^(1/3
)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*d+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))*g-1/3/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))*d+1/3/b*ln(b*x^3+a)*h-1/3*e*ln(b*x^3+a)/a-d/a/x+e*ln(x)/
a-1/2*c/a/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08235, size = 393, normalized size = 1.51

$$\frac{e\log\left(|x|\right)}{a}+\frac{(ah-be)\log\left(\left|bx^3+a\right|\right)}{3\,ab}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] e*log(abs(x))/a + 1/3*(a*h - b*e)*log(abs(b*x^3 + a))/(a*b) - 1/3*sqrt(3)*(
(-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)
^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2)
 + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*g*(-a/b)^(1/3) + a*b^2*c - a^2*b*f)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2) - 1
/6*((-a*b^2)^(1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*
b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

3.411 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=276

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{6a^{5/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-\ldots}{\sqrt{3}\ldots}\right.\ldots}{\ldots}$$

```
[Out] -c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3
)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2 - ((b^(1/3)*(b*d - a*g) - a^(
1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b^(1/3)
*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(6*a^(5/3)*b^(2/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.435676, antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -c/(3*a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3
)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2 - ((b^(1/3)*(b*d - a*g) - a^(
1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*d - a
*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(6*a^(5/3)*b^(1/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}dx = \int \left(\frac{c}{ax^4}+\frac{d}{ax^3}+\frac{e}{ax^2}+\frac{-bc+af}{a^2x}+\frac{-a(bd-ag)-a(be-ah)x+b(bc-af)x^2}{a^2\left(a+bx^3\right)}\right)$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{\int \frac{-a(bd-ag)-a(be-ah)x+b(bc-af)x^2}{a+bx^3}dx}{a^2}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{\int \frac{-a(bd-ag)-a(be-ah)x}{a+bx^3}dx}{a^2}+\frac{(b(bc-af))}{a^2}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}+\frac{\int \frac{\sqrt[3]{a}\left(-2a\sqrt[3]{b}(\right.}{}}{}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(bd-ag-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}}-\frac{(bc-}{}$$

Mathematica [A] time = 0.439707, size = 264, normalized size = 0.96

$$-\frac{-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{b^{2/3}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{}\right.}{b^{2/3}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

```
[Out] -((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d)
- a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (2*a^(1/3)*(b^(4/3)*d - a^(1/3)
*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3
)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3])/(6*a^2)
```

Maple [B] time = 0.006, size = 442, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*g-1/3/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^
(1/3))*d-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g+1/6/a/
(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/3/b/(1/b*a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g-1/3/a/(1/b*a)^(2/3)*3^(1
```

```
/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/3/b/(1/b*a)^(1/3)*ln(x+(1
/b*a)^(1/3))*h+1/3/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/6/b/(1/b*a)^(1/3
)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h-1/6/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))*e+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))*h-1/3/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))*e-1/3/a*ln(b*x^3+a)*f+1/3/a^2*b*ln(b*x^3+a)*c-e/a/x-1/3*
c/a/x^3-1/2*d/a/x^2+1/a*ln(x)*f-1/a^2*ln(x)*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0965, size = 423, normalized size = 1.53

$$\frac{\left(bc-af\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^2}-\frac{\left(bc-af\right)\log\left(\left|x\right|\right)}{a^2}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah-\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\cdots\right.}{3\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="giac"
)
```

```
[Out] 1/3*(b*c - a*f)*log(abs(b*x^3 + a))/a^2 - (b*c - a*f)*log(abs(x))/a^2 - 1/3
*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h
- (-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))
/(a^2*b^2) - 1/6*((-a*b^2)^(1/3)*b^2*d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2
/3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2
*b^2) - 1/3*(a^4*b*h*(-a/b)^(1/3) - a^3*b^2*(-a/b)^(1/3)*e - a^3*b^2*d + a^
4*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(6*a*x^2*e + 3
*a*d*x + 2*a*c)/(a^2*x^3)
```

3.412 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=337

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}b^{10/3}}$$

```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*
x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) -
 ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*
a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3))
 + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.717088, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{18\sqrt[3]{a}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{9\sqrt[3]{a}b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```


```
[Out] ((b*e - 2*a*h)*x)/b^3 + (f*x^2)/(2*b^2) + (g*x^3)/(3*b^2) + (h*x^4)/(4*b^2)
 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*
x^3)) - ((2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(10/3)) -
 ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)
*x])/(9*a^(1/3)*b^(10/3)) + ((b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*
a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(1/3)*b^(10/3))
 + ((b*d - 2*a*g)*Log[a + b*x^3])/(3*b^3)
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3b^3\left(a+bx^3\right)} - \int \frac{a^2(be-ah)-2ab(bc-af)x-3ab(ba}{}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3b^3\left(a+bx^3\right)} - \frac{\int\left(-3a(be-2ah)-3abfx\right.}{}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-\right.}{3b^3\left(a+bx^3\right)}$$

$$= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-\right.}{3b^3\left(a+bx^3\right)}$$

Mathematica [A] time = 0.397708, size = 334, normalized size = 0.99

$$-\frac{12b^{2/3}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{a+bx^3} + \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(4a^{2/3}b^{4/3}e-7a^{5/3}\sqrt[3]{b}h-5abf+2b^2c\right)}{\sqrt[3]{a}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-4a^{2/3}b^{4/3}e+7a^{5/3}\sqrt[3]{b}h+\right.}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3 + 9*b^(5/
3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))
/(a + b*x^3) - (4*Sqrt[3]*(2*b^2*c - 4*a^(2/3)*b^(4/3)*e - 5*a*b*f + 7*a^(5
/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-
2*b^2*c - 4*a^(2/3)*b^(4/3)*e + 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
+ b^(1/3)*x])/a^(1/3) + (2*(2*b^2*c + 4*a^(2/3)*b^(4/3)*e - 5*a*b*f - 7*a^(
5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + 1
2*b^(2/3)*(b*d - 2*a*g)*Log[a + b*x^3])/(36*b^(11/3))
```

Maple [B] time = 0.011, size = 562, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] -7/18/b^4*a^2*h/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*g*x
^3/b^2+1/4*h*x^4/b^2+1/2*f*x^2/b^2-2/b^3*a*h*x-1/3/b*x^2/(b*x^3+a)*c-2/9/b^
2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9/b^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/
b*a)^(2/3))*e-1/3/b^3/(b*x^3+a)*a^2*g+1/3/b^2/(b*x^3+a)*d*a-2/3/b^3*ln(b*x^
3+a)*a*g+5/9/b^3*a*f/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18/b^3*a*f/(1/b*a)
^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b^2*a*x^2/(b*x^3+a)*f+1/3/
b^2*x*a/(b*x^3+a)*e-4/9/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
1/b*a)^(1/3)*x-1))*e-4/9/b^3*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e+1/b^2*x*
e+1/3/b^2*ln(b*x^3+a)*d+2/9/b^2*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*
(2/(1/b*a)^(1/3)*x-1))+7/9/b^4*a^2*h/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-5/9/
b^3*a*f*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3
/b^3/(b*x^3+a)*a^2*h*x+7/9/b^4*a^2*h/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09132, size = 512, normalized size = 1.52

$$\frac{\left(bd-2\,ag\right)\log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{\sqrt{3}\left(7\,\left(-ab^2\right)^{\frac{1}{3}}a^2h-4\,\left(-ab^2\right)^{\frac{1}{3}}abe-2\,\left(-ab^2\right)^{\frac{2}{3}}bc+5\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/3*(b*d - 2*a*g)*log(abs(b*x^3 + a))/b^3 + 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a
^2*h - 4*(-a*b^2)^(1/3)*a*b*e - 2*(-a*b^2)^(2/3)*b*c + 5*(-a*b^2)^(2/3)*a*f
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) + 1/3*(a*b*
d - a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h - a*b*e)*x)/((b*x^3 + a)*b^3) + 1/
18*(7*(-a*b^2)^(1/3)*a^2*h - 4*(-a*b^2)^(1/3)*a*b*e + 2*(-a*b^2)^(2/3)*b*c
- 5*(-a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) -
1/9*(2*b^6*c*(-a/b)^(1/3) - 5*a*b^5*f*(-a/b)^(1/3) + 7*a^2*b^4*h - 4*a*b^5*
e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/12*(3*b^6*h*x^4 + 4*
b^6*g*x^3 + 6*b^6*f*x^2 - 24*a*b^5*h*x + 12*b^6*x*e)/b^8
```

3.413 $\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=311

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}-\frac{\text{ta}}{}$$

```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)
*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d -
4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d
 - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c -
 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.639639, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)}{9a^{2/3}b^{8/3}}-\frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```


```
[Out] (f*x)/b^2 + (g*x^2)/(2*b^2) + (h*x^3)/(3*b^2) - (x*(b*c - a*f + (b*d - a*g)
*x + (b*e - a*h)*x^2))/(3*b^2*(a + b*x^3)) - ((b^(4/3)*c + 2*a^(1/3)*b*d -
4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(3*Sqrt[3]*a^(2/3)*b^(8/3)) + ((b^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d
 - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*c -
 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(18*a^(2/3)*b^(8/3)) + ((b*e - 2*a*h)*Log[a + b*x^3])/(3*b^3)
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x-3ab(be-ah)x^2}{a+bx^3}}{3ab^3}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)} - \frac{\int \left(-3abf-3abgx-3abhx^2-\frac{ab}{}\right.}{3}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\int \frac{ab(bc-4af)+ab(}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\int \frac{ab(bc-4af)+ab(}{a+bx^3}}{3ab^3}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{(be-2ah)\log}{3b^3}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{b}(bc-4af)\right.}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}+\frac{\left(\sqrt[3]{b}(bc-4af)\right.}{}$$

$$= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3b^2\left(a+bx^3\right)}-\frac{\left(b^{4/3}c+2\sqrt[3]{a}bc\right.}{}$$

Mathematica [A] time = 0.212374, size = 294, normalized size = 0.95

$$\frac{-\frac{6\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{a+bx^3}-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{4/3}g-2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{a^{2/3}}+}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
x*(f + g*x))))/(a + b*x^3) + (2*Sqrt[3]*b^(1/3)*(-(b^(4/3)*c) - 2*a^(1/3)*b
*d + 4*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(2/3) + (2*b^(1/3)*(b^(4/3)*c - 2*a^(1/3)*b*d - 4*a*b^(1/3)*f + 5*a^(
4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - (b^(1/3)*(b^(4/3)*c - 2*a^(1/3)
*b*d - 4*a*b^(1/3)*f + 5*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(2/3) + 6*(b*e - 2*a*h)*Log[a + b*x^3])/(18*b^3)
```

Maple [B] time = 0.011, size = 533, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*h*x^3/b^2+1/2*g*x^2/b^2+f*x/b^2+1/3/b^2/(b*x^3+a)*x^2*a*g-1/3*x^2*d/(b*
x^3+a)/b+1/3/b^2*a*x/(b*x^3+a)*f-1/3/b*x/(b*x^3+a)*c-1/3/b^3/(b*x^3+a)*a^2*
h+1/3/b^2*a/(b*x^3+a)*e-4/9/b^3*a*f/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/9/b
^3*a*f/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/b^3*a*f/(1/b
*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/9/b^2*c/(1/b*
a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/18/b^2*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))+1/9/b^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))+5/9/b^3*a*g/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18/b^3*a*g/
(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/b^3*a*g*3^(1/2)/(1/
b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2/(1/b*a)^(1/3)*
ln(x+(1/b*a)^(1/3))*d+1/9/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))*d+2/9/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1))*d-2/3/b^3*ln(b*x^3+a)*a*h+1/3/b^2*ln(b*x^3+a)*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08641, size = 475, normalized size = 1.53

$$-\frac{(2\,ah-be)\log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\,\left(-ab^2\right)^{\frac{1}{3}}abf-2\,\left(-ab^2\right)^{\frac{2}{3}}bd+5\,\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*(2*a*h - b*e)*log(abs(b*x^3 + a))/b^3 + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^
2*c - 4*(-a*b^2)^(1/3)*a*b*f - 2*(-a*b^2)^(2/3)*b*d + 5*(-a*b^2)^(2/3)*a*g)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/3*(a^2*h
 + (b^2*d - a*b*g)*x^2 - a*b*e + (b^2*c - a*b*f)*x)/((b*x^3 + a)*b^3) + 1/1
8*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*f + 2*(-a*b^2)^(2/3)*b*d - 5
*(-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) - 1/9
*(2*b^4*d*(-a/b)^(1/3) - 5*a*b^3*g*(-a/b)^(1/3) + b^4*c - 4*a*b^3*f)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 1/6*(2*b^4*h*x^3 + 3*b^4*g*x^2
+ 6*b^4*f*x)/b^6
```

3.414 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=290

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{18a^{2/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{9a^{2/3}b^{8/3}}-$$

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*
x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e - 4*a*b^(1/3)*g - 5*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2
/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/
3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2
*b*e - 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*
b^(8/3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.497935, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1823, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}-4ag+bd\right)}{18a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)}{9a^{2/3}b^{8/3}}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (4*g*x)/(3*b^2) + (5*h*x^2)/(6*b^2) - (c + d*x + e*x^2 + f*x^3 + g*x^4 + h*
x^5)/(3*b*(a + b*x^3)) - ((b^(4/3)*d + 2*a^(1/3)*b*e - 4*a*b^(1/3)*g - 5*a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2
/3)*b^(8/3)) + ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/
3) + b^(1/3)*x])/(9*a^(2/3)*b^(8/3)) - ((b*d - 4*a*g - (a^(1/3)*(2*b*e - 5*
a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(2/3)*
b^(7/3)) + (f*Log[a + b*x^3])/(3*b^2)
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2 \left(c + dx + ex^2 + fx^3 + gx^4 + hx^5\right)}{\left(a + bx^3\right)^2} dx = -\frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{a+bx^3} dx}{3b}$$

$$= -\frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{\int \left(\frac{4g}{b} + \frac{5hx}{b} + \frac{bd-4ag+(2be-5ah)x+3bfx^2}{b(a+bx^3)}\right.}{3b}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{\int \frac{bd-4ag+(2be-5ah)x+3bfx}{a+bx^3}}{3b^2}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{\int \frac{bd-4ag+(2be-5ah)x}{a+bx^3} dx}{3b^2} +$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{f \log \left(a + bx^3\right)}{3b^2} + \int$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{\left(bd - 4ag - \frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right.}{9a^{2/3}b}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} + \frac{\left(bd - 4ag - \frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}\right.}{9a^{2/3}b}$$

$$= \frac{4gx}{3b^2} + \frac{5hx^2}{6b^2} - \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{3b \left(a + bx^3\right)} - \frac{\left(b^{4/3}d + 2\sqrt[3]{a}be - 4a\sqrt[3]{b}\right.}{3\sqrt{}}$$

Mathematica [A] time = 0.198048, size = 280, normalized size = 0.97

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}h-2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5a^{4/3}h-2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{2/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h-2\sqrt[3]{a}be+4\right.}{a^{2/3}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f
+ x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d) - 2*a^(1/3)*b*e + 4
*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
(2/3) + (2*(b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^
(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g +
 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) + 6*b
^(2/3)*f*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [B] time = 0.01, size = 506, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/2*h*x^2/b^2+g*x/b^2+1/3/b^2/(b*x^3+a)*x^2*a*h-1/3/b*x^2/(b*x^3+a)*e+1/3/b
^2/(b*x^3+a)*a*g*x-1/3/b*x/(b*x^3+a)*d+1/3*a/b^2/(b*x^3+a)*f-1/3/b/(b*x^3+a
)*c-4/9/b^3*a*g/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+2/9/b^3*a*g/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/b^3*a*g/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^
(1/3))*d-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/9
/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d+5/9/
b^3*a*h/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-5/18/b^3*a*h/(1/b*a)^(1/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/b^3*a*h*3^(1/2)/(1/b*a)^(1/3)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3)
)+1/9/b^2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*e*3
^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*f*ln(b*x
^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 11.4732, size = 27702, normalized size = 95.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] 1/36*(18*b*h*x^5 + 36*b*g*x^4 - 6*(2*b*e - 5*a*h)*x^2 - 2*(b^3*x^3 + a*b^2)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*
f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h
+ (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*
h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)
*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1
/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*
g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^
2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h
^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a
^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4
*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)*log(-8*a*b^3*d
*e^2 + 3*a*b^3*d^2*f - 18*a^2*b^2*e*f^2 + 48*a^3*b*f*g^2 - 1/4*(2*a^2*b^6*e
 - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 2
0*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*
e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^
```

```
3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h
 + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(3
2*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g
^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(
1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h
+ (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*
h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)
*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)
^2 - 50*(a^3*b*d - 4*a^4*g)*h^2 + 1/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b
^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))
/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(
a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^
3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^
4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 2
4*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*
d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 -
9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d
^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^
2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4
*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^
2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(
a^2*b^8))^(1/3) - 6*f/b^2) + 8*(4*a^2*b^2*e^2 - 3*a^2*b^2*d*f)*g + 5*(8*a^2
*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e*g)*h - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^
3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*
h^2 - 125*a^4*h^3)*x) - 12*b*c + 12*a*f - 12*(b*d - 4*a*g)*x + (18*b*f*x^3
+ (b^3*x^3 + a*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e
 + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^
2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*
a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e
^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 -
2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (
16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8
))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*
g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*
a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*
b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*
f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h
)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/
b^2) + 18*a*f - 3*sqrt(1/3)*(b^3*x^3 + a*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*
b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*
h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*
g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2
*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3
*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9
*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54
*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^
7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*
g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^
3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*
f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*
g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)
/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)
*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^
2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b
```

```
^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(
9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d
*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f
^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7)
 - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^
3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3
+ 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*
g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)
*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128
*a*b*e*g - 80*(a*b*d - 4*a^2*g)*h)/(a*b^5)))*log(8*a*b^3*d*e^2 - 3*a*b^3*d^
2*f + 18*a^2*b^2*e*f^2 - 48*a^3*b*f*g^2 + 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(
2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^
2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h +
(9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*
d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^
2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h
+ 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a
^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3
)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g
- 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*
b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3
)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3
*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e
^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2 + 50*(a^3*b*d
- 4*a^4*g)*h^2 - 1/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b
^3*g^2 + 30*a^3*b^3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^
2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*
(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3
 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*
b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h
^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*
h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^
2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20
*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3
- 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150
*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3
- 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 1
5*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) -
 6*f/b^2) - 8*(4*a^2*b^2*e^2 - 3*a^2*b^2*d*f)*g - 5*(8*a^2*b^2*d*e + 9*a^3*
b*f^2 - 32*a^3*b*e*g)*h - 2*(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^
2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h
^3)*x + 3/4*sqrt(1/3)*(2*a*b^5*d^2 + 12*a^2*b^4*e*f - 16*a^2*b^4*d*g + 32*a
^3*b^3*g^2 - 30*a^3*b^3*f*h + (2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-
I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*
h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g -
 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b
^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)
/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*
b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^
3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1
)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/
(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a
^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b
^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 -
24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6
*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2))*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)
/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)
*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^
```

2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b
^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(
9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d
*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f
^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7)
 - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^
3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3
+ 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*
g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)
*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(
a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a
*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2
- 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8
) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*
f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e
*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3
/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) -
 (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3
- 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 +
125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g
+ 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a
*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a
*b*e*g - 80*(a*b*d - 4*a^2*g)*h)/(a*b^5))) + (18*b*f*x^3 + (b^3*x^3 + a*b^2
)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9
*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h
 + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b
^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e
*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g
*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d
)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(
1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e
*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a
^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*
h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*
a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(
4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2) + 18*a*f + 3*
sqrt(1/3)*(b^3*x^3 + a*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b
^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^
3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7)
- (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3
 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 +
 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g
 + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*
a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^
2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*
a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e
^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 -
2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (
16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8
))^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4
 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/
b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) -
(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 -
 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 1
25*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g +
 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*
b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*
d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*

b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2
*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*
(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16
*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))
^(1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a*b*e*g - 80*(a*b
*d - 4*a^2*g)*h)/(a*b^5)))*log(8*a*b^3*d*e^2 - 3*a*b^3*d^2*f + 18*a^2*b^2*e
*f^2 - 48*a^3*b*f*g^2 + 1/4*(2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)
*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5
*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2
*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(
a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b
+ 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3
- 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*
(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a
*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3
*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4
*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24
*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d
^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2)^2 + 50*(a^3*b*d - 4*a^4*g)*h^2 - 1
/2*(a*b^5*d^2 - 12*a^2*b^4*e*f - 8*a^2*b^4*d*g + 16*a^3*b^3*g^2 + 30*a^3*b^
3*f*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h
 + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^
2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 1
2*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^
3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 9
0*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f
*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2
- 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g +
 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125
*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*
h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2
- 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(1/3) - 6*f/b^2) - 8*(4*a
^2*b^2*e^2 - 3*a^2*b^2*d*f)*g - 5*(8*a^2*b^2*d*e + 9*a^3*b*f^2 - 32*a^3*b*e
*g)*h - 2*(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a
^3*b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)*x - 3/4*sqrt(1
/3)*(2*a*b^5*d^2 + 12*a^2*b^4*e*f - 16*a^2*b^4*d*g + 32*a^3*b^3*g^2 - 30*a^
3*b^3*f*h + (2*a^2*b^6*e - 5*a^3*b^5*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*
f^2/b^4 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(
54*f^3/b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*
b^7) - (b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*
b*g^3 - 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*
d^3 + 125*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*
e*f*g + 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^
2*g)*a*b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*
(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3
 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*
b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h
^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*
h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^
2*b^8))^(1/3) - 6*f/b^2))*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4
 - (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/
b^6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) -
(b^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 -
 60*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 1
25*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g +
 20*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*
b^3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*

```
d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*
b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2
*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*
(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16
*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))
^(1/3) - 6*f/b^2)^2*a*b^5 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*f^2/b^4 -
 (2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)/(a*b^5))/(54*f^3/b^
6 - 9*(2*b^2*d*e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b
^4*d^3 + 8*a*b^3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 6
0*a^2*b^2*e^2*h + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125
*a^4*h^3 - 2*(32*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 2
0*e^2*h + (16*g^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^
3)/(a^2*b^8))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*f^3/b^6 - 9*(2*b^2*d*
e + 20*a^2*g*h + (9*f^2 - 8*e*g - 5*d*h)*a*b)*f/(a*b^7) - (b^4*d^3 + 8*a*b^
3*e^3 - 12*a*b^3*d^2*g + 48*a^2*b^2*d*g^2 - 64*a^3*b*g^3 - 60*a^2*b^2*e^2*h
 + 150*a^3*b*e*h^2 - 125*a^4*h^3)/(a^2*b^8) + (b^4*d^3 + 125*a^4*h^3 - 2*(3
2*g^3 - 90*f*g*h + 75*e*h^2)*a^3*b + 3*(9*f^3 - 24*e*f*g + 20*e^2*h + (16*g
^2 - 15*f*h)*d)*a^2*b^2 - 2*(4*e^3 - 9*d*e*f + 6*d^2*g)*a*b^3)/(a^2*b^8))^(
1/3) - 6*f/b^2)*a*b^3*f + 32*b^2*d*e + 36*a*b*f^2 - 128*a*b*e*g - 80*(a*b*d
 - 4*a^2*g)*h)/(a*b^5))))/(b^3*x^3 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09203, size = 444, normalized size = 1.53

$$\frac{f\log\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{(ah-be)x^2-bc+af-\left(bd-ag\right)x}{3\left(bx^3+a\right)b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d-4\left(-ab^2\right)^{\frac{1}{3}}abg+5\left(-ab^2\right)^{\frac{2}{3}}ah-2\left(-ab^2\right.\right.}{9\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/3*f*log(abs(b*x^3 + a))/b^2 + 1/3*((a*h - b*e)*x^2 - b*c + a*f - (b*d - a
*g)*x)/((b*x^3 + a)*b^2) + 1/9*sqrt(3)*((-a*b^2)^(1/3)*b^2*d - 4*(-a*b^2)^(
1/3)*a*b*g + 5*(-a*b^2)^(2/3)*a*h - 2*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/
b^4 + 1/18*((-a*b^2)^(1/3)*b^2*d - 4*(-a*b^2)^(1/3)*a*b*g - 5*(-a*b^2)^(2/3
)*a*h + 2*(-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b
^4) + 1/9*(5*a*b^3*h*(-a/b)^(1/3) - 2*b^4*(-a/b)^(1/3)*e - b^4*d + 4*a*b^3*
g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

3.415 $\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}-\tan$$

```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*
b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(7
/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/
3)*x])/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h
))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (
g*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.505355, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1828, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{18a^{4/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(2af+bc)-a^{2/3}(be-4ah)\right)}{9a^{4/3}b^{7/3}}-\tan$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (h*x)/b^2 - (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a*
b^2*(a + b*x^3)) - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3)*h)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(7
/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b^(1/
3)*x])/(9*a^(4/3)*b^(7/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h
))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(7/3)) + (
g*Log[a + b*x^3])/(3*b^2)
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^2}\,dx = -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\int \frac{-a(be-ah)-b(bc+2af)x-3abgx^2-3a}{a+bx^3}}{3ab^2}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\int\left(-3ah-\frac{a(be-4ah)+b(bc+2af)x}{a+bx^3}\right.}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} + \frac{\int \frac{a(be-4ah)+b(bc+2af)x+3abg}{a+bx^3}}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} + \frac{\int \frac{a(be-4ah)+b(bc+2af)x}{a+bx^3}dx}{3ab^2}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} + \frac{g\log\left(a+bx^3\right)}{3b^2} + \frac{\int \frac{\sqrt[3]{a}(}{}}{}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be\right.}{9a^{4/3}}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be\right.}{9a^{4/3}}$$

$$= \frac{hx}{b^2} - \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3ab^2\left(a+bx^3\right)} - \frac{\left(b^{5/3}c+a^{2/3}be+2ab^{2/3}f\right.}{3\sqrt{3}}$$

Mathematica [A] time = 0.196368, size = 285, normalized size = 0.99

$$\frac{\frac{6b^{2/3}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{a\left(a+bx^3\right)} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}b^{4/3}e+4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-a^{2/3}b^{4/3}e+4a^{5/3}\sqrt[3]{b}h+2abf+b^2c\right)}{a^{4/3}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + a^(2/3)*b^(4/3)*e + 2*a*b*f -
4*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) -
 (2*(b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[a^(1/3)
 + b^(1/3)*x])/a^(4/3) + ((b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*
b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) + 6*b^(2
/3)*g*Log[a + b*x^3])/(18*b^(8/3))
```

Maple [B] time = 0.01, size = 502, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] h*x/b^2-1/3/b*x^2/(b*x^3+a)*f+1/3*x^2/a/(b*x^3+a)*c+1/3/b^2/(b*x^3+a)*a*h*x
-1/3/b*x/(b*x^3+a)*e+1/3/b^2/(b*x^3+a)*a*g-1/3/b/(b*x^3+a)*d-4/9/b^3*a/(1/b
*a)^(2/3)*ln(x+(1/b*a)^(1/3))*h+1/9/b^2*e/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))
+2/9/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h-1/18/b^2*e
/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-4/9/b^3*a/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/9/b^2*e/(1/b*a)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2/(1/b*a)^(1/3)*
ln(x+(1/b*a)^(1/3))*f-1/9/b/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/9/b^2/(
1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/18/b/a/(1/b*a)^(1/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+2/9/b^2*3^(1/2)/(1/b*a)^(1/3)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+1/9/b/a*3^(1/2)/(1/b*a)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*g*ln(b*x^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 13.0278, size = 27753, normalized size = 96.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] 1/36*(36*a*b*h*x^4 - 12*a*b*d + 12*a^2*g + 12*(b^2*c - a*b*f)*x^2 - 2*(a*b^
3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9
*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*
e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2
*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^
2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 6
4*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*
e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*
f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*
b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*
e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(
9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*
h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^
2)*log(-2*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 8*a^3*b^2*e*f^2 + 3*a^3*b^2*e^2*g
 + 48*a^5*g*h^2 - 1/4*(a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^
2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*
b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
```

+ 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
 - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*
a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
- 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3
/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6
*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^
3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2
*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2 - 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 + 1/2*(
a^3*b^4*e^2 - 8*a^4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*
g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*
a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8
*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a
*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b
*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*
(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g
*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(
e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12
*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5
*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g
*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (
e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) + 8*(a^2*b^
3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - (b^5*c^3 + a^2*b^3*e
^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h +
48*a^4*b*e*h^2 - 64*a^5*h^3)*x) - 12*(a*b*e - 4*a^2*h)*x + (18*a*b*g*x^3 +
18*a^2*g + (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4
 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3
/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6
*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^
3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2
*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c
*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^
2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e
^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6
*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7)
)^(1/3) - 6*g/b^2) - 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3
)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2
*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e
*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*
a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*
h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*
h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e
^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3
) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b
)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 +
 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
- (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a
^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 -
 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f -
2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(
e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12
*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5
*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g

```
*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (
e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(
3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*
b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2
+ 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7)
 - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*
a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
- 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 3
2*a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4)))*log(2*a*b^4*c^
2*e + 8*a^2*b^3*c*e*f + 8*a^3*b^2*e*f^2 - 3*a^3*b^2*e^2*g - 48*a^5*g*h^2 +
1/4*(a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 -
(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^
6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (
b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 -
12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*
b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 -
 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^
3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e
+ (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b
^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*
h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*
a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^
2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(
1/3) - 6*g/b^2)^2 + 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 1/2*(a^3*b^4*e^2 - 8*a^
4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c
*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
 - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^
2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^
3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
+ 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3
 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*
g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8
*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
(b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4
*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3
*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) - 8*(a^2*b^3*c^2 + 4*a^3*b^2*
c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - 2*(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2
*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 -
 64*a^5*h^3)*x + 3/4*sqrt(1/3)*(2*a^3*b^4*e^2 - 16*a^4*b^3*e*h + 32*a^5*b^2
*h^2 + (a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4
 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3
/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3
 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6
*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^
3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2
*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c
*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^
2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e
^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6
*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7)
)^(1/3) - 6*g/b^2) + 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*sqrt(-((2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*
h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
- 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2
*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3
```

)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 32*a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4))) + (18*a*b*g*x^3 + 18*a^2*g + (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) + 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2

+ 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7
) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)
*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2
- 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e +
32*a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4)))*log(2*a*b^4*c
^2*e + 8*a^2*b^3*c*e*f + 8*a^3*b^2*e*f^2 - 3*a^3*b^2*e^2*g - 48*a^5*g*h^2 +
1/4*(a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^4 -
(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^3/b
^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) -
(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 -
12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a
*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3
- 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b
^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*c*e
+ (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*
b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2
*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64
*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e
^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^
(1/3) - 6*g/b^2)^2 + 9*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 1/2*(a^3*b^4*e^2 - 8*a
^4*b^3*e*h + 16*a^5*b^2*h^2 - 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*
c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*
f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a
^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h
^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
+ 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^
3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)
*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 +
8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
(b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^
4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 -
3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2) - 8*(a^2*b^3*c^2 + 4*a^3*b^2
*c*f + 4*a^4*b*f^2 - 3*a^4*b*e*g)*h - 2*(b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^
2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2
- 64*a^5*h^3)*x - 3/4*sqrt(1/3)*(2*a^3*b^4*e^2 - 16*a^4*b^3*e*h + 32*a^5*b^
2*h^2 + (a^3*b^6*c + 2*a^4*b^5*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*g^2/b^
4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)/(a^2*b^4))/(54*g^
3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6)
- (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^
3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 +
6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f
^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^
2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*g^3/b^6 - 9*(b^2*
c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a
^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*
e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f +
64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g +
6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7
))^(1/3) - 6*g/b^2) + 6*(a^3*b^4*c + 2*a^4*b^3*f)*g)*sqrt(-((2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c
*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f
- 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^
2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^
3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
+ 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3
- 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)*

```
g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8
*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
(b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^4
*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 - 3
*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)^2*a^2*b^4 + 12*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(9*g^2/b^4 - (b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*
c*h)*a*b)/(a^2*b^4))/(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*
f - 2*c*h)*a*b)*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a
^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h
^3)/(a^4*b^7) - (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h
 + 16*e*h^2)*a^4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^
3 - 3*(4*f^2 - 3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
 + 1)*(54*g^3/b^6 - 9*(b^2*c*e + (9*g^2 - 8*f*h)*a^2 + 2*(e*f - 2*c*h)*a*b)
*g/(a^2*b^6) - (b^5*c^3 + a^2*b^3*e^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 +
8*a^3*b^2*f^3 - 12*a^3*b^2*e^2*h + 48*a^4*b*e*h^2 - 64*a^5*h^3)/(a^4*b^7) -
 (b^5*c^3 + 6*a*b^4*c^2*f + 64*a^5*h^3 - 3*(9*g^3 - 24*f*g*h + 16*e*h^2)*a^
4*b + 2*(4*f^3 - 9*e*f*g + 6*e^2*h + 18*c*g*h)*a^3*b^2 - (e^3 - 3*(4*f^2 -
3*e*g)*c)*a^2*b^3)/(a^4*b^7))^(1/3) - 6*g/b^2)*a^2*b^2*g + 16*b^2*c*e + 32*
a*b*e*f + 36*a^2*g^2 - 64*(a*b*c + 2*a^2*f)*h)/(a^2*b^4))))/(a*b^3*x^3 + a^
2*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08823, size = 451, normalized size = 1.56

$$\frac{hx}{b^2} + \frac{g\log\left(\left|bx^3+a\right|\right)}{3\,b^2} - \frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{1}{3}}a^2h - \left(-ab^2\right)^{\frac{1}{3}}abe + \left(-ab^2\right)^{\frac{2}{3}}bc + 2\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3} - \frac{abd -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] h*x/b^2 + 1/3*g*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*a^2
*h - (-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c + 2*(-a*b^2)^(2/3)*a*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) - 1/3*(a*b*d -
a^2*g - (b^2*c - a*b*f)*x^2 - (a^2*h - a*b*e)*x)/((b*x^3 + a)*a*b^2) - 1/18
*(4*(-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c - 2*(-
a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/9*
(a*b^5*c*(-a/b)^(1/3) + 2*a^2*b^4*f*(-a/b)^(1/3) - 4*a^3*b^3*h + a^2*b^4*e)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^5)
```

3.416 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{9a^{5/3}b^{5/3}}-$$

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2
*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*
c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5
/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b
^2)
```

Rubi [A] time = 0.369973, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1858, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{9a^{5/3}b^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```


```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) - ((2
*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + ((b^(1/3)*(2*b*
c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(5
/3)) - ((b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)) + (h*Log[a + b*x^3])/(3*b
^2)
```


Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx = \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x-3abhx^2}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} - \frac{\int \frac{-b(2bc+af)-b(bd+2ag)x}{a+bx^3}\,dx}{3ab^2} + \frac{h\int \frac{x^2}{a+bx^3}}{b}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} + \frac{h\log\left(a+bx^3\right)}{3b^2} - \frac{\int \frac{\sqrt[3]{a}\left(-2b^{4/3}(2bc+af)\right.}{}}{}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log}{9a^{5/3}b^{5/3}}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log}{9a^{5/3}b^{5/3}}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3ab\left(a+bx^3\right)} - \frac{\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\text{ta}}{3\sqrt{3}a^{5/3}b^{5/3}}$$

Mathematica [A] time = 0.17552, size = 268, normalized size = 0.97

$$\frac{\frac{6\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{a\left(a+bx^3\right)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}g+\sqrt[3]{a}bd-a\sqrt[3]{b}f-2b^{4/3}c\right)}{a^{5/3}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{4/3}g-\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{a^{5/3}}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

```
[Out] ((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a*(a + b*x^3)) - (2
*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(2*b^(4/3)*
c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(5
/3) + (b^(1/3)*(-2*b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*x^3])/
(18*b^2)
```

Maple [B] time = 0.008, size = 462, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] (-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b*x^3+a)+1/
9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+2/9*c/a/b/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-1
/9*c/a/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/b^2/(1/b*a
```

```
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/9*c/a/b/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2/(1/b*a)^
(1/3)*ln(x+(1/b*a)^(1/3))*g-1/9*d/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/9
/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g+1/18*d/a/b/(1/b*
a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/b^2*3^(1/2)/(1/b*a)^(1/3
)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+1/9*d/a*3^(1/2)/b/(1/b*a)^(1/
3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*h*ln(b*x^3+a)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 12.3701, size = 27752, normalized size = 100.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas"
)
```

```
[Out] -1/36*(12*a*b*e - 12*a^2*h - 12*(b^2*d - a*b*g)*x^2 + 2*(a*b^3*x^3 + a^2*b^
2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g +
9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*
a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b
^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 1
2*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 -
 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g
 - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*
h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*
c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*
a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b +
(f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c
)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)*log(4*a*b^4
*c*d^2 + 2*a^2*b^3*d^2*f + 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*
c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2
 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2
*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4
*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^
3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a
^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
+ 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a
*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f
^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) +
(8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(
4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12
```

```
*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2 + 8*(2*a^3*b^2*c + a^4*b*f)*g^
2 + 9*(a^4*b*d + 2*a^5*g)*h^2 - 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^
3*f^2 - 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h
^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b
^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a
*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f
^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) +
(8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(
4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12
*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 -
 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6)
+ (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6
*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^
5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*
a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^
5*b^6))^(1/3) - 6*h/b^2) + 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g - 3*(4*a^2*b^3
*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + (8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2
*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8
*a^4*b*g^3)*x) - 12*(b^2*c - a*b*f)*x - (18*a*b*h*x^3 + 18*a^2*h + (a*b^3*x
^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a
^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b
^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^
4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^
2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 -
 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2
 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f
*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3
+ 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b
*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*
h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2
 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)
+ 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a
^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*
g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2
*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5
) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h -
 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
- 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b
^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b
^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 2
7*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)
*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)
/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(
a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c
*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^
2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^
5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h
- 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/
b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*
b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^
3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 +
27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h
)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4
)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d*f + 36*a^
```

```
3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4)))*log(-4*a*b^4*c*d^2 - 2*a^2*
b^3*d^2*f - 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^
3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g
)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*
c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5)
 + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h -
3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 -
 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^
6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^
6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
+ 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27
*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/
(a^5*b^6))^(1/3) - 6*h/b^2)^2 - 8*(2*a^3*b^2*c + a^4*b*f)*g^2 - 9*(a^4*b*d
+ 2*a^5*g)*h^2 + 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^3*f^2 - 6*(a^4*
b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3
*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^
6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^
6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
+ 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27
*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/
(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d +
 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 +
a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g
+ 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^
3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^
2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) -
 6*h/b^2) - 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g + 3*(4*a^2*b^3*c^2 + 4*a^3*b^
2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2*f + 6*a^2*b^
3*c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8*a^4*b*g^3)*x
 + 3/4*sqrt(1/3)*(8*a^2*b^5*c^2 + 8*a^3*b^4*c*f + 2*a^4*b^3*f^2 + (a^4*b^5*
d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d +
2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(
2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8
*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2
*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^
3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*
b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^
6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*
b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d
^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^
3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f
*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (
f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^
2) + 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*
(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a
^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*
g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2
*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5
) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h -
 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
- 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b
^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b
^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 2
7*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)
*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)
```

/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(
a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c
*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^
2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^
5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h
- 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3
 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/
b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*
b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^
3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 +
27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h
)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4
)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d*f + 36*a^
3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4))) - (18*a*b*h*x^3 + 18*a^2*h
+ (a*b^3*x^3 + a^2*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2*b^3
*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^
6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^
6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3
+ 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27
*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*
d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/
(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c*d +
 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 +
a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g
+ 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^
3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^
2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) -
 6*h/b^2) - 3*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2)*sqrt(-((2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)
*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (
d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f +
 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3
)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b
^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2
)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b
^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^
2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2
*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sq
rt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g
)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 +
(d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f
+ 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^
3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*
b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1
)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^
2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2
+ a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*
b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g
^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^
2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d
*f + 36*a^3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4)))*log(-4*a*b^4*c*d^
2 - 2*a^2*b^3*d^2*f - 1/4*(a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt
(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*
a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d

*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3) /(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 3 6*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^ 3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)* (54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2) *h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^ 5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2* f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2 - 8*(2*a^3*b^2*c + a^4*b*f)*g^2 - 9 *(a^4*b*d + 2*a^5*g)*h^2 + 1/2*(4*a^2*b^5*c^2 + 4*a^3*b^4*c*f + a^4*b^3*f^2 - 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^ 4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/ (54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2) *h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^ 5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2* f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2 *b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8* b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2* b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b ^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2) - 8*(2*a^2*b^3*c*d + a^3*b^2*d*f)*g + 3*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a*b^4*d^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^2*b^3*d^2*g + 12*a^3*b^2*d*g^2 + 8*a^ 4*b*g^3)*x - 3/4*sqrt(1/3)*(8*a^2*b^5*c^2 + 8*a^3*b^4*c*f + 2*a^4*b^3*f^2 + (a^4*b^5*d + 2*a^5*b^4*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(9*h^2/b^4 - (2* b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3 /b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3 *b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f ^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f* h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^ 4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(54*h^3/b^6 - 9*(2*b^3*c* d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2 *g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4 *g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6* (d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2) + 6*(a^4*b^3*d + 2*a^5*b^2*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqr t(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g) *a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b ^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1) *(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*b ^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g^ 2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^2 *f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)^2*a^3*b^4 + 12*(2*(1/2)^(2/3)*(-I*sq rt(3) + 1)*(9*h^2/b^4 - (2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)/(a^3*b^4))/(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f

```
+ 6*a^2*b^2*c*f^2 + a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^
3)/(a^5*b^5) + (8*b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 +
 36*c*g*h - 3*(4*g^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*
b^3 - (d^3 - 12*c^2*f)*a*b^4)/(a^5*b^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1
)*(54*h^3/b^6 - 9*(2*b^3*c*d + 2*a^2*b*f*g + 9*a^3*h^2 + (d*f + 4*c*g)*a*b^
2)*h/(a^3*b^6) + (8*b^4*c^3 + a*b^3*d^3 + 12*a*b^3*c^2*f + 6*a^2*b^2*c*f^2
+ a^3*b*f^3 + 6*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 + 8*a^4*g^3)/(a^5*b^5) + (8*
b^5*c^3 + 27*a^5*h^3 - 2*(4*g^3 - 9*f*g*h)*a^4*b + (f^3 + 36*c*g*h - 3*(4*g
^2 - 3*f*h)*d)*a^3*b^2 - 6*(d^2*g - (f^2 + 3*d*h)*c)*a^2*b^3 - (d^3 - 12*c^
2*f)*a*b^4)/(a^5*b^6))^(1/3) - 6*h/b^2)*a^3*b^2*h + 32*b^3*c*d + 16*a*b^2*d
*f + 36*a^3*h^2 + 32*(2*a*b^2*c + a^2*b*f)*g)/(a^3*b^4))))/(a*b^3*x^3 + a^2
*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.08394, size = 429, normalized size = 1.55

$$\frac{h\log\left(\left|bx^3+a\right|\right)}{3\,b^2}+\frac{\left(bd-ag\right)x^2+\left(bc-af\right)x+\frac{a^2h-abe}{b}}{3\left(bx^3+a\right)ab}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}b^2c+\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd-2\left(-ab^2\right)^{\frac{2}{3}}\right.}{9\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*h*log(abs(b*x^3 + a))/b^2 + 1/3*((b*d - a*g)*x^2 + (b*c - a*f)*x + (a^2
*h - a*b*e)/b)/((b*x^3 + a)*a*b) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^2*c + (-
a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d - 2*(-a*b^2)^(2/3)*a*g)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/18*(2*(-a*b^2)^(1/
3)*b^2*c + (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d + 2*(-a*b^2)^(2/3)*a*g
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/9*(a*b^3*d*(-a/b)^
(1/3) + 2*a^2*b^2*g*(-a/b)^(1/3) + 2*a*b^3*c + a^2*b^2*f)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

3.417 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{18a^{5/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}}{}$$

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x
^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Lo
g[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e +
2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(5/3)
) - (c*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.559415, antiderivative size = 287, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ah+be)}{\sqrt[3]{b}}+ag+2bd\right)}{18a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+2bd)-\sqrt[3]{a}(2ah+be)\right)}{9a^{5/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*x
^3)) - ((2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)) + (c*Lo
g[x])/a^2 + ((b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*h))*Log[a^(1/3) +
b^(1/3)*x])/(9*a^(5/3)*b^(5/3)) - ((2*b*d + a*g - (a^(1/3)*(b*e + 2*a*h))/b
^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3)
) - (c*Log[a + b*x^3])/(3*a^2)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}dx = \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-b(2bd+ag)x-b(be+2ah)x^2}{x\left(a+bx^3\right)}dx}{3ab^2}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax}+\frac{b\left(-a(2bd+ag)-a(be+2ah)x+3\right.}{a\left(a+bx^3\right)}\right.}{3ab^2}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2bd+ag)-a(be+2ah)x+3}{a+bx^3}}{3a^2b}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{\int \frac{-a(2bd+ag)-a(be+2ah)x}{a+bx^3}d}{3a^2b}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} - \frac{c\log\left(a+bx^3\right)}{3a^2} - \frac{\int \frac{\sqrt[3]{a}}{}}{}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right.}{9a^{5/3}b^4}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{c\log(x)}{a^2} + \frac{\left(2bd+ag-\frac{\sqrt[3]{a}(be+2ah)}{\sqrt[3]{b}}\right.}{9a^{5/3}b^4}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)}{3\sqrt{3}a^{5/3}b^{5/3}}$$

Mathematica [A] time = 0.197893, size = 269, normalized size = 0.93

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}h+\sqrt[3]{a}be-a\sqrt[3]{b}g-2b^{4/3}d\right)}{b^{5/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-2a^{4/3}h-\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{b^{5/3}}-\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+\right.}{b^{5/3}}}{18a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)) - (2
*Sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 18*c*Log[x] + (2*a^(1/
3)*(2*b^(4/3)*d - a^(1/3)*b*e + a*b^(1/3)*g - 2*a^(4/3)*h)*Log[a^(1/3) + b^
(1/3)*x])/b^(5/3) + (a^(1/3)*(-2*b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g + 2*
a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 6*c*Lo
g[a + b*x^3])/(18*a^2)
```

Maple [B] time = 0.012, size = 507, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/3/(b*x^3+a)/b*x^2*h+1/3/a*x^2/(b*x^3+a)*e-1/3/(b*x^3+a)/b*x*g+1/3/a*x/(b
*x^3+a)*d-1/3/b/(b*x^3+a)*f+1/3/a/(b*x^3+a)*c+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1
/b*a)^(1/3))*g+2/9/b/a*d/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/18/b^2/(1/b*a)
^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/9/b/a*d/(1/b*a)^(2/3)*ln(x
^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+2/9/b/a*d/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-2/9/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*h
-1/9/a/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+1/9/b^2/(1/b*a)^(1/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h+1/18/a/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*e+2/9/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/
b*a)^(1/3)*x-1))*h+1/9/a/b*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))*e-1/3*c*ln(b*x^3+a)/a^2+c*ln(x)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 156.896, size = 28260, normalized size = 97.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="frica
s")
```

```
[Out] 1/324*(108*a*b*c - 108*a^2*f + 108*(a*b*e - a^2*h)*x^2 - 2*(a^2*b^2*x^3 + a
^3*b)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
 (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^
2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 +
a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h
+ 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 -
(g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^
2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1
/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e +
2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^
3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3
*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 1
2*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*
(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54
*c/a^2)*log(12*b^4*c*d^2 + 9*b^4*c^2*e + 4*a*b^3*d*e^2 + 3*a^2*b^2*c*g^2 +
1/324*(a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 +
 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 +
 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b
^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^
```

3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*
(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g
*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c
*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(
9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/
1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3
+ 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*
c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*
b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*
b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2 + 8*(2*a^3*b*d + a^4*g)*h^2 - 1/18*(4*a
^2*b^4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c*h)*
((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g
+ 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e
+ 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*
e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a
^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 -
12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g +
9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) +
81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*
g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12
*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h
^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^
2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g +
4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2
) + 2*(6*a*b^3*c*d + a^2*b^2*e^2)*g + 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e + 4*a^
3*b*e*g)*h + (8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^
3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x) + 108*(a*b*d - a
^2*g)*x - (162*b^2*c*x^3 + 162*a*b*c - (a^2*b^2*x^3 + a^3*b)*((-I*sqrt(3) +
1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b
)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
(e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3
*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8
*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4
b - 6(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h
)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3)
+ 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4
*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g +
6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3
)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(
d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2
b^3 - 2(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2) - 3*sqrt(1/3
)*(a^2*b^2*x^3 + a^3*b)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 +
2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 +
1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^
3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3
*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(
27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*
h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*
d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9
*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1
458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 +
6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c
^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b
^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b
^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((-I*sqrt(3) + 1)*(9*c^2/a
^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))
/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h
)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a
^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a

```
^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^
2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3
 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*
c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)
*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d
*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5)
- 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*
h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*
d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*a^2*b^3*c + 2916*b^3*c^2
 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g)*h)/(a^4*b^3))
)*log(-12*b^4*c*d^2 - 9*b^4*c^2*e - 4*a*b^3*d*e^2 - 3*a^2*b^2*c*g^2 - 1/324
*(a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*
b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/16
2*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) +
 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g
^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b
^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a
^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)
*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3
*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*
(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a
^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 +
 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 +
 (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/
(a^6*b^5))^(1/3) + 54*c/a^2)^2 - 8*(2*a^3*b*d + a^4*g)*h^2 + 1/18*(4*a^2*b^
4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c*h)*((-I*
sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d
*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a
^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 +
 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*
e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e
*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*
g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I
*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
 (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^
3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 +
8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^
4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*
h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2) - 2
*(6*a*b^3*c*d + a^2*b^2*e^2)*g - 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e + 4*a^3*b*e
*g)*h + 2*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b
*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x + 1/108*sqrt(1/3)*(7
2*a^2*b^4*d^2 - 54*a^2*b^4*c*e + 72*a^3*b^3*d*g + 18*a^4*b^2*g^2 - 108*a^3*
b^3*c*h + (a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c
^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a
^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a
^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
+ a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1
458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3
*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 -
 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/1
62*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3)
+ 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*
g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*
b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*
a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e
)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4
- (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-
1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a
```

^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*
b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*
b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 -
e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 -
2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3
/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/
(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^
2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1
/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h -
3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3
- 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((-I*sqrt(3
) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^
2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h
+ (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*
b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2
+ 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*
a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*
d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(
3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g
+ 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*
g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*
h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b -
6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*
a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*a^2*b^3*c
+ 2916*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g
)*h)/(a^4*b^3))) - (162*b^2*c*x^3 + 162*a*b*c - (a^2*b^2*x^3 + a^3*b)*((-I*
sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d
*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a
^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 +
12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*
e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e
*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*
g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I
sqrt(3) + 1)(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h +
(e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^
3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 +
8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^
4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*
h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2) + 3
sqrt(1/3)(a^2*b^2*x^3 + a^3*b)*sqrt(-(((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b
^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c
^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*
c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*
g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) -
1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h
- 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d
^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 +
1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b
^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^
3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*
(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g
*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c
*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((-I*sqrt(3) + 1)
*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(
a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*
g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^
2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^
4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b
- 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c

```
)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1
)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*
h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*
a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(
a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g
^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^
3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*a^2*b^3*c + 291
6*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*d + a^3*g)*h)/(
a^4*b^3)))*log(-12*b^4*c*d^2 - 9*b^4*c^2*e - 4*a*b^3*d*e^2 - 3*a^2*b^2*c*g^
2 - 1/324*(a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c
^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a
^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a
^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
+ a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1
458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3
*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 -
 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/1
62*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3)
+ 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*
g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*
b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*
a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e
)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2 - 8*(2*a^3*b*d + a^4*g)*h^2 + 1/18*
(4*a^2*b^4*d^2 + 6*a^2*b^4*c*e + 4*a^3*b^3*d*g + a^4*b^2*g^2 + 12*a^3*b^3*c
*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (
e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*
d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*
b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h +
12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g
^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*
g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3
) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*
a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3
+ 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b
*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*
e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e
*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c
/a^2) - 2*(6*a*b^3*c*d + a^2*b^2*e^2)*g - 2*(9*a*b^3*c^2 + 8*a^2*b^2*d*e +
4*a^3*b*e*g)*h + 2*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^
2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)*x - 1/108*sqr
t(1/3)*(72*a^2*b^4*d^2 - 54*a^2*b^4*c*e + 72*a^3*b^3*d*g + 18*a^4*b^2*g^2 -
 108*a^3*b^3*c*h + (a^4*b^4*e + 2*a^5*b^3*h)*((-I*sqrt(3) + 1)*(9*c^2/a^4 -
 (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4*b^3))/(-1
/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^
2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b
^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b
^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 -
e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2
*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*c^3/
a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(
a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2
 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/
1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h -
3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3
- 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9
*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*a^2*b)/(a^4
*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g +
 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g
 + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h
```

```
^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6
*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a
^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81*(I*sqrt(3) + 1)*(
-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g + 4*d*h)*
a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a*b^3*d^2*g + 6*a^2
*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2 + 8*a^4*h^3)/(a^5
*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)*a^4*b - 6*(d*g^2
- e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4*d*h)*c)*a^2*b^3 -
 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)^2*a^4*b^3 - 108*((
-I*sqrt(3) + 1)*(9*c^2/a^4 - (9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*h + (e*g +
4*d*h)*a^2*b)/(a^4*b^3))/(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e +
2*a^3*g*h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^
3 + 12*a*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3
*b*e*h^2 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 1
2*e*h^2)*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*
(e*g + 4*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 81
*(I*sqrt(3) + 1)*(-1/27*c^3/a^6 + 1/162*(9*b^3*c^2 + 2*a*b^2*d*e + 2*a^3*g*
h + (e*g + 4*d*h)*a^2*b)*c/(a^6*b^3) + 1/1458*(8*b^4*d^3 + a*b^3*e^3 + 12*a
*b^3*d^2*g + 6*a^2*b^2*d*g^2 + a^3*b*g^3 + 6*a^2*b^2*e^2*h + 12*a^3*b*e*h^2
 + 8*a^4*h^3)/(a^5*b^5) - 1/1458*(27*b^5*c^3 + 8*a^5*h^3 - (g^3 - 12*e*h^2)
*a^4*b - 6*(d*g^2 - e^2*h - 3*c*g*h)*a^3*b^2 + (e^3 - 12*d^2*g + 9*(e*g + 4
*d*h)*c)*a^2*b^3 - 2*(4*d^3 - 9*c*d*e)*a*b^4)/(a^6*b^5))^(1/3) + 54*c/a^2)*
a^2*b^3*c + 2916*b^3*c^2 + 2592*a*b^2*d*e + 1296*a^2*b*e*g + 2592*(2*a^2*b*
d + a^3*g)*h)/(a^4*b^3))) + 324*(b^2*c*x^3 + a*b*c)*log(x))/(a^2*b^2*x^3 +
a^3*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08568, size = 452, normalized size = 1.56

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{c\log\left(|x|\right)}{a^2}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}b^2d+\left(-ab^2\right)^{\frac{1}{3}}abg-2\left(-ab^2\right)^{\frac{2}{3}}ah-\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/9*sqrt(3)*(2*(-a*b^2
)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g - 2*(-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3
)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/
3*(a*b*c - a^2*f - (a^2*h - a*b*e)*x^2 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^
2*b) + 1/18*(2*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g + 2*(-a*b^2)^(2/
```

```
3)*a*h + (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*
b^3) - 1/9*(2*a^4*b^2*h*(-a/b)^(1/3) + a^3*b^3*(-a/b)^(1/3)*e + 2*a^3*b^3*d
 + a^4*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

3.418 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=301

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{9a^{7/3}b^{4/3}}+\frac{\text{ta}}{}$$

```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3
*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b
^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) +
a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*
a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.593431, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{18a^{7/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{9a^{7/3}b^{4/3}}+\frac{\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(c/(a^2*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3
*a^2*b*(a + b*x^3)) + ((4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)
*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b
^(4/3)) + (d*Log[x])/a^2 + ((b^(2/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(4/3)) - ((b^(2/3)*(4*b*c - a*f) +
a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*
a^(7/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a^2)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-b(2be+ah)x^2+b^2\left(\frac{bc}{a}-f\right)x^3}{x^2\left(a+bx^3\right)}}{3ab^2}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} - \frac{\int \left(-\frac{3b^2c}{ax^2}-\frac{3b^2d}{ax}+\frac{b\left(-a(2be+ah)+b(4bc\right.}{a\left(a+bx\right.}\right.}{3ab^2}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{\int \frac{-a(2be+ah)+b(4}{a+}}{3}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{\int \frac{-a(2be+ah)+b(4}{a+bx^3}}{3a^2b}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^3\right)}{3a^2}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\left(b^{2/3}(4bc-af\right.}{}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{d\log(x)}{a^2} + \frac{\left(b^{2/3}(4bc-af\right.}{}$$

$$= -\frac{c}{a^2x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{3a^2b\left(a+bx^3\right)} + \frac{\left(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f\right.}{3\sqrt{3}a}$$

Mathematica [A] time = 0.298581, size = 285, normalized size = 0.95

$$-\frac{\frac{6a\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{b\left(a+bx^3\right)} + \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{2/3}be+a^{5/3}h-ab^{2/3}f+4b^{5/3}c\right)}{b^{4/3}} - \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{2/3}be+a^{5/3}h-ab^{2/3}f+4b^{5/3}c\right)}{b^{4/3}}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))/(b
*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^(2/3)*b*e + a*b^(2/3
)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/3) - 18*
a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3
)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*
b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/b^(4/3) + 6*a*d*Log[a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.014, size = 517, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2/(b*x^3+a)*f-1/3*b/a^2*x^2/(b*x^3+a)*c-1/3/(b*x^3+a)/b*x*h+1/3/a*x
/(b*x^3+a)*e-1/3/(b*x^3+a)/b*g+1/3/a/(b*x^3+a)*d+1/9/b^2/(1/b*a)^(2/3)*ln(x
+(1/b*a)^(1/3))*h+2/9/a/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-1/18/b^2/(1/b
*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h-1/9/a/b/(1/b*a)^(2/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+2/9/a/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/9/a*f/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))+4/9/a^2*c/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*f/b/(1/b*a)^(1/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-2/9/a^2*c/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^
(1/3)*x+(1/b*a)^(2/3))+1/9/a*f*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))-4/9/a^2*c*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))-1/3*d*ln(b*x^3+a)/a^2+d*ln(x)/a^2-1/a^2*c/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10244, size = 473, normalized size = 1.57

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{d\log\left(|x|\right)}{a^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h+2\left(-ab^2\right)^{\frac{1}{3}}abe+4\left(-ab^2\right)^{\frac{2}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 + 1/9*sqrt(3)*((-a*b^2)^
(1/3)*a^2*h + 2*(-a*b^2)^(1/3)*a*b*e + 4*(-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3
)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) - 1/
3*(4*b^2*c*x^3 - a*b*f*x^3 + a^2*h*x^2 - a*b*x^2*e - a*b*d*x + a^2*g*x + 3*
a*b*c)/((b*x^4 + a*x)*a^2*b) + 1/18*((-a*b^2)^(1/3)*a^2*h + 2*(-a*b^2)^(1/3
)*a*b*e - 4*(-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(a^3*b^2) + 1/9*(4*a^2*b^4*c*(-a/b)^(1/3) - a^3*b^3*f*(-
a/b)^(1/3) - a^4*b^2*h - 2*a^3*b^3*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^5*b^3)
```

3.419 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=306

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{18a^{8/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{9a^{8/3}b^{2/3}}+$$

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^
2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f - a
^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d
 - a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((b^(1/3)*(5*b*c -
 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(8/3)*b^(2/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] time = 0.576867, antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}-2af+5bc\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - d/(a^2*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^
2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*c + 4*a^(1/3)*b*d - 2*a*b^(1/3)*f - a
^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
8/3)*b^(2/3)) + (e*Log[x])/a^2 - ((b^(1/3)*(5*b*c - 2*a*f) - a^(1/3)*(4*b*d
 - a*g))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^(2/3)) + ((5*b*c - 2*a*f -
(a^(1/3)*(4*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(8/3)*b^(1/3)) - (e*Log[a + b*x^3])/(3*a^2)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+2b^2\left(\frac{bc}{a}-f\right)x^3+b^2\left(\frac{b}{}\right.}{x^3\left(a+bx^3\right)}}{3ab^2}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^2c}{ax^3}-\frac{3b^2d}{ax^2}-\frac{3b^2e}{ax}+\frac{b^2(5bc-2af}{a}\right.}{3ab^2}$$

$$= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{e\log(x)}{a^2}-\frac{\int \frac{5bc-2a}{}}{}$$

$$= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{e\log(x)}{a^2}-\frac{\int \frac{5bc-2a}{}}{}$$

$$= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{e\log(x)}{a^2}-\frac{e\log\left(a\right.}{3a}$$

$$= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{e\log(x)}{a^2}-\frac{\left(5bc-\right.}{}$$

$$= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{e\log(x)}{a^2}-\frac{\left(5bc-\right.}{}$$

$$= -\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{3a^2\left(a+bx^3\right)}+\frac{\left(5b^{4/3}c+4\sqrt[3]{a}bd-\right.}{}$$

Mathematica [A] time = 0.488182, size = 292, normalized size = 0.95

$$-\frac{\frac{6a\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{b\left(a+bx^3\right)}-\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g-4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{b^{2/3}}+\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}g-4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/}\right.}{b^{2/3}}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f
+ g*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*c - 4*a^(1/3)*b*
d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^
(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(5*b^(4/3
)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/(18*a^3)
```

Maple [B] time = 0.013, size = 527, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a/(b*x^3+a)*x^2*g-1/3/a^2*b*x^2/(b*x^3+a)*d+1/3/a*x/(b*x^3+a)*f-1/3*b/a
^2*x/(b*x^3+a)*c-1/3/(b*x^3+a)/b*h+1/3/a/(b*x^3+a)*e-5/9/a^2*c/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))+5/18/a^2*c/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))-5/9/a^2*c/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/
3)*x-1))+2/9/a*f/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/9/a*f/b/(1/b*a)^(2/3
)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/9/a*f/b/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+4/9/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(
1/3))*d-2/9/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-4/9/a
^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-1/9/a*
g/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/18/a*g/b/(1/b*a)^(1/3)*ln(x^2-(1/b*
a)^(1/3)*x+(1/b*a)^(2/3))+1/9/a*g*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2
)*(2/(1/b*a)^(1/3)*x-1))-1/3*e*ln(b*x^3+a)/a^2-d/a^2/x-1/2*c/a^2/x^2+e*ln(x
)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 121.35, size = 28467, normalized size = 93.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] -1/324*(108*(4*b^2*d - a*b*g)*x^4 + 324*a*b*d*x + 54*(5*b^2*c - 2*a*b*f)*x^
3 + 162*a*b*c - 108*(a*b*e - a^2*h)*x^2 + 2*(a^2*b^2*x^5 + a^3*b*x^2)*((-I*
sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)
*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*
d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^
3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^
2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g
 + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a
^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*s
qrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f
 - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c
^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 -
 a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g +
6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*
b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*
log(-160*a*b^3*c*d^2 + 75*a*b^3*c^2*e - 36*a^2*b^2*d*e^2 + 12*a^3*b*e*f^2 -
 1/324*(4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d +
 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(
```

20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(1
25*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^
3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^
4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f
 + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*
f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*
b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*
b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 -
 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c
^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f +
16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*
a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2 - 2*(5*a^3*b*c - 2*a^4*f)*g^2 - 1/18*
(25*a^3*b^3*c^2 - 24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e
g)((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f
 - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9
*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 -
 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a
^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3
- 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*
e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3)
+ 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^
2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 15
0*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*
b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9
*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g
)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 5
4*e/a^2) + 4*(16*a^2*b^2*d^2 - 15*a^2*b^2*c*e)*f + (80*a^2*b^2*c*d + 9*a^3*
b*e^2 - 32*a^3*b*d*f)*g - (125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 6
0*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3
)*x) + (162*b^2*e*x^5 + 162*a*b*e*x^2 - (a^2*b^2*x^5 + a^3*b*x^2)*((-I*sqrt
(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b
)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f
- 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^
2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 -
a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6
*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b
^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(
3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5
*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f
 + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4
*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*
g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2
- 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) - 3*
sqrt(1/3)*(a^2*b^2*x^5 + a^3*b*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (
20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a
^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b)
 - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2
- 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/
1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^
3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d
*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6
+ 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) -
1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8
*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/145
8*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 -
 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e
+ 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3)
+ 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a
^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*

c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f
+ 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*
g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g
^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 -
2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) +
1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g
)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 6
0*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3
)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)
*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*
(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e
+ 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g
)/(a^5*b)))*log(160*a*b^3*c*d^2 - 75*a*b^3*c^2*e + 36*a^2*b^2*d*e^2 - 12*a^
3*b*e*f^2 + 1/324*(4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (2
0*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^
6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b)
- 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 -
8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1
458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3
- 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*
e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 +
1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1
/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*
a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458
*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 -
24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e +
75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2 + 2*(5*a^3*b*c - 2*a^4*f)*
g^2 + 1/18*(25*a^3*b^3*c^2 - 24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2
+ 6*a^5*b*e*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*
e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a
^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*
a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d
^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3
- 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*
(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b
^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*
f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b
^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*
g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2
*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*
f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2)
)^(1/3) + 54*e/a^2) - 4*(16*a^2*b^2*d^2 - 15*a^2*b^2*c*e)*f - (80*a^2*b^2*c
*d + 9*a^3*b*e^2 - 32*a^3*b*d*f)*g - 2*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*
b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*
g^2 - a^4*g^3)*x + 1/108*sqrt(1/3)*(450*a^3*b^3*c^2 + 216*a^4*b^2*d*e - 360
*a^4*b^2*c*f + 72*a^5*b*f^2 - 54*a^5*b*e*g - (4*a^6*b^2*d - a^7*b*g)*((-I*s
qrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*
a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d
*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3
*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2
- a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g
+ 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^
2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sq
rt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f
- 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^
2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 -
a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6
*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b
^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2))*

sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g)/(a^5*b))) + (162*b^2*e*x^5 + 162*a*b*e*x^2 - (a^2*b^2*x^5 + a^3*b*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) + 3*sqrt(1/3)*(a^2*b^2*x^5 + a^3*b*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*

f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*
b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e
*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^
2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(2
0*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(12
5*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3
 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4
*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f
 + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f
)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^
2 - 10368*a*b*d*f - 1296*(5*a*b*c - 2*a^2*f)*g)/(a^5*b)))*log(160*a*b^3*c*d
^2 - 75*a*b^3*c^2*e + 36*a^2*b^2*d*e^2 - 12*a^3*b*e*f^2 + 1/324*(4*a^6*b^2*
d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^
2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2
*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*
b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2
*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 -
2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4
*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2
))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*
g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3
*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g
 + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(
4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^
2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^
(1/3) + 54*e/a^2)^2 + 2*(5*a^3*b*c - 2*a^4*f)*g^2 + 1/18*(25*a^3*b^3*c^2 -
24*a^4*b^2*d*e - 20*a^4*b^2*c*f + 4*a^5*b*f^2 + 6*a^5*b*e*g)*((-I*sqrt(3) +
 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^
5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c
*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f +
 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g
^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^
2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 -
2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) +
1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)
*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60
*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)
/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*
a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(
32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2) - 4*(16*a
^2*b^2*d^2 - 15*a^2*b^2*c*e)*f - (80*a^2*b^2*c*d + 9*a^3*b*e^2 - 32*a^3*b*d
*f)*g - 2*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2
 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)*x - 1/108*sqrt
(1/3)*(450*a^3*b^3*c^2 + 216*a^4*b^2*d*e - 360*a^4*b^2*c*f + 72*a^5*b*f^2 -
 54*a^5*b*e*g - (4*a^6*b^2*d - a^7*b*g)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*
b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-1/27*e^3/a^6
 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) -
1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8
*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/145
8*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 -
 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e
 + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1
/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1
458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^
3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(
125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24
*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 7
5*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2))*sqrt(-(((-I*sqrt(3) + 1)*(9*e
^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)/(a^5*b))/(-

```
1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)
*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*
b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8
*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b
 + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^
3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/2
7*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*g)*a*b)*e/
(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3*c^2*f + 60*a^2*b^2
*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2 - a^4*g^3)/(a^8*b^
2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g + 6*d*g^2)*a^3*b +
3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^2*b^2 - 2*(32*d^3 -
 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2)^2*a^5*b - 108*((-
I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d*f - 5*c*
g)*a*b)/(a^5*b))/(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 -
8*d*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*
b^3*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*
g^2 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f
*g + 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)
*a^2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 81*(I
*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b^2*c*d + 2*a^2*f*g + (9*e^2 - 8*d
*f - 5*c*g)*a*b)*e/(a^7*b) - 1/1458*(125*b^4*c^3 + 64*a*b^3*d^3 - 150*a*b^3
*c^2*f + 60*a^2*b^2*c*f^2 - 8*a^3*b*f^3 - 48*a^2*b^2*d^2*g + 12*a^3*b*d*g^2
 - a^4*g^3)/(a^8*b^2) - 1/1458*(125*b^4*c^3 + a^4*g^3 - 2*(4*f^3 - 9*e*f*g
+ 6*d*g^2)*a^3*b + 3*(9*e^3 - 24*d*e*f + 16*d^2*g + 5*(4*f^2 - 3*e*g)*c)*a^
2*b^2 - 2*(32*d^3 - 90*c*d*e + 75*c^2*f)*a*b^3)/(a^8*b^2))^(1/3) + 54*e/a^2
)*a^3*b*e + 25920*b^2*c*d + 2916*a*b*e^2 - 10368*a*b*d*f - 1296*(5*a*b*c -
2*a^2*f)*g)/(a^5*b))) - 324*(b^2*e*x^5 + a*b*e*x^2)*log(x))/(a^2*b^2*x^5 +
a^3*b*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07838, size = 483, normalized size = 1.58

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{e\log\left(|x|\right)}{a^2}-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}b^2c-2\left(-ab^2\right)^{\frac{1}{3}}abf-4\left(-ab^2\right)^{\frac{2}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right.\right.}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/9*sqrt(3)*(5*(-a*b^2
)^(1/3)*b^2*c - 2*(-a*b^2)^(1/3)*a*b*f - 4*(-a*b^2)^(2/3)*b*d + (-a*b^2)^(2
```

```
/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) -
1/18*(5*(-a*b^2)^(1/3)*b^2*c - 2*(-a*b^2)^(1/3)*a*b*f + 4*(-a*b^2)^(2/3)*b*
d - (-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
+ 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) - a^3*b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^
3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d -
a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*b*c + 2*(a^2*h - a*b
*e)*x^2)/((b*x^3 + a)*a^2*b*x^2)
```

3.420 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=338

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{18a^{8/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{9a^{8/3}b^{2/3}}+$$

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*x
- b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b*e
 - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*
(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/
3)*b^(2/3)) + ((b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(2/3)) + ((2*b*c - a*f)
*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.727317, antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4be-ah)}{\sqrt[3]{b}}-2ag+5bd\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)}{9a^{8/3}b^{2/3}}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -c/(3*a^2*x^3) - d/(2*a^2*x^2) - e/(a^2*x) - (x*(b*d - a*g + (b*e - a*h)*x
- b*((b*c)/a - f)*x^2))/(3*a^2*(a + b*x^3)) + ((5*b^(4/3)*d + 4*a^(1/3)*b*e
 - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(8/3)*b^(2/3)) - ((2*b*c - a*f)*Log[x])/a^3 - ((b^(1/3)*
(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/
3)*b^(2/3)) + ((5*b*d - 2*a*g - (a^(1/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)*b^(1/3)) + ((2*b*c - a*f)
*Log[a + b*x^3])/(3*a^3)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}dx = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int \frac{-3b^2c-3b^2dx-3b^2ex^2+3b^2\left(\frac{bc}{a}-f\right)x^3+2b^2}{x^4\left(a+bx^3\right)}}{3ab^2}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int\left(-\frac{3b^2c}{ax^4}-\frac{3b^2d}{ax^3}-\frac{3b^2e}{ax^2}-\frac{3b^2(-2bc+}{a^2x}\right.}{}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-a}{a}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-a}{a}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-a}{a}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-a}{a}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} - \frac{(2bc-a}{a}$$

$$= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2\left(a+bx^3\right)} + \frac{\left(5b^{4/3}d\right.}{}$$

Mathematica [A] time = 0.454282, size = 303, normalized size = 0.9

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{2/3}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}h-4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h-4\sqrt[3]{}\right.}{b^{2/3}}}{18a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e*x)) + 6*a
*(f + x*(g + h*x))))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*d - 4*a^(
1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(2/3) + 18*(-2*b*c + a*f)*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d - 4*a^
(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) +
(a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*(2*b*c - a*f)*Log[a + b
*x^3])/(18*a^3)
```

Maple [B] time = 0.014, size = 561, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*b*x/(b*x^3+a)*d+4/9/a^2*e/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-2/9/a^
2*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/2*d/a^2/x^2-e/a^2
/x+2/9/a*g/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)
)-1/3/a^2*x^2/(b*x^3+a)*b*e-1/3/a^2*ln(b*x^3+a)*f+1/3/a/(b*x^3+a)*f+1/a^2*l
n(x)*f+1/3/a/(b*x^3+a)*x^2*h+1/3/a/(b*x^3+a)*g*x-1/3*c/x^3/a^2-1/9/a*g/b/(1
/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/9/a*h/b/(1/b*a)^(1/3)*l
n(x+(1/b*a)^(1/3))+1/18/a*h/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))-5/9/a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-
1))*d+1/9/a*h*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))+2/9/a*g/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-4/9/a^2*e*3^(1/2)/(1/b*a)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-5/9/a^2/(1/b*a)^(2/3)*ln(x+
(1/b*a)^(1/3))*d+5/18/a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3
))*d-1/3/a^2*b/(b*x^3+a)*c-2*b*c*ln(x)/a^3+2/3*b*c*ln(b*x^3+a)/a^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09594, size = 520, normalized size = 1.54

$$\frac{(2\,bc-af)\log\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{(2\,bc-af)\log\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{1}{3}}b^2d-2\,\left(-ab^2\right)^{\frac{1}{3}}abg+\left(-ab^2\right)^{\frac{2}{3}}ah-4\,\left(-ab^2\right)^{\frac{2}{3}}be\right)}{9\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="gia
c")
```

```
[Out] 1/3*(2*b*c - a*f)*log(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*log(abs(x))/a^3 -
 1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*d - 2*(-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2
/3)*a*h - 4*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^3*b^2) - 1/18*(5*(-a*b^2)^(1/3)*b^2*d - 2*(-a*b^2)^(1/3)*a*b*
g - (-a*b^2)^(2/3)*a*h + 4*(-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/(a^3*b^2) - 1/9*(a^5*b*h*(-a/b)^(1/3) - 4*a^4*b^2*(-a/b)^(1/3)
*e - 5*a^4*b^2*d + 2*a^5*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*
b) + 1/6*(2*(a^2*h - 4*a*b*e)*x^5 - (5*a*b*d - 2*a^2*g)*x^4 - 6*a^2*x^2*e -
 3*a^2*d*x - 2*(2*a*b*c - a^2*f)*x^3 - 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

3.421 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=345

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}}-\,\mathrm{ta}$$

```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^
3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d
- 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b
^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e -
7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c +
5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.891265, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {1828, 1858, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{54a^{4/3}b^{10/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(5af+bc)-2a^{2/3}(be-7ah)\right)}{27a^{4/3}b^{10/3}}-\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```


```
[Out] (h*x)/b^3 + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^
3*(a + b*x^3)^2) - (x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d
- 3*a*g)*x^2))/(18*a*b^3*(a + b*x^3)) - ((b^(5/3)*c + 2*a^(2/3)*b*e + 5*a*b
^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(9*Sqrt[3]*a^(4/3)*b^(10/3)) - ((b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e -
7*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(10/3)) + ((b^(2/3)*(b*c +
5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(4/3)*b^(10/3)) + (g*Log[a + b*x^3])/(3*b^3)
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{\int \frac{a^2(be-ah)-2ab(bc-af)x-3ab(bd-ag}{}}{}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(bc-4a\right.}{18ab^3\left(a+\right.}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(bc-4a\right.}{18ab^3\left(a+\right.}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(b\right.}{18ab^3}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(b\right.}{18ab^3}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(b\right.}{18ab^3}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(b\right.}{18ab^3}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(b\right.}{18ab^3}$$

$$= \frac{hx}{b^3} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6b^3\left(a+bx^3\right)^2} - \frac{x\left(a(7be-13ah)-2b(b\right.}{18ab^3}$$

Mathematica [A] time = 0.305199, size = 342, normalized size = 0.99

$$\frac{-\frac{9b^{2/3}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{\left(a+bx^3\right)^2}+\frac{3b^{2/3}\left(a^2(12g+13hx)-ab(6d+x(7e+8fx))+2b^2cx^2\right)}{a\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^{2/3}b^{4/3}e+14a^{5/3}\sqrt[3]{b}h+5abf+b^2\right.}{a^{4/3}}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(2/3)*h*x - (9*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f
*x))))/(a + b*x^3)^2 + (3*b^(2/3)*(2*b^2*c*x^2 + a^2*(12*g + 13*h*x) - a*b*
(6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + 2*a^(2/3)*b
^(4/3)*e + 5*a*b*f - 14*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/a^(4/3) - (2*(b^2*c - 2*a^(2/3)*b^(4/3)*e + 5*a*b*f + 14*a^(5/
3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - 2*a^(2/3)*b^(4/
```

```
3)*e + 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/a^(4/3) + 18*b^(2/3)*g*Log[a + b*x^3])/(54*b^(11/3))
```

Maple [B] time = 0.011, size = 619, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] -4/9/(b*x^3+a)^2/b*x^5*f-5/27/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f+5/54/
b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+13/18/b^2/(b*x^3+
a)^2*x^4*a*h+2/3/b^2/(b*x^3+a)^2*x^3*a*g+2/27/b^3/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+5/9/b^3/(b*x^3+a)^2*a^2*h*x-14/27/
b^4*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*h-2/9/b^2/(b*x^3+a)^2*a*e*x+h*x/b^3
-1/3/b/(b*x^3+a)^2*d*x^3+1/2/b^3/(b*x^3+a)^2*a^2*g-1/6/b^2/(b*x^3+a)^2*a*d+
5/27/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f-
5/18*a/(b*x^3+a)^2/b^2*x^2*f+7/27/b^4*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))*h-1/27/b^2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c+1/54/b^2/
a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-14/27/b^4*a/(1/b*a)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/3*g*ln(b*x^3+a
)/b^3-7/18/b/(b*x^3+a)^2*x^4*e+2/27/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e
-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+1/9/(b*x^3+
a)^2/a*x^5*c-1/18/b/(b*x^3+a)^2*x^2*c+1/27/b^2/a*3^(1/2)/(1/b*a)^(1/3)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 34.1865, size = 30286, normalized size = 87.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/108*(108*a*b^2*h*x^7 + 12*(b^3*c - 4*a*b^2*f)*x^5 - 42*(a*b^2*e - 7*a^2*b
*h)*x^4 - 18*a^2*b*d + 54*a^3*g - 36*(a*b^2*d - 2*a^2*b*g)*x^3 - 6*(a*b^2*c
 + 5*a^2*b*f)*x^2 - 2*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*
(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*
f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h
```

)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*
a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176
*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*
a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g
+ 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)
/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*
c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3
 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 16
8*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 +
15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b +
 (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2
 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*log(-4*a*b^4*c^2*e - 4
0*a^2*b^3*c*e*f - 100*a^3*b^2*e*f^2 + 36*a^3*b^2*e^2*g + 1764*a^5*g*h^2 - 1
/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 -
(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(145
8*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*
g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2
+ 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a
^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*
h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^
2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3
)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2
*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2
*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*
h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 -
 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2
*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^1
0))^(1/3) - 18*g/b^3)^2 - 81*(a^3*b^2*c + 5*a^4*b*f)*g^2 + (2*a^3*b^5*e^2 -
 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*b^4*f)*g)*(2*(1/2)^
(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2
*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 -
70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3
 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h
+ 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f +
 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*
e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^
2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(
2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b
^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^
3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*
c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a
^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(
25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 28*(a^2*b^3*c^
2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - (b^5*c^3 + 8*a^2*b^3*
e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2
*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x) - 24*(a^2*b*e - 7*a^3*h)*x + (54*a
*b^2*g*x^6 + 108*a^2*b*g*x^3 + 54*a^3*g + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*
b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 7
0*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*
e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 +
 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*
a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15
*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (
125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 -
 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*
g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/
(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 +
125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4
*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h

```
+ 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2
- (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) -
3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f -
 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a
^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b
^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^
4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5
*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 1
68*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a
^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e
 + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 +
8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a
^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*
a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (1
25*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 -
18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(
2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*
(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 7
0*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3
+ 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h +
 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f +
2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e
*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2
*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2
*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^
5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3
 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c
^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^
4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(2
5*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^
2*c*e + 160*a*b*e*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6)))*lo
g(4*a*b^4*c^2*e + 40*a^2*b^3*c*e*f + 100*a^3*b^2*e*f^2 - 36*a^3*b^2*e^2*g -
 1764*a^5*g*h^2 + 1/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a
*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5
*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f
+ 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2
 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*
(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h
+ 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^
2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3
*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^
2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2
*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 -
270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c
)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2 + 81*(a^3*b^2*c + 5*a^4*b*f)*g^2
 - (2*a^3*b^5*e^2 - 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*
b^4*f)*g)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^
2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b
^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*
c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 -
 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3
 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*
b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*
f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*
b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f
```

```
^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)
/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f
*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3
*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^
3) - 28*(a^2*b^3*c^2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - 2*
(b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*
f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x + 3/4*sqrt(1/3
)*(8*a^3*b^5*e^2 - 112*a^4*b^4*e*h + 392*a^5*b^3*h^2 + (a^3*b^8*c + 5*a^4*b
^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 -
70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c
*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3
+ 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168
*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 1
5*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b +
(125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2
- 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458
*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g
/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 +
 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^
4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h
 + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2
 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) +
 18*(a^3*b^5*c + 5*a^4*b^4*f)*g)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81
*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2
*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7
*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2
*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*
a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3
 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*
g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f
*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 1
5*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 11
76*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 274
4*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*
g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^
3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a
*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5
*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f
+ 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2
 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*
(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h
+ 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^
2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3
*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^
2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2
*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 -
270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c
)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160*a*b*e
*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6))) + (54*a*b^2*g*x^6 +
 108*a^2*b*g*x^3 + 54*a^3*g + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2
)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 +
 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2
- 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e
^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*
h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f
 + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 27
```

0*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*
a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27
*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) -
(b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*
f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^
5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)
*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3
*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 3*sqrt(1/3)*
(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)
/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*
f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 7
5*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 -
2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(24
3*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 3
78*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1
/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 -
70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^
3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h
+ 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f
+ 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270
*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a
^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c
*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 +
2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c
^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*
e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3
- 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e
^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b
^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (
81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^
2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b
^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^
4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f
^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e
*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160*
a*b*e*f + 324*a^2*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6)))*log(4*a*b^4*c^
2*e + 40*a^2*b^3*c*e*f + 100*a^3*b^2*e*f^2 - 36*a^3*b^2*e^2*g - 1764*a^5*g*
h^2 + 1/4*(a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2
/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6
))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h
)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3
*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*
h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 6
30*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)
*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*
a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*
b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a
^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^
5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g +
168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(
a^4*b^10))^(1/3) - 18*g/b^3)^2 + 81*(a^3*b^2*c + 5*a^4*b*f)*g^2 - (2*a^3*b^
5*e^2 - 28*a^4*b^4*e*h + 98*a^5*b^3*h^2 - 9*(a^3*b^5*c + 5*a^4*b^4*f)*g)*(2
(1/2)^(2/3)(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*
a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81
*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*
b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2

```
*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*
c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3
 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g
)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9
 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^
9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3
*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10)
- (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e
*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^
3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) - 28*(a^2
*b^3*c^2 + 10*a^3*b^2*c*f + 25*a^4*b*f^2 - 18*a^4*b*e*g)*h - 2*(b^5*c^3 + 8
*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^
3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)*x - 3/4*sqrt(1/3)*(8*a^3*b^5
*e^2 - 112*a^4*b^4*e*h + 392*a^5*b^3*h^2 + (a^3*b^8*c + 5*a^4*b^7*f)*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2
+ 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2
 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*
e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2
*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*
f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 2
70*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)
*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 2
7*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) -
 (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2
*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b
^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2
)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 -
3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) - 18*g/b^3) + 18*(a^3*b^5
*c + 5*a^4*b^4*f)*g)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2/b^6 - (
2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6))/(1458
*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)*g
/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3*c*f^2 +
 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*h^3)/(a^
4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 630*f*g*h
 + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)*a^3*b^2
 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/2)^(1/3)
*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*
(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*
f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h
^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 -
3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*
h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10
))^(1/3) - 18*g/b^3)^2*a^2*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*g^2
/b^6 - (2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h)*a*b)/(a^2*b^6
))/(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*a^2 + 2*(5*e*f - 7*c*h
)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*b^4*c^2*f + 75*a^2*b^3
*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a^4*b*e*h^2 - 2744*a^5*
h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^5*h^3 - 3*(243*g^3 - 6
30*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g + 168*e^2*h + 378*c*g*h)
*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(a^4*b^10))^(1/3) + (1/
2)^(1/3)*(I*sqrt(3) + 1)*(1458*g^3/b^9 - 27*(2*b^2*c*e + (81*g^2 - 70*f*h)*
a^2 + 2*(5*e*f - 7*c*h)*a*b)*g/(a^2*b^9) - (b^5*c^3 + 8*a^2*b^3*e^3 + 15*a*
b^4*c^2*f + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 168*a^3*b^2*e^2*h + 1176*a
^4*b*e*h^2 - 2744*a^5*h^3)/(a^4*b^10) - (b^5*c^3 + 15*a*b^4*c^2*f + 2744*a^
5*h^3 - 3*(243*g^3 - 630*f*g*h + 392*e*h^2)*a^4*b + (125*f^3 - 270*e*f*g +
168*e^2*h + 378*c*g*h)*a^3*b^2 - (8*e^3 - 3*(25*f^2 - 18*e*g)*c)*a^2*b^3)/(
a^4*b^10))^(1/3) - 18*g/b^3)*a^2*b^3*g + 32*b^2*c*e + 160*a*b*e*f + 324*a^2
*g^2 - 224*(a*b*c + 5*a^2*f)*h)/(a^2*b^6))))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a
^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08362, size = 541, normalized size = 1.57

$$\frac{hx}{b^3}+\frac{g\log\left(\left|bx^3+a\right|\right)}{3\,b^3}-\frac{\sqrt{3}\left(14\,\left(-ab^2\right)^{\frac{1}{3}}a^2h-2\,\left(-ab^2\right)^{\frac{1}{3}}abe+\left(-ab^2\right)^{\frac{2}{3}}bc+5\,\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] h*x/b^3 + 1/3*g*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*a
^2*h - 2*(-a*b^2)^(1/3)*a*b*e + (-a*b^2)^(2/3)*b*c + 5*(-a*b^2)^(2/3)*a*f)*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) - 1/54*(14*
(-a*b^2)^(1/3)*a^2*h - 2*(-a*b^2)^(1/3)*a*b*e - (-a*b^2)^(2/3)*b*c - 5*(-a*
b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/18*(
2*(b^3*c - 4*a*b^2*f)*x^5 + (13*a^2*b*h - 7*a*b^2*e)*x^4 - 3*a^2*b*d + 9*a^
3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 + 2*(5*a^3*h
- 2*a^2*b*e)*x)/((b*x^3 + a)^2*a*b^3) - 1/27*(a*b^6*c*(-a/b)^(1/3) + 5*a^2*
b^5*f*(-a/b)^(1/3) - 14*a^3*b^4*h + 2*a^2*b^5*e)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a^3*b^7)
```

3.422 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=325

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan}{}$$

```
[Out] -(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*x^3)^2) +
(x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a +
b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((
b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*Log[a
+ b*x^3])/(3*b^3)
```

Rubi [A] time = 0.642445, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1828, 1858, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```


```
[Out] -(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*b^2*(a + b*x^3)^2) +
(x*(b*c - 7*a*f + 2*(b*d - 4*a*g)*x + 3*(b*e - 3*a*h)*x^2))/(18*a*b^2*(a +
b*x^3)) - ((b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((
b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(5/3)*b^(8/3)) - ((b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d + 5*a*g))*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3)) + (h*Log[a
+ b*x^3])/(3*b^3)
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} - \frac{\int \frac{-ab(bc-af)-2ab(bd-ag)x-3ab(be-ah)x^2 \cdots}{\left(a+bx^3\right)^2}}{6ab^3}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-4ag)x+3(b \cdots\right.}{18ab^2\left(a+bx^3\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-4ag)x+3(b \cdots\right.}{18ab^2\left(a+bx^3\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-4ag)x+3(b \cdots\right.}{18ab^2\left(a+bx^3\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-4ag)x+3(b \cdots\right.}{18ab^2\left(a+bx^3\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-4ag)x+3(b \cdots\right.}{18ab^2\left(a+bx^3\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6b^2\left(a+bx^3\right)^2} + \frac{x\left(bc-7af+2(bd-4ag)x+3(b \cdots\right.}{18ab^2\left(a+bx^3\right)}$$

Mathematica [A] time = 0.253258, size = 315, normalized size = 0.97

$$\frac{-\frac{9\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{\left(a+bx^3\right)^2}+\frac{36a^2h-3ab(6e+x(7f+8gx))+3b^2x(c+2dx)}{a\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}g+\sqrt[3]{a}bd-2a\sqrt[3]{b}f-b^{4/3}c\right)}{a^{5/3}}+\frac{2\sqrt[3]{b}\log\left(\cdots\right.}{}}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^3)^2 + (36
*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*g*x)))/(a*(a + b*x^3
)) - (2*Sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3
)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(b^(
4/3)*c - a^(1/3)*b*d + 2*a*b^(1/3)*f - 5*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x
])/a^(5/3) + (b^(1/3)*(-(b^(4/3)*c) + a^(1/3)*b*d - 2*a*b^(1/3)*f + 5*a^(4/
3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 18*h*Log[a
+ b*x^3])/(54*b^3)
```

Maple [A] time = 0.012, size = 515, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e)/b^2*x^3-
1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3*a*h-b*e)/b^3)/(b*x^
3+a)^2+2/27/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+1/27/b^2/a/(1/b*a)^(2/3
)*ln(x+(1/b*a)^(1/3))*c-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))*f-1/54/b^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*
c+2/27/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
f+1/27/b^2/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)
)*c-5/27/b^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*g-1/27/b^2/a/(1/b*a)^(1/3)*l
n(x+(1/b*a)^(1/3))*d+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^
(2/3))*g+1/54/b^2/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+5
/27/b^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+1
/27/b^2/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d
+1/3*h*ln(b*x^3+a)/b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 28.7256, size = 29083, normalized size = 89.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/108*(12*(b^3*d - 4*a*b^2*g)*x^5 + 6*(b^3*c - 7*a*b^2*f)*x^4 - 18*a^2*b*e
+ 54*a^3*h - 36*(a*b^2*e - 2*a^2*b*h)*x^3 - 6*(a*b^2*d + 5*a^2*b*g)*x^2 - 2
*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*
h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^
3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b
^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(
a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 +
135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c
)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3
) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5
*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^
2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a
^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 1
35*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)
*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*log(2*a*b^4*
c*d^2 + 4*a^2*b^3*d^2*f + 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^
3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3
*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
```

```
 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*
a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g -
(4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*
c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
+ 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a
^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (
4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/
b^3)^2 + 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 + 81*(a^4*b*d + 5*a^5*g)*h^2 - 1/2*
(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4*d + 5*a^5*b^3*g)
*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g +
 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d
 + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^
3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2
*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 -
 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*
a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/
(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d
+ 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3
 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*
d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 -
5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a
^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(
a^5*b^9))^(1/3) - 18*h/b^3) + 20*(a^2*b^3*c*d + 2*a^3*b^2*d*f)*g - 9*(a^2*b
^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + (b^5*c^3 + a*b^4*d^3 + 6*a*b^4*c^
2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*g + 75*a^3*b^2*d*g^
2 + 125*a^4*b*g^3)*x) - 12*(a*b^2*c + 2*a^2*b*f)*x + (54*a*b^2*h*x^6 + 108*
a^2*b*h*x^3 + 54*a^3*h + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2
*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g
+ 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 +
6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*
b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*
f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d
^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3)
 + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g +
 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6
*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b
*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f
*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^
2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3)
- 18*h/b^3) + 3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*
h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^
2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^
3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g
^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
- 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9
))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2
*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3
*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^
3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 -
 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9)
)^(1/3) - 18*h/b^3)^2*a^3*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/
b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^
6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*
```

g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c
*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*
b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*
c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^
2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g
)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*
f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b
^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c
*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2
*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*b
^3*c*d + 32*a*b^2*d*f + 324*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6)
))*log(-2*a*b^4*c*d^2 - 4*a^2*b^3*d^2*f - 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a
^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b
^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
+ 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*
g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^
9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^
2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^
3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g
^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
- 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9
))^(1/3) - 18*h/b^3)^2 - 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 81*(a^4*b*d + 5*a
^5*g)*h^2 + 1/2*(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4*
d + 5*a^5*b^3*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d
+ 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b
^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^
3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*
f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3
 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g
^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 -
6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^
9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3
*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f
^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3
+ 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^
2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6
*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3) - 20*(a^2*b^3*c*d + 2*a^3*b^2*d
*f)*g + 9*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + 2*(b^5*c^3 + a*b^
4*d^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*g
 + 75*a^3*b^2*d*g^2 + 125*a^4*b*g^3)*x + 3/4*sqrt(1/3)*(2*a^2*b^6*c^2 + 8*a
^3*b^5*c*f + 8*a^4*b^4*f^2 + (a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5
*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*
c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
+ 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a
^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (
4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)
^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*
h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c
^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 +
 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^
4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4
*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b

^3) + 18*(a^4*b^4*d + 5*a^5*b^3*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1
)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^
2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*
d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12
*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*
g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*
f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*
d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*
sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d
*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*
a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g
^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f
^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d
*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)^2*a^3*
b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*
f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^
3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b
^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^
2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*
h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h
)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*
b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3
*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^
4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2
*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h
^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)
*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b
^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*b^3*c*d + 32*a*b^2*d*f + 32
4*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6))) + (54*a*b^2*h*x^6 + 108
*a^2*b*h*x^3 + 54*a^3*h + (a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*(2*(1/2)^(2
/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (
2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g
 + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 +
 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3
*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54
*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*
d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3
) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g
+ 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 +
6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*
b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*
f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d
^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3)
 - 18*h/b^3) - 3*sqrt(1/3)*(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)*sqrt(-((2*
(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a
^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b
^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
+ 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*
g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^
9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^
2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^
3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g +
 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g
^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
- 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9
))^(1/3) - 18*h/b^3)^2*a^3*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2
/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b

^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c
*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*
c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5
*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135
*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a
^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +
 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*
g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c
*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*
b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*
c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^
2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*
b^3*c*d + 32*a*b^2*d*f + 324*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6
)))*log(-2*a*b^4*c*d^2 - 4*a^2*b^3*d^2*f - 1/4*(a^4*b^7*d + 5*a^5*b^6*g)*(2
(1/2)^(2/3)(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^
3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*
a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*
b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
 + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25
*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^
2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b
^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a
^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b
^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g
+ 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*
g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2
 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^
9))^(1/3) - 18*h/b^3)^2 - 50*(a^3*b^2*c + 2*a^4*b*f)*g^2 - 81*(a^4*b*d + 5*
a^5*g)*h^2 + 1/2*(a^2*b^6*c^2 + 4*a^3*b^5*c*f + 4*a^4*b^4*f^2 - 18*(a^4*b^4
*d + 5*a^5*b^3*g)*h)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d
 + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/
b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a
^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b
*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^
3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*
g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 -
 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b
^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^
3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*
f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3
 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g
^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 -
6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3) - 20*(a^2*b^3*c*d + 2*a^3*b^2*
d*f)*g + 9*(a^2*b^3*c^2 + 4*a^3*b^2*c*f + 4*a^4*b*f^2)*h + 2*(b^5*c^3 + a*b
^4*d^3 + 6*a*b^4*c^2*f + 12*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 + 15*a^2*b^3*d^2*
g + 75*a^3*b^2*d*g^2 + 125*a^4*b*g^3)*x - 3/4*sqrt(1/3)*(2*a^2*b^6*c^2 + 8*
a^3*b^5*c*f + 8*a^4*b^4*f^2 + (a^4*b^7*d + 5*a^5*b^6*g)*(2*(1/2)^(2/3)*(-I*
sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f +
5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^
3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3
*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*
a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g -
(4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2
)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3
*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*
c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2
+ 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a
^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (

```
4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/
b^3) + 18*(a^4*b^4*d + 5*a^5*b^3*g)*h)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b
^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2
*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 1
2*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4
*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8
*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9
*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*
d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12
*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*
g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*
f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*
d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)^2*a^3
*b^6 + 36*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(81*h^2/b^6 - (b^3*c*d + 10*a^2*b
*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)/(a^3*b^6))/(1458*h^3/b^9 - 27*(b
^3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (
b^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a
^2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5
*h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*
h)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a
*b^4)/(a^5*b^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(1458*h^3/b^9 - 27*(b^
3*c*d + 10*a^2*b*f*g + 81*a^3*h^2 + (2*d*f + 5*c*g)*a*b^2)*h/(a^3*b^9) + (b
^4*c^3 + a*b^3*d^3 + 6*a*b^3*c^2*f + 12*a^2*b^2*c*f^2 + 8*a^3*b*f^3 + 15*a^
2*b^2*d^2*g + 75*a^3*b*d*g^2 + 125*a^4*g^3)/(a^5*b^8) + (b^5*c^3 + 729*a^5*
h^3 - 5*(25*g^3 - 54*f*g*h)*a^4*b + (8*f^3 + 135*c*g*h - 3*(25*g^2 - 18*f*h
)*d)*a^3*b^2 - 3*(5*d^2*g - (4*f^2 + 9*d*h)*c)*a^2*b^3 - (d^3 - 6*c^2*f)*a*
b^4)/(a^5*b^9))^(1/3) - 18*h/b^3)*a^3*b^3*h + 16*b^3*c*d + 32*a*b^2*d*f + 3
24*a^3*h^2 + 80*(a*b^2*c + 2*a^2*b*f)*g)/(a^3*b^6))))/(a*b^5*x^6 + 2*a^2*b^
4*x^3 + a^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08958, size = 512, normalized size = 1.58

$$\frac{h\log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c+2\left(-ab^2\right)^{\frac{1}{3}}abf-\left(-ab^2\right)^{\frac{2}{3}}bd-5\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4}+\frac{2\left(b^2d-4\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] 1/3*h*log(abs(b*x^3 + a))/b^3 + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b^2*c + 2*(-a*
b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d - 5*(-a*b^2)^(2/3)*a*g)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/18*(2*(b^2*d - 4*a*b
*g)*x^5 + (b^2*c - 7*a*b*f)*x^4 + 6*(2*a^2*h - a*b*e)*x^3 - (a*b*d + 5*a^2*
g)*x^2 - 2*(a*b*c + 2*a^2*f)*x + 3*(3*a^3*h - a^2*b*e)/b)/((b*x^3 + a)^2*a*
b^2) + 1/54*((-a*b^2)^(1/3)*b^2*c + 2*(-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)
*b*d + 5*(-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*
b^4) - 1/27*(a*b^4*d*(-a/b)^(1/3) + 5*a^2*b^3*g*(-a/b)^(1/3) + a*b^4*c + 2*
a^2*b^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^5)
```

3.423 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}}{}$$

```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) -
(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d
 + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g)
 - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) -
((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```

Rubi [A] time = 0.430108, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1823, 1858, 1860, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{54a^{5/3}b^{8/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(2ag+bd)-\sqrt[3]{a}(5ah+be)\right)}{27a^{5/3}b^{8/3}}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```


```
[Out] (x*(b*d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(18*a*b^2*(a + b*x^3)) -
(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(6*b*(a + b*x^3)^2) - ((b^(4/3)*d
 + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(8/3)) + ((b^(1/3)*(b*d + 2*a*g)
 - a^(1/3)*(b*e + 5*a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(8/3)) -
((b^(1/3)*(b*d + 2*a*g) - a^(1/3)*(b*e + 5*a*h))*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(8/3))
```


Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx = -\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2} + \frac{\int \frac{d+2ex+3fx^2+4gx^3+5hx^4}{\left(a+bx^3\right)^2}dx}{6b}$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2} - \int$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2} - \int$$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2} + ($$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2} + ($$

$$= \frac{x\left(bd-4ag+(2be-5ah)x+3bfx^2\right)}{18ab^2\left(a+bx^3\right)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2} - ($$

Mathematica [A] time = 0.234061, size = 287, normalized size = 0.97

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{4/3}h+\sqrt[3]{a}be-2a\sqrt[3]{b}g-b^{4/3}d\right)}{a^{5/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-5a^{4/3}h-\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{a^{5/3}} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b\right.}{a^{5/3}}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (
3*b^(2/3)*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x))))/(a*(a + b*x^3)) -
(2*Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(4/3)*d - a^(1/3)*b*e
+ 2*a*b^(1/3)*g - 5*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-(b^(4
/3)*d) + a^(1/3)*b*e - 2*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(8/3))
```

Maple [A] time = 0.011, size = 490, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3*f*x^3/b-1/18*(5*a*h+
b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(b*x^3+a)^2+2/27/b^3/
```

```
(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*g+1/27/b^2/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(
1/3))*d-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/54
/b^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+2/27/b^3/(1/b*
a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+1/27/b^2/a/(1/
b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-5/27/b^3/(1/
b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*h-1/27/a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3
))*e+5/54/b^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h+1/54/a/
b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+5/27/b^3*3^(1/2)/
(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/27/a/b^2*3^(1/2
)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 19.2287, size = 15354, normalized size = 51.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] -1/108*(36*a*b*f*x^3 - 12*(b^2*e - 4*a*b*h)*x^5 - 6*(b^2*d - 7*a*b*g)*x^4 +
 18*a*b*c + 18*a^2*f + 6*(a*b*e + 5*a^2*h)*x^2 + 2*(a*b^4*x^6 + 2*a^2*b^3*x
^3 + a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
 - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) +
 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a
^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^
4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*
b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))*log(2*a*b^3*d*e^2 + 4*a^2*b
^2*e^2*g + 1/4*(a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4
*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*
b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^
3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2
*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g
+ 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2
*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 1
25*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b +
 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^
2 + 50*(a^3*b*d + 2*a^4*g)*h^2 - 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b
^3*g^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
+ 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*
a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*
```

```
(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(
1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a
^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g
^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
- 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
(e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) + 20*(a^2*b^2*d*e + 2*a^3*b*e*g)*
h + (b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 +
 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)*x) + 12*(a*b*d + 2*a^2*g)
*x - ((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a
^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4
*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*
d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e
*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
)) + 3*sqrt(1/3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*sqrt(-(((1/2)^(1/3)*
(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 +
8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) +
(b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a
^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e +
 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*
b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h
 + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^
3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3
)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^
2*g)*h)/(a^3*b^5)))*log(-2*a*b^3*d*e^2 - 4*a^2*b^2*e^2*g - 1/4*(a^4*b^6*e +
 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*
d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2
+ 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*
b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)
 - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) +
 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a
^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^
4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*
b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2 - 50*(a^3*b*d + 2*a^4*g)*
h^2 + 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*d*g + 4*a^4*b^3*g^2)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*
b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d
^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2
 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^
2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^
3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*
a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75
*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5
*b^8))^(1/3))) - 20*(a^2*b^2*d*e + 2*a^3*b*e*g)*h + 2*(b^4*d^3 + a*b^3*e^3
+ 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^
3*b*e*h^2 + 125*a^4*h^3)*x + 3/4*sqrt(1/3)*(2*a^2*b^5*d^2 + 8*a^3*b^4*d*g +
 8*a^4*b^3*g^2 + (a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b
^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^
2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*
h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d
^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*
g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d
^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 +
 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b
 + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))
))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*
```

```
g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 12
5*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b +
3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2
*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/
(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b
*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^
3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2
- (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b
*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3*b^5))) - ((a*b^4*x^6 + 2*a^2*b^3*x^3 +
a^3*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
 + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125
*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3
*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*
(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(
a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*
g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
 (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) - 3*sqrt(1/3)*(a*b^4*x^6 + 2*a^2
*b^3*x^3 + a^3*b^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e
^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75
*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 7
5*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^
5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*
(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^
2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a
^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 -
5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3)))^2*a^3*b^5 + 16
*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3*b^5)))*log(-2*a*b^3*d*
e^2 - 4*a^2*b^2*e^2*g - 1/4*(a^4*b^6*e + 5*a^5*b^5*h)*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*
g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
 (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*
g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3
+ 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^
3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e
*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b
^8))^(1/3)))^2 - 50*(a^3*b*d + 2*a^4*g)*h^2 + 1/2*(a^2*b^5*d^2 + 4*a^3*b^4*
d*g + 4*a^4*b^3*g^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6
*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b
*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^
2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8)
)^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sq
rt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^
2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8
) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*
h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))) - 20*(a^2*b^2*d*e +
2*a^3*b*e*g)*h + 2*(b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2
+ 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)*x - 3/4*sq
rt(1/3)*(2*a^2*b^5*d^2 + 8*a^3*b^4*d*g + 8*a^4*b^3*g^2 + (a^4*b^6*e + 5*a^5
*b^5*h)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g
+ 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*
a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*
(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(
1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h + (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a
^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g
^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3
- 125*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 -
(e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
```

```
 1)*((b^4*d^3 + a*b^3*e^3 + 6*a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3
+ 15*a^2*b^2*e^2*h + 75*a^3*b*e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 1
25*a^4*h^3 + (8*g^3 - 75*e*h^2)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^
3 - 6*d^2*g)*a*b^3)/(a^5*b^8))^(1/3) - 2*(1/2)^(2/3)*(b^2*d*e + 10*a^2*g*h
+ (2*e*g + 5*d*h)*a*b)*(-I*sqrt(3) + 1)/(a^3*b^5*((b^4*d^3 + a*b^3*e^3 + 6*
a*b^3*d^2*g + 12*a^2*b^2*d*g^2 + 8*a^3*b*g^3 + 15*a^2*b^2*e^2*h + 75*a^3*b*
e*h^2 + 125*a^4*h^3)/(a^5*b^8) + (b^4*d^3 - 125*a^4*h^3 + (8*g^3 - 75*e*h^2
)*a^3*b + 3*(4*d*g^2 - 5*e^2*h)*a^2*b^2 - (e^3 - 6*d^2*g)*a*b^3)/(a^5*b^8))
^(1/3)))^2*a^3*b^5 + 16*b^2*d*e + 32*a*b*e*g + 80*(a*b*d + 2*a^2*g)*h)/(a^3
*b^5))))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12213, size = 454, normalized size = 1.53

$$-\frac{\left(5\,ah\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+bd+2\,ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2d+2\left(-ab^2\right)^{\frac{1}{3}}abg-5\left(-ab^2\right)^{\frac{2}{3}}ah-\left(-\right.\right.}{27\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/27*(5*a*h*(-a/b)^(1/3) + b*(-a/b)^(1/3)*e + b*d + 2*a*g)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^2*b^2) + 1/27*sqrt(3)*((-a*b^2)^(1/3)*b^2*d + 2
*(-a*b^2)^(1/3)*a*b*g - 5*(-a*b^2)^(2/3)*a*h - (-a*b^2)^(2/3)*b*e)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) - 1/18*(8*a*b*h*x^5
 - 2*b^2*x^5*e - b^2*d*x^4 + 7*a*b*g*x^4 + 6*a*b*f*x^3 + 5*a^2*h*x^2 + a*b*
x^2*e + 2*a*b*d*x + 4*a^2*g*x + 3*a*b*c + 3*a^2*f)/((b*x^3 + a)^2*a*b^2) +
1/54*((-a*b^2)^(1/3)*b^2*d + 2*(-a*b^2)^(1/3)*a*b*g + 5*(-a*b^2)^(2/3)*a*h
+ (-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.424 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=323

$$\frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{18a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}}-\text{lo}$$

```
[Out] -(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b^2*(a + b*
x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2))
/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a
^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(
7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b
*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^
(7/3))
```

Rubi [A] time = 0.481828, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1828, 1858, 1860, 31, 634, 617, 204, 628}

$$\frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{18a^2b^2\left(a+bx^3\right)}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{54a^{7/3}b^{7/3}}-\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] -(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a*b^2*(a + b*
x^3)^2) + (x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*x^2))
/(18*a^2*b^2*(a + b*x^3)) - ((2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a
^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(
7/3)*b^(7/3)) - ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2*a*h))*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(7/3)*b^(7/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b
*e + 2*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^
(7/3))
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
```

```
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}dx = -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} - \frac{\int \frac{-a(be-ah)-2b(2bc+af)x-3b(bc}{\left(a+bx^3\right)^2}}{6ab^2}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2b(2bc+\right.}{18a^2b^2\left(a\right.}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2b(2bc+\right.}{18a^2b^2\left(a\right.}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2b(2bc+\right.}{18a^2b^2\left(a\right.}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2b(2bc+\right.}{18a^2b^2\left(a\right.}$$

$$= -\frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6ab^2\left(a+bx^3\right)^2} + \frac{x\left(a(be-7ah)+2b(2bc+\right.}{18a^2b^2\left(a\right.}$$

Mathematica [A] time = 0.299634, size = 297, normalized size = 0.92

$$\frac{\frac{9a^{4/3}\sqrt[3]{b}\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{\left(a+bx^3\right)^2} - \frac{3\sqrt[3]{a}\sqrt[3]{b}\left(a^2(6g+7hx)-abx(e+2fx)-4b^2cx^2\right)}{a+bx^3} + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-a^{2/3}be-2a^{5/3}h+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

```
[Out] ((-3*a^(1/3)*b^(1/3)*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g + 7*h*x))
)/(a + b*x^3) + (9*a^(4/3)*b^(1/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*
(e + f*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*(2*b^(5/3)*c + a^(2/3)*b*e + a*b^(2/
3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*b^(
5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f + 2*a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x]
+ (2*b^(5/3)*c - a^(2/3)*b*e + a*b^(2/3)*f - 2*a^(5/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(7/3))
```

Maple [A] time = 0.011, size = 498, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3*g*x^3/b-1/18*(a*f-7*b
*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b*x^3+a)^2+2/27/b^3/(
1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*h+1/27/a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1
```

```
/3))*e-1/27/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h-1/54/
a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e+2/27/b^3/(1/b*a
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h+1/27/a/b^2/(1/b
*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e-1/27/a/b^2/(1
/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*f-2/27/b/a^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))*c+1/54/a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f+1/27
/b/a^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+1/27/a/b^2*3^(
1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+2/27/b/a^2*3
^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxim
a")

[Out] Exception raised: ValueError
```

Fricas [C] time = 22.2304, size = 15553, normalized size = 48.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="frica
s")

[Out] -1/108*(36*a^2*b*g*x^3 - 12*(2*b^3*c + a*b^2*f)*x^5 - 6*(a*b^2*e - 7*a^2*b*
h)*x^4 + 18*a^2*b*d + 18*a^3*g - 6*(7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b^4*x
^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^
2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*
h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 1
2*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b
^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)
*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f
+ 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*
h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 +
 (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))*log(
8*a*b^4*c^2*e + 8*a^2*b^3*c*e*f + 2*a^3*b^2*e*f^2 + 1/4*(2*a^5*b^6*c + a^6*
b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^
2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*
a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h
^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) -
2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/
(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3
*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5
*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^
2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2 - 1/2*(a^4*b^4*e^2 + 4*a^
5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b
^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h +
 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a
^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)
```

/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) + 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + (8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x) + 12*(a^2*b*e + 2*a^3*h)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4)))*log(-8*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 2*a^3*b^2*e*f^2 - 1/4*(2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2 + 1/2*(a^4*b^4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) - 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*(8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x + 3/4*sqrt(1/3)*(2*a^4*b^4*e^2 + 8*a^5*b^3*e*h + 8*a^6*b^2*h^2 + (2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e

```
^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*
a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b
*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^
7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(
-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2
*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a
^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 -
 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 +
 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4))) - ((a^2*b^4*
x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a
^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2
*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f -
12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*
b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h
)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f
 + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5
*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3
+ (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3))) - 3
*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*s
qrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 +
a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*
b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3
*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e
 + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a
^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2
*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f -
12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*
b^3)/(a^7*b^7))^(1/3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c +
 a^2*f)*h)/(a^4*b^4)))*log(-8*a*b^4*c^2*e - 8*a^2*b^3*c*e*f - 2*a^3*b^2*e*f
^2 - 1/4*(2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5*c^3
 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2
*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*
f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*
a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f + 4
*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c
^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8
*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*
h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3)))
^2 + 1/2*(a^4*b^4*e^2 + 4*a^5*b^3*e*h + 4*a^6*b^2*h^2)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3
*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5
*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^
2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e +
2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*
b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h
+ 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*
a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3
)/(a^7*b^7))^(1/3))) - 4*(4*a^2*b^3*c^2 + 4*a^3*b^2*c*f + a^4*b*f^2)*h + 2*
(8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 +
 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)*x - 3/4*sqrt(1/3)*(2*a^4*b^4
*e^2 + 8*a^5*b^3*e*h + 8*a^6*b^2*h^2 + (2*a^5*b^6*c + a^6*b^5*f)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c
*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7
) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2
*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*
b^2*c*e + 2*a^2*f*h + (e*f + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*
c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*
b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c
^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^
```

```
2)*a^2*b^3)/(a^7*b^7))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^5
*c^3 + a^2*b^3*e^3 + 12*a*b^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3
*b^2*e^2*h + 12*a^4*b*e*h^2 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*
c^2*f - 12*a^4*b*e*h^2 - 8*a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f
^2)*a^2*b^3)/(a^7*b^7))^(1/3) - 2*(1/2)^(2/3)*(2*b^2*c*e + 2*a^2*f*h + (e*f
 + 4*c*h)*a*b)*(-I*sqrt(3) + 1)/(a^4*b^4*((8*b^5*c^3 + a^2*b^3*e^3 + 12*a*b
^4*c^2*f + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 + 6*a^3*b^2*e^2*h + 12*a^4*b*e*h^2
 + 8*a^5*h^3)/(a^7*b^7) - (8*b^5*c^3 + 12*a*b^4*c^2*f - 12*a^4*b*e*h^2 - 8*
a^5*h^3 + (f^3 - 6*e^2*h)*a^3*b^2 - (e^3 - 6*c*f^2)*a^2*b^3)/(a^7*b^7))^(1/
3)))^2*a^4*b^4 + 32*b^2*c*e + 16*a*b*e*f + 32*(2*a*b*c + a^2*f)*h)/(a^4*b^4
))))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09139, size = 481, normalized size = 1.49

$$-\frac{\left(2\,b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2h+abe\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^2}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}a^2h+\left(-ab^2\right)^{\frac{1}{3}}abe-2\,\left(-ab^2\right)^{\frac{2}{3}}b\right.}{27\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/27*(2*b^2*c*(-a/b)^(1/3) + a*b*f*(-a/b)^(1/3) + 2*a^2*h + a*b*e)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) + 1/27*sqrt(3)*(2*(-a*b^2)^(1/3)
*a^2*h + (-a*b^2)^(1/3)*a*b*e - 2*(-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/54*(2*(
-a*b^2)^(1/3)*a^2*h + (-a*b^2)^(1/3)*a*b*e + 2*(-a*b^2)^(2/3)*b*c + (-a*b^2
)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(4*b
^3*c*x^5 + 2*a*b^2*f*x^5 - 7*a^2*b*h*x^4 + a*b^2*x^4*e - 6*a^2*b*g*x^3 + 7*
a*b^2*c*x^2 - a^2*b*f*x^2 - 4*a^3*h*x - 2*a^2*b*x*e - 3*a^2*b*d - 3*a^3*g)/
((b*x^3 + a)^2*a^2*b^2)
```

3.425 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3} dx$

Optimal. Leaf size=313

$$-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (
3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b
*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b
^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rubi [A] time = 0.429386, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1858, 1854, 1860, 31, 634, 617, 204, 628}

$$-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{18a^2b^2(a+bx^3)}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(af+5bc)-\sqrt[3]{a}(ag+2bd)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) - (
3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(18*a^2*b^2*(a + b
*x^3)) - ((5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(5/3)) + ((b
^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^(1/3) + b^(1/3)*x])/(27
*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*c + a*f) - a^(1/3)*(2*b*d + a*g))*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(5/3))
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx = \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{\int \frac{-b(5bc+af)-2b(2bd+ag)x-3b(be+ah)x^2}{(a+bx^3)^2}\,dx}{6ab^2}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+a}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+a}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+a}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+a}{18a^2b^2\left(a+bx^3\right)}$$

$$= \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6ab\left(a+bx^3\right)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+a}{18a^2b^2\left(a+bx^3\right)}$$

Mathematica [A] time = 0.237521, size = 295, normalized size = 0.94

$$\frac{\frac{9a^{5/3}\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{(a+bx^3)^2} + \frac{3a^{2/3}\left(-6a^2h+abx(f+2gx)+b^2x(5c+4dx)\right)}{a+bx^3} + \sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}g+2\sqrt[3]{a}bd-a\sqrt[3]{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

```
[Out] ((3*a^(2/3)*(-6*a^2*h + b^2*x*(5*c + 4*d*x) + a*b*x*(f + 2*g*x)))/(a + b*x^
3) + (9*a^(5/3)*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x
^3)^2 - 2*Sqrt[3]*b^(1/3)*(5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a^(4
/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*b^(4/3)*c
 - 2*a^(1/3)*b*d + a*b^(1/3)*f - a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + b^(1
/3)*(-5*b^(4/3)*c + 2*a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^2)
```

Maple [A] time = 0.008, size = 506, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3*h*x^3/b-1/18*(a*g-7*b
*d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b*x^3+a)^2+1/27/a/b^2
/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*f+5/27*c/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a
```

```
)^(1/3))-1/54/a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*f-5
/54*c/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/27/a/b^2/
(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*f+5/27*c/a^
2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/27/b^
2/a/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*g-2/27*d/a^2/b/(1/b*a)^(1/3)*ln(x+(1/
b*a)^(1/3))+1/54/b^2/a/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*
g+1/27*d/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/27/b^2
/a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*g+2/27*d
/a^2*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 20.8148, size = 15406, normalized size = 49.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas"
)
```

```
[Out] -1/108*(36*a^2*b*h*x^3 - 12*(2*b^3*d + a*b^2*g)*x^5 - 6*(5*b^3*c + a*b^2*f)
*x^4 + 18*a^2*b*e + 18*a^3*h - 6*(7*a*b^2*d - a^2*b*g)*x^2 + 2*(a^2*b^4*x^6
 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*
a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*
g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*
d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a
^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)
*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 1
5*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(
a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*
d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))*log(40*a*b^3*
c*d^2 + 8*a^2*b^2*d^2*f + 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^
2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*
b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g
+ 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*
g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8
*b^5))^(1/3)))^2 + 2*(5*a^3*b*c + a^4*f)*g^2 - 1/2*(25*a^3*b^4*c^2 + 10*a^4
*b^3*c*f + a^5*b^2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^
3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g +
6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^
2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b
```

```
^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I
*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^
2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*
b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*
g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) + 8*(5*a^2*b^2*c*
d + a^3*b*d*f)*g + (125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2
*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x) - 12*(4
*a*b^2*c - a^2*b*f)*x - ((a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^
2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5)
 + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a
^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2
*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c
^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b
^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f
^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*
b^3)/(a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^
2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b
^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2
 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(
5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2
*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/
(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^
4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (
8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a*
b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3)))*log(-40*a*b^3*c*d^2 - 8*a^2*b^2
*d^2*f - 1/4*(2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a
^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f
)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f +
5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^
3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 +
 a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5
*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2
 - 2*(5*a^3*b*c + a^4*f)*g^2 + 1/2*(25*a^3*b^4*c^2 + 10*a^4*b^3*c*f + a^5*b^
2*f^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*
c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a
^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c
*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1
/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^
5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3
*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c
^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d
^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) - 8*(5*a^2*b^2*c*d + a^3*b*d*f)*g
 + 2*(125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*
f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)*x + 3/4*sqrt(1/3)*(50*a^3
*b^4*c^2 + 20*a^4*b^3*c*f + 2*a^5*b^2*f^2 + (2*a^6*b^4*d + a^7*b^3*g)*((1/2
)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a
^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8
*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2
*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(1
0*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*
b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*
a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*
f)*a*b^3)/(a^8*b^5))^(1/3))))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4
*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2
```

```
*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 +
(f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*
a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*
c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*
c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a
^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c
*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^
5*b^3 + 160*b^2*c*d + 32*a*b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3))) - ((
a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b
^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a
^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3
+ (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f
)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f +
5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^
3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 +
 a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5
*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3))) -
3*sqrt(1/3)*(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)*sqrt(-(((1/2)^(1/3)*(I*
sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^
2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (12
5*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2
 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d +
 a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8
*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2
*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6
*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(
a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a*b*d*f + 16*(5*a*b*c + a^2*
f)*g)/(a^5*b^3)))*log(-40*a*b^3*c*d^2 - 8*a^2*b^2*d^2*f - 1/4*(2*a^6*b^4*d
+ a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*
a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g
^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b +
3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)
- 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) +
1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2
 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125
*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2
- (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2 - 2*(5*a^3*b*c + a^4*f)*g^
2 + 1/2*(25*a^3*b^4*c^2 + 10*a^4*b^3*c*f + a^5*b^2*f^2)*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^
2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*
b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g
+ 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*
g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8
*b^5))^(1/3))) - 8*(5*a^2*b^2*c*d + a^3*b*d*f)*g + 2*(125*b^4*c^3 + 8*a*b^3
*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6
*a^3*b*d*g^2 + a^4*g^3)*x - 3/4*sqrt(1/3)*(50*a^3*b^4*c^2 + 20*a^4*b^3*c*f
+ 2*a^5*b^2*f^2 + (2*a^6*b^4*d + a^7*b^3*g)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((
125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 +
 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4
*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*
c^2*f)*a*b^3)/(a^8*b^5))^(1/3) - 2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d
*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75
*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*
g^2 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b +
 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)
)))*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*
```

```
b^3*c^2*f + 15*a^2*b^2*c*f^2 + a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2
 + a^4*g^3)/(a^8*b^5) + (125*b^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*
(5*c*f^2 - 4*d^2*g)*a^2*b^2 - (8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3) -
2*(1/2)^(2/3)*(10*b^2*c*d + a^2*f*g + (2*d*f + 5*c*g)*a*b)*(-I*sqrt(3) + 1)
/(a^5*b^3*((125*b^4*c^3 + 8*a*b^3*d^3 + 75*a*b^3*c^2*f + 15*a^2*b^2*c*f^2 +
 a^3*b*f^3 + 12*a^2*b^2*d^2*g + 6*a^3*b*d*g^2 + a^4*g^3)/(a^8*b^5) + (125*b
^4*c^3 - a^4*g^3 + (f^3 - 6*d*g^2)*a^3*b + 3*(5*c*f^2 - 4*d^2*g)*a^2*b^2 -
(8*d^3 - 75*c^2*f)*a*b^3)/(a^8*b^5))^(1/3)))^2*a^5*b^3 + 160*b^2*c*d + 32*a
*b*d*f + 16*(5*a*b*c + a^2*f)*g)/(a^5*b^3))))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3
+ a^4*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08976, size = 467, normalized size = 1.49

$$-\frac{\left(2\,bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{3}b}+\frac{\sqrt{3}\left(5\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}c+\left(-ab^{2}\right)^{\frac{1}{3}}abf-2\,\left(-ab^{2}\right)^{\frac{2}{3}}bd-(-\right.}{27\,a^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b*d*(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/27*sqrt(3)*(5*(-a*b^2)^(1/3)*b^2*c + (
-a*b^2)^(1/3)*a*b*f - 2*(-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*g)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/54*(5*(-a*b^2)^(1
/3)*b^2*c + (-a*b^2)^(1/3)*a*b*f + 2*(-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*
g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1/18*(4*b^3*d*x^5 +
 2*a*b^2*g*x^5 + 5*b^3*c*x^4 + a*b^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x^2
- a^2*b*g*x^2 + 8*a*b^2*c*x - 2*a^2*b*f*x - 3*a^3*h - 3*a^2*b*e)/((b*x^3 +
a)^2*a^2*b^2)
```

3.426 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{54a^{8/3}b^{5/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{27a^{8/3}b^{5/3}}-$$

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x
^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2)
)/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8
/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e +
a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((b^(1/3)*(5*b*d + a
*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(54*a^(8/3)*b^(5/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.723069, antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ah+2be)}{\sqrt[3]{b}}+ag+5bd\right)}{54a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(ag+5bd)-\sqrt[3]{a}(ah+2be)\right)}{27a^{8/3}b^{5/3}}-\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] (x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*x
^3)^2) + (x*(a*(5*b*d + a*g) + 2*a*(2*b*e + a*h)*x - 3*b*(3*b*c - a*f)*x^2)
)/(18*a^3*b*(a + b*x^3)) - ((5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^
(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8
/3)*b^(5/3)) + (c*Log[x])/a^3 + ((b^(1/3)*(5*b*d + a*g) - a^(1/3)*(2*b*e +
a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(5/3)) - ((5*b*d + a*g - (a^(
1/3)*(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(54*a^(8/3)*b^(4/3)) - (c*Log[a + b*x^3])/(3*a^3)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-b(5bd+ag)x-2b(2be+ah)x^2+3b}{x\left(a+bx^3\right)^2}}{6ab^2}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

$$= \frac{x\left(a(bd-ag)+a(be-ah)x-b(bc-af)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5bd+ag)+2a(2be+ah)\right.}{18a^3b\left(a+bx\right.}$$

Mathematica [A] time = 0.274023, size = 311, normalized size = 0.9

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{4/3}h+2\sqrt[3]{a}be-a\sqrt[3]{b}g-5b^{4/3}d\right)}{b^{5/3}} + \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{4/3}(-h)-2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{b^{5/3}} - \frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}h+2\right.}{b^{5/3}}}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((3*a*(6*b*c + b*x*(5*d + 4*e*x) + a*x*(g + 2*h*x)))/(b*(a + b*x^3)) - (9*a
^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)^2) - (2*S
qrt[3]*a^(1/3)*(5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 54*c*Log[x] + (2*a^(1/3)
*(5*b^(4/3)*d - 2*a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(5/3) + (a^(1/3)*(-5*b^(4/3)*d + 2*a^(1/3)*b*e - a*b^(1/3)*g + a^
(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 18*c*Log
[a + b*x^3])/(54*a^3)
```

Maple [B] time = 0.017, size = 618, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/27/a/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
g-1/3*c*ln(b*x^3+a)/a^3+1/2/a/(b*x^3+a)^2*c+2/9/a^2/(b*x^3+a)^2*x^5*b*e+7/1
8/a/(b*x^3+a)^2*x^2*e+1/9/a/(b*x^3+a)^2*x^5*h+1/18/a/(b*x^3+a)^2*x^4*g-1/18
/(b*x^3+a)^2/b*x^2*h-1/9/(b*x^3+a)^2/b*x*g+5/18/a^2/(b*x^3+a)^2*b*x^4*d+1/2
7/a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*g-1/54/a/b^2/(1/b*a)^(2/3)*ln(x^2
-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*g-1/27/a/b^2/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))*h+1/54/a/b^2/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*h+1/27
/a/b^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*h-1/
6/b/(b*x^3+a)^2*f-5/54/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*d+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+4/9/a/(b*x^3+a)^2*x*
d-2/27/a^2/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*e+2/27/a^2/b*3^(1/2)/(1/b*a)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*e+1/3/a^2/(b*x^3+a)^2*b*c*
x^3+c*ln(x)/a^3+5/27/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))*d+1/27/a^2/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09791, size = 529, normalized size = 1.52

$$-\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{c\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}b^2d+\left(-ab^2\right)^{\frac{1}{3}}abg-\left(-ab^2\right)^{\frac{2}{3}}ah-2\left(-ab^2\right)^{\frac{2}{3}}be\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac"
)
```

```
[Out] -1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b^
2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*a*h - 2*(-a*b^2)^(2/
3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1
/54*(5*(-a*b^2)^(1/3)*b^2*d + (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*a*h + 2
*(-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^3) + 1
/18*(6*a*b^2*c*x^3 + 2*(a^2*b*h + 2*a*b^2*e)*x^5 + (5*a*b^2*d + a^2*b*g)*x^
4 + 9*a^2*b*c - 3*a^3*f - (a^3*h - 7*a^2*b*e)*x^2 + 2*(4*a^2*b*d - a^3*g)*x
)/((b*x^3 + a)^2*a^3*b) - 1/27*(a^5*b^2*h*(-a/b)^(1/3) + 2*a^4*b^3*(-a/b)^(
1/3)*e + 5*a^4*b^3*d + a^5*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(
a^7*b^3)
```

3.427 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=362

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+$$

```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6
*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(
3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e
- 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a
*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)
) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a
^3)
```

Rubi [A] time = 0.829961, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{54a^{10/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{27a^{10/3}b^{4/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```



```
[Out] -(c/(a^3*x)) + (x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6
*a^2*b*(a + b*x^3)^2) + (x*(a*(5*b*e + a*h) - 2*b*(5*b*c - 2*a*f)*x - 3*b*(
3*b*d - a*g)*x^2))/(18*a^3*b*(a + b*x^3)) + ((14*b^(5/3)*c - 5*a^(2/3)*b*e
- 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(10/3)*b^(4/3)) + (d*Log[x])/a^3 + ((2*b^(2/3)*(7*b*c - a
*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(4/3)
) - ((2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3)*b^(4/3)) - (d*Log[a + b*x^3])/(3*a
^3)
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}dx = \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-b(5be+ah)x^2+4b^2\left(\frac{bc}{a}-f\right)x^3}{x^2\left(a+bx^3\right)^2}}{6ab^2}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-2af)x-\right.}{18a^3b\left(a+bx^3\right)}$$

$$= \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-2af)x-\right.}{18a^3b\left(a+bx^3\right)}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-\right.}{18a^3b\left(a\right.}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-\right.}{18a^3b\left(a\right.}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-\right.}{18a^3b\left(a\right.}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-\right.}{18a^3b\left(a\right.}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-\right.}{18a^3b\left(a\right.}$$

$$= -\frac{c}{a^3x} + \frac{x\left(a(be-ah)-b(bc-af)x-b(bd-ag)x^2\right)}{6a^2b\left(a+bx^3\right)^2} + \frac{x\left(a(5be+ah)-2b(5bc-\right.}{18a^3b\left(a\right.}$$

Mathematica [A] time = 0.569985, size = 336, normalized size = 0.93

$$-\frac{\frac{9a^2\left(a^2(g+hx)-ab(d+x(e+fx))+b^2cx^2\right)}{b\left(a+bx^3\right)^2} - \frac{3a\left(a^2hx+ab(6d+x(5e+4fx))-10b^2cx^2\right)}{b\left(a+bx^3\right)} + \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5a^{2/3}be+a^{5/3}h-2ab^{2/3}f+14b^{5/3}c\right)}{b^{4/3}} - \frac{2a^{2/3}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] -((54*a*c)/x + (9*a^2*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))))/
(b*(a + b*x^3)^2) - (3*a*(a^2*h*x - 10*b^2*c*x^2 + a*b*(6*d + x*(5*e + 4*f*
x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-14*b^(5/3)*c + 5*a^(2/3)*b*e +
 2*a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(4/3) - 54*a*d*Log[x] - (2*a^(2/3)*(14*b^(5/3)*c + 5*a^(2/3)*b*e - 2*a*b^(2
/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(14*b^(5/3)
*c + 5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/b^(4/3) + 18*a*d*Log[a + b*x^3])/(54*a^4)
```

Maple [B] time = 0.016, size = 622, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x)
```

```
[Out] 1/27/a/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
h+14/27/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*c-7/27/a^3/(1/b*a)^(1/3)*ln(x
^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c+d*ln(x)/a^3-1/3*d*ln(b*x^3+a)/a^3-5/9/a
^3/(b*x^3+a)^2*b^2*x^5*c+5/18/a^2/(b*x^3+a)^2*x^4*b*e+2/9/a^2/(b*x^3+a)^2*x
^5*b*f-5/54/a^2/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*e-c/a
^3/x+5/27/a^2/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))*e+5/27/a^2/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*e-2/27/a^2*f/b/(1/b*a)^
(1/3)*ln(x+(1/b*a)^(1/3))+1/27/a^2*f/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))+7/18/a/(b*x^3+a)^2*x^2*f-1/9/(b*x^3+a)^2/b*x*h+1/18/a/(b*x^
3+a)^2*x^4*h-1/6/(b*x^3+a)^2/b*g+4/9/a/(b*x^3+a)^2*x*e+1/2/a/(b*x^3+a)^2*d-
14/27/a^3*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
-13/18/a^2/(b*x^3+a)^2*b*x^2*c+2/27/a^2*f*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/54/a/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1
/3)*x+(1/b*a)^(2/3))*h+1/27/a/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*h+1/3/a
^2/(b*x^3+a)^2*b*d*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09166, size = 556, normalized size = 1.54

$$-\frac{d\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{d\log\left(|x|\right)}{a^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}a^2h+5\left(-ab^2\right)^{\frac{1}{3}}abe+14\left(-ab^2\right)^{\frac{2}{3}}bc-2\left(-ab^2\right)^{\frac{2}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 + 1/27*sqrt(3)*((-a*b^2)
^(1/3)*a^2*h + 5*(-a*b^2)^(1/3)*a*b*e + 14*(-a*b^2)^(2/3)*b*c - 2*(-a*b^2)^
(2/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2)
+ 1/54*((-a*b^2)^(1/3)*a^2*h + 5*(-a*b^2)^(1/3)*a*b*e - 14*(-a*b^2)^(2/3)*b
*c + 2*(-a*b^2)^(2/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^
2) + 1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (a^2*b*h + 5*a*b^2*e
)*x^5 - 18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 - 2*(a^3*h - 4*a^2*b*e)*x^
2 + 3*(3*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b*x) + 1/27*(14*a^3*b^4*c*(
-a/b)^(1/3) - 2*a^4*b^3*f*(-a/b)^(1/3) - a^5*b^2*h - 5*a^4*b^3*e)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

3.428 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=360

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{54a^{11/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}$$

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^
2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*(3
*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d -
5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a
*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/
3)) + ((5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(2/3)) - (e*Log[a + b*x^3])/
(3*a^3)
```

Rubi [A] time = 0.813874, antiderivative size = 357, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}-5af+20bc\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -c/(2*a^3*x^2) - d/(a^3*x) - (x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^
2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*c - 5*a*f + 2*(5*b*d - 2*a*g)*x + 3*(3
*b*e - a*h)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(4/3)*c + 14*a^(1/3)*b*d -
5*a*b^(1/3)*f - 2*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) + (e*Log[x])/a^3 - ((5*b^(1/3)*(4*b*c - a
*f) - 2*a^(1/3)*(7*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/
3)) + ((20*b*c - 5*a*f - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3)) - (e*Log[a + b*x^3])
/(3*a^3)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+5b^2\left(\frac{bc}{a}-f\right)x^3+4b^2(}{x^3\left(a+bx^3\right)^2}}{6ab^2}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(5bd-2ag)x+3\right.}{18a^3\left(a+bx^3\right)}$$

$$= -\frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(5bd-2ag)x+3\right.}{18a^3\left(a+bx^3\right)}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(\right.}{18}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(\right.}{18}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(\right.}{18}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(\right.}{18}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(\right.}{18}$$

$$= -\frac{c}{2a^3x^2} - \frac{d}{a^3x} - \frac{x\left(bc-af+(bd-ag)x+(be-ah)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bc-5af+2(\right.}{18}$$

Mathematica [A] time = 0.606096, size = 337, normalized size = 0.94

$$-\frac{\frac{9a^2\left(a^2h-ab(e+x(f+gx))+b^2x(c+dx)\right)}{b\left(a+bx^3\right)^2} - \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{b^{2/3}} + \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{4/3}g-14\sqrt[3]{a}bd-5a\sqrt[3]{b}\right.}{b^{2/3}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] -((27*a*c)/x^2 + (54*a*d)/x - (3*a*(6*a*e - b*x*(11*c + 10*d*x) + a*x*(5*f
+ 4*g*x)))/(a + b*x^3) + (9*a^2*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f +
g*x))))/(b*(a + b*x^3)^2) + (2*Sqrt[3]*a^(1/3)*(-20*b^(4/3)*c - 14*a^(1/3)*
b*d + 5*a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(2/3) - 54*a*e*Log[x] + (2*a^(1/3)*(20*b^(4/3)*c - 14*a^(1/3)*b*d -
5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)*(
20*b^(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*a*e*Log[a + b*x^3])/(54*a^4)
```

Maple [B] time = 0.017, size = 626, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -20/27/a^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*
c-11/18/a^3/(b*x^3+a)^2*b^2*x^4*c+e*ln(x)/a^3-1/3*e*ln(b*x^3+a)/a^3+2/27/a^
2*g*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-5/9/a
^3/(b*x^3+a)^2*x^5*b^2*d-7/9/a^2/(b*x^3+a)^2*b*x*c-14/27/a^3*3^(1/2)/(1/b*a
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-d/a^3/x-1/2*c/a^3/x^2-1
3/18/a^2/(b*x^3+a)^2*b*x^2*d-2/27/a^2*g/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))
+1/27/a^2*g/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-20/27/a^3
/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c+10/27/a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)
^(1/3)*x+(1/b*a)^(2/3))*c+14/27/a^3/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))*d-7/2
7/a^3/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/6/(b*x^3+a)^2
/b*h+2/9/a^2/(b*x^3+a)^2*x^5*b*g+4/9/a/(b*x^3+a)^2*f*x+7/18/a/(b*x^3+a)^2*x
^2*g+1/2/a/(b*x^3+a)^2*e+5/18/a^2/(b*x^3+a)^2*x^4*b*f+5/27/a^2*f/b/(1/b*a)^
(2/3)*ln(x+(1/b*a)^(1/3))-5/54/a^2*f/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x
+(1/b*a)^(2/3))+5/27/a^2*f/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/b*a)^(1/3)*x-1))+1/3*b/a^2/(b*x^3+a)^2*e*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 139.296, size = 31293, normalized size = 86.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2916*(972*a*b^2*e*x^5 - 648*(7*b^3*d - a*b^2*g)*x^7 - 810*(4*b^3*c - a*b^
2*f)*x^6 - 2916*a^2*b*d*x - 1134*(7*a*b^2*d - a^2*b*g)*x^4 - 1458*a^2*b*c -
 1296*(4*a*b^2*c - a^2*b*f)*x^3 + 486*(3*a^2*b*e - a^3*h)*x^2 - 2*(a^3*b^3*
x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c
*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 +
 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10
*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*
b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^
```

3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a *b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2 *b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^ 3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630* d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d* e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*log(-7840*a*b^3*c*d^2 + 3600*a*b^3*c^2*e - 1134*a^2*b^2*d*e^2 + 225*a^3*b*e*f^2 - 1/1458*(7*a^8*b ^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2* c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(80 00*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a ^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1 /39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8* (343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a *b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168* a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a* b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2 - 40*(4*a^3*b*c - a^4*f)*g^2 - 1/54*(400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*c*f + 25*a^6*b*f^2 + 36*a^6 *b*e*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10 *a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c ^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243 *e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(- 1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2 *f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d* g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(2 5*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^ 11*b^2))^(1/3) + 486*e/a^3) + 40*(49*a^2*b^2*d^2 - 45*a^2*b^2*c*e)*f + 2*(1 120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*g - (8000*b^4*c^3 + 2744*a* b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2* b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x) - (1458*b^3*e*x^8 + 2916*a*b^2* e*x^5 + 1458*a^2*b*e*x^2 - (a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*s qrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40 *c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81 *e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^ 3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^ 2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e *f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10 *b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2* b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^ 3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g

)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3)
 + 486*e/a^3) - 3*sqrt(1/3)*(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*sqrt(
-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*
d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f
*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2
744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 117
6*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b
^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 -
 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945
*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e
^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)
*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1
500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 -
8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*
e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2
- 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2
))^(1/3) + 486*e/a^3)^2*a^7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^
2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^
9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a
^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a
^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4
*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g
 + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*
e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1
/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g
 + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 274
4*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*
a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4
*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 6
30*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c
*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b
^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7
*b)))*log(7840*a*b^3*c*d^2 - 3600*a*b^3*c^2*e + 1134*a^2*b^2*d*e^2 - 225*a^
3*b*e*f^2 + 1/1458*(7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 -
(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27
e^3/a^9 + 1/1458(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*
b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f +
 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2
- 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 27
0*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^
2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b
^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*
a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^
3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3
- 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8
000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*
e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3
- 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2 + 40*(4*a^
3*b*c - a^4*f)*g^2 + 1/54*(400*a^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*
c*f + 25*a^6*b*f^2 + 36*a^6*b*e*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2
*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9
 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^
10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^
2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*
g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g
+ 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e
*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/
3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g
+ (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744

```
*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a
^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*
c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 63
0*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*
d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3) - 40*(49*a^2*b^2*d^2
 - 45*a^2*b^2*c*e)*f - 2*(1120*a^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*
g - 2*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^
2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x + 1
/486*sqrt(1/3)*(10800*a^4*b^3*c^2 + 3402*a^5*b^2*d*e - 5400*a^5*b^2*c*f + 6
75*a^6*b*f^2 - 486*a^6*b*e*g - (7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(8
1*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^
7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f
- 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a
*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*
a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (
125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g
 + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*
b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b
^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*
(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 12
5*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2)
- 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3
*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 -
 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3))
*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2
 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10
*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c
^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3
 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(
8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243
*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3
 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-
1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g
)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2
*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*
g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3
- 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(2
5*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^
11*b^2))^(1/3) + 486*e/a^3)^2*a^7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (
280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*
e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b
)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f +
1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 -
 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270
*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2
 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^
2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a
^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3
 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 -
 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(80
00*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e
^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 -
 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 326
5920*b^2*c*d + 236196*a*b*e^2 - 816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g
)/(a^7*b))) - (1458*b^3*e*x^8 + 2916*a*b^2*e*x^5 + 1458*a^2*b*e*x^2 - (a^3*
b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b
^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a
^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(
a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*
```

```
a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^
4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*
g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18
*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(
1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*
g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 27
44*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176
*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^
4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 -
630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*
c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3) + 3*sqrt(1/3)*(a^3
*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*sqrt(-(((-I*sqrt(3) + 1)*(81*e^2/a^6
 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/
27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*
a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f
 + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^
2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 -
270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*
f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11
*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 1
0*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*
c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^
3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*
(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(24
3*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^
3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2*a^7*b -
972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 7
0*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2
*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 +
 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1
176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000
*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3
 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 9
45*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27
*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*
b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f +
 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2
- 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 27
0*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^
2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b
^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 - 816480
*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b)))*log(7840*a*b^3*c*d^2 - 360
0*a*b^3*c^2*e + 1134*a^2*b^2*d*e^2 - 225*a^3*b*e*f^2 + 1/1458*(7*a^8*b^2*d
- a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*
e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d +
 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^
4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*
f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/3936
6*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(
243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*
d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)
*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*
c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*
c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b
*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f
^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20
*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/
(a^11*b^2))^(1/3) + 486*e/a^3)^2 + 40*(4*a^3*b*c - a^4*f)*g^2 + 1/54*(400*a
^4*b^3*c^2 - 252*a^5*b^2*d*e - 200*a^5*b^2*c*f + 25*a^6*b*f^2 + 36*a^6*b*e*
```

```
g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70
*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*
f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 +
2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 11
76*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*
b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3
- 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 94
5*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*
e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b
)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f +
1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 -
 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270
*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2
 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^
2))^(1/3) + 486*e/a^3) - 40*(49*a^2*b^2*d^2 - 45*a^2*b^2*c*e)*f - 2*(1120*a
^2*b^2*c*d + 81*a^3*b*e^2 - 280*a^3*b*d*f)*g - 2*(8000*b^4*c^3 + 2744*a*b^3
*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2
*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)*x - 1/486*sqrt(1/3)*(10800*a^4*b^3*c^
2 + 3402*a^5*b^2*d*e - 5400*a^5*b^2*c*f + 675*a^6*b*f^2 - 486*a^6*b*e*g - (
7*a^8*b^2*d - a^9*b*g)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^
2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(2
80*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39
366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2
- 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b
^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)
*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b
^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*
sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 -
70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 -
 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g
 + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*
g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 39
2*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^
2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3))*sqrt(-(((-I*sqrt(3) + 1)*(81*e^
2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b)
)/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40
*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3
*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*
b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*
f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 2
0*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)
/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b^2*c
*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(800
0*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^
3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/
39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b +
 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(
343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 486*e/a^3)^2*a^
7*b - 972*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b^2*c*d + 10*a^2*f*g + (81*e
^2 - 70*d*f - 40*c*g)*a*b)/(a^7*b))/(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d +
10*a^2*f*g + (81*e^2 - 70*d*f - 40*c*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4
*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f
^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366
*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(2
43*e^3 - 630*d*e*f + 392*d^2*g + 20*(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d
^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(a^11*b^2))^(1/3) + 729*(I*sqrt(3) + 1)*
(-1/27*e^3/a^9 + 1/1458*(280*b^2*c*d + 10*a^2*f*g + (81*e^2 - 70*d*f - 40*c
*g)*a*b)*e/(a^10*b) - 1/39366*(8000*b^4*c^3 + 2744*a*b^3*d^3 - 6000*a*b^3*c
^2*f + 1500*a^2*b^2*c*f^2 - 125*a^3*b*f^3 - 1176*a^2*b^2*d^2*g + 168*a^3*b*
```

```
d*g^2 - 8*a^4*g^3)/(a^11*b^2) - 1/39366*(8000*b^4*c^3 + 8*a^4*g^3 - (125*f^
3 - 270*e*f*g + 168*d*g^2)*a^3*b + 3*(243*e^3 - 630*d*e*f + 392*d^2*g + 20*
(25*f^2 - 18*e*g)*c)*a^2*b^2 - 8*(343*d^3 - 945*c*d*e + 750*c^2*f)*a*b^3)/(
a^11*b^2))^(1/3) + 486*e/a^3)*a^4*b*e + 3265920*b^2*c*d + 236196*a*b*e^2 -
816480*a*b*d*f - 116640*(4*a*b*c - a^2*f)*g)/(a^7*b))) + 2916*(b^3*e*x^8 +
2*a*b^2*e*x^5 + a^2*b*e*x^2)*log(x))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x
^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11512, size = 568, normalized size = 1.58

$$-\frac{e\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{e\log\left(|x|\right)}{a^3}-\frac{\sqrt{3}\left(20\left(-ab^2\right)^{\frac{1}{3}}b^2c-5\left(-ab^2\right)^{\frac{1}{3}}abf-14\left(-ab^2\right)^{\frac{2}{3}}bd+2\left(-ab^2\right)^{\frac{2}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\right.\right.}{3\left(-\frac{a}{b}\right)}\right.}{27\,a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="gia
c")
```

```
[Out] -1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 1/27*sqrt(3)*(20*(-a*b
^2)^(1/3)*b^2*c - 5*(-a*b^2)^(1/3)*a*b*f - 14*(-a*b^2)^(2/3)*b*d + 2*(-a*b^
2)^(2/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^
2) - 1/54*(20*(-a*b^2)^(1/3)*b^2*c - 5*(-a*b^2)^(1/3)*a*b*f + 14*(-a*b^2)^(
2/3)*b*d - 2*(-a*b^2)^(2/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
a^4*b^2) - 1/18*(28*b^3*d*x^7 - 4*a*b^2*g*x^7 + 20*b^3*c*x^6 - 5*a*b^2*f*x^
6 - 6*a*b^2*x^5*e + 49*a*b^2*d*x^4 - 7*a^2*b*g*x^4 + 32*a*b^2*c*x^3 - 8*a^2
*b*f*x^3 + 3*a^3*h*x^2 - 9*a^2*b*x^2*e + 18*a^2*b*d*x + 9*a^2*b*c)/((b*x^4
+ a*x)^2*a^3*b) + 1/27*(14*a^3*b^2*d*(-a/b)^(1/3) - 2*a^4*b*g*(-a/b)^(1/3)
+ 20*a^3*b^2*c - 5*a^4*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.429 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=395

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{54a^{11/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+$$

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*x
- b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5*b
*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(
4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f
)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^
(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((5*b^(1/3)*(4*b*d - a*g) - 2*a
^(1/3)*(7*b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a
^(11/3)*b^(2/3)) + ((3*b*c - a*f)*Log[a + b*x^3])/(3*a^4)
```

Rubi [A] time = 1.00616, antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1829, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}}-5ag+20bd\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)}{27a^{11/3}b^{2/3}}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -c/(3*a^3*x^3) - d/(2*a^3*x^2) - e/(a^3*x) - (x*(b*d - a*g + (b*e - a*h)*x
- b*((b*c)/a - f)*x^2))/(6*a^2*(a + b*x^3)^2) - (x*(11*b*d - 5*a*g + 2*(5*b
*e - 2*a*h)*x - 3*b*((5*b*c)/a - 3*f)*x^2))/(18*a^3*(a + b*x^3)) + ((20*b^(
4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(2/3)) - ((3*b*c - a*f
)*Log[x])/a^4 - ((5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^
(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(2/3)) + ((20*b*d - 5*a*g - (2*a^(1/3)*(
7*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*
a^(11/3)*b^(1/3)) + ((3*b*c - a*f)*Log[a + b*x^3])/(3*a^4)
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
```

```
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int \frac{-6b^2c-6b^2dx-6b^2ex^2+6b^2\left(\frac{bc}{a}-f\right)x^3+5b^2}{x^4(a+b}}{6a}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-2ah)x-\right.}{18a^3\left(a+bx^3\right)}$$

$$= -\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd-5ag+2(5be-2ah)x-\right.}{18a^3\left(a+bx^3\right)}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd\right.}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd\right.}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd\right.}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd\right.}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd\right.}{}$$

$$= -\frac{c}{3a^3x^3} - \frac{d}{2a^3x^2} - \frac{e}{a^3x} - \frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{x\left(11bd\right.}{}$$

Mathematica [A] time = 0.590882, size = 352, normalized size = 0.89

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(2a^{4/3}h-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(2a^{4/3}h-14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{b^{2/3}} + \frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(-\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*f - b*x*(1
1*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*(-9*b*(c + x*(d + e*
x)) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (2*Sqrt[3]*a^(1/3)*(20*b^(4/3
)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*(20*b
^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3) + b^(1
/3)*x])/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g +
 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*
```

```
(3*b*c - a*f)*Log[a + b*x^3])/(54*a^4)
```

Maple [B] time = 0.017, size = 680, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x)
```

```
[Out] 1/2/a/(b*x^3+a)^2*f-1/3/a^3*ln(b*x^3+a)*f+1/a^3*ln(x)*f+2/27/a^2*h*3^(1/2)/
b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+5/27/a^2*g/b/(1/b
*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))-1/3*c/a^3/x^3+5
/27/a^2*g/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-5/54/a^2*g/b/(1/b*a)^(2/3)*ln
(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-5/9/a^3/(b*x^3+a)^2*x^5*e*b^2-20/27/a^3
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-11/18/a^
3/(b*x^3+a)^2*x^4*b^2*d-1/2*d/a^3/x^2-e/a^3/x-5/6/a^2*b/(b*x^3+a)^2*c-7/9/a
^2/(b*x^3+a)^2*b*x*d-13/18/a^2/(b*x^3+a)^2*x^2*b*e-14/27/a^3*e*3^(1/2)/(1/b
*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+14/27/a^3*e/(1/b*a)^(1/
3)*ln(x+(1/b*a)^(1/3))-7/27/a^3*e/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))-20/27/a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+10/27/a^3/(1/b*a)^
(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d+7/18/a/(b*x^3+a)^2*x^2*h+4/9/
a/(b*x^3+a)^2*g*x+5/18/a^2/(b*x^3+a)^2*x^4*b*g+2/9/a^2/(b*x^3+a)^2*x^5*b*h-
2/27/a^2*h/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/3/a^2/(b*x^3+a)^2*x^3*b*f+
1/27/a^2*h/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-3*b*c*ln(x
)/a^4+b*c*ln(b*x^3+a)/a^4-2/3/a^3*b^2/(b*x^3+a)^2*c*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)

[Out] Timed out
```

Giac [A] time = 1.09566, size = 612, normalized size = 1.55

$$\frac{(3\,bc-af)\log\left(\left|bx^3+a\right|\right)}{3\,a^4}-\frac{(3\,bc-af)\log\left(|x|\right)}{a^4}-\frac{\sqrt{3}\left(20\,\left(-ab^2\right)^{\frac{1}{3}}b^2d-5\,\left(-ab^2\right)^{\frac{1}{3}}abg+2\,\left(-ab^2\right)^{\frac{2}{3}}ah-14\,\left(-ab^2\right)^{\frac{2}{3}}\right.}{27\,a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="gia
c")

[Out] 1/3*(3*b*c - a*f)*log(abs(b*x^3 + a))/a^4 - (3*b*c - a*f)*log(abs(x))/a^4 -
 1/27*sqrt(3)*(20*(-a*b^2)^(1/3)*b^2*d - 5*(-a*b^2)^(1/3)*a*b*g + 2*(-a*b^2
)^(2/3)*a*h - 14*(-a*b^2)^(2/3)*b*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a^4*b^2) - 1/54*(20*(-a*b^2)^(1/3)*b^2*d - 5*(-a*b^2)^(1/3
)*a*b*g - 2*(-a*b^2)^(2/3)*a*h + 14*(-a*b^2)^(2/3)*b*e)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/(a^4*b^2) - 1/27*(2*a^6*b*h*(-a/b)^(1/3) - 14*a^5*b^2
*(-a/b)^(1/3)*e - 20*a^5*b^2*d + 5*a^6*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^9*b) + 1/18*(4*(a^2*b*h - 7*a*b^2*e)*x^8 - 5*(4*a*b^2*d - a^2*b
*g)*x^7 - 6*(3*a*b^2*c - a^2*b*f)*x^6 + 7*(a^3*h - 7*a^2*b*e)*x^5 - 18*a^3*
x^2*e - 9*a^3*d*x - 8*(4*a^2*b*d - a^3*g)*x^4 - 6*a^3*c - 9*(3*a^2*b*c - a^
3*f)*x^3)/((b*x^3 + a)^2*a^4*x^3)
```

3.430 $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=583

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+$$

```
[Out] (-4*a*e*Sqrt[a + b*x^3])/(9*b^2) + (2*c*x*Sqrt[a + b*x^3])/(5*b) + (2*d*x^2
*Sqrt[a + b*x^3])/(7*b) + (2*e*x^3*Sqrt[a + b*x^3])/(9*b) - (8*a*d*Sqrt[a +
 b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[
2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^
(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.733102, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1888, 1594, 1886, 261, 1878, 218, 1877}

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-4*a*e*Sqrt[a + b*x^3])/(9*b^2) + (2*c*x*Sqrt[a + b*x^3])/(5*b) + (2*d*x^2
*Sqrt[a + b*x^3])/(7*b) + (2*e*x^3*Sqrt[a + b*x^3])/(9*b) - (8*a*d*Sqrt[a +
 b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[
2 - Sqrt[3]]*a^(4/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^
(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^3 \left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx = \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{2\int \frac{-3aex^2+\frac{9}{2}bcx^3+\frac{9}{2}bdx^4}{\sqrt{a+bx^3}}dx}{9b}$$

$$= \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{2\int \frac{x^2\left(-3ae+\frac{9bcx}{2}+\frac{9}{2}bdx^2\right)}{\sqrt{a+bx^3}}dx}{9b}$$

$$= \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{4\int \frac{-9abdx-\frac{21}{2}abex^2+\frac{63}{4}b^2cx^3}{\sqrt{a+bx^3}}dx}{63b^2}$$

$$= \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{4\int \frac{x\left(-9abd-\frac{21}{2}abex+\frac{63}{4}b^2cx^2\right)}{\sqrt{a+bx^3}}dx}{63b^2}$$

$$= \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{8\int \frac{-\frac{63}{4}ab^2c-\frac{45}{2}ab^2dx-\frac{105}{4}ab^2ex^2}{\sqrt{a+bx^3}}dx}{315b^3}$$

$$= \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} + \frac{8\int \frac{-\frac{63}{4}ab^2c-\frac{45}{2}ab^2dx}{\sqrt{a+bx^3}}dx}{315b^3} - \frac{(2ae)\int \frac{x^2}{\sqrt{a+bx^3}}dx}{3b}$$

$$= -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{(4ad)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{7b^{4/3}} -$$

$$= -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

Mathematica [C] time = 0.128637, size = 132, normalized size = 0.23

$$\frac{-2\left(a+bx^3\right)(70ae-bx(63c+5x(9d+7ex)))-126abcx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-90abdx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{315b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*(a + b*x^3)*(70*a*e - b*x*(63*c + 5*x*(9*d + 7*e*x))) - 126*a*b*c*x*Sqr
t[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 90*a*b*d*
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(31
5*b^2*Sqrt[a + b*x^3])
```

Maple [A] time = 0.016, size = 793, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] e*(2/9/b*x^3*(b*x^3+a)^(1/2)-4/9/b^2*a*(b*x^3+a)^(1/2))+d*(2/7/b*x^2*(b*x^3
+a)^(1/2)+8/21*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/
```

```
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2
*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/
3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2
*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+c*(2/5/b*
x*(b*x^3+a)^(1/2)+4/15*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1
/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2
),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+dx+c)x^3}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^5+dx^4+cx^3}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4 + c*x^3)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.13196, size = 129, normalized size = 0.22

$$e\left(\begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2}+\frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b\neq 0\\ \frac{x^6}{6\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}+\frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b)
, Ne(b, 0)), (x**6/(6*sqrt(a)), True)) + c*x**4*gamma(4/3)*hyper((1/2, 4/3)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + d*x**5*gamma(5
/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/
3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^3}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

3.431 $\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=560

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*c*Sqrt[a + b*x^3])/(3*b) + (2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*e*x^2*Sqrt
[a + b*x^3])/(7*b) - (8*a*e*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*d - 10*(1 - Sqrt[3])*a^(1/3)*e)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1
/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.484352, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1888, 1594, 1886, 261, 1878, 218, 1877}

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*c*Sqrt[a + b*x^3])/(3*b) + (2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*e*x^2*Sqrt
[a + b*x^3])/(7*b) - (8*a*e*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (4*Sqrt[2 + Sqrt[3]]*a*(7*b^(1/3)*d - 10*(1 - Sqrt[3])*a^(1/3)*e)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*3^(1
/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}dx = \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{-2aex+\frac{7}{2}bcx^2+\frac{7}{2}bdx^3}{\sqrt{a+bx^3}}dx}{7b}$$
$$= \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{2\int \frac{x\left(-2ae+\frac{7bcx}{2}+\frac{7}{2}bdx^2\right)}{\sqrt{a+bx^3}}dx}{7b}$$
$$= \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{-\frac{7}{2}abd-5abex+\frac{35}{4}b^2cx^2}{\sqrt{a+bx^3}}dx}{35b^2}$$
$$= \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} + \frac{4\int \frac{-\frac{7}{2}abd-5abex}{\sqrt{a+bx^3}}dx}{35b^2} + c\int \frac{x^2}{\sqrt{a+bx^3}}dx$$
$$= \frac{2c\sqrt{a+bx^3}}{3b} + \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} - \frac{(4ae)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{7b^{4/3}} - \frac{\left(2a\left(7\sqrt[3]{b}d-10\right.\right.}{}$$
$$= \frac{2c\sqrt{a+bx^3}}{3b} + \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2ex^2\sqrt{a+bx^3}}{7b} - \frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

Mathematica [C] time = 0.0790605, size = 121, normalized size = 0.22

$$\frac{2\left(a+bx^3\right)(35c+3x(7d+5ex))-42adx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-30aex^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{105b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)*(35*c + 3*x*(7*d + 5*e*x)) - 42*a*d*x*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 30*a*e*x^2*Sqrt[1 + (b*x^3)/
a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(105*b*Sqrt[a + b*x^3])
```

Maple [A] time = 0.007, size = 773, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] e*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(
1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/
```

```
3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/
2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))
^(1/2))))+d*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(
-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/3*c*(b*x^3+a)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,\sqrt{bx^3+ac}}{3\,b}+\int\frac{ex^4+dx^3}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*c/b + integrate((e*x^4 + d*x^3)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^4+dx^3+cx^2}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.87018, size = 107, normalized size = 0.19

$$c\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}+\frac{ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] c*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^2}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a), x)
```

3.432 $\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=537

$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} -$$

```
[Out] (2*d*Sqrt[a + b*x^3])/(3*b) + (2*e*x*Sqrt[a + b*x^3])/(5*b) + (2*c*Sqrt[a +
 b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1
 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.322531, antiderivative size = 537, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1888, 1886, 261, 1878, 218, 1877}

$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*d*Sqrt[a + b*x^3])/(3*b) + (2*e*x*Sqrt[a + b*x^3])/(5*b) + (2*c*Sqrt[a +
 b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1
 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
```

```
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx &= \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{2\int \frac{-ae+\frac{5bcx}{2}+\frac{5}{2}bdx^2}{\sqrt{a+bx^3}}\,dx}{5b} \\
&= \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{2\int \frac{-ae+\frac{5bcx}{2}}{\sqrt{a+bx^3}}\,dx}{5b} + d\int \frac{x^2}{\sqrt{a+bx^3}}\,dx \\
&= \frac{2d\sqrt{a+bx^3}}{3b} + \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{c\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} - \frac{\left(\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+2a^{2/3}e\right)\right)\int \frac{1}{\sqrt{a+bx^3}}}{5b} \\
&= \frac{2d\sqrt{a+bx^3}}{3b} + \frac{2ex\sqrt{a+bx^3}}{5b} + \frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}}{((1}}}{b^{2/3}\sqrt{\frac{}{((}}}
\end{aligned}
$$

Mathematica [C] time = 0.0621461, size = 114, normalized size = 0.21

$$
\frac{15bcx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4\left(a+bx^3\right)(5d+3ex)-12aex\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{30b\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

```
[Out] (4*(5*d + 3*e*x)*(a + b*x^3) - 12*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(30*b*Sqrt[a + b*x^3])
```

Maple [A] time = 0.006, size = 753, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] e*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I/b^2*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2)))+2/3*d*(b*x^3+a)^(1/2)/b-2/3*I*c*3^(1/2)/b*(-b^2
*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
```

```
/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+dx+c)x}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^3+dx^2+cx}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2 + c*x)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.68915, size = 107, normalized size = 0.2

$$d\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right)+\frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] d*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+dx+c)x}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

3.433 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=509

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.173025, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\cdots}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(3*b) + (2*d*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c - (1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
```

```
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx &= e\int \frac{x^2}{\sqrt{a+bx^3}}\,dx+\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx\\ &= \frac{2e\sqrt{a+bx^3}}{3b}+\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}+\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx\\ &= \frac{2e\sqrt{a+bx^3}}{3b}+\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}\end{aligned}$$

Mathematica [C] time = 0.045915, size = 107, normalized size = 0.21

$$\frac{6bcx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3bdx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4e\left(a+bx^3\right)}{6b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

```
[Out] (4*e*(a + b*x^3) + 6*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2
/3, 5/3, -((b*x^3)/a)])/(6*b*Sqrt[a + b*x^3])
```

Maple [A] time = 0.004, size = 735, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((
x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(
-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
)+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b
^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
-2/3*I*c*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2
*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + dx + c}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 1.98094, size = 105, normalized size = 0.21

$$e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b = 0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True))
 + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.434 $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=518

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*
ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.204874, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*e*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*c*
ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*
a^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - (1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
```

```
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = c\int \frac{1}{x\sqrt{a+bx^3}}\,dx + \int \frac{d+ex}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{1}{3}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) + \frac{e\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \left(d - \frac{(1-\sqrt{3})\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$= \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

Mathematica [C] time = 0.141124, size = 128, normalized size = 0.25

$$-\frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{dx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}} + \frac{ex^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^3)
/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqrt[a + b*x^3] + (e*x^
2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sq
rt[a + b*x^3])
```

Maple [A] time = 0.007, size = 740, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*e*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*I*d*3^(1/2)/b*(-b^
2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
```

```
2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/3*c*arctanh((b*x^3+a)^(
1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\left(ex^2+dx+c\right)}{bx^4+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b*x^4 + a*x), x)
```

Sympy [A] time = 3.33502, size = 105, normalized size = 0.2

$$-\frac{2c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}+\frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + d*x*gamma(1/3)*hyper((
1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + e*x**
2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

3.435 $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=547

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{\cdots}}{\cdots}$$

```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) + (b^(1/3)*c*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 +
Sqrt[3]]*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rubi [A] time = 0.342628, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```


```
[Out] -((c*Sqrt[a + b*x^3])/(a*x)) + (b^(1/3)*c*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (Sqrt[2 +
Sqrt[3]]*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```


Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{ax} - \frac{\int \frac{-2ad-2aex-bcx^2}{x\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} - \frac{\int \frac{-2ae-bcx}{\sqrt{a+bx^3}}\,dx}{2a} + d\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\left(b^{2/3}c\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a} + \frac{1}{3}d\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) - \frac{1}{2}\left(\frac{\left(1-\sqrt{3}\right)b^{2/3}c}{a^{2/3}}\right.$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\right.}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b}}$$

$$= -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}}{((1}}}{2a^{2/3}\sqrt{\frac{}{(}}}$$

Mathematica [C] time = 0.123586, size = 126, normalized size = 0.23

$$-\frac{c\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{ex\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/a
]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(x*Sqrt[a + b*x^3]) + (e
*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqrt
[a + b*x^3]
```

Maple [A] time = 0.007, size = 759, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*e*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
```

```
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2
*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-2/
3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+c*(-1/a*(b*x^3+a)^(1/2)/x-1/3*
I/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}\left(ex^2 + dx + c\right)}{bx^5 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b*x^5 + a*x^2), x)
```

Sympy [A] time = 2.94802, size = 107, normalized size = 0.2

$$\frac{c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)} - \frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) - 2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + e*x*
```

```
gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*g
amma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + ax^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

3.436 $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=569

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \ldots$$

```
[Out] -(c*Sqrt[a + b*x^3])/(2*a*x^2) - (d*Sqrt[a + b*x^3])/(a*x) + (b^(1/3)*d*Sqr
t[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*e*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*c + 2*(1 - Sqrt[3])
*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.463536, antiderivative size = 569, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```


```
[Out] -(c*Sqrt[a + b*x^3])/(2*a*x^2) - (d*Sqrt[a + b*x^3])/(a*x) + (b^(1/3)*d*Sqr
t[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*e*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) - (Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*c + 2*(1 - Sqrt[3])
*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{\int \frac{-4ad-4aex+bcx^2}{x^2\sqrt{a+bx^3}}\,dx}{4a}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\int \frac{8a^2e-2abcx+4abdx^2}{x\sqrt{a+bx^3}}\,dx}{8a^2}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\int \frac{-2abc+4abdx}{\sqrt{a+bx^3}}\,dx}{8a^2} + e\int \frac{1}{x\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\left(b^{2/3}d\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a} - \frac{\left(b^{2/3}\left(\sqrt[3]{b}c+2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\right)\int \frac{1}{\sqrt{a+bx}}}{4a}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\right.}{\left(\left(1+\sqrt{3}\right.\right.}}}$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\right.}{}$$

Mathematica [C] time = 0.193753, size = 131, normalized size = 0.23

$$-\frac{c\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt{a+bx^3}} - \frac{d\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/a
]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(2*x^2*Sqrt[a + b*x^3])
- (d*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(
x*Sqrt[a + b*x^3])
```

Maple [A] time = 0.009, size = 778, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+c*(-1/2/a*(b*x^3+a)^(1/2)/x
^2+1/6*I/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
```

```
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2
*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d
*(-1/a*(b*x^3+a)^(1/2)/x-1/3*I/a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x
-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-
b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^
2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}\left(ex^2 + dx + c\right)}{bx^6 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b*x^6 + a*x^3), x)
```

Sympy [A] time = 2.96634, size = 112, normalized size = 0.2

$$\frac{c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)} - \frac{2e\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] c*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + d*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(3*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

3.437 $\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=594

$$-\frac{16\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt[a + b*x^3
])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2*Sqrt[a + b*x^3])/(7
*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (1
6*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.640867, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1828, 1888, 1886, 261, 1878, 218, 1877}

$$-\frac{16\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{40\sqrt{2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*c*Sqrt[a + b*x^3
])/(3*b^2) + (2*d*x*Sqrt[a + b*x^3])/(5*b^2) + (2*e*x^2*Sqrt[a + b*x^3])/(7
*b^2) - (80*a*e*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (40*Sqrt[2 - Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (1
6*Sqrt[2 + Sqrt[3]]*a*(14*b^(1/3)*d - 25*(1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(105*3^(1/4)*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^5\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}} - \frac{2\int \frac{a^2bd+2a^2bex-3ab^2cx^2-\frac{3}{2}ab^2dx^3-\frac{3}{2}ab^2ex^4}{\sqrt{a+bx^3}}\,dx}{3ab^3}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{4\int \frac{\frac{7}{2}a^2b^2d+10a^2b^2ex-\frac{21}{2}ab^3cx^2-\frac{21}{4}ab^3dx^3}{\sqrt{a+bx^3}}\,dx}{21ab^4}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{8\int \frac{14a^2b^3d+25a^2b^3ex-\frac{105}{4}ab^4cx^2}{\sqrt{a+bx^3}}\,dx}{105ab^5}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{8\int \frac{14a^2b^3d+25a^2b^3ex}{\sqrt{a+bx^3}}\,dx}{105ab^5} + \frac{(2c)\int \frac{x^2}{\sqrt{a+bx^3}}}{b}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{(40ae)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}}{21b^{7/3}}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}} + \frac{4c\sqrt{a+bx^3}}{3b^2} + \frac{2dx\sqrt{a+bx^3}}{5b^2} + \frac{2ex^2\sqrt{a+bx^3}}{7b^2} - \frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

Mathematica [C] time = 0.111454, size = 134, normalized size = 0.23

$$\frac{2\left(-56adx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+150aex^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+70ac+56adx-150aex^2+35bcx^3+21bd\right.}{105b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(70*a*c + 56*a*d*x - 150*a*e*x^2 + 35*b*c*x^3 + 21*b*d*x^4 + 15*b*e*x^5
- 56*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a
)] + 150*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*
x^3)/a)]))/(105*b^2*Sqrt[a + b*x^3])
```

Maple [A] time = 0.02, size = 836, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3/b^2*x^2*a/((x^3+1/b*a)*b)^(1/2)+2/7/b^2*x^2*(b*x^3+a)^(1/2)+80/63*I/
b^3*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/
```

```
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+d*(2/3/b^2*x*a/((x^3+1/b*a
)*b)^(1/2)+2/5/b^2*x*(b*x^3+a)^(1/2)+32/45*I/b^3*a*3^(1/2)*(-b^2*a)^(1/3)*(
I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a
)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+c*(2/3/b^2*a/((x^3+1/b*a)*b)^(1/2)+2
/3/b^2*(b*x^3+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{3}c\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}b^2}\right)+\int\frac{\left(ex^7+dx^6\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) + integrate((e*x^7 +
d*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ex^7+dx^6+cx^5\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^7 + d*x^6 + c*x^5)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2
), x)
```

Sympy [A] time = 27.0308, size = 129, normalized size = 0.22

$$c\left(\begin{cases}\frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for}\ b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{dx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{ex^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + d*x**7*gamma(7/3)*hyper((3/2, 7/3)
, (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3)) + e*x**8*gamm
a(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gam
ma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^5}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2), x)
```

3.438 $\int \frac{x^4\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=574

$$-\frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt[a + b*x^3
])/(3*b^2) + (2*e*x*Sqrt[a + b*x^3])/(5*b^2) + (8*c*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1 - Sqrt[3])
*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.46911, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1828, 1888, 1886, 261, 1878, 218, 1877}

$$-\frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) + (4*d*Sqrt[a + b*x^3
])/(3*b^2) + (2*e*x*Sqrt[a + b*x^3])/(5*b^2) + (8*c*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*c*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2 + Sqrt[3]]*a^(1/3)*(5*(1 - Sqrt[3])
*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{2\int\frac{a^2e-2abcx-3abdx^2-\frac{3}{2}abex^3}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$=\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{2ex\sqrt{a+bx^3}}{5b^2}-\frac{4\int\frac{4a^2be-5ab^2cx-\frac{15}{2}ab^2dx^2}{\sqrt{a+bx^3}}\,dx}{15ab^3}$$

$$=\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{2ex\sqrt{a+bx^3}}{5b^2}-\frac{4\int\frac{4a^2be-5ab^2cx}{\sqrt{a+bx^3}}\,dx}{15ab^3}+\frac{(2d)\int\frac{x^2}{\sqrt{a+bx^3}}\,dx}{b}$$

$$=\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}+\frac{2ex\sqrt{a+bx^3}}{5b^2}+\frac{(4c)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}}-\frac{\left(4\sqrt[3]{a}\left(5\left(1\right.\right.\right.}{}$$

$$=\frac{2x\left(ae-bcx-bdx^2\right)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2}+\frac{2ex\sqrt{a+bx^3}}{5b^2}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4\sqrt{2-}}{}$$

Mathematica [C] time = 0.102964, size = 127, normalized size = 0.22

$$\frac{2\left(-15bcx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-8aex\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+10ad+8aex+15bcx^2+5bdx^3+3bex^4\right)}{15b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(10*a*d + 8*a*e*x + 15*b*c*x^2 + 5*b*d*x^3 + 3*b*e*x^4 - 8*a*e*x*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 15*b*c*x^2*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(15*b^2*
Sqrt[a + b*x^3])
```

Maple [A] time = 0.009, size = 817, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3/b^2*x*a/((x^3+1/b*a)*b)^(1/2)+2/5/b^2*x*(b*x^3+a)^(1/2)+32/45*I/b^3*
a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(2/3/b^2
*a/((x^3+1/b*a)*b)^(1/2)+2/3/b^2*(b*x^3+a)^(1/2))+c*(-2/3/b*x^2/((x^3+1/b*a
```

```
)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a
)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^4}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^6 + dx^5 + cx^4\right)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2
), x)
```

Sympy [A] time = 18.1381, size = 129, normalized size = 0.22

$$d\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**5*gamma(5/3)*hyper((3/2, 5/3)
, (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3)) + e*x**7*gamma(
7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma
```

```
(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^4}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

3.439 $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=542

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2}\cdots}{\cdots}$$

```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*x^3])/(3*b
^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 +
 Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rubi [A] time = 0.327242, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1828, 1886, 261, 1878, 218, 1877}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```


```
[Out] (-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (4*e*Sqrt[a + b*x^3])/(3*b
^2) + (8*d*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
 - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*d*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 +
 Sqrt[3]]*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} - \frac{2\int \frac{-abc-2abdx-3abex^2}{\sqrt{a+bx^3}}\,dx}{3ab^2}$$

$$= -\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} - \frac{2\int \frac{-abc-2abdx}{\sqrt{a+bx^3}}\,dx}{3ab^2} + \frac{(2e)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{b}$$

$$= -\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{(4d)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} + \frac{\left(2\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\right)\int \frac{}{}}{3b^{4/3}}$$

$$= -\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} + \frac{4e\sqrt{a+bx^3}}{3b^2} + \frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3^{3/}}$$

Mathematica [C] time = 0.0950857, size = 118, normalized size = 0.22

$$\frac{2\left(bcx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-3bdx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+2ae-bcx+3bdx^2+bex^3\right)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(2*a*e - b*c*x + 3*b*d*x^2 + b*e*x^3 + b*c*x*Sqrt[1 + (b*x^3)/a]*Hyperge
ometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hyp
ergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(3*b^2*Sqrt[a + b*x^3])
```

Maple [A] time = 0.007, size = 800, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3/b^2*a/((x^3+1/b*a)*b)^(1/2)+2/3/b^2*(b*x^3+a)^(1/2))+d*(-2/3/b*x^2/(
(x^3+1/b*a)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1
/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^
2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1
/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+c
*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2
/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
```

```
*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x^3}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^5 + dx^4 + cx^3)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2
), x)
```

Sympy [A] time = 13.5041, size = 129, normalized size = 0.24

$$e\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
 Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**4*gamma(4/3)*hyper((4/3, 3/2)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + d*x**5*gamma(
5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(
8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+dx+c)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

3.440 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=522

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\cdots$$

```
[Out] (-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (8*e*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - 2*(1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.26164, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1823, 1878, 218, 1877}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (8*e*Sqrt[a + b*x^3])/(3*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*e*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4
)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(b^(1/3)*d - 2*(1 - Sqrt
[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
```

```
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} + \frac{2\int \frac{d+2ex}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} + \frac{(4e)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} + \frac{\left(2\left(d-\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}} + \frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

Mathematica [C] time = 0.0880882, size = 107, normalized size = 0.2

$$\frac{2dx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-2\left(3ex^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+c+x(d-3ex)\right)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] -
 2*(c + x*(d - 3*e*x) + 3*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)]))/(3*b*Sqrt[a + b*x^3])
```

Maple [B] time = 0.007, size = 779, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(-2/3/b*x^2/((x^3+1/b*a)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+d*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^
(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*c/b/(b*x^3+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,c}{3\,\sqrt{bx^3+ab}} + \int \frac{\left(ex^4+dx^3\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*c/(sqrt(b*x^3 + a)*b) + integrate((e*x^4 + d*x^3)*sqrt(b*x^3 + a)/(b^2
*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^4+dx^3+cx^2\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2
), x)
```

Sympy [A] time = 11.5607, size = 109, normalized size = 0.21

$$c\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{ex^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] c*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + dx + c\right)x^2}{\left(bx^3 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2), x)
```

3.441 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}} dx$

Optimal. Leaf size=561

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+b^{2/3}\left(c-\sqrt{3}c\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+ -$$

```
[Out] (-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^
3])/(3*a*b) - (2*c*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*
Sqrt[2 + Sqrt[3]]*(b^(2/3)*(c - Sqrt[3]*c) + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(4/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.321337, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1828, 1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+b^{2/3}\left(c-\sqrt{3}c\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) - (2*d*Sqrt[a + b*x^
3])/(3*a*b) - (2*c*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*c*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*
Sqrt[2 + Sqrt[3]]*(b^(2/3)*(c - Sqrt[3]*c) + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(4/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}dx = -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-ae+\frac{bcx}{2}+\frac{3}{2}bdx^2}{\sqrt{a+bx^3}}dx}{3ab}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-ae+\frac{bcx}{2}}{\sqrt{a+bx^3}}dx}{3ab} - \frac{d\int \frac{x^2}{\sqrt{a+bx^3}}dx}{a}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{c\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{3a\sqrt[3]{b}} + \frac{\left(\frac{(1-\sqrt{3})b^{2/3}c}{a^{2/3}}+2e\right)\int \frac{1}{\sqrt{a+bx^3}}dx}{3b}$$

$$= -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt{2-\sqrt{3}}c\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Mathematica [C] time = 0.0642084, size = 108, normalized size = 0.19

$$\frac{3bcx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+4aex\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-4a(d+ex)}{6ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-4*a*(d + e*x) + 4*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4
/3, -((b*x^3)/a)] + 3*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/
2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

Maple [A] time = 0.005, size = 782, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(-2/3/b*x/((x^3+1/b*a)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a
)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3)))^(1/2)))-2/3*d/b/(b*x^3+a)^(1/2)+c*(2/3*x^2/a/((x^3+
1/b*a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)
-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-
b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/
```

```
2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^
(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-
b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1
/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3 + a}\,(ex^3 + dx^2 + cx)}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^3 + d*x^2 + c*x)/(b^2*x^6 + 2*a*b*x^3 + a^2),
 x)
```

Sympy [A] time = 11.1238, size = 109, normalized size = 0.19

$$d\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, \frac{3}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3}, \frac{3}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] d*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + dx + c)x}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

3.442 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=532

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\cdots}}{\cdots}$$

```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
(2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*d*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 -
Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.24596, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1854, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
(2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (Sqrt[2 - Sqrt[3]]*d*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*c + (1 -
Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
```

```
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{c}{2}+\frac{dx}{2}}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= -\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} - \frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} + \frac{\left(c+\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= -\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}} + \frac{\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2\cdots}}\cdots}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}$$

Mathematica [C] time = 0.0474435, size = 109, normalized size = 0.2

$$\frac{2bcx\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3bdx^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-4ae+4bcx}{6ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

```
[Out] (-4*a*e + 4*b*c*x + 2*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

Maple [A] time = 0.004, size = 785, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*e/b/(b*x^3+a)^(1/2)+d*(2/3*x^2/a/((x^3+1/b*a)*b)^(1/2)+2/9*I/a*3^(1/2)
/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+c*(2/3*x/a/((x^3+1/b*a)*b)^(1/2)-2/9*
I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(
1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}\left(ex^2 + dx + c\right)}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 11.2642, size = 107, normalized size = 0.2

$$e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

```
[Out] e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True
)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.443 $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=579

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\cdots}}{\cdots}$$

```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt[a + b*x^3
])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 -
Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.399942, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1829, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```


```
[Out] (2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*c*Sqrt[a + b*x^3
])/(3*a^2) - (2*e*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)) + (Sqrt[2 -
Sqrt[3]]*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b^(1
/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
```

```
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{3bc}{2}-\frac{bdx}{2}+\frac{1}{2}bex^2-\frac{3b^2cx^3}{2a}}{x\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{bd}{2}+\frac{bex}{2}-\frac{3b^2cx^2}{2a}}{\sqrt{a+bx^3}}\,dx}{3ab} + \frac{c\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{a}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{bd}{2}+\frac{bex}{2}}{\sqrt{a+bx^3}}\,dx}{3ab} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a} + \frac{(bc)\int \frac{x^2}{\sqrt{a+bx^3}}\,dx}{a^2}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{3a^2} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab} - \frac{e\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,d}{3a\sqrt[3]{b}}$$

$$= \frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}} + \frac{2c\sqrt{a+bx^3}}{3a^2} - \frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2c\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} + \frac{\sqrt{2-}}{}$$

Mathematica [C] time = 0.10284, size = 119, normalized size = 0.21

$$\frac{4c\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)+x\left(2d\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+3ex\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+4d\right)}{6a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (4*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] + x*(4*d + 2*d*Sqrt[1 +
 (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*e*x*Sqrt[1 +
 (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(6*a*Sqrt[a +
b*x^3])
```

Maple [A] time = 0.007, size = 810, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3*x^2/a/((x^3+1/b*a)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1
/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*
3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^
(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2))))+d*(2/3*x/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-b^2*a)^(1
/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+c*(2/3/a/((x^3+1/b*a)*b)^(1/2)-
2/3/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + dx + c}{\left(bx^3 + a\right)^{\frac{3}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}\left(ex^2 + dx + c\right)}{b^2x^7 + 2\,abx^4 + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b^2*x^7 + 2*a*b*x^4 + a^2*x), x
)
```

Sympy [A] time = 22.4442, size = 265, normalized size = 0.46

$$c\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{dx\Gamma\left(\frac{1}{3}\right)2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3)) + d*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + e*x**2*gamma(2/3)*hyper((2/3,
3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

3.444 $\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-$$

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt[a + b*x^3
])/(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c*Sqrt[a + b*x^3])/(3
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a^(5/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.559812, antiderivative size = 607, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1829, 1835, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-5\sqrt{2-}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + (2*d*Sqrt[a + b*x^3
])/(3*a^2) - (c*Sqrt[a + b*x^3])/(a^2*x) + (5*b^(1/3)*c*Sqrt[a + b*x^3])/(3
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*d*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/(3*a^(3/2)) - (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*c*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3]) - (Sqrt[2 + Sqrt[3]]*(5*(1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a^(5/3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{2\int \frac{-\frac{3bc}{2}-\frac{3bdx}{2}-\frac{1}{2}bex^2-\frac{b^2cx^3}{2a}-\frac{3b^2dx^4}{2a}}{x^2\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{3abd+abex+\frac{5}{2}b^2cx^2+3b^2dx^3}{x\sqrt{a+bx^3}}\,dx}{3a^2b}$$

$$= \frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx+3b^2dx^2}{\sqrt{a+bx^3}}\,dx}{3a^2b} + \frac{d\int \frac{1}{x\sqrt{a+bx^3}}\,dx}{a}$$

$$= \frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\int \frac{abe+\frac{5}{2}b^2cx}{\sqrt{a+bx^3}}\,dx}{3a^2b} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a} + \frac{(bd)\int \cdots}{a}$$

$$= \frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}} + \frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{\left(5b^{2/3}c\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{6a^2} + \frac{(2d)\,\mathrm{Subst}\cdots}{\cdots}$$

$$= \frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}} + \frac{2d\sqrt{a+bx^3}}{3a^2} - \frac{c\sqrt{a+bx^3}}{a^2x} + \frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2d\tanh^{-1}\left(\cdots\right)}{3a^{3/2}}$$

Mathematica [C] time = 0.0984323, size = 121, normalized size = 0.2

$$\frac{-3c\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a}\right)+2dx\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)+ex^2\left(\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+2\right)}{3ax\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] - 3*c*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[-1/3, 3/2, 2/3, -((b*x^3)/a)] + e*x^2*(2 + Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*x*Sqrt[a +
 b*x^3])
```

Maple [A] time = 0.008, size = 825, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x)
```

```
[Out] e*(2/3*x/a/((x^3+1/b*a)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2
/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(2/3/a/((x^3+1/b*a)*b)^(1/2)-2/3/a^(3/2)*a
rctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-2/3*b*x^2/a^2/((x^3+1/b*a)*b)^(1/2)-1/
a^2*(b*x^3+a)^(1/2)/x-5/9*I/a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-
1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b
^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+
1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2
*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\left(ex^2+dx+c\right)}{b^2x^8+2\,abx^5+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*(e*x^2 + d*x + c)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2),
 x)
```

Sympy [A] time = 34.6304, size = 267, normalized size = 0.44

$$d\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{c\Gamma\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*
x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) +
1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/
2) + 3*a**(7/2)*b*x**3)) + c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3)) + e*x*gamma(1/3)*hyper((1/3, 3
/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.445 $\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx$

Optimal. Leaf size=733

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-8af)-1\right.}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (-4*a^2*e*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*Sqrt[a + b*x^
3])/(935*b^2) + (6*a*(19*b*d - 10*a*g)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2
*a*e*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*f*x^4*Sqrt[a + b*x^3])/(187*b) + (6
*a*g*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*b*d - 10*a*g)*Sqrt[a + b*x^
3])/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*Sqrt[a + b*
x^3]*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/6
92835 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 10*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 8*
a*f) - 1870*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rubi [A] time = 1.90966, antiderivative size = 733, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1\right.\right.}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (-4*a^2*e*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*c - 8*a*f)*x*Sqrt[a + b*x^
3])/(935*b^2) + (6*a*(19*b*d - 10*a*g)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2
*a*e*x^3*Sqrt[a + b*x^3])/(45*b) + (6*a*f*x^4*Sqrt[a + b*x^3])/(187*b) + (6
*a*g*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*b*d - 10*a*g)*Sqrt[a + b*x^
3])/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^3*Sqrt[a + b*
x^3]*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))/6
92835 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*d - 10*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1729*b^(1/3)*(17*b*c - 8*
```

```
a*f) - 1870*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

```
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465\right.}{692835}$$

$$= \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+\right.}{692835}$$

$$= \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295\right.}{}$$

$$= \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}\left(\right.}{}$$

$$= \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \frac{6agx^5\sqrt{a+bx^3}}{247b} + \frac{2x^3\sqrt{a+bx^3}\left(\right.}{}$$

$$= \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \cdots$$

$$= \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a+bx^3}}{45b} + \frac{6afx^4\sqrt{a+bx^3}}{187b} + \cdots$$

$$= \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a}}{45b}$$

$$= \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2aex^3\sqrt{a}}{45b}$$

$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a}}{1729b^2}$$

$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2} + \frac{6a(19bd-10ag)x^2\sqrt{a}}{1729b^2}$$

Mathematica [C] time = 0.299319, size = 172, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(-\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(a(92378e+90x(988f+935gx))-3bx\left(62985c+11x\left(4845d+13x\left(323e+285fx\right.\right.\right.\right.\right.}{2078505b^2\sqrt{\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(a*(92378*e + 90*x*(9
88*f + 935*g*x)) - 3*b*x*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f*x +
255*g*x^2))))) + 11115*a*(-17*b*c + 8*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4
/3, -((b*x^3)/a)] + 8415*a*(-19*b*d + 10*a*g)*x^2*Hypergeometric2F1[-1/2, 2
/3, 5/3, -((b*x^3)/a)]))/(2078505*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.019, size = 1674, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] g*(2/19*x^8*(b*x^3+a)^(1/2)+6/247/b*a*x^5*(b*x^3+a)^(1/2)-60/1729*a^2/b^2*x
^2*(b*x^3+a)^(1/2)-80/1729*I*a^3/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b
^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/
b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*
(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2
))))+f*(2/17*x^7*(b*x^3+a)^(1/2)+6/187/b*a*x^4*(b*x^3+a)^(1/2)-48/935*a^2/b
^2*x*(b*x^3+a)^(1/2)-32/935*I*a^3/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-
b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2
)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3)))^(1/2)))+e*(2/15*x^6*(b*x^3+a)^(1/2)+2/45/b*a*x^3*(b*x^3+a
)^(1/2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+d*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*
x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*
(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2
))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-
b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
))+c*(2/11*x^4*(b*x^3+a)^(1/2)+6/55/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^
(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^7 + fx^6 + ex^5 + dx^4 + cx^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((g*x^7 + f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 5.10566, size = 238, normalized size = 0.32

$$\frac{\sqrt{a}cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}gx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\right.}{3\Gamma\left(\frac{11}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + sqrt(a)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*f*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*g*
x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(11/3)) + e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqr
t(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6
/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

3.446 $\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx$

Optimal. Leaf size=681

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag+1547bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8*a*g)*x*Sq
rt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*f*x^3*
Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a + b*x^3])/(187*b) - (24*a^2*e*S
qrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*S
qrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x
^5))/109395 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)
*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.42152, antiderivative size = 681, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag+1547bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```


```
[Out] (2*a*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*(17*b*d - 8*a*g)*x*Sq
rt[a + b*x^3])/(935*b^2) + (6*a*e*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*f*x^3*
Sqrt[a + b*x^3])/(45*b) + (6*a*g*x^4*Sqrt[a + b*x^3])/(187*b) - (24*a^2*e*S
qrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*S
qrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6435*g*x
^5))/109395 + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 1870*(1 - Sqrt[3])*a^(1/3)
*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
```

```
 - 4*Sqrt[3]])/(85085*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395} + \frac{1}{2}$$

$$= \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293f\right.}{109395}$$

$$= \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+\right.}{1093}$$

$$= \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}\left(12\right.}{}$$

$$= \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4\sqrt{a+bx^3}}{187b} + \frac{2x^2\sqrt{a+bx^3}\left(12\right.}{}$$

$$= \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4}{1}$$

$$= \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} + \frac{2afx^3\sqrt{a+bx^3}}{45b} + \frac{6agx^4}{1}$$

$$= \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} +$$

$$= \frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2} + \frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2} + \frac{6aex^2\sqrt{a+bx^3}}{91b} +$$

Mathematica [C] time = 0.253586, size = 158, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(-\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(26a(187f+180gx)-b\left(12155c+9945dx+33x^2(255e+13x(17f+15gx))\right)\right)+585ax\right.}{109395b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(26*a*(187*f + 180*g*
x) - b*(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x))))) + 585
*a*(-17*b*d + 8*a*g)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] - 84
15*a*b*e*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(109395*b^2*
Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.008, size = 1197, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] g*(2/17*x^7*(b*x^3+a)^(1/2)+6/187/b*a*x^4*(b*x^3+a)^(1/2)-48/935*a^2/b^2*x*
(b*x^3+a)^(1/2)-32/935*I*a^3/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x
-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)))+f*(2/15*x^6*(b*x^3+a)^(1/2)+2/45/b*a*x^3*(b*x^3+a)^(1/
2)-4/45*a^2/b^2*(b*x^3+a)^(1/2))+e*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*x^2*(
b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/
b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2
*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/
b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a
)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+d*
(2/11*x^4*(b*x^3+a)^(1/2)+6/55/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)
*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/9*c/b*(b*x^3+a)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}c}{9\,b}+\int\left(gx^6+fx^5+ex^4+dx^3\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*c/b + integrate((g*x^6 + f*x^5 + e*x^4 + d*x^3)*sqrt(
b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^6 + fx^5 + ex^4 + dx^3 + cx^2\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((g*x^6 + f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.52438, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}gx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + c\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b = \\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{other}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + sqrt(a)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*g*x**7*gamma(7/3)*hyper((
-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + c*Piecewis
e((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + f*Piec
ewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b
) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2, x)
```

3.447 $\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx$

Optimal. Leaf size=667

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right)\cdots\right)}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a + b*x^3])/
(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^3*Sqrt[a + b*x^3])/(
45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e
*x^3 + 3465*f*x^4 + 3003*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/
3)*(13*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
^(4/3)*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rubi [A] time = 1.04731, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1826, 1836, 1888, 1886, 261, 1878, 218, 1877}

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{bx}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\cdots\right)}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a*(5*b*d - 2*a*g)*Sqrt[a + b*x^3])/(45*b^2) + (6*a*e*x*Sqrt[a + b*x^3])/
(55*b) + (6*a*f*x^2*Sqrt[a + b*x^3])/(91*b) + (2*a*g*x^3*Sqrt[a + b*x^3])/(
45*b) + (6*a*(13*b*c - 4*a*f)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) + (2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e
*x^3 + 3465*f*x^4 + 3003*g*x^5))/45045 - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/
3)*(13*b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
^(4/3)*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(5/3)*S
```

```
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045} + \frac{1}{2}$$

$$= \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465f\right.}{45045}$$

$$= \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2\right.}{450}$$

$$= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}\left(64\right.}{}$$

$$= \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{a+bx^3}}{45b} + \frac{2x\sqrt{a+bx^3}\left(64\right.}{}$$

$$= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{}}{45}$$

$$= \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2} + \frac{6aex\sqrt{a+bx^3}}{55b} + \frac{6afx^2\sqrt{a+bx^3}}{91b} + \frac{2agx^3\sqrt{}}{45}$$

Mathematica [C] time = 0.193972, size = 143, normalized size = 0.21

$$\frac{\sqrt{a+bx^3}\left(495bx^2(13bc-4af)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-4\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(286ag-b\left(715d+585ex+495fx^2+429g\right.\right.\right.}{12870b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(286*a*g - b*(715*d +
585*e*x + 495*f*x^2 + 429*g*x^3)) - 2340*a*b*e*x*Hypergeometric2F1[-1/2, 1/
3, 4/3, -((b*x^3)/a)] + 495*b*(13*b*c - 4*a*f)*x^2*Hypergeometric2F1[-1/2,
2/3, 5/3, -((b*x^3)/a)]))/(12870*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.006, size = 1311, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] g*(2/15*x^6*(b*x^3+a)^(1/2)+2/45/b*a*x^3*(b*x^3+a)^(1/2)-4/45*a^2/b^2*(b*x^
3+a)^(1/2))+f*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*x^2*(b*x^3+a)^(1/2)+8/91*I
/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+e*(2/11*x^4*(b*x^3+a)^(
1/2)+6/55/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/9*d/b*(b*x^3+a)^(3/2)+c*(2/7*x^2*(b*x^
3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a
)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^5 + fx^4 + ex^3 + dx^2 + cx\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((g*x^5 + f*x^4 + e*x^3 + d*x^2 + c*x)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.30689, size = 223, normalized size = 0.33

$$\frac{\sqrt{a}cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + d\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for} \\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{oth}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + sqrt(a)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*f*x**5*gamma(5/3)*hyper((
-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + d*Piecewise(
(sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + g*Piecew
ise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b)
+ 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

3.448 $\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=639

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(11bc-2af)-55\left(1\right.\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b) + (6*a*g*x
^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt[a + b*x^3])/(91*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(9009*c*x +
6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(4/3)*(13*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)
*(13*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.719342, antiderivative size = 639, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1853, 1888, 1886, 261, 1878, 218, 1877}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{}\right.}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```


```
[Out] (2*a*e*Sqrt[a + b*x^3])/(9*b) + (6*a*f*x*Sqrt[a + b*x^3])/(55*b) + (6*a*g*x
^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*b*d - 4*a*g)*Sqrt[a + b*x^3])/(91*b^(
5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a + b*x^3]*(9009*c*x +
6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/45045 - (3*3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(4/3)*(13*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)
*(13*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5005*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}+\frac{1}{2}(3a)$$

$$= \frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3\right.}{45045}$$

$$= \frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005\right.}{45045}$$

$$= \frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005\right.}{45045}$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{2\sqrt{a+bx^3}\left(9009cx+6\right.}{}$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b}+\frac{6afx\sqrt{a+bx^3}}{55b}+\frac{6agx^2\sqrt{a+bx^3}}{91b}+\frac{6a(13bd-4ag)\sqrt{a+}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

Mathematica [C] time = 0.148057, size = 135, normalized size = 0.21

$$\frac{\sqrt{a+bx^3}\left(234x(11bc-2af)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+99x^2(13bd-4ag)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(143e+9\right.}{2574b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(143*e + 9*x*(13*f + 11
*g*x)) + 234*(11*b*c - 2*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)
/a)] + 99*(13*b*d - 4*a*g)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2574*b*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.005, size = 1557, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x)
```

```
[Out] g*(2/13*x^5*(b*x^3+a)^(1/2)+6/91/b*a*x^2*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(
1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a
)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+
```

```
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(2/11*x^4*(b*x^3+a)^(1/2)+6/55/b*
a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)))+2/9*e/b*(b*x^3+a)^(3/2)+d*(2/7*x^2*(b*x^3+a)^(1/2)-2
/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+c*(2/5*x*(b*x^3+a)^(1/2
)-2/5*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/
(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^
2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.90194, size = 194, normalized size = 0.3

$$\frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right.}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*f*x**4*gamma(4/3)*hyper((-1/
2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*g*x**5*
gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/
3)) + e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b),
 True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

3.449 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

Optimal. Leaf size=620

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}\right.\right.}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b) + (6*a*e*S
qrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a
 + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465
*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rubi [A] time = 0.547109, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1826, 1832, 266, 63, 208, 1888, 1886, 261, 1878, 218, 1877}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|-7}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (2*a*f*Sqrt[a + b*x^3])/(9*b) + (6*a*g*x*Sqrt[a + b*x^3])/(55*b) + (6*a*e*S
qrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqrt[a
 + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465
*x) - (2*Sqrt[a]*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a
*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(385*b^(4/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
```

```
^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx = \frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}+\frac{1}{2}(3a)\int$$

$$= \frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}+\frac{1}{2}(3a)\int$$

$$= \frac{6agx\sqrt{a+bx^3}}{55b}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+31\right.}{3465x}$$

$$= \frac{6agx\sqrt{a+bx^3}}{55b}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+31\right.}{3465x}$$

$$= \frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex\right.}{3465x}$$

$$= \frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+}}{}$$

Mathematica [C] time = 0.317007, size = 185, normalized size = 0.3

$$\frac{4\sqrt{\frac{bx^3}{a}+1}\left(\sqrt{a+bx^3}\left(11af+9agx+33bc+11bfx^3+9bgx^4\right)-33\sqrt{a}bc\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+18x\sqrt{a+bx^3}(11bd-2ag}{198b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(33*b*c + 11*a*f + 9*a*g*x + 11*b*f
*x^3 + 9*b*g*x^4) - 33*Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 18*(
11*b*d - 2*a*g)*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^
3)/a)] + 99*b*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*
x^3)/a)])/(198*b*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.008, size = 1118, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x)
```

```
[Out] g*(2/11*x^4*(b*x^3+a)^(1/2)+6/55/b*a*x*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/
2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/9*f/b*(b*x^3+a)
^(3/2)+e*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(
1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))))+d*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+c*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arcta
nh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

Sympy [A] time = 9.2548, size = 235, normalized size = 0.38

$$-\frac{2\sqrt{a}c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}gx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*d*x*gamma(1/3)*h
yper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a
)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(5/3)) + sqrt(a)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**
3) + 1)) + 2*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + f*Piecewise((sqr
t(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

3.450 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

Optimal. Leaf size=638

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(2af+7bc)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b*c + 2*a*f
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqr
t[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x
^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.649673, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1826, 1835, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(2af+7bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*a*g*Sqrt[a + b*x^3])/(9*b) - (3*c*Sqrt[a + b*x^3])/x + (3*(7*b*c + 2*a*f
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*Sqr
t[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))/(315*x
^2) - (2*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 2*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(35*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
```

```
)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx = \frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}+\frac{1}{2}(3a)\int \frac{2c+\frac{2d}{3}\cdots}{\cdots}$$

$$= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}-\cdots$$

$$= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}-\cdots$$

$$= -\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}-\cdots$$

$$= \frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x}+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx\cdots\right.}{315x^2}$$

$$= \frac{2ag\sqrt{a+bx^3}}{9b}-\frac{3c\sqrt{a+bx^3}}{x}+\frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2\sqrt{a+bx^3}\left(3\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.244137, size = 211, normalized size = 0.33

$$-\frac{c\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a}+1}}+\frac{2}{3}d\left(\sqrt{a+bx^3}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{ex\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+\frac{fx^2\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*g*(a + b*x^3)^(3/2))/(9*b) + (2*d*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]]))/3 - (c*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3
, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (e*x*Sqrt[a + b*x^3]*Hyperg
eometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a] + (f*x^2*Sqr
t[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (
b*x^3)/a])
```

Maple [B] time = 0.009, size = 1248, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] 2/9*g/b*(b*x^3+a)^(3/2)+f*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-b^2*
a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^
(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2))))+e*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(2/3*(b*x^3+a)^(1/2)-
2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-(b*x^3+a)^(1/2)/x-I*3^(1/
2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

Sympy [A] time = 5.50739, size = 236, normalized size = 0.37

$$\frac{\sqrt{a}c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}d\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}fx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3)) - 2*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(
a)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + sqrt(a)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3)
 + 1)) + 2*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + g*Piecewise((sqrt(
a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

3.451 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

Optimal. Leaf size=640

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(4af+5bc)-10\left(1-\right.\right.}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b*d + 2*a*g
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqr
t[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5))/(105*x
^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) - 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d +
2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.764714, antiderivative size = 640, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(4af+5bc)-10\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```


```
[Out] (3*c*Sqrt[a + b*x^3])/(2*x^2) - (3*d*Sqrt[a + b*x^3])/x + (3*(7*b*d + 2*a*g
)*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqr
t[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5))/(105*x
^3) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(7*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2
 + Sqrt[3]]*(7*b^(1/3)*(5*b*c + 4*a*f) - 10*(1 - Sqrt[3])*a^(1/3)*(7*b*d +
2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(70*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
```

```
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx = -\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}+\frac{1}{2}(3a)\int \frac{-2c}{}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}-$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21f\right.}{105x^3}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21f\right.}{105x^3}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21f\right.}{105x^3}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}+\frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{a+bx^3}\,(}{}$$

$$= \frac{3c\sqrt{a+bx^3}}{2x^2}-\frac{3d\sqrt{a+bx^3}}{x}+\frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{a+bx^3}\,(}{}$$

Mathematica [C] time = 0.410574, size = 218, normalized size = 0.34

$$\frac{x\left(x\left(4e\sqrt{\frac{bx^3}{a}+1}\left(\sqrt{a+bx^3}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+6fx\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+3gx^2\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3}\right.\right.\right.}{6x^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (-3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + x*
(-6*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)] + x*
(4*e*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]]) + 6*f*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x
^3)/a)] + 3*g*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^
3)/a)])))/(6*x^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1529, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] g*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b
^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/
b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*
(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2
))))+f*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2)))+e*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x
^3+a)^(1/2)/a^(1/2)))+c*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1
/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)
*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-
b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

Sympy [A] time = 6.01137, size = 255, normalized size = 0.4

$$\frac{\sqrt{a}c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*e*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*g*x**2*gamma(2/3)*hyper((-1/2,
2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*e/(3*sqrt(b)*x
**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

3.452 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

Optimal. Leaf size=637

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+5bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*e*Sqrt[a +
 b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))
/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) -
(3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqr
t[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rubi [A] time = 0.844394, antiderivative size = 637, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+5bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```



```
[Out] (c*Sqrt[a + b*x^3])/(3*x^3) + (3*d*Sqrt[a + b*x^3])/(2*x^2) - (3*e*Sqrt[a +
 b*x^3])/x + (3*b^(1/3)*e*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x) - (2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))
/(15*x^4) - ((b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) -
(3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqr
t[2 + Sqrt[3]]*(5*b*d - 10*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
```

```
qrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx = -\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}+\frac{1}{2}(3a)\int \frac{-\frac{2c}{3}-2d\ldots}{\ldots}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}-\frac{1}{4}\int \ldots$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-\ldots\right)}{15x^4}\ldots$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-\ldots\right)}{15x^4}\ldots$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-\ldots\right)}{15x^4}\ldots$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-\ldots\right)}{15x^4}\ldots$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}-\frac{2\sqrt{a}\ldots}{\ldots}$$

$$= \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}-\frac{2\sqrt{a}\ldots}{\ldots}$$

Mathematica [C] time = 0.386874, size = 254, normalized size = 0.4

$$-\frac{bc\left(\frac{a+bx^3}{bx^3}+\sqrt{\frac{bx^3}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)\right)}{3\sqrt{a+bx^3}}-\frac{d\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt{\frac{bx^3}{a}+1}}-\frac{e\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a}+1}}+\frac{2}{3}\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (2*f*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/3 - (b*c
*((a + b*x^3)/(b*x^3) + Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]]))/
(3*Sqrt[a + b*x^3]) - (d*Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3,
 -((b*x^3)/a)])/(2*x^2*Sqrt[1 + (b*x^3)/a]) - (e*Sqrt[a + b*x^3]*Hypergeome
tric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (g*x*Sqrt
[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^
3)/a]
```

Maple [B] time = 0.01, size = 1114, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] g*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)))+f*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)
^(1/2)/a^(1/2)))+c*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/
a^(1/2))/a^(1/2))+d*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1/3)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

Sympy [A] time = 6.89922, size = 265, normalized size = 0.42

$$\frac{\sqrt{a}d\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}gx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + sqrt(a)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*f*asinh(sqrt(a)/(sqrt(
b)*x**(3/2)))/3 + sqrt(a)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**
3) + 1)) - sqrt(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*f*x**(3/
2)/(3*sqrt(a/(b*x**3) + 1)) - b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt
(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

3.453 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

Optimal. Leaf size=694

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(8af+bc)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3*e*Sqrt[a + b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x^5) - ((b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 1.08425, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(8af+bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]

[Out] (3*c*Sqrt[a + b*x^3])/(20*x^4) + (d*Sqrt[a + b*x^3])/(3*x^3) + (3*e*Sqrt[a + b*x^3])/(2*x^2) - (3*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(b*c + 8*a*f)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15*x^5) - ((b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(4*a^(2/3)*b^(1/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/

```
(8*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx = -\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}+\frac{1}{2}(3a)\int \frac{-\frac{2c}{5}-\frac{2dx}{3}-\cdots}{x^5\cdots}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}-\frac{3}{16}\int \cdots$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx\cdots\right.}{15x^5}\cdots$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3\cdots\right.}{15x^5}\cdots$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}-\frac{2\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}-\frac{2\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}-\frac{2\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}(b\cdots}{8a\left(\left(1\cdots\right.\right.}$$

$$= \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}(b\cdots}{8a\left(\left(1\cdots\right.\right.}$$

Mathematica [C] time = 0.387049, size = 253, normalized size = 0.36

$$\frac{-3c\left(a+bx^3\right)\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)-4dx\sqrt{\frac{bx^3}{a}+1}\left(bx^3\sqrt{\frac{bx^3}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+a+bx^3\right)-6ex^2\left(a+bx^3\right)\,{}_2F_1\left(\cdots\right.}{12x^4\sqrt{a+bx^3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (8*g*x^4*Sqrt[a + b*x^3]*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3] - Sqrt[a]*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 4*d*x*Sqrt[1 + (b*x^3)/a]*(a + b*x^3 + b*x
^3*Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]]) - 3*c*(a + b*x^3)*Hype
rgeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] - 6*e*x^2*(a + b*x^3)*Hyperge
ometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] - 12*f*x^3*(a + b*x^3)*Hypergeome
tric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(12*x^4*Sqrt[a + b*x^3]*Sqrt[1 + (b
*x^3)/a])
```

Maple [B] time = 0.01, size = 1286, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x)
```

```
[Out] g*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/
3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+c*(-1
/4*(b*x^3+a)^(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-b^2*a)
^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1
/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3)))^(1/2))))+e*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*
a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a
)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(-(b*x^3+a)^(1/2)/x-I*3^
(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

Sympy [A] time = 7.31502, size = 274, normalized size = 0.39

$$\frac{\sqrt{a}c\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}g\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + sqrt(a)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*f*gamma(-1/3)*hyper
((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(
a)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a*g/(3*sqrt(b)*x**(3/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*g
*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/
(3*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

3.454 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

Optimal. Leaf size=652

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(2\sqrt[3]{b}(bc-10af)+5\left(1-\right.\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt[a + b
*x^3])/60 - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d*Sqrt[a + b*x^3])/(8
*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*
3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3
^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[3]
)*a^(1/3)*(b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.787092, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(2\sqrt[3]{b}(bc-10af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```


```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt[a + b
*x^3])/60 - (3*b*c*Sqrt[a + b*x^3])/(20*a*x^2) - (3*b*d*Sqrt[a + b*x^3])/(8
*a*x) + (3*b^(1/3)*(b*d + 8*a*g)*Sqrt[a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) - (b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (3*
3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*d + 8*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3
^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(2*b^(1/3)*(b*c - 10*a*f) + 5*(1 - Sqrt[3]
)*a^(1/3)*(b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
```

```
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{1}{2}(3b)\int \frac{-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}}{x^3\sqrt{a+}}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}+\frac{(3b)\int}{}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a}}{8a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a}}{8a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a}}{8a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a}}{8a}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a}}{8a}$$

Mathematica [C] time = 0.260194, size = 180, normalized size = 0.28

$$-\frac{\sqrt{a+bx^3}\left(12ac\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)+5x\left(3ad\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)+2x\left(2ae\sqrt{\frac{bx^3}{a}+1}+2bex^3\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)\right.\right.\right.}{60ax^5\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(12*a*c*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)]
 + 5*x*(3*a*d*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 2*x*(2*a*
e*Sqrt[1 + (b*x^3)/a] + 2*b*e*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]] + 3*a*f*x*Hy
pergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 6*a*g*x^2*Hypergeometric2F
1[-1/2, -1/3, 2/3, -((b*x^3)/a)]))))/(60*a*x^5*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1571, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x)
```

```
[Out] e*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
+c*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b*3^(1/2
)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(-1/4*(b*x^3+a)^
(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+f*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1/3)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b
^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+g*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-b^2*a
)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

Sympy [A] time = 6.55081, size = 240, normalized size = 0.37

$$\frac{\sqrt{a}c\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3}, -\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, -\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(-\frac{1}{2}\right.}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*f*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sq
rt(a)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(
3*x*gamma(2/3)) - sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*e*asinh(s
qrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

3.455 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

Optimal. Leaf size=659

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag+2bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sqrt[a +
 b*x^3])/60 - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d*Sqrt[a + b*x^3])/(2
0*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*e*Sqrt[a + b*x^3])/
(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(12*a^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.979823, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag+2bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sqrt[a +
 b*x^3])/60 - (b*c*Sqrt[a + b*x^3])/(12*a*x^3) - (3*b*d*Sqrt[a + b*x^3])/(2
0*a*x^2) - (3*b*e*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*e*Sqrt[a + b*x^3])/
(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*c - 4*a*f)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(12*a^(3/2)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*e*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d + 5*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
```

```
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{1}{2}(3b)\int \frac{-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}}{x^4\sqrt{a+}}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}+\frac{b\int \frac{\frac{6ad}{5}+}{}}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+}}{20ax^2}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+}}{20ax^2}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+}}{20ax^2}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+}}{20ax^2}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+}}{20ax^2}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+}}{20ax^2}$$

Mathematica [C] time = 0.424939, size = 211, normalized size = 0.32

$$-\frac{\sqrt{a+bx^3}\left(5x\left(2x\left(6a^2f\left(a\sqrt{\frac{bx^3}{a}+1}+bx^3\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)\right)+9a^3gx\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)+4b^2cx^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}\cdots}\right.\right.\right.}{180a^3x^5\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(36*a^3*d*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a
)] + 5*x*(9*a^3*e*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 2*x*(
6*a^2*f*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]]) + 9*a^
3*g*x*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 4*b^2*c*x^3*(a + b
*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/a]))))
/(180*a^3*x^5*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.008, size = 1180, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] f*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
+d*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b*3^(1/2
)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(-1/4*(b*x^3+a)^
(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+g*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-b^2*a)^(1/3)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b
^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+c*(-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b/a*(b
*x^3+a)^(1/2)/x^3+1/12/a^(3/2)*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x)
```

Sympy [A] time = 9.38378, size = 304, normalized size = 0.46

$$\frac{\sqrt{a}d\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}e\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt{a}g\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}-\frac{ac}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + sqrt(a)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*g*gamma(-2/3)*hyp
er((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) - a*
c/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*c/(4*x**(9/2)*sqrt(a
/(b*x**3) + 1)) - sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/
(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2))
)/(3*sqrt(a)) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x)
```

3.456 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx$

Optimal. Leaf size=711

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*Sqrt[a
 + b*x^3])/420 - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (b*d*Sqrt[a + b*x^3])
/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x^2) + (3*b*(5*b*c - 14*a*f)*Sq
rt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(
112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*
(5*b*c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/
3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(5/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rubi [A] time = 1.12302, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|-7-}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)*Sqrt[a
 + b*x^3])/420 - (3*b*c*Sqrt[a + b*x^3])/(56*a*x^4) - (b*d*Sqrt[a + b*x^3])
/(12*a*x^3) - (3*b*e*Sqrt[a + b*x^3])/(20*a*x^2) + (3*b*(5*b*c - 14*a*f)*Sq
rt[a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*b*c - 14*a*f)*Sqrt[a + b*x^3])/(
112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (b*(b*d - 4*a*g)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/(12*a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*
(5*b*c - 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/
3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
```

```
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(5/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

```
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{1}{2}(3b)\int \frac{-\frac{c}{7}-\frac{d}{6}\ldots}{\ldots}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}+\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{56ax^4}-\ldots$$

Mathematica [C] time = 0.452092, size = 213, normalized size = 0.3

$$-\frac{\sqrt{a+bx^3}\left(7x^2\left(5x\left(9a^3f\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)+12a^2gx\left(a\sqrt{\frac{bx^3}{a}+1}+bx^3\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)\right)+8b^2dx^4\left(a+bx^3\right)\ldots\right.\right.\right.}{1260a^3x^7\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(180*a^3*c*Hypergeometric2F1[-7/3, -1/2, -4/3, -((b*x^3)/
a)] + 7*x^2*(36*a^3*e*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 5
*x*(12*a^2*g*x*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]])
 + 9*a^3*f*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 8*b^2*d*x^4*
(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/
a]))))/(1260*a^3*x^7*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.01, size = 1376, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x)
```

```
[Out] g*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
+e*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b*3^(1/2
)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(-1/4*(b*x^3+a)^
(1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+d*(-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b/a*(b*x^3+a)^(1/2)/x^3+1/12
/a^(3/2)*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/7/x^7*(b*x^3+a)^(1/2)-
3/56*b/a*(b*x^3+a)^(1/2)/x^4+15/112/a^2*b^2*(b*x^3+a)^(1/2)/x+5/112*I/a^2*b
^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx^4 + fx^3 + ex^2 + dx + c)\sqrt{bx^3 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

Sympy [A] time = 10.0289, size = 308, normalized size = 0.43

$$\frac{\sqrt{a}c\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}e\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}f\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}-\frac{ad}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + sqrt(a)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*f*gamma(-4/3)*hyp
er((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) -
a*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*d/(4*x**(9/2)*sqrt
(a/(b*x**3) + 1)) - sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*
d/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(3*sqrt(a)) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

3.457 $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

Optimal. Leaf size=743

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\right.\right.}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*Sqr
t[a + b*x^3])/840 - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5) - (3*b*d*Sqrt[a + b*
x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*a*x^3) + (3*b*(7*b*c - 16*a*f)
*Sqrt[a + b*x^3])/(320*a^2*x^2) + (3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(1
12*a^2*x) - (3*b^(4/3)*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*
a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16*a
*f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^2*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.33135, antiderivative size = 743, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{bx}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{}\right.}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x^4)*Sqr
t[a + b*x^3])/840 - (3*b*c*Sqrt[a + b*x^3])/(80*a*x^5) - (3*b*d*Sqrt[a + b*
x^3])/(56*a*x^4) - (b*e*Sqrt[a + b*x^3])/(12*a*x^3) + (3*b*(7*b*c - 16*a*f)
*Sqrt[a + b*x^3])/(320*a^2*x^2) + (3*b*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(1
12*a^2*x) - (3*b^(4/3)*(5*b*d - 14*a*g)*Sqrt[a + b*x^3])/(112*a^2*((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)) + (b^2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*
a^(3/2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*b*d - 14*a*g)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(4/3)*(7*b^(1/3)*(7*b*c - 16*a
*f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt
```

```
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a^2*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

```
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{1}{2}(3b)\int -\frac{c}{8}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}-\frac{3bc\sqrt{a+bx^3}}{80ax^5}$$

Mathematica [C] time = 0.190994, size = 192, normalized size = 0.26

$$-\frac{\sqrt{a+bx^3}\left(14x^3\left(36a^3f\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)+45a^3gx\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)+40b^2ex^5\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{3}{2},\right.\right.\right.}{2520a^3x^8\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(315*a^3*c*Hypergeometric2F1[-8/3, -1/2, -5/3, -((b*x^3)/
a)] + 360*a^3*d*x*Hypergeometric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)] + 14*x^
3*(36*a^3*f*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 45*a^3*g*x*
Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 40*b^2*e*x^5*(a + b*x^3
)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/a])))/(252
0*a^3*x^8*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1679, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x)
```

```
[Out] f*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x^2+1/20*I/a*b*3^(1/2)
*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+g*(-1/4*(b*x^3+a)^(
1/2)/x^4-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I/a*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1
/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*
3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^
(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2))))+c*(-1/8*(b*x^3+a)^(1/2)/x^8-3/80*b/a*(b*x^3+a)^(1/2)/x^5+21/32
0/a^2*b^2*(b*x^3+a)^(1/2)/x^2-7/320*I/a^2*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2
*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b/a*(b*x
^3+a)^(1/2)/x^3+1/12/a^(3/2)*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/7/
x^7*(b*x^3+a)^(1/2)-3/56*b/a*(b*x^3+a)^(1/2)/x^4+15/112/a^2*b^2*(b*x^3+a)^(
1/2)/x+5/112*I/a^2*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2
*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/
3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2
*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\sqrt{bx^3 + a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="frica
s")
```

```
[Out] integral((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

Sympy [A] time = 9.38585, size = 304, normalized size = 0.41

$$\frac{\sqrt{a}c\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\ -\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{\sqrt{a}d\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\ -\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{\sqrt{a}f\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{\sqrt{a}g\Gamma\left(-\frac{4}{3}\right){}_2F_1}{3x^4\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + sqrt(a)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*f*gamma(-5/3)*hyp
er((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) +
sqrt(a)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) - a*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - s
qrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e/(12*a*x**(3/2)*sqrt
(a/(b*x**3) + 1)) + b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

3.458 $\int x^3 \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) dx$

Optimal. Leaf size=791

$$-\frac{36\, 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(23bc-8af)-\right.}{37182145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (-4*a^3*e*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x*Sqrt[a +
b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*Sqrt[a + b*x^3])/(8645*b^
2) + (2*a^2*e*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*f*x^4*Sqrt[a + b*x^3])
/(4301*b) + (54*a^2*g*x^5*Sqrt[a + b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a
*g)*Sqrt[a + b*x^3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*x^3*(a + b*x^3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575*
f*x^4 + 156009*g*x^5))/3900225 + (2*a*x^3*Sqrt[a + b*x^3]*(8947575*c*x + 67
74075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5))/185910725 + (
108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*b*d - 2*a*g)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]) - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*(23*b*c - 8*a*f) - 8
602*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 2.11123, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$-\frac{36\, 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(\right.\right.}{37182145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```


```
[Out] (-4*a^3*e*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*c - 8*a*f)*x*Sqrt[a +
b*x^3])/(21505*b^2) + (54*a^2*(5*b*d - 2*a*g)*x^2*Sqrt[a + b*x^3])/(8645*b^
2) + (2*a^2*e*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*f*x^4*Sqrt[a + b*x^3])
/(4301*b) + (54*a^2*g*x^5*Sqrt[a + b*x^3])/(6175*b) - (216*a^3*(5*b*d - 2*a
*g)*Sqrt[a + b*x^3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (
2*x^3*(a + b*x^3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 169575*
f*x^4 + 156009*g*x^5))/3900225 + (2*a*x^3*Sqrt[a + b*x^3]*(8947575*c*x + 67
74075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5))/185910725 + (
108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*b*d - 2*a*g)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
```

```
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]) - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(1729*b^(1/3)*(23*b*c - 8*a*f) - 8
602*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(37182145*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
```

```
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx\right.}{3900225}$$

$$= \frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx\right.}{3900225}$$

$$= \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+18\right.}{3900225}$$

$$= \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425\right.}{}$$

$$= \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(}{}$$

$$= \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+bx^3}}{4301b} + \frac{54a^2gx^5\sqrt{a+bx^3}}{6175b} + \frac{2x^3(}{}$$

$$= \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+b}}{4301b}$$

$$= \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2ex^3\sqrt{a+bx^3}}{105b} + \frac{54a^2fx^4\sqrt{a+b}}{4301b}$$

$$= \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2}{}$$

$$= \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{2a^2}{}$$

$$= -\frac{4a^3e\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)}{8645b}$$

$$= -\frac{4a^3e\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2(5bd-2ag)}{8645b}$$

Mathematica [C] time = 0.52666, size = 179, normalized size = 0.23

$$\frac{2\sqrt{a+bx^3}\left(9975a^2x(8af-23bc)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+41055a^2x^2(2ag-5bd)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}}\right.}{3900225b^2\sqrt{\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(10*a*(7429*e + 21*
x*(380*f + 391*g*x)) - b*x*(229425*c + 17*x*(12075*d + 19*x*(575*e + 525*f*
x + 483*g*x^2))))) + 9975*a^2*(-23*b*c + 8*a*f)*x*Hypergeometric2F1[-3/2, 1
/3, 4/3, -((b*x^3)/a)] + 41055*a^2*(-5*b*d + 2*a*g)*x^2*Hypergeometric2F1[-
```

```
3/2, 2/3, 5/3, -((b*x^3)/a)]))/(3900225*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.017, size = 1764, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] g*(2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175/b*a^2*x
^5*(b*x^3+a)^(1/2)-108/8645*a^3/b^2*x^2*(b*x^3+a)^(1/2)-144/8645*I*a^4/b^3*
3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(2/23*b*x^10*(b*x^3+a)^(1/2)+5
2/391*a*x^7*(b*x^3+a)^(1/2)+54/4301/b*a^2*x^4*(b*x^3+a)^(1/2)-432/21505*a^3
/b^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2
/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(
b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2
))+d*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2
*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2
)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/
b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2))))+c*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*
a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^{10} + bfx^9 + bex^8 + \left(bd + ag\right)x^7 + aex^5 + \left(bc + af\right)x^6 + adx^4 + acx^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^10 + b*f*x^9 + b*e*x^8 + (b*d + a*g)*x^7 + a*e*x^5 + (b*c +
 a*f)*x^6 + a*d*x^4 + a*c*x^3)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 10.7682, size = 512, normalized size = 0.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + a**(3/2)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*f*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(3/2
)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3)) + sqrt(a)*b*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**8*gamma(8/3)*hyper
((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*
b*f*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(13/3)) + sqrt(a)*b*g*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(14/3)) + a*e*Piecewise((-4*a**2*sqrt
(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a +
 b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*e*Piecewise((16*a**3*sq
rt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x
**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt
(a)*x**9/9, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

3.459 $\int x^2 \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) dx$

Optimal. Leaf size=742

$$\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-}{(1+}\right.\right.}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*d - 8*a*g
)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a + b*x^3])/(1729*b)
+ (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*g*x^4*Sqrt[a + b*x^3])/(4
301*b) - (216*a^3*e*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*
x^3 + 37145*f*x^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(743642
9*c*x + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/111
546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 + (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e
 - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(37182145*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.55222, antiderivative size = 742, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1836, 1888, 1594, 1886, 261, 1878, 218, 1877}

$$\frac{2a^2\sqrt{a+bx^3}(7bc-2af)}{105b^2}+\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-\right.}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*a^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*(23*b*d - 8*a*g
)*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*e*x^2*Sqrt[a + b*x^3])/(1729*b)
+ (2*a^2*f*x^3*Sqrt[a + b*x^3])/(105*b) + (54*a^2*g*x^4*Sqrt[a + b*x^3])/(4
301*b) - (216*a^3*e*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*
x^3 + 37145*f*x^4 + 33915*g*x^5))/780045 + (2*a*x^2*Sqrt[a + b*x^3]*(743642
9*c*x + 5368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/111
546435 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*e*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

```
 + (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e
 - 1729*(23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(37182145*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
```

```
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) dx = \frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003cx+45885dx^2+41055ex^3+37145fx^4+339\right.}{780045}$$

$$= \frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003cx+45885dx^2+41055ex^3+37145fx^4+339\right.}{780045}$$

$$= \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003cx+45885dx^2+41055e\right.}{780045}$$

$$= \frac{2a^2fx^3\sqrt{a+bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003cx+\right.}{}$$

$$= \frac{54a^2ex^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a+bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2 \left(a+\right.}{}$$

$$= \frac{54a^2ex^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a+bx^3}}{105b} + \frac{54a^2gx^4\sqrt{a+bx^3}}{4301b} + \frac{2x^2 \left(a+\right.}{}$$

$$= \frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a+bx^3}}{105b}$$

$$= \frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2fx^3\sqrt{a+bx^3}}{105b}$$

$$= \frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{}}{1729}$$

$$= \frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2ex^2\sqrt{}}{1729}$$

Mathematica [C] time = 0.349519, size = 162, normalized size = 0.22

$$\frac{2\left(1995a^3x\sqrt{\frac{bx^3}{a}+1}(8ag-23bd)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-41055a^3bex^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+\left(a+bx^3\right)^3(-38a(39\right.}{780045b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (2*((a + b*x^3)^3*(52003*b*c - 38*a*(391*f + 420*g*x) + 5*b*x*(9177*d + 17*
x*(483*e + 19*x*(23*f + 21*g*x)))) + 1995*a^3*(-23*b*d + 8*a*g)*x*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] - 41055*a^3*b*e*
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(
780045*b^2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.008, size = 1269, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] g*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301/b*a^2*x
^4*(b*x^3+a)^(1/2)-432/21505*a^3/b^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3*
3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(2/21*b*x^
9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*(b*x^3+a)^(1
/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+e*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x
^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1
/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(
-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+d*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187
*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1
/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/15*c/b*(b*x^3+
a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}c}{15\,b}+\int\left(bgx^9+bfx^8+bex^7+afx^5+\left(bd+ag\right)x^6+aex^4+adx^3\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxim
a")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*c/b + integrate((b*g*x^9 + b*f*x^8 + b*e*x^7 + a*f*x
^5 + (b*d + a*g)*x^6 + a*e*x^4 + a*d*x^3)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^9+bfx^8+bex^7+\left(bd+ag\right)x^6+aex^4+\left(bc+af\right)x^5+adx^3+acx^2\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^9 + b*f*x^8 + b*e*x^7 + (b*d + a*g)*x^6 + a*e*x^4 + (b*c +
a*f)*x^5 + a*d*x^3 + a*c*x^2)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 9.44158, size = 525, normalized size = 0.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + a**(3/2)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*g*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)
*b*d*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(10/3)) + sqrt(a)*b*e*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*g*x**10*gamma(10/3)*hy
per((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)) + a*c*
Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
+ a*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x
**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)
) + b*c*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b
*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, Tru
e)) + b*f*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt
(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a
 + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2, x)
```

3.460 $\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)dx$

Optimal. Leaf size=723

$$-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1}{(1}\right.\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*Sqrt[a + b*
x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*g*x^3*Sqrt
[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*f)*Sqrt[a + b*x^3])/(1729*b^(5
/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*
x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x
*Sqrt[a + b*x^3]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 +
 138567*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*c - 4
*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(345
8*a^(2/3)*b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rubi [A] time = 1.23865, antiderivative size = 723, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1826, 1836, 1888, 1886, 261, 1878, 218, 1877}

$$-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```


```
[Out] (2*a^2*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/(105*b^2) + (54*a^2*e*x*Sqrt[a + b*
x^3])/(935*b) + (54*a^2*f*x^2*Sqrt[a + b*x^3])/(1729*b) + (2*a^2*g*x^3*Sqrt
[a + b*x^3])/(105*b) + (54*a^2*(19*b*c - 4*a*f)*Sqrt[a + b*x^3])/(1729*b^(5
/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x*(a + b*x^3)^(3/2)*(33915*c*
x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^4 + 20995*g*x^5))/440895 + (2*a*x
*Sqrt[a + b*x^3]*(479655*c*x + 323323*d*x^2 + 233415*e*x^3 + 176715*f*x^4 +
 138567*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*b*c - 4
*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(345
```

```
8*a^(2/3)*b^(1/3)*e + 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1616615*b^(5/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
```

```
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20\right.}{440895}$$

$$= \frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20\right.}{440895}$$

$$= \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex\right.}{440895}$$

$$= \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+\right.}{}$$

$$= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x\left(a+\right.}{}$$

$$= \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} + \frac{2a^2gx^3\sqrt{a+bx^3}}{105b} + \frac{2x\left(a+\right.}{}$$

$$= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} +$$

$$= \frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2} + \frac{54a^2ex\sqrt{a+bx^3}}{935b} + \frac{54a^2fx^2\sqrt{a+bx^3}}{1729b} +$$

Mathematica [C] time = 0.272086, size = 148, normalized size = 0.2

$$-\frac{\sqrt{a+bx^3}\left(7980a^2bex\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+1785abx^2(4af-19bc)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}+1}(646a\right.}{67830b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] -(Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-2261*b*d + 646*a*g
 - 5*b*x*(399*e + 17*x*(21*f + 19*g*x))) + 7980*a^2*b*e*x*Hypergeometric2F1
[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 1785*a*b*(-19*b*c + 4*a*f)*x^2*Hypergeomet
ric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(67830*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.006, size = 1383, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] g*(2/21*b*x^9*(b*x^3+a)^(1/2)+16/105*a*x^6*(b*x^3+a)^(1/2)+2/105/b*a^2*x^3*
(b*x^3+a)^(1/2)-4/105*a^3/b^2*(b*x^3+a)^(1/2))+f*(2/19*b*x^8*(b*x^3+a)^(1/2
)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*x^2*(b*x^3+a)^(1/2)+72/1729*I/
b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+e*(2/17*b*x^7*(b*x^3+a)^
(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(b*x^3+a)^(1/2)+36/935*I/
b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)
^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+2/15
*d/b*(b*x^3+a)^(5/2)+c*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1
/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(
1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bgx^{8}+bfx^{7}+bex^{6}+\left(bd+ag\right)x^{5}+aex^{3}+\left(bc+af\right)x^{4}+adx^{2}+acx\right)\sqrt{bx^{3}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas"
)
```

```
[Out] integral((b*g*x^8 + b*f*x^7 + b*e*x^6 + (b*d + a*g)*x^5 + a*e*x^3 + (b*c +
a*f)*x^4 + a*d*x^2 + a*c*x)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 8.80445, size = 525, normalized size = 0.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + a**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*f*x**5*gamma(5/3)*hype
r((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b
*c*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3)) + sqrt(a)*b*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*f*x**8*gamma(8/3)*hyper((-
1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*d*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + a*g*P
iecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(4
5*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*d
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b
*g*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*
x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x*
*3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^{4}+fx^{3}+ex^{2}+dx+c\right)\left(bx^{3}+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

3.461 $\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

Optimal. Leaf size=694

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-2af)-9\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*d - 4*a*g)*Sqrt[a +
b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)
^(3/2)*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))
/692835 + (2*a*Sqrt[a + b*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 +
233415*f*x^4 + 176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/
3)*(19*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]
]*a^2*(1729*b^(1/3)*(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d -
4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.899239, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1853, 1888, 1886, 261, 1878, 218, 1877}

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\right.\right.}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```


```
[Out] (2*a^2*e*Sqrt[a + b*x^3])/(15*b) + (54*a^2*f*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*g*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*b*d - 4*a*g)*Sqrt[a +
b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(a + b*x^3)
^(3/2)*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4 + 36465*g*x^5))
/692835 + (2*a*Sqrt[a + b*x^3]*(793611*c*x + 479655*d*x^2 + 323323*e*x^3 +
233415*f*x^4 + 176715*g*x^5))/4849845 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/
3)*(19*b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]
]*a^2*(1729*b^(1/3)*(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d -
4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
```

```
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(1616615*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1853

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[(Coeff[Pq, x, i]*x^(i + 1))/(n*p + i + 1),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*
x^i)/(n*p + i + 1), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
```

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)dx = \frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx\right.}{692835}$$

$$= \frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx\right.}{692835}$$

$$= \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+\right.}{692835}$$

$$= \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+5329\right.}{}$$

$$= \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+5329\right.}{}$$

$$= \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{2\left(a+bx^3\right)^{3/2}}{}$$

$$= \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19bd-}{1729b^{5/3}\left(\left(1+\right.\right.}$$

Mathematica [C] time = 0.177176, size = 139, normalized size = 0.2

$$\frac{\sqrt{a+bx^3}\left(-570ax(2af-17bc)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-255ax^2(4ag-19bd)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}+1}(32\right.}{9690b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(323*e + 15*x*(19*f +
 17*g*x)) - 570*a*(-17*b*c + 2*a*f)*x*Hypergeometric2F1[-3/2, 1/3, 4/3, -((
b*x^3)/a)] - 255*a*(-19*b*d + 4*a*g)*x^2*Hypergeometric2F1[-3/2, 2/3, 5/3,
-((b*x^3)/a)]))/(9690*b*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.006, size = 1629, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

```
[Out] g*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729/b*a^2*x^
2*(b*x^3+a)^(1/2)+72/1729*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^
2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*
((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b
*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/
2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(
-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
)))+f*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2
*x*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^
2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*
((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3)))^(1/2)))+2/15*e/b*(b*x^3+a)^(5/2)+d*(2/13*b*x^5*(b*x^3+a)^(1
/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I
*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3
^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3)))^(1/2))))+c*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55
*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b
^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a
)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bgx^7 + bfx^6 + bex^5 + \left(bd + ag\right)x^4 + aex^2 + \left(bc + af\right)x^3 + adx + ac\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 7.32464, size = 444, normalized size = 0.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*f*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*g*x
**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(8/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5
/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*f*x**7*ga
mma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/
3)) + sqrt(a)*b*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(11/3)) + a*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2
*(a + b*x**3)**(3/2)/(9*b), True)) + b*e*Piecewise((-4*a**2*sqrt(a + b*x**3
)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15
, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

3.462 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

Optimal. Leaf size=676

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-935\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-182ag+1547bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3}}{(1+\sqrt{3}}\right.\right.}{85085b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
+ (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 +
6435*g*x^5))/(109395*x) + (2*a*Sqrt[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 2
5245*e*x^3 + 17017*f*x^4 + 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.707064, antiderivative size = 676, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1826, 1832, 266, 63, 208, 1888, 1886, 261, 1878, 218, 1877}

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-935\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-182ag+1547bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{85085b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```


```
[Out] (2*a^2*f*Sqrt[a + b*x^3])/(15*b) + (54*a^2*g*x*Sqrt[a + b*x^3])/(935*b) + (
54*a^2*e*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
+ (2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 +
6435*g*x^5))/(109395*x) + (2*a*Sqrt[a + b*x^3]*(85085*c*x + 41769*d*x^2 + 2
5245*e*x^3 + 17017*f*x^4 + 12285*g*x^5))/(255255*x) - (2*a^(3/2)*c*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*e*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(1547*b*d - 935*(1 -
Sqrt[3])*a^(1/3)*b^(2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
```

```
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(85085*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$= \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$= \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$= \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7\right.}{109395x}$$

$$= \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7\right.}{109395x}$$

$$= \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945\right.}{1}$$

$$= \frac{2a^2f\sqrt{a+bx^3}}{15b} + \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} +$$

Mathematica [C] time = 0.439483, size = 215, normalized size = 0.32

$$\frac{4\sqrt{\frac{bx^3}{a}+1}\left(\sqrt{a+bx^3}\left(a^2(51f+45gx)+2ab\left(170c+51fx^3+45gx^4\right)+b^2x^3\left(85c+51fx^3+45gx^4\right)\right)-255a^{3/2}bc\tanh^{-1}\right.}{1530b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

```
[Out] (4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(a^2*(51*f + 45*g*x) + b^2*x^3*(85*
c + 51*f*x^3 + 45*g*x^4) + 2*a*b*(170*c + 51*f*x^3 + 45*g*x^4)) - 255*a^(3/
2)*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 90*a*(-17*b*d + 2*a*g)*x*Sqrt[a
+ b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 765*a*b*e*x^2*Sq
rt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(1530*b*Sqrt
[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1188, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x)
```

```
[Out] g*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935/b*a^2*x*(
b*x^3+a)^(1/2)+36/935*I/b^2*a^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-
1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3)))^(1/2)))+2/15*f/b*(b*x^3+a)^(5/2)+e*(2/13*b*x^5*(b*x^3+a)^(1/2)+
32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(
1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/
2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))
^(1/2))))+d*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a
^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a
)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1
/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+c*(2/9*
b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(
1/2)/a^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas"
)
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x, x)
```

Sympy [A] time = 21.3669, size = 473, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

```
[Out] -2*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*d*x*gamma(1/3)
*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(
3/2)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(5/3)) + a**(3/2)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*e*
x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(8/3)) + sqrt(a)*b*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**2*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*
x**3) + 1)) + 2*a*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*f*Piecewi
se((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*c*P
iecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) +
 b*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x*
*3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

3.463 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

Optimal. Leaf size=692

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e-55\left(1-\sqrt{3}\right)(2af+13bc)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x) + (27*a*(
13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 + 2457*e*x^3 + 1485*f
*x^4 + 1001*g*x^5))/(15015*x^2) + (2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x
^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13
*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3
)*(182*a^(2/3)*b^(1/3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rubi [A] time = 0.790546, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {1826, 1835, 1832, 266, 63, 208, 1886, 261, 1878, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(182a^{2/3}\sqrt[3]{b}e-55\left(1-\sqrt{3}\right)(2af+13bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```


```
[Out] (2*a^2*g*Sqrt[a + b*x^3])/(15*b) - (27*a*c*Sqrt[a + b*x^3])/(7*x) + (27*a*(
13*b*c + 2*a*f)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) + (2*a*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 + 2457*e*x^3 + 1485*f
*x^4 + 1001*g*x^5))/(15015*x^2) + (2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x
^2 + 4095*e*x^3 + 3465*f*x^4 + 3003*g*x^5))/(45045*x^2) - (2*a^(3/2)*d*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13
*b*c + 2*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(4/3
)*(182*a^(2/3)*b^(1/3)*e - 55*(1 - Sqrt[3])*(13*b*c + 2*a*f))*(a^(1/3) + b^
```

```
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5005*b^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$$= \frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5\right)}{15015x^2} +$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+148\right.}{15015x^2}$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+148\right.}{15015x^2}$$

$$= -\frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+148\right.}{15015x^2}$$

$$= \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{2a\sqrt{a+bx^3}\left(19305cx+5005dx^2+\right.}{1501}$$

$$= \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{27a(13bc+2af)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2a}{}$$

Mathematica [C] time = 0.340123, size = 224, normalized size = 0.32

$$\frac{2}{9}d\left(\sqrt{a+bx^3}\left(4a+bx^3\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)-\frac{ac\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a}+1}}+\frac{aex\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\right.}{\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

```
[Out] (2*g*(a + b*x^3)^(5/2))/(15*b) + (2*d*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^
(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/9 - (a*c*Sqrt[a + b*x^3]*Hypergeom
etric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (a*e*x*S
qrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b
*x^3)/a] + (a*f*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*
x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1317, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x)
```

```
[Out] 2/15*g/b*(b*x^3+a)^(5/2)+f*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a
)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*
a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3
)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*
a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/
(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+e*(2/11*b*
x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-b^2*a
)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(2/9*b*x^3*(b*x^3+a)^(1/2
)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-a
*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-b^2*a)^(1/3)
*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2
*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/
```

```
2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^2, x)
```

Sympy [A] time = 10.2973, size = 474, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) - 2*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**
(3/2)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) + a**(3/2)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**2*gamma(2/3)*hyper((-1
/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*e*x*
*4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(7/3)) + sqrt(a)*b*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3
) + 1)) + 2*a*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*g*Piecewise((
sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*d*Piece
wise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*g
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

3.464 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

Optimal. Leaf size=694

$$\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(4af+11bc)-11\right.}{10010b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x) + (27*a*
(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2 - 5005*e*x^3 - 2457
*f*x^4 - 1485*g*x^5))/(15015*x^3) + (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d
*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(
13*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(9
1*b^(1/3)*(11*b*c + 4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10010*b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.887517, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(4af+11bc)-110\left(1-\sqrt{}\right.\right.}{10010b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```


```
[Out] (27*a*c*Sqrt[a + b*x^3])/(10*x^2) - (27*a*d*Sqrt[a + b*x^3])/(7*x) + (27*a*
(13*b*d + 2*a*g)*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)) - (2*a*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2 - 5005*e*x^3 - 2457
*f*x^4 - 1485*g*x^5))/(15015*x^3) + (2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d
*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465*g*x^5))/(45045*x^3) - (2*a^(3/2)*e*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(
13*b*d + 2*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(9
1*b^(1/3)*(11*b*c + 4*a*f) - 110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g))*(a
```

```
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10010*b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$$= -\frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5\right.}{15015x^3}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457\right.}{15015x^3}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2\right.}{1501}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2\right.}{1501}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} - \frac{2a\sqrt{a+bx^3}\left(27027cx-19305dx^2\right.}{1501}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a}{}$$

Mathematica [C] time = 0.355531, size = 232, normalized size = 0.33

$$\frac{4ex^2\sqrt{\frac{bx^3}{a}+1}\left(\sqrt{a+bx^3}\left(4a+bx^3\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)-9ac\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)-18adx\sqrt{a+bx^3}\,{}_2}{18x^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

```
[Out] (4*e*x^2*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 9*a*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3
/2, -2/3, 1/3, -((b*x^3)/a)] - 18*a*d*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-
3/2, -1/3, 2/3, -((b*x^3)/a)] + 18*a*f*x^3*Sqrt[a + b*x^3]*Hypergeometric2F
1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 9*a*g*x^4*Sqrt[a + b*x^3]*Hypergeometric2
F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(18*x^2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1613, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x)
```

```
[Out] g*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/
2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a
)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55
*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2
*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*(
(x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/
2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3)))^(1/2)))+e*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2
/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/2*a*(b*x^3+a)^(1/2)/x^2+
2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x
-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1
/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3)))^(1/2)))+d*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*
I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)
^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(
```

```
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^3, x)
```

Sympy [A] time = 10.5008, size = 462, normalized size = 0.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

```
[Out] a**(3/2)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + a**(3/2)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*e*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + a**(3/2)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*g*x**2*gamma(2/3)*hyper((-
1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x
*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4
/3)) + sqrt(a)*b*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*f*x**4*gamma(4/3)*hyper((-1/2, 4/3)
, (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*g*x**5*gamma
(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) +
 2*a**2*e/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*e*x**(3/2
)/(3*sqrt(a/(b*x**3) + 1)) + b*e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

3.465 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

Optimal. Leaf size=692

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag+77bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\ldots\right)}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (27*a*e*Sqr
t[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/(7*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*e*
x^3 - 385*f*x^4 - 189*g*x^5))/(1155*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x +
 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b
*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*
b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(770*b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.956585, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag+77bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7\ldots\right)}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```


```
[Out] (a*c*Sqrt[a + b*x^3])/x^3 + (27*a*d*Sqrt[a + b*x^3])/(10*x^2) - (27*a*e*Sqr
t[a + b*x^3])/(7*x) + (27*a*b^(1/3)*e*Sqrt[a + b*x^3])/(7*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)) - (2*a*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*e*
x^3 - 385*f*x^4 - 189*g*x^5))/(1155*x^4) + (2*(a + b*x^3)^(3/2)*(1155*c*x +
 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x^5))/(3465*x^4) - (Sqrt[a]*(3*b
*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(4/3)*b^(1/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(77*
b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
```

```
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(770*b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
```

```
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}+\frac{1}{2}(\cdots$$

$$= -\frac{2a\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5\right)}{1155x^4}+\cdots$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}-\frac{2a\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex^3-385fx^4-\cdots\right.}{1155x^4}\cdots$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{2a\sqrt{a+bx^3}\left(1155cx+2079dx^2-14\cdots\right.}{1155x^4}\cdots$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}-\frac{2a\sqrt{a+bx^3}\left(1155c\cdots\right.}{\cdots}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}-\frac{2a\sqrt{a+bx^3}\left(1155c\cdots\right.}{\cdots}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}-\frac{2a\sqrt{a+bx^3}\left(1155c\cdots\right.}{\cdots}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}+\frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}\cdots}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}$$

$$= \frac{ac\sqrt{a+bx^3}}{x^3}+\frac{27ad\sqrt{a+bx^3}}{10x^2}-\frac{27ae\sqrt{a+bx^3}}{7x}+\frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}\cdots}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}$$

Mathematica [C] time = 0.611858, size = 243, normalized size = 0.35

$$\frac{4x^2\sqrt{\frac{bx^3}{a}+1}\left(5a^2f\left(\sqrt{a+bx^3}\left(4a+bx^3\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+3bc\left(a+bx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)\right)-45a^3d\sqrt{\cdots}}{90a^2x^2\sqrt{\frac{bx^3}{a}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

```
[Out] (-45*a^3*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)]
 - 90*a^3*e*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/
a)] + 90*a^3*g*x^3*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x
^3)/a)] + 4*x^2*Sqrt[1 + (b*x^3)/a]*(5*a^2*f*(Sqrt[a + b*x^3]*(4*a + b*x^3)
 - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*c*(a + b*x^3)^(5/2)*Hy
pergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(90*a^2*x^2*Sqrt[1 + (b*x^3)/
a])
```

Maple [B] time = 0.01, size = 1193, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x)
```

```
[Out] g*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)
/b*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(2/9*b*x^3*(b*x
^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/
2)))+c*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh(
(b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(
1/2)-9/10*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3)
)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-
b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
)+e*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-b^2*a
)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + \left(bd + ag\right)x^4 + aex^2 + \left(bc + af\right)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^4, x)
```

Sympy [A] time = 11.9664, size = 484, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

```
[Out] a**(3/2)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + a**(3/2)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*f*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + a**(3/2)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*
x**(3/2))) + sqrt(a)*b*d*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*e*x**2*gamma(2/3)*hyper((-1/2, 2
/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*g*x**4*ga
mma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)
) + 2*a**2*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*c*sqrt(a
/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*c/(3*x**(3/2)*sqrt(a/(b*x**3) + 1
)) + 2*a*sqrt(b)*f*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*c*x**(3/2
)/(3*sqrt(a/(b*x**3) + 1)) + b*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(
a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

3.466 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

Optimal. Leaf size=741

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(8af+7bc)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (27*a*e*Sqr
t[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*x) + (27*
b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4
- 35*g*x^5))/(105*x^5) + (2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x
^3 + 45*f*x^4 + 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7
*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3
)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.24017, antiderivative size = 741, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(8af+7bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```


```
[Out] (27*a*c*Sqrt[a + b*x^3])/(20*x^4) + (a*d*Sqrt[a + b*x^3])/x^3 + (27*a*e*Sqr
t[a + b*x^3])/(10*x^2) - (27*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*x) + (27*
b^(1/3)*(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (2*a*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 - 135*f*x^4
- 35*g*x^5))/(105*x^5) + (2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x
^3 + 45*f*x^4 + 35*g*x^5))/(315*x^5) - (Sqrt[a]*(3*b*d + 2*a*g)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/3 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7
*b*c + 8*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(1/3
```

```
)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}dx = \frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}+\frac{1}{2}(9a)\int$$

$$= -\frac{2a\sqrt{a+bx^3}\left(189cx+105dx^2+189ex^3-135fx^4-35gx^5\right)}{105x^5}+\frac{2\left(a+\right.}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}-\frac{2a\sqrt{a+bx^3}\left(189cx+105dx^2+189ex^3-135fx^4-\right.}{105x^5}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}-\frac{2a\sqrt{a+bx^3}\left(189cx+105dx^2+189e\right.}{105x^5}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{2a\sqrt{a+bx^3}\left(189cx\right.}{}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+}}{56x}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+}}{56x}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+}}{56x}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+}}{56x}$$

$$= \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+}}{56x}$$

Mathematica [C] time = 0.602643, size = 246, normalized size = 0.33

$$\frac{-45a^3c\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)+4x^3\left(2x\sqrt{\frac{bx^3}{a}+1}\left(5a^2g\left(\sqrt{a+bx^3}\left(4a+bx^3\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+3bd\left(\right.\right.\right.}{180a^2x^4\sqrt{\frac{b}{}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

```
[Out] (-45*a^3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)
] - 90*a^3*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^
3)/a)] + 4*x^3*(-45*a^3*f*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3
, -((b*x^3)/a)] + 2*x*Sqrt[1 + (b*x^3)/a]*(5*a^2*g*(Sqrt[a + b*x^3]*(4*a +
b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*d*(a + b*x^3)^(5
/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])))/(180*a^2*x^4*Sqrt[1 +
(b*x^3)/a])
```

Maple [B] time = 0.01, size = 1342, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x)
```

```
[Out] g*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x
^3+a)^(1/2)/a^(1/2)))+d*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a
^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11
/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1
/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^
2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1
/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+e
*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2
*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(-a*(b*x^3+a)^(1/2)/x+2
/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x
-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)
*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-
b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

```
+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^
2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^5, x)
```

Sympy [A] time = 12.1667, size = 495, normalized size = 0.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

```
[Out] a**(3/2)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + a**(3/2)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*f*gamma(-1/3)*hy
per((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a*
*(3/2)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*b*c*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a
)*b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*e*x*gamma(1/3)*hyper((-
1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*f*x
**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(5/3)) + 2*a**2*g/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d*
sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*d/(3*x**(3/2)*sqrt(a/(b*x**
3) + 1)) + 2*a*sqrt(b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*d*x
**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*g*Piecewise((sqrt(a)*x**3/3, Eq(b, 0))
```

```
, (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

3.467 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

Optimal. Leaf size=689

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\bigl(14\sqrt[3]{b}(2af+bc)-5\bigl(}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x) + (27*b^
(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a +
 b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 1
26*f*x^4 - 180*g*x^5))/(140*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]] - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f)
- 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.920922, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {14, 1825, 1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\bigl(14\sqrt[3]{b}(2af+bc)-5\left(1-\sqrt{3}\right)}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```


```
[Out] (27*b*c*Sqrt[a + b*x^3])/(20*x^2) - (27*b*d*Sqrt[a + b*x^3])/(8*x) + (27*b^
(1/3)*(7*b*d + 8*a*g)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a +
 b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^3 - 1
26*f*x^4 - 180*g*x^5))/(140*x^3) - Sqrt[a]*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]] - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*b*d + 8*a*g)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(14*b^(1/3)*(b*c + 2*a*f)
- 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
```

```
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(280*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)(a+bx^3)^{3/2} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}}{}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}\,(252cx}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{}}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} + \frac{27\sqrt[3]{b}(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{60}\left(\frac{12c}{x^5}\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x} + \frac{27\sqrt[3]{b}(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{60}\left(\frac{12c}{x^5}\right.$$

Mathematica [C] time = 0.243595, size = 191, normalized size = 0.28

$$\frac{\sqrt{a+bx^3}\left(-12a^3c\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)-15a^3dx\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)-30a^3fx^3\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)-60a^3gx^4\,{}_2F_1\right.}{60a^2x^5\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-12*a^3*c*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a
)] - 15*a^3*d*x*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)] - 30*a^3*
f*x^3*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)] - 60*a^3*g*x^4*Hyper
geometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 8*b*e*x^5*(a + b*x^3)^2*Sqrt[
1 + (b*x^3)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(60*a^2*x^5*
Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.009, size = 1606, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x)
```

```
[Out] e*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^
3+a)^(1/2)/a^(1/2)))+c*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/
x^2-9/20*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b
^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
+d*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2))))+f*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^
3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)
^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2
)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2)))+g*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(
-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(
(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^
2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + \left(bd + ag\right)x^4 + aex^2 + \left(bc + af\right)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^6, x)
```

Sympy [A] time = 11.7593, size = 476, normalized size = 0.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*f*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
 a**(3/2)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) + sqrt(a)*b*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b
*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a
)*b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*f*x*gamma(1/3)*hyper((-
1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*g*x
**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(5/3)) - a*sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*e/(3*
x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*e*x**(3/2)/(3*sqrt(a/(b*x**3) +
 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

3.468 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

Optimal. Leaf size=692

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+2bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2) - (27*b*e
*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3])/(8*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3
 + (30*g)/x^2)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^
2 - 45*e*x^3 - 20*f*x^4 - 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)
*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5
*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.00417, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {14, 1825, 1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+2bd\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] (b*c*Sqrt[a + b*x^3])/(4*x^3) + (27*b*d*Sqrt[a + b*x^3])/(20*x^2) - (27*b*e
*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*e*Sqrt[a + b*x^3])/(8*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3
 + (30*g)/x^2)*(a + b*x^3)^(3/2))/60 - (b*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^
2 - 45*e*x^3 - 20*f*x^4 - 18*g*x^5))/(20*x^4) - (b*(b*c + 4*a*f)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)
*b^(4/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(2*b*d - 5
*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
```

```
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(40*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a+bx^3)^{3/2} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}\,(10cx+}{}$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{a}}{}$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}\right.$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}$$

$$= \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}$$

Mathematica [C] time = 0.507349, size = 240, normalized size = 0.35

$$\frac{-\frac{12a^2d\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{x^5} - \frac{15a^2e\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{x^4} + \frac{8bf(a+bx^3)^3\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)}{a^2} - \frac{30a^2g\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{x^2} -}{60\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

```
[Out] ((-15*b*c*(a + b*x^3))/x^3 - (10*c*(a + b*x^3)^2)/x^6 - 15*b^2*c*Sqrt[1 + (
b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (12*a^2*d*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (15*a^2*e*Sqrt[1 + (b*
x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/x^4 - (30*a^2*g*
Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/x^2 +
 (8*b*f*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/a^2)/(
60*Sqrt[a + b*x^3])
```

Maple [B] time = 0.01, size = 1196, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x)
```

```
[Out] f*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^
3+a)^(1/2)/a^(1/2)))+d*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/
x^2-9/20*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b
^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
+e*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2))))+g*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^
3+a)^(1/2)-9/10*I*a*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)
^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2
)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^
(1/2)))+c*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*ar
ctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + \left(bd + ag\right)x^4 + aex^2 + \left(bc + af\right)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^7, x)
```

Sympy [A] time = 15.5393, size = 524, normalized size = 0.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + a**(3/2)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*g*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
 sqrt(a)*b*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*f*asinh(sqrt(a)/(
sqrt(b)*x**(3/2))) + sqrt(a)*b*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**2*c/(6*sqrt(b)*x**(15/2)*sqrt(a
/(b*x**3) + 1)) - a*sqrt(b)*c/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)
*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*f/(3*x**(3/2)*sqrt(a/(b*
x**3) + 1)) - b**(3/2)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*c/(12
*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*f*x**(3/2)/(3*sqrt(a/(b*x**3)
+ 1)) - b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7, x)
```

3.469 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

Optimal. Leaf size=746

$$\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(14af+bc)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots}\right.\right.}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3) + (27*b*
e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*
x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 +
(140*g)/x^3)*(a + b*x^3)^(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2
 + 252*e*x^3 - 315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4
/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b
^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rubi [A] time = 1.28367, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {14, 1825, 1826, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(14af+bc)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|-7\cdots}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] (27*b*c*Sqrt[a + b*x^3])/(280*x^4) + (b*d*Sqrt[a + b*x^3])/(4*x^3) + (27*b*
e*Sqrt[a + b*x^3])/(20*x^2) - (27*b*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*
x) + (27*b^(4/3)*(b*c + 14*a*f)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 +
(140*g)/x^3)*(a + b*x^3)^(3/2))/420 - (b*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2
 + 252*e*x^3 - 315*f*x^4 - 140*g*x^5))/(140*x^5) - (b*(b*d + 4*a*g)*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4
/3)*(b*c + 14*a*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
```

```
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b
^(4/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(560*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2}-\frac{1}{2}(9b)\int \frac{\sqrt{a+\cdots}}{}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2}-\frac{b\sqrt{a+bx^3}\left(36\cdots\right.}{}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\left(a+bx^3\right)^{3/2}-\cdots$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\cdots\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}\cdots\right.$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+b\cdots}}{112ax}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+b\cdots}}{112ax}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+b\cdots}}{112ax}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+b\cdots}}{112ax}$$

$$= \frac{27bc\sqrt{a+bx^3}}{280x^4}+\frac{bd\sqrt{a+bx^3}}{4x^3}+\frac{27be\sqrt{a+bx^3}}{20x^2}-\frac{27b(bc+14af)\sqrt{a+b\cdots}}{112ax}$$

Mathematica [C] time = 0.781448, size = 240, normalized size = 0.32

$$\frac{-\frac{60a^2c\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{x^7}-\frac{84a^2e\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{x^5}-\frac{105a^2f\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{x^4}+\frac{56bg\left(a+bx^3\right)^3\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\cdots\right)}{a^2}\cdots}{420\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

```
[Out] ((-105*b*d*(a + b*x^3))/x^3 - (70*d*(a + b*x^3)^2)/x^6 - 105*b^2*d*Sqrt[1 +
 (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (60*a^2*c*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/x^7 - (84*a^2*e*Sqrt[1 + (
b*x^3)/a]*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (105*a^2
*f*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/x
^4 + (56*b*g*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/a
^2)/(420*Sqrt[a + b*x^3])
```

Maple [B] time = 0.009, size = 1375, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x)
```

```
[Out] g*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^
3+a)^(1/2)/a^(1/2)))+e*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/
x^2-9/20*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))
/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b
^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
+f*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*
a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)
^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3)))^(1/2))))+d*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3
+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+c*(-1/7*a*(
b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)
/x-9/112*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1
/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3
/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac\right)\sqrt{bx^3 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^8, x)
```

Sympy [A] time = 16.4573, size = 536, normalized size = 0.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + a**(3/2)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*f*gamma(-4/3)*
hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
 + sqrt(a)*b*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*e*gamma(-2/3)*hyper((-2/3, -1/2), (
1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*f*gamma(-1
/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
- sqrt(a)*b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2*d/(6*sqrt(b)*x**(15/
2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) -
a*sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*g/(3*x**(3/2)*s
qrt(a/(b*x**3) + 1)) - b**(3/2)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3
/2)*d/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*g*x**(3/2)/(3*sqrt(a/
(b*x**3) + 1)) - b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

3.470 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

Optimal. Leaf size=705

$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(bc-16af)+2\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g)/x)*Sqrt[
a + b*x^3])/560 - (27*b^2*c*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*d*Sqrt[a
 + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*a*g)*Sqrt[a + b*x^3])/(112*a*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)
/x^6 + (168*f)/x^5 + (210*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3
)*(b*d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(
4/3)*(7*b^(1/3)*(b*c - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.00792, antiderivative size = 705, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(bc-16af)+20\left(1-\right.\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(b*((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^3 + (252*f)/x^2 + (630*g)/x)*Sqrt[
a + b*x^3])/560 - (27*b^2*c*Sqrt[a + b*x^3])/(320*a*x^2) - (27*b^2*d*Sqrt[a
 + b*x^3])/(112*a*x) + (27*b^(4/3)*(b*d + 14*a*g)*Sqrt[a + b*x^3])/(112*a*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((105*c)/x^8 + (120*d)/x^7 + (140*e)
/x^6 + (168*f)/x^5 + (210*g)/x^4)*(a + b*x^3)^(3/2))/840 - (b^2*e*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a]) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3
)*(b*d + 14*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(
4/3)*(7*b^(1/3)*(b*c - 16*a*f) + 20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))*(
```

```
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2240*a*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2}-\frac{1}{2}(9b)\int$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{1}{840}\left(\frac{105c}{x^8}\right.$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+}}{320ax^2}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+}}{320ax^2}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+}}{320ax^2}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+}}{320ax^2}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+}}{320ax^2}$$

$$= -\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}-\frac{27b^2c\sqrt{a+}}{320ax^2}$$

Mathematica [C] time = 0.460713, size = 202, normalized size = 0.29

$$-\frac{\sqrt{a+bx^3}\left(2x\left(7x\left(5\left(3a^2gx^2\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)+3b^2ex^6\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+ae\left(2a+5bx^3\right)\sqrt{\frac{bx^3}{a}+1}\right)+12a^2fx\,{}_2\cdots\right.\right.\right.}{840ax^8\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(105*a^2*c*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/
a)] + 2*x*(60*a^2*d*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 7*x
*(12*a^2*f*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)] + 5*(a*e*(2*
a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*e*x^6*ArcTanh[Sqrt[1 + (b*x^3)/a]]
 + 3*a^2*g*x^2*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])))))/(840*
a*x^8*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.01, size = 1663, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x)
```

```
[Out] f*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+g*(-1/4*a*(b*x^3+a)^
(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2
/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))
^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^
(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1
/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
)^(1/2))))+c*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320
/a*b^2*(b*x^3+a)^(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b
*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(
1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2)))+e*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)
^(1/2)/x^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+d*(-1/7*a*(b*x
^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)/x-
9/112*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/
```

```
2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3)
)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="maxim
a")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="frica
s")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^9, x)
```

Sympy [A] time = 15.5129, size = 527, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)
```

```
[Out] a**(3/2)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + a**(3/2)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*f*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
 + a**(3/2)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-5/3)*hyper((-5/3, -1/2), (-
2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*gamma(-
```

```
4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-
1/3)) + sqrt(a)*b*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*g*gamma(-1/3)*hyper((-1/2, -1/3)
, (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - a**2*e/(6*sqrt(b)*x*
*(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1
)) - b**(3/2)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*e/(12*x**(3/2)
*sqrt(a/(b*x**3) + 1)) - b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="giac"
)
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

3.471 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

Optimal. Leaf size=714

$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag+7bd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756*g)/x^2)*S
qrt[a + b*x^3])/1680 - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*d*Sqrt[
a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(7/3
)*e*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((280*c
)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3)^
(3/2))/2520 + (b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3
/2)) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9
*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2/3
)*e - 112*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.12299, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{b^2(bc-6af)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag+7b\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```


```
[Out] -(b*((140*c)/x^6 + (189*d)/x^5 + (270*e)/x^4 + (420*f)/x^3 + (756*g)/x^2)*S
qrt[a + b*x^3])/1680 - (b^2*c*Sqrt[a + b*x^3])/(24*a*x^3) - (27*b^2*d*Sqrt[
a + b*x^3])/(320*a*x^2) - (27*b^2*e*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(7/3
)*e*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((280*c
)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/x^5)*(a + b*x^3)^
(3/2))/2520 + (b^2*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3
/2)) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*e*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9
*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(5/3)*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)*b^(2/3
```

```
)*e - 112*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(2240*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
```

```
Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = -\frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\,(-$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\right.}{25}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \left(\frac{280c}{x^9}+\right.$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d}{32}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d}{32}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d}{32}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d}{32}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d}{32}$$

$$= -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d}{32}$$

Mathematica [C] time = 0.644999, size = 226, normalized size = 0.32

$$-\frac{\sqrt{a+bx^3}\left(2x\left(7x\left(5a^3f\left(3b^2x^6\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+a(2a+5bx^3)\sqrt{\frac{bx^3}{a}+1}\right)+12a^5gx\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)-8b^3cx^6\right.\right.\right.}{840a^4x^8\sqrt{\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(105*a^5*d*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/
a)] + 2*x*(60*a^5*e*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 7*x
*(5*a^3*f*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*ArcTanh[Sqrt[1
 + (b*x^3)/a]]) + 12*a^5*g*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/
a)] - 8*b^3*c*x^6*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/2,
4, 7/2, 1 + (b*x^3)/a]))))/(840*a^4*x^8*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.025, size = 1273, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x)
```

```
[Out] g*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*
(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3
^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+d*(-1/8*a*(b*x^3+a)^
(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x^3+a)^(1/2)/x^2+9/32
0*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-
3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*
(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x
^3+a)^(1/2)/a^(1/2))/a^(1/2))+e*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+
a)^(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)/x-9/112*I/a*b^2*3^(1/2)*(-b^2*a)^
(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3)))^(1/2))))+c*(-1/9*a*(b*x^3+a)^(1/2)/x^9-7/36*b*(b*x^3+a)^(1
/2)/x^6-1/24/a*b^2*(b*x^3+a)^(1/2)/x^3+1/24/a^(3/2)*b^3*arctanh((b*x^3+a)^(
1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^{7} + bfx^{6} + bex^{5} + \left(bd + ag\right)x^{4} + aex^{2} + \left(bc + af\right)x^{3} + adx + ac\right)\sqrt{bx^{3} + a}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^10, x)
```

Sympy [A] time = 23.7195, size = 573, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)
```

```
[Out] a**(3/2)*d*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + a**(3/2)*e*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*g*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
 + sqrt(a)*b*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*e*gamma(-4/3)*hyper((-4/3, -1/2), (
-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*g*gamma(
-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1
/3)) - a**2*c/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*f/(6*sqrt(b
)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*c/(36*x**(15/2)*sqrt(a/(b*
x**3) + 1)) - a*sqrt(b)*f/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*c
/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*f*sqrt(a/(b*x**3) + 1)/(3*x*
*(3/2)) - b**(3/2)*f/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*c/(24*a*
x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*f*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(
4*sqrt(a)) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4 + fx^3 + ex^2 + dx + c\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10, x)
```

3.472 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{11}}\,dx$

Optimal. Leaf size=764

$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots}\right.\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420*g)/x^3)*S
qrt[a + b*x^3])/1680 - (27*b^2*c*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*d*Sqr
t[a + b*x^3])/(24*a*x^3) - (27*b^2*e*Sqrt[a + b*x^3])/(320*a*x^2) + (27*b^2
*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*Sqr
t[a + b*x^3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((252*c)/x^1
0 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3/2
))/2520 + (b^2*(b*d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2))
 + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 - Sq
rt[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(2240*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.33014, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7\cdots\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -(b*((108*c)/x^7 + (140*d)/x^6 + (189*e)/x^5 + (270*f)/x^4 + (420*g)/x^3)*S
qrt[a + b*x^3])/1680 - (27*b^2*c*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*d*Sqr
t[a + b*x^3])/(24*a*x^3) - (27*b^2*e*Sqrt[a + b*x^3])/(320*a*x^2) + (27*b^2
*(b*c - 4*a*f)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*c - 4*a*f)*Sqr
t[a + b*x^3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((252*c)/x^1
0 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)/x^6)*(a + b*x^3)^(3/2
))/2520 + (b^2*(b*d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2))
 + (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*c - 4*a*f)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*
```

```
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3]) - (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 - Sq
rt[3])*(b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(2240*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{11}}dx = -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)\left(a+bx^3\right)^{3/2}}{2520} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx^3}\left(-\cdots\right.}{\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\cdots\right.}{25\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{\left(\frac{252}{x^{10}}\cdots\right.}{\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

$$= -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\cdots}{2\cdots}$$

Mathematica [C] time = 0.506918, size = 227, normalized size = 0.3

$$-\frac{\sqrt{a+bx^3}\left(2x^3\left(35a^3gx\left(3b^2x^6\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+a\left(2a+5bx^3\right)\sqrt{\frac{bx^3}{a}+1}\right)+60a^5f\,{}_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)-56b^3dx^7\cdots\right.\right.}{840a^4x^{10}\sqrt{\frac{bx^3}{a}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

```
[Out] -(Sqrt[a + b*x^3]*(84*a^5*c*Hypergeometric2F1[-10/3, -3/2, -7/3, -((b*x^3)/
a)] + 105*a^5*e*x^2*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] + 2*x
^3*(35*a^3*g*x*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*ArcTanh[S
qrt[1 + (b*x^3)/a]]) + 60*a^5*f*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^
3)/a)] - 56*b^3*d*x^7*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5
/2, 4, 7/2, 1 + (b*x^3)/a])))/(840*a^4*x^10*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.008, size = 1470, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x)
```

```
[Out] e*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x
^3+a)^(1/2)/x^2+9/320*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/
b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b
^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3)))^(1/2)))+g*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-
1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+f*(-1/7*a*(b*x^3+a)^(1/2)
/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112/a*b^2*(b*x^3+a)^(1/2)/x-9/112*I/a*b
^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^
(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1
/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))+d*(-1/9*a*(b*x^3+a)^(1/2)/x^9
-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24/a*b^2*(b*x^3+a)^(1/2)/x^3+1/24/a^(3/2)*b^3
*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+c*(-1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b
*(b*x^3+a)^(1/2)/x^7-27/1120/a*b^2*(b*x^3+a)^(1/2)/x^4+27/448/a^2*b^3*(b*x^
3+a)^(1/2)/x+9/448*I/a^2*b^3*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b
*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(
-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*
a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b
*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)
^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="maxi
ma")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bgx^7+bfx^6+bex^5+\left(bd+ag\right)x^4+aex^2+\left(bc+af\right)x^3+adx+ac\right)\sqrt{bx^3+a}}{x^{11}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^11, x)
```

Sympy [A] time = 24.3077, size = 576, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)
```

```
[Out] a**(3/2)*c*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*e*gamma(-8/3)*hyper((-8/3, -1/2), (-5
/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*f*gamma(-7/
3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/
3)) + sqrt(a)*b*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*e*gamma(-5/3)*hyper((-5/3, -1/2)
, (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*f*gam
ma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gam
ma(-1/3)) - a**2*d/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*g/(6*s
qrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*d/(36*x**(15/2)*sqrt(
a/(b*x**3) + 1)) - a*sqrt(b)*g/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3
/2)*d/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*g*sqrt(a/(b*x**3) + 1)/
(3*x**(3/2)) - b**(3/2)*g/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(5/2)*d/(
24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2
)))/(4*sqrt(a)) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

3.473 $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

Optimal. Leaf size=796

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)b^3}{24a^{3/2}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

```
[Out] -(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (2970*g)/x^
4)*Sqrt[a + b*x^3])/18480 - (27*b^2*c*Sqrt[a + b*x^3])/(1760*a*x^5) - (27*b
^2*d*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*e*Sqrt[a + b*x^3])/(24*a*x^3) + (
27*b^2*(7*b*c - 22*a*f)*Sqrt[a + b*x^3])/(7040*a^2*x^2) + (27*b^2*(b*d - 4*
a*g)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*Sqrt[a + b*x^
3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (2772
*d)/x^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/2))/2
7720 + (b^3*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2)) + (27*3^(1/4)*
Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4
)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1 - Sqrt[3])
*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(49280*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 1.53375, antiderivative size = 796, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 1825, 1835, 1832, 266, 63, 208, 1878, 218, 1877}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)b^3}{24a^{3/2}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\frac{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{bx^3+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] -(b*((945*c)/x^8 + (1188*d)/x^7 + (1540*e)/x^6 + (2079*f)/x^5 + (2970*g)/x^
4)*Sqrt[a + b*x^3])/18480 - (27*b^2*c*Sqrt[a + b*x^3])/(1760*a*x^5) - (27*b
^2*d*Sqrt[a + b*x^3])/(1120*a*x^4) - (b^2*e*Sqrt[a + b*x^3])/(24*a*x^3) + (
27*b^2*(7*b*c - 22*a*f)*Sqrt[a + b*x^3])/(7040*a^2*x^2) + (27*b^2*(b*d - 4*
a*g)*Sqrt[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(b*d - 4*a*g)*Sqrt[a + b*x^
3])/(448*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (((2520*c)/x^11 + (2772
*d)/x^10 + (3080*e)/x^9 + (3465*f)/x^8 + (3960*g)/x^7)*(a + b*x^3)^(3/2))/2
7720 + (b^3*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(3/2)) + (27*3^(1/4)*
Sqrt[2 - Sqrt[3]]*b^(7/3)*(b*d - 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
```

```
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4
)*Sqrt[2 + Sqrt[3]]*b^(7/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1 - Sqrt[3])
*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(49280*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1878

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx = -\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)\left(a+bx^3\right)^{3/2}}{27720} - \frac{1}{2}(9b)\int \frac{\sqrt{a+bx}}{}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{30}{}\right.}{}$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

$$= -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480} - \frac{27b^2c\sqrt{a+bx^3}}{1760ax^5} -$$

Mathematica [C] time = 0.400484, size = 194, normalized size = 0.24

$$\frac{\sqrt{a+bx^3}\left(11x^3\left(-105a^5f\,{}_2F_1\left(-\frac{8}{3},-\frac{3}{2};-\frac{5}{3};-\frac{bx^3}{a}\right)-120a^5gx\,{}_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)+112b^3ex^8\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{5}{2}\right.\right.\right.}{9240a^4x^{11}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-840*a^5*c*Hypergeometric2F1[-11/3, -3/2, -8/3, -((b*x^3)
/a)] - 924*a^5*d*x*Hypergeometric2F1[-10/3, -3/2, -7/3, -((b*x^3)/a)] + 11*
x^3*(-105*a^5*f*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] - 120*a^5
*g*x*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 112*b^3*e*x^8*(a +
 b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^3)/a]
)))/(9240*a^4*x^11*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.025, size = 1773, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x)
```

```
[Out] c*(-1/11*a*(b*x^3+a)^(1/2)/x^11-25/176*b*(b*x^3+a)^(1/2)/x^8-27/1760/a*b^2*
(b*x^3+a)^(1/2)/x^5+189/7040/a^2*b^3*(b*x^3+a)^(1/2)/x^2-63/7040*I/a^2*b^3*
3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/
3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(
-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+f*(-1/8*a*(b
*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320/a*b^2*(b*x^3+a)^(1/2)/
x^2+9/320*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(
1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/
b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2)))+g*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^(1/2)/x^4-27/112/a*b^
2*(b*x^3+a)^(1/2)/x-9/112*I/a*b^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*
a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((
x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2
)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(
-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
)+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b
^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
)+e*(-1/9*a*(b*x^3+a)^(1/2)/x^9-7/36*b*(b*x^3+a)^(1/2)/x^6-1/24/a*b^2*(b*x^
3+a)^(1/2)/x^3+1/24/a^(3/2)*b^3*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+d*(-1/10*
a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(1/2)/x^7-27/1120/a*b^2*(b*x^3+a)
^(1/2)/x^4+27/448/a^2*b^3*(b*x^3+a)^(1/2)/x+9/448*I/a^2*b^3*3^(1/2)*(-b^2*a
)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*
b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/
3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="maxi
ma")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bgx^7 + bfx^6 + bex^5 + (bd + ag)x^4 + aex^2 + (bc + af)x^3 + adx + ac)\sqrt{bx^3 + a}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x^7 + b*f*x^6 + b*e*x^5 + (b*d + a*g)*x^4 + a*e*x^2 + (b*c +
a*f)*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a)/x^12, x)
```

Sympy [A] time = 22.8427, size = 541, normalized size = 0.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/3)*hyper((-11/3, -1/2), (-8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**11*gamma(-8/3)) + a**(3/2)*d*gamma(-10/3)*hyper((-10/3, -1/2), (
-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*f*gamma(
-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(
-5/3)) + a**(3/2)*g*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-8/3)*hyper((-8/3, -1/
2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a)*b*d*g
amma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*g
amma(-4/3)) + sqrt(a)*b*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*g*gamma(-4/3)*hyper((-4/
3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) - a**2*e/
(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*e/(36*x**(15/2)*s
qrt(a/(b*x**3) + 1)) - 17*b**(3/2)*e/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b
**(5/2)*e/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**3*e*asinh(sqrt(a)/(sqrt
(b)*x**(3/2)))/(24*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx^4 + fx^3 + ex^2 + dx + c)(bx^3 + a)^{\frac{3}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="giac
")
```

```
[Out] integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

3.474 $\int (c+dx+ex^2)(a+bx^3)^p\,dx$

Optimal. Leaf size=102

$$\frac{cx\left(a+bx^3\right)^{p+1}{}_2F_1\left(1,p+\frac{4}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a}+\frac{dx^2\left(a+bx^3\right)^{p+1}{}_2F_1\left(1,p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}+\frac{e\left(a+bx^3\right)^{p+1}}{3b(p+1)}$$

```
[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^(1 + p)*Hypergeome
tric2F1[1, 4/3 + p, 4/3, -((b*x^3)/a)])/a + (d*x^2*(a + b*x^3)^(1 + p)*Hype
rgeometric2F1[1, 5/3 + p, 5/3, -((b*x^3)/a)])/(2*a)
```

Rubi [A] time = 0.0762832, antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1886, 261, 1893, 246, 245, 365, 364}

$$cx\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{1}{2}dx^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{e\left(a+bx^3\right)^{p+1}}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^p*Hypergeometric2F
1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (d*x^2*(a + b*x^3)^p*Hyp
ergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p)
```

Rule 1886

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx &= e\int x^2\left(a+bx^3\right)^p\,dx+\int (c+dx)\left(a+bx^3\right)^p\,dx\\ &= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\int \left(c\left(a+bx^3\right)^p+dx\left(a+bx^3\right)^p\right)dx\\ &= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+c\int \left(a+bx^3\right)^p\,dx+d\int x\left(a+bx^3\right)^p\,dx\\ &= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\left(c\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int \left(1+\frac{bx^3}{a}\right)^p dx+\left(d\left(a+bx^3\right)^p\left(1+\right.\right.\\ &= \frac{e\left(a+bx^3\right)^{1+p}}{3b(1+p)}+cx\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{1}{2}dx^2\left(a+bx^3\right)^p\end{aligned}$$

Mathematica [A] time = 0.0616251, size = 114, normalized size = 1.12

$$\frac{\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(6bc(p+1)x\,{}_2F_1\left(\frac{1}{3},-p;\frac{4}{3};-\frac{bx^3}{a}\right)+3bd(p+1)x^2\,{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+2e\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^p\right)}{6b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(2*e*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x*Hyperge
ometric2F1[1/3, -p, 4/3, -((b*x^3)/a)] + 3*b*d*(1 + p)*x^2*Hypergeometric2F
1[2/3, -p, 5/3, -((b*x^3)/a)]))/(6*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + dx + c\right)\left(bx^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^2 + dx + c\right)\left(bx^3 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

Sympy [A] time = 91.6781, size = 112, normalized size = 1.1

$$\frac{a^p cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -p \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^p dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, -p \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + e\left(\begin{cases} \frac{a^p x^3}{3} & \text{for}\ b = 0 \\ \frac{\begin{cases} \frac{\left(a+bx^3\right)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a + bx^3\right) & \text{otherwise} \end{cases}}{3b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + a**p*d*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(5/3)) + e*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piecewis
e(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(3*b
), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + dx + c\right)\left(bx^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

3.475 $\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx$

Optimal. Leaf size=107

$$\frac{cx^2\left(a+bx^3\right)^{p+1}{}_2F_1\left(1,p+\frac{5}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}+\frac{d\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{ex^4\left(a+bx^3\right)^{p+1}{}_2F_1\left(1,p+\frac{7}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}$$

```
[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^(1 + p)*Hypergeo
metric2F1[1, 5/3 + p, 5/3, -((b*x^3)/a)])/(2*a) + (e*x^4*(a + b*x^3)^(1 + p
)*Hypergeometric2F1[1, 7/3 + p, 7/3, -((b*x^3)/a)])/(4*a)
```

Rubi [A] time = 0.0911636, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1893, 365, 364, 261}

$$\frac{1}{2}cx^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{d\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{1}{4}ex^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^p*Hypergeometric
2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p) + (e*x^4*(a + b*x^3)
^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p)
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx=\int\left(cx\left(a+bx^3\right)^p+dx^2\left(a+bx^3\right)^p+ex^3\left(a+bx^3\right)^p\right)dx$$
$$=c\int x\left(a+bx^3\right)^p\,dx+d\int x^2\left(a+bx^3\right)^p\,dx+e\int x^3\left(a+bx^3\right)^p\,dx$$
$$=\frac{d\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\left(c\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x\left(1+\frac{bx^3}{a}\right)^p\,dx+\left(e\left(a+bx^3\right)^p\left(1+\right.\right.$$
$$=\frac{d\left(a+bx^3\right)^{1+p}}{3b(1+p)}+\frac{1}{2}cx^2\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{1}{4}ex^4\left(a+bx^3\right)$$

Mathematica [A] time = 0.0620941, size = 116, normalized size = 1.08

$$\frac{\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(6bc(p+1)x^2\,{}_2F_1\left(\frac{2}{3},-p;\frac{5}{3};-\frac{bx^3}{a}\right)+4d\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^p+3be(p+1)x^4\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)\right)}{12b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(4*d*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x^2*Hyper
geometric2F1[2/3, -p, 5/3, -((b*x^3)/a)] + 3*b*e*(1 + p)*x^4*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)]))/(12*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int x\left(ex^2+dx+c\right)\left(bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(ex^2+dx+c\right)\left(bx^3+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ex^3+dx^2+cx\right)\left(bx^3+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x^2 + c*x)*(b*x^3 + a)^p, x)
```

Sympy [A] time = 145.318, size = 114, normalized size = 1.07

$$\frac{a^p c x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, -p\\ \frac{5}{3}\end{matrix}\middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{a^p e x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{4}{3}, -p\\ \frac{7}{3}\end{matrix}\middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + d\left(\begin{cases}\frac{a^p x^3}{3} & \text{for } b = 0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p \neq -1\\ \log\left(a + bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] a**p*c*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(5/3)) + a**p*e*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + d*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/(
3*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + dx + c\right)\left(bx^3 + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

3.476 $\int x^2 \left(c+dx+ex^2\right) \left(a+bx^3\right)^p \, dx$

Optimal. Leaf size=107

$$\frac{c\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{dx^4\left(a+bx^3\right)^{p+1}\,{}_2F_1\left(1,p+\frac{7}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}+\frac{ex^5\left(a+bx^3\right)^{p+1}\,{}_2F_1\left(1,p+\frac{8}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5a}$$

```
[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^(1 + p)*Hypergeo
metric2F1[1, 7/3 + p, 7/3, -((b*x^3)/a)])/(4*a) + (e*x^5*(a + b*x^3)^(1 + p
)*Hypergeometric2F1[1, 8/3 + p, 8/3, -((b*x^3)/a)])/(5*a)
```

Rubi [A] time = 0.106132, antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1893, 261, 365, 364}

$$\frac{c\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{1}{4}dx^4\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)+\frac{1}{5}ex^5\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{5}{3},-p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] (c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^p*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p) + (e*x^5*(a + b*x^3)
^p*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)])/(5*(1 + (b*x^3)/a)^p)
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^p\,dx = \int\left(cx^2\left(a+bx^3\right)^p+dx^3\left(a+bx^3\right)^p+ex^4\left(a+bx^3\right)^p\right)dx$$
$$= c\int x^2\left(a+bx^3\right)^p\,dx + d\int x^3\left(a+bx^3\right)^p\,dx + e\int x^4\left(a+bx^3\right)^p\,dx$$
$$= \frac{c\left(a+bx^3\right)^{1+p}}{3b(1+p)} + \left(d\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}\right)\int x^3\left(1+\frac{bx^3}{a}\right)^p\,dx + \left(e\left(a+bx^3\right)^p\right.$$
$$= \frac{c\left(a+bx^3\right)^{1+p}}{3b(1+p)} + \frac{1}{4}dx^4\left(a+bx^3\right)^p\left(1+\frac{bx^3}{a}\right)^{-p}{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right) + \frac{1}{5}ex^5\left(a+\right.$$

Mathematica [A] time = 0.0762696, size = 116, normalized size = 1.08

$$\frac{\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\left(20c\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^p+15bd(p+1)x^4\,{}_2F_1\left(\frac{4}{3},-p;\frac{7}{3};-\frac{bx^3}{a}\right)+12be(p+1)x^5\,{}_2F_1\left(\frac{5}{3},-p;\frac{8}{3};-\frac{bx^3}{a}\right)\right)}{60b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^p*(20*c*(a + b*x^3)*(1 + (b*x^3)/a)^p + 15*b*d*(1 + p)*x^4*Hyp
ergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)] + 12*b*e*(1 + p)*x^5*Hypergeomet
ric2F1[5/3, -p, 8/3, -((b*x^3)/a)]))/(60*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int x^2\left(ex^2+dx+c\right)\left(bx^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

```
[Out] int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^4+dx^3+cx^2\right)\left(bx^3+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^4 + d*x^3 + c*x^2)*(b*x^3 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + dx + c\right)\left(bx^3 + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2, x)
```

3.477 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx$

Optimal. Leaf size=68

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rubi [A] time = 0.0444891, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1850}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx &= \int \left(ac+adx+aex^2+afx^3+bcx^4+bdx^5+bex^6+bfx^7\right)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8\end{aligned}$$

Mathematica [A] time = 0.0053242, size = 68, normalized size = 1.

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6
 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Maple [A] time = 0.001, size = 55, normalized size = 0.8

$$acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{afx^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}+\frac{bfx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x
^7+1/8*b*f*x^8
```

Maxima [A] time = 0.916526, size = 73, normalized size = 1.07

$$\frac{1}{8}bfx^8 + \frac{1}{7}bex^7 + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A] time = 1.38129, size = 142, normalized size = 2.09

$$\frac{1}{8}x^8fb + \frac{1}{7}x^7eb + \frac{1}{6}x^6db + \frac{1}{5}x^5cb + \frac{1}{4}x^4fa + \frac{1}{3}x^3ea + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*f*b + 1/7*x^7*e*b + 1/6*x^6*d*b + 1/5*x^5*c*b + 1/4*x^4*f*a + 1/3*x
^3*e*a + 1/2*x^2*d*a + x*c*a
```

Sympy [A] time = 0.06287, size = 63, normalized size = 0.93

$$acx + \frac{adx^2}{2} + \frac{aex^3}{3} + \frac{afx^4}{4} + \frac{bcx^5}{5} + \frac{bdx^6}{6} + \frac{bex^7}{7} + \frac{bfx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*
e*x**7/7 + b*f*x**8/8
```

Giac [A] time = 1.08806, size = 76, normalized size = 1.12

$$\frac{1}{8}bfx^8 + \frac{1}{7}bx^7e + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}ax^3e + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*f*x^8 + 1/7*b*x^7*e + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*a
*x^3*e + 1/2*a*d*x^2 + a*c*x
```

3.478 $\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4) \, dx$

Optimal. Leaf size=73

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rubi [A] time = 0.0641713, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1820}

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3 (c + dx + ex^2 + fx^3) (a + bx^4) \, dx &= \int (acx^3 + adx^4 + aex^5 + afx^6 + bcx^7 + bdx^8 + bex^9 + bfx^{10}) \, dx \\ &= \frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}\end{aligned}$$

Mathematica [A] time = 0.0029445, size = 73, normalized size = 1.

$$\frac{1}{4}acx^4 + \frac{1}{5}adx^5 + \frac{1}{6}aex^6 + \frac{1}{7}afx^7 + \frac{1}{8}bcx^8 + \frac{1}{9}bdx^9 + \frac{1}{10}bex^{10} + \frac{1}{11}bfx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

```
[Out] (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d*
x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Maple [A] time = 0., size = 58, normalized size = 0.8

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{afx^7}{7} + \frac{bcx^8}{8} + \frac{bdx^9}{9} + \frac{bex^{10}}{10} + \frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

```
[Out] 1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/1
0*b*e*x^10+1/11*b*f*x^11
```

Maxima [A] time = 0.918607, size = 77, normalized size = 1.05

$$\frac{1}{11}bfx^{11} + \frac{1}{10}bex^{10} + \frac{1}{9}bdx^9 + \frac{1}{8}bcx^8 + \frac{1}{7}afx^7 + \frac{1}{6}aex^6 + \frac{1}{5}adx^5 + \frac{1}{4}acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Fricas [A] time = 1.43254, size = 155, normalized size = 2.12

$$\frac{1}{11}x^{11}fb + \frac{1}{10}x^{10}eb + \frac{1}{9}x^9db + \frac{1}{8}x^8cb + \frac{1}{7}x^7fa + \frac{1}{6}x^6ea + \frac{1}{5}x^5da + \frac{1}{4}x^4ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*b + 1/10*x^10*e*b + 1/9*x^9*d*b + 1/8*x^8*c*b + 1/7*x^7*f*a + 1
/6*x^6*e*a + 1/5*x^5*d*a + 1/4*x^4*c*a
```

Sympy [A] time = 0.065535, size = 66, normalized size = 0.9

$$\frac{acx^4}{4} + \frac{adx^5}{5} + \frac{aex^6}{6} + \frac{afx^7}{7} + \frac{bcx^8}{8} + \frac{bdx^9}{9} + \frac{bex^{10}}{10} + \frac{bfx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

```
[Out] a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/9
 + b*e*x**10/10 + b*f*x**11/11
```

Giac [A] time = 1.05782, size = 80, normalized size = 1.1

$$\frac{1}{11}bfx^{11} + \frac{1}{10}bx^{10}e + \frac{1}{9}bdx^9 + \frac{1}{8}bcx^8 + \frac{1}{7}afx^7 + \frac{1}{6}ax^6e + \frac{1}{5}adx^5 + \frac{1}{4}acx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*b*f*x^11 + 1/10*b*x^10*e + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 + 1
/6*a*x^6*e + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

3.479 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=109

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rubi [A] time = 0.0744559, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1657}

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 +
 (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f*(
a + b*x^4)^3)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx &= \frac{f\left(a+bx^4\right)^3}{12b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^2\,dx\\ &= \frac{f\left(a+bx^4\right)^3}{12b}+\int \left(a^2c+a^2dx+a^2ex^2+2abcx^4+2abdx^5+2abex^6+b^2cx^8\right.\\ &= a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{1}\end{aligned}$$

Mathematica [A] time = 0.0036873, size = 124, normalized size = 1.14

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5 +
 (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Maple [A] time = 0., size = 103, normalized size = 0.9

$$\frac{b^2fx^{12}}{12}+\frac{b^2ex^{11}}{11}+\frac{b^2dx^{10}}{10}+\frac{b^2cx^9}{9}+\frac{fabx^8}{4}+\frac{2\,abex^7}{7}+\frac{abdx^6}{3}+\frac{2\,abcx^5}{5}+\frac{fa^2x^4}{4}+\frac{a^2ex^3}{3}+\frac{a^2dx^2}{2}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*f*a*b*x^8
+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*f*a^2*x^4+1/3*a^2*e*x^3+1/2*
a^2*d*x^2+a^2*c*x
```

Maxima [A] time = 0.914583, size = 138, normalized size = 1.27

$$\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Fricas [A] time = 1.48887, size = 258, normalized size = 2.37

$$\frac{1}{12}x^{12}fb^2+\frac{1}{11}x^{11}eb^2+\frac{1}{10}x^{10}db^2+\frac{1}{9}x^9cb^2+\frac{1}{4}x^8fba+\frac{2}{7}x^7eba+\frac{1}{3}x^6dba+\frac{2}{5}x^5cba+\frac{1}{4}x^4fa^2+\frac{1}{3}x^3ea^2+\frac{1}{2}x^2da^2+xc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f*b^2 + 1/11*x^11*e*b^2 + 1/10*x^10*d*b^2 + 1/9*x^9*c*b^2 + 1/4*x
^8*f*b*a + 2/7*x^7*e*b*a + 1/3*x^6*d*b*a + 2/5*x^5*c*b*a + 1/4*x^4*f*a^2 +
1/3*x^3*e*a^2 + 1/2*x^2*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.074751, size = 121, normalized size = 1.11

$$a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{a^2fx^4}{4} + \frac{2abcx^5}{5} + \frac{abdx^6}{3} + \frac{2abex^7}{7} + \frac{abfx^8}{4} + \frac{b^2cx^9}{9} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11} + \frac{b^2fx^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5 +
 a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**1
0/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Giac [A] time = 1.08465, size = 142, normalized size = 1.3

$$\frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2x^{11}e + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abx^7e + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2x^3e + \frac{1}{2}a^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*f*x^12 + 1/11*b^2*x^11*e + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a
*b*f*x^8 + 2/7*a*b*x^7*e + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4 +
1/3*a^2*x^3*e + 1/2*a^2*d*x^2 + a^2*c*x
```

3.480 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^2 \, dx$

Optimal. Leaf size=114

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c\left(a+bx^4\right)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^
10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15
)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rubi [A] time = 0.0837339, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1582, 1850}

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c\left(a+bx^4\right)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x^
10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15
)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx &= \frac{c\left(a+bx^4\right)^3}{12b}+\int \left(a+bx^4\right)^2\left(-cx^3+x^3\left(c+dx+ex^2+fx^3\right)\right)\,dx\\ &= \frac{c\left(a+bx^4\right)^3}{12b}+\int \left(a^2dx^4+a^2ex^5+a^2fx^6+2abdx^8+2abex^9+2abfx^{10}+b^2\right.\\ &= \frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\end{aligned}$$

Mathematica [A] time = 0.0041538, size = 129, normalized size = 1.13

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{1}{4}abcx^8+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{12}b^2cx^{12}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

```
[Out] (a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8)
/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/12
 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15
```

Maple [A] time = 0.001, size = 106, normalized size = 0.9

$$\frac{b^2fx^{15}}{15}+\frac{b^2ex^{14}}{14}+\frac{b^2dx^{13}}{13}+\frac{b^2cx^{12}}{12}+\frac{2\,abfx^{11}}{11}+\frac{abex^{10}}{5}+\frac{2\,abdx^9}{9}+\frac{abcx^8}{4}+\frac{a^2fx^7}{7}+\frac{a^2ex^6}{6}+\frac{a^2dx^5}{5}+\frac{a^2cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

```
[Out] 1/15*b^2*f*x^15+1/14*b^2*e*x^14+1/13*b^2*d*x^13+1/12*b^2*c*x^12+2/11*a*b*f*
x^11+1/5*a*b*e*x^10+2/9*a*b*d*x^9+1/4*a*b*c*x^8+1/7*a^2*f*x^7+1/6*a^2*e*x^6
+1/5*a^2*d*x^5+1/4*a^2*c*x^4
```

Maxima [A] time = 0.91656, size = 142, normalized size = 1.25

$$\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Fricas [A] time = 1.43756, size = 273, normalized size = 2.39

$$\frac{1}{15}x^{15}fb^2+\frac{1}{14}x^{14}eb^2+\frac{1}{13}x^{13}db^2+\frac{1}{12}x^{12}cb^2+\frac{2}{11}x^{11}fba+\frac{1}{5}x^{10}eba+\frac{2}{9}x^9dba+\frac{1}{4}x^8cba+\frac{1}{7}x^7fa^2+\frac{1}{6}x^6ea^2+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*f*b^2 + 1/14*x^14*e*b^2 + 1/13*x^13*d*b^2 + 1/12*x^12*c*b^2 + 2/1
1*x^11*f*b*a + 1/5*x^10*e*b*a + 2/9*x^9*d*b*a + 1/4*x^8*c*b*a + 1/7*x^7*f*a
^2 + 1/6*x^6*e*a^2 + 1/5*x^5*d*a^2 + 1/4*x^4*c*a^2
```

Sympy [A] time = 0.079195, size = 124, normalized size = 1.09

$$\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{a^2fx^7}{7}+\frac{abcx^8}{4}+\frac{2abdx^9}{9}+\frac{abex^{10}}{5}+\frac{2abfx^{11}}{11}+\frac{b^2cx^{12}}{12}+\frac{b^2dx^{13}}{13}+\frac{b^2ex^{14}}{14}+\frac{b^2fx^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

```
[Out] a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8/
4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 + b
**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15
```

Giac [A] time = 1.08274, size = 146, normalized size = 1.28

$$\frac{1}{15}b^2fx^{15}+\frac{1}{14}b^2x^{14}e+\frac{1}{13}b^2dx^{13}+\frac{1}{12}b^2cx^{12}+\frac{2}{11}abfx^{11}+\frac{1}{5}abx^{10}e+\frac{2}{9}abdx^9+\frac{1}{4}abcx^8+\frac{1}{7}a^2fx^7+\frac{1}{6}a^2x^6e+\frac{1}{5}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^2*f*x^15 + 1/14*b^2*x^14*e + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/1
1*a*b*f*x^11 + 1/5*a*b*x^10*e + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f*x
^7 + 1/6*a^2*x^6*e + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

3.481 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=151

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b$$

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rubi [A] time = 0.108231, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1657}

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6)
/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e
*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a + b
*x^4)^4)/(16*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(c+dx+ex^2\right)\left(a+bx^4\right)^3\,dx\\ &= \frac{f\left(a+bx^4\right)^4}{16b}+\int \left(a^3c+a^3dx+a^3ex^2+3a^2bcx^4+3a^2bdx^5+3a^2bex^6+3ab\right.\\ &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab\end{aligned}$$

Mathematica [A] time = 0.0049483, size = 180, normalized size = 1.19

$$\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/5
 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9)/
3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*c*x
^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Maple [A] time = 0., size = 151, normalized size = 1.

$$\frac{b^3fx^{16}}{16}+\frac{b^3ex^{15}}{15}+\frac{b^3dx^{14}}{14}+\frac{b^3cx^{13}}{13}+\frac{ab^2fx^{12}}{4}+\frac{3\,ab^2ex^{11}}{11}+\frac{3\,ab^2dx^{10}}{10}+\frac{ab^2cx^9}{3}+\frac{3\,fba^2x^8}{8}+\frac{3\,a^2bex^7}{7}+\frac{a^2bdx^6}{2}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/16*b^3*f*x^16+1/15*b^3*e*x^15+1/14*b^3*d*x^14+1/13*b^3*c*x^13+1/4*a*b^2*f
*x^12+3/11*a*b^2*e*x^11+3/10*a*b^2*d*x^10+1/3*a*b^2*c*x^9+3/8*f*b*a^2*x^8+3
/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/4*f*a^3*x^4+1/3*a^3*e*x^3+
1/2*a^3*d*x^2+a^3*c*x
```

Maxima [A] time = 0.910649, size = 203, normalized size = 1.34

$$\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Fricas [A] time = 1.38173, size = 375, normalized size = 2.48

$$\frac{1}{16}x^{16}fb^3+\frac{1}{15}x^{15}eb^3+\frac{1}{14}x^{14}db^3+\frac{1}{13}x^{13}cb^3+\frac{1}{4}x^{12}fb^2a+\frac{3}{11}x^{11}eb^2a+\frac{3}{10}x^{10}db^2a+\frac{1}{3}x^9cb^2a+\frac{3}{8}x^8fba^2+\frac{3}{7}x^7eba^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*f*b^3 + 1/15*x^15*e*b^3 + 1/14*x^14*d*b^3 + 1/13*x^13*c*b^3 + 1/4
*x^12*f*b^2*a + 3/11*x^11*e*b^2*a + 3/10*x^10*d*b^2*a + 1/3*x^9*c*b^2*a + 3
```

```
/8*x^8*f*b*a^2 + 3/7*x^7*e*b*a^2 + 1/2*x^6*d*b*a^2 + 3/5*x^5*c*b*a^2 + 1/4*
x^4*f*a^3 + 1/3*x^3*e*a^3 + 1/2*x^2*d*a^3 + x*c*a^3
```

Sympy [A] time = 0.087819, size = 180, normalized size = 1.19

$$a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2f}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5/
5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**9
/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3*c*
x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Giac [A] time = 1.06258, size = 208, normalized size = 1.38

$$\frac{1}{16}\,b^3fx^{16} + \frac{1}{15}\,b^3x^{15}e + \frac{1}{14}\,b^3dx^{14} + \frac{1}{13}\,b^3cx^{13} + \frac{1}{4}\,ab^2fx^{12} + \frac{3}{11}\,ab^2x^{11}e + \frac{3}{10}\,ab^2dx^{10} + \frac{1}{3}\,ab^2cx^9 + \frac{3}{8}\,a^2bfx^8 + \frac{3}{7}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*f*x^16 + 1/15*b^3*x^15*e + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/4
*a*b^2*f*x^12 + 3/11*a*b^2*x^11*e + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3
/8*a^2*b*f*x^8 + 3/7*a^2*b*x^7*e + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/4*
a^3*f*x^4 + 1/3*a^3*x^3*e + 1/2*a^3*d*x^2 + a^3*c*x
```

3.482 $\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^3 dx$

Optimal. Leaf size=156

$$\frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfx^{11} + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{3}{13}ab^2dx^{13} + \frac{3}{14}ab^2ex^{14} + \frac{1}{5}ab^2fx^{15} + \frac{c\left(a + bx^4\right)^4}{16b}$$

```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*
e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/1
4 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
+ (c*(a + b*x^4)^4)/(16*b)
```

Rubi [A] time = 0.112745, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1582, 1850}

$$\frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfx^{11} + \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{3}{13}ab^2dx^{13} + \frac{3}{14}ab^2ex^{14} + \frac{1}{5}ab^2fx^{15} + \frac{c\left(a + bx^4\right)^4}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```


```
[Out] (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b*
e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/1
4 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
+ (c*(a + b*x^4)^4)/(16*b)
```


Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x^3 \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^3 dx = \frac{c\left(a + bx^4\right)^4}{16b} + \int \left(a + bx^4\right)^3 \left(-cx^3 + x^3\left(c + dx + ex^2 + fx^3\right)\right) dx$$

$$= \frac{c\left(a + bx^4\right)^4}{16b} + \int \left(a^3dx^4 + a^3ex^5 + a^3fx^6 + 3a^2bdx^8 + 3a^2bex^9 + 3a^2bfx^{10} + \right.$$

$$= \frac{1}{5}a^3dx^5 + \frac{1}{6}a^3ex^6 + \frac{1}{7}a^3fx^7 + \frac{1}{3}a^2bdx^9 + \frac{3}{10}a^2bex^{10} + \frac{3}{11}a^2bfx^{11} + \frac{3}{13}ab^2dx$$

Mathematica [A] time = 0.0052431, size = 185, normalized size = 1.19

$$\frac{3}{8}a^2bcx^8+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{4}ab^2cx^{12}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

```
[Out] (a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c*
x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a*b
^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15)/5
 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19
```

Maple [A] time = 0.001, size = 154, normalized size = 1.

$$\frac{b^3fx^{19}}{19}+\frac{b^3ex^{18}}{18}+\frac{b^3dx^{17}}{17}+\frac{b^3cx^{16}}{16}+\frac{ab^2fx^{15}}{5}+\frac{3\,ab^2ex^{14}}{14}+\frac{3\,ab^2dx^{13}}{13}+\frac{acb^2x^{12}}{4}+\frac{3\,a^2bfx^{11}}{11}+\frac{3\,a^2bex^{10}}{10}+\frac{a^2bdx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

```
[Out] 1/19*b^3*f*x^19+1/18*b^3*e*x^18+1/17*b^3*d*x^17+1/16*b^3*c*x^16+1/5*a*b^2*f
*x^15+3/14*a*b^2*e*x^14+3/13*a*b^2*d*x^13+1/4*a*c*b^2*x^12+3/11*a^2*b*f*x^1
1+3/10*a^2*b*e*x^10+1/3*a^2*b*d*x^9+3/8*b*a^2*c*x^8+1/7*a^3*f*x^7+1/6*a^3*e
*x^6+1/5*a^3*d*x^5+1/4*a^3*c*x^4
```

Maxima [A] time = 0.916298, size = 207, normalized size = 1.33

$$\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Fricas [A] time = 1.44069, size = 390, normalized size = 2.5

$$\frac{1}{19}x^{19}fb^3+\frac{1}{18}x^{18}eb^3+\frac{1}{17}x^{17}db^3+\frac{1}{16}x^{16}cb^3+\frac{1}{5}x^{15}fb^2a+\frac{3}{14}x^{14}eb^2a+\frac{3}{13}x^{13}db^2a+\frac{1}{4}x^{12}cb^2a+\frac{3}{11}x^{11}fba^2+\frac{3}{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*f*b^3 + 1/18*x^18*e*b^3 + 1/17*x^17*d*b^3 + 1/16*x^16*c*b^3 + 1/5
*x^15*f*b^2*a + 3/14*x^14*e*b^2*a + 3/13*x^13*d*b^2*a + 1/4*x^12*c*b^2*a +
```

```
3/11*x^11*f*b*a^2 + 3/10*x^10*e*b*a^2 + 1/3*x^9*d*b*a^2 + 3/8*x^8*c*b*a^2 +
 1/7*x^7*f*a^3 + 1/6*x^6*e*a^3 + 1/5*x^5*d*a^3 + 1/4*x^4*c*a^3
```

Sympy [A] time = 0.084306, size = 184, normalized size = 1.18

$$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

```
[Out] a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c*
x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*b*
*2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**15/5
 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/19
```

Giac [A] time = 1.07847, size = 212, normalized size = 1.36

$$\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3x^{18}e+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2x^{14}e+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^3*f*x^19 + 1/18*b^3*x^18*e + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/5
*a*b^2*f*x^15 + 3/14*a*b^2*x^14*e + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12 +
3/11*a^2*b*f*x^11 + 3/10*a^2*b*x^10*e + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^8 +
 1/7*a^3*f*x^7 + 1/6*a^3*x^6*e + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.483 $\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx$

Optimal. Leaf size=193

$$\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab$$

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^
6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*
x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*
x^4)^5)/(20*b)
```

Rubi [A] time = 0.15563, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1582, 1657}

$$\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```


```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x^
6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*
a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*
x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a + b*
x^4)^5)/(20*b)
```


Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx &= \frac{f(a+bx^4)^5}{20b}+\int (c+dx+ex^2)(a+bx^4)^4\,dx\\ &= \frac{f(a+bx^4)^5}{20b}+\int \left(a^4c+a^4dx+a^4ex^2+4a^3bcx^4+4a^3bdx^5+4a^3bex^6+6a^2\right.\\ &= a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2\end{aligned}$$

Mathematica [A] time = 0.0055653, size = 236, normalized size = 1.22

$$\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{2}a^2b^2fx^{12}+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^3bfx^8+a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4e$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/5
 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c*x
^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 +
 (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^3*f*
x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f*x^20
)/20
```

Maple [A] time = 0.001, size = 199, normalized size = 1.

$$\frac{fb^4x^{20}}{20}+\frac{b^4ex^{19}}{19}+\frac{b^4dx^{18}}{18}+\frac{b^4cx^{17}}{17}+\frac{fab^3x^{16}}{4}+\frac{4\,ab^3ex^{15}}{15}+\frac{2\,ab^3dx^{14}}{7}+\frac{4\,ab^3cx^{13}}{13}+\frac{fb^2a^2x^{12}}{2}+\frac{6\,a^2b^2ex^{11}}{11}+\frac{3\,a^2b^2c}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

```
[Out] 1/20*f*b^4*x^20+1/19*b^4*e*x^19+1/18*b^4*d*x^18+1/17*b^4*c*x^17+1/4*f*a*b^3
*x^16+4/15*a*b^3*e*x^15+2/7*a*b^3*d*x^14+4/13*a*b^3*c*x^13+1/2*f*b^2*a^2*x^
12+6/11*a^2*b^2*e*x^11+3/5*a^2*b^2*d*x^10+2/3*a^2*b^2*c*x^9+1/2*b*f*a^3*x^8
+4/7*a^3*b*e*x^7+2/3*a^3*b*d*x^6+4/5*a^3*b*c*x^5+1/4*a^4*f*x^4+1/3*a^4*e*x^
3+1/2*a^4*d*x^2+a^4*c*x
```

Maxima [A] time = 0.931103, size = 267, normalized size = 1.38

$$\frac{1}{20}b^4fx^{20}+\frac{1}{19}b^4ex^{19}+\frac{1}{18}b^4dx^{18}+\frac{1}{17}b^4cx^{17}+\frac{1}{4}ab^3fx^{16}+\frac{4}{15}ab^3ex^{15}+\frac{2}{7}ab^3dx^{14}+\frac{4}{13}ab^3cx^{13}+\frac{1}{2}a^2b^2fx^{12}+\frac{6}{11}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Fricas [A] time = 1.48056, size = 490, normalized size = 2.54

$$\frac{1}{20}x^{20}fb^4+\frac{1}{19}x^{19}eb^4+\frac{1}{18}x^{18}db^4+\frac{1}{17}x^{17}cb^4+\frac{1}{4}x^{16}fb^3a+\frac{4}{15}x^{15}eb^3a+\frac{2}{7}x^{14}db^3a+\frac{4}{13}x^{13}cb^3a+\frac{1}{2}x^{12}fb^2a^2+\frac{6}{11}x^{11}eb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*f*b^4 + 1/19*x^19*e*b^4 + 1/18*x^18*d*b^4 + 1/17*x^17*c*b^4 + 1/4
*x^16*f*b^3*a + 4/15*x^15*e*b^3*a + 2/7*x^14*d*b^3*a + 4/13*x^13*c*b^3*a +
1/2*x^12*f*b^2*a^2 + 6/11*x^11*e*b^2*a^2 + 3/5*x^10*d*b^2*a^2 + 2/3*x^9*c*b
^2*a^2 + 1/2*x^8*f*b*a^3 + 4/7*x^7*e*b*a^3 + 2/3*x^6*d*b*a^3 + 4/5*x^5*c*b*
a^3 + 1/4*x^4*f*a^4 + 1/3*x^3*e*a^4 + 1/2*x^2*d*a^4 + x*c*a^4
```

Sympy [A] time = 0.089925, size = 241, normalized size = 1.25

$$a^4cx + \frac{a^4dx^2}{2} + \frac{a^4ex^3}{3} + \frac{a^4fx^4}{4} + \frac{4a^3bcx^5}{5} + \frac{2a^3bdx^6}{3} + \frac{4a^3bex^7}{7} + \frac{a^3bfx^8}{2} + \frac{2a^2b^2cx^9}{3} + \frac{3a^2b^2dx^{10}}{5} + \frac{6a^2b^2ex^{11}}{11} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5/
5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2*c
*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x**1
2/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 + a*b*
*3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b**4*f
*x**20/20
```

Giac [A] time = 1.10567, size = 274, normalized size = 1.42

$$\frac{1}{20}b^4fx^{20} + \frac{1}{19}b^4x^{19}e + \frac{1}{18}b^4dx^{18} + \frac{1}{17}b^4cx^{17} + \frac{1}{4}ab^3fx^{16} + \frac{4}{15}ab^3x^{15}e + \frac{2}{7}ab^3dx^{14} + \frac{4}{13}ab^3cx^{13} + \frac{1}{2}a^2b^2fx^{12} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/20*b^4*f*x^20 + 1/19*b^4*x^19*e + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/4
*a*b^3*f*x^16 + 4/15*a*b^3*x^15*e + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13 +
1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*x^11*e + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*b^2
*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*x^7*e + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*x^3*e + 1/2*a^4*d*x^2 + a^4*c*x
```

3.484 $\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx$

Optimal. Leaf size=198

$$\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{17}ab^3dx^{17}+$$

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*
b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^
14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (
4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 +
(c*(a + b*x^4)^5)/(20*b)
```

Rubi [A] time = 0.150432, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1582, 1850}

$$\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{17}ab^3dx^{17}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3*
b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^
14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (
4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 +
(c*(a + b*x^4)^5)/(20*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx&=\frac{c\left(a+bx^4\right)^5}{20b}+\int\left(a+bx^4\right)^4\left(-cx^3+x^3\left(c+dx+ex^2+fx^3\right)\right)dx\\&=\frac{c\left(a+bx^4\right)^5}{20b}+\int\left(a^4dx^4+a^4ex^5+a^4fx^6+4a^3bdx^8+4a^3bex^9+4a^3bfx^{10}+\right.\\&=\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx\end{aligned}$$

Mathematica [A] time = 0.0062771, size = 241, normalized size = 1.22

$$\frac{1}{2}a^2b^2cx^{12} + \frac{6}{13}a^2b^2dx^{13} + \frac{3}{7}a^2b^2ex^{14} + \frac{2}{5}a^2b^2fx^{15} + \frac{1}{2}a^3bcx^8 + \frac{4}{9}a^3bdx^9 + \frac{2}{5}a^3bex^{10} + \frac{4}{11}a^3bfx^{11} + \frac{1}{4}a^4cx^4 + \frac{1}{5}a^4d$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

```
[Out] (a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x^
8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^2*
b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f
*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9 + (4
*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (
b^4*f*x^23)/23
```

Maple [A] time = 0.001, size = 202, normalized size = 1.

$$\frac{b^4fx^{23}}{23} + \frac{b^4ex^{22}}{22} + \frac{b^4dx^{21}}{21} + \frac{cb^4x^{20}}{20} + \frac{4\,ab^3fx^{19}}{19} + \frac{2\,ab^3ex^{18}}{9} + \frac{4\,ab^3dx^{17}}{17} + \frac{acb^3x^{16}}{4} + \frac{2\,a^2b^2fx^{15}}{5} + \frac{3\,a^2b^2ex^{14}}{7} + \frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

```
[Out] 1/23*b^4*f*x^23+1/22*b^4*e*x^22+1/21*b^4*d*x^21+1/20*c*b^4*x^20+4/19*a*b^3*
f*x^19+2/9*a*b^3*e*x^18+4/17*a*b^3*d*x^17+1/4*a*c*b^3*x^16+2/5*a^2*b^2*f*x^
15+3/7*a^2*b^2*e*x^14+6/13*a^2*b^2*d*x^13+1/2*a^2*b^2*c*x^12+4/11*a^3*b*f*x
^11+2/5*a^3*b*e*x^10+4/9*a^3*b*d*x^9+1/2*c*a^3*b*x^8+1/7*a^4*f*x^7+1/6*a^4*
e*x^6+1/5*a^4*d*x^5+1/4*a^4*c*x^4
```

Maxima [A] time = 0.920182, size = 271, normalized size = 1.37

$$\frac{1}{23}\,b^4fx^{23} + \frac{1}{22}\,b^4ex^{22} + \frac{1}{21}\,b^4dx^{21} + \frac{1}{20}\,b^4cx^{20} + \frac{4}{19}\,ab^3fx^{19} + \frac{2}{9}\,ab^3ex^{18} + \frac{4}{17}\,ab^3dx^{17} + \frac{1}{4}\,ab^3cx^{16} + \frac{2}{5}\,a^2b^2fx^{15} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

Fricas [A] time = 1.49758, size = 504, normalized size = 2.55

$$\frac{1}{23}x^{23}fb^4 + \frac{1}{22}x^{22}eb^4 + \frac{1}{21}x^{21}db^4 + \frac{1}{20}x^{20}cb^4 + \frac{4}{19}x^{19}fb^3a + \frac{2}{9}x^{18}eb^3a + \frac{4}{17}x^{17}db^3a + \frac{1}{4}x^{16}cb^3a + \frac{2}{5}x^{15}fb^2a^2 + \frac{3}{7}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*f*b^4 + 1/22*x^22*e*b^4 + 1/21*x^21*d*b^4 + 1/20*x^20*c*b^4 + 4/1
9*x^19*f*b^3*a + 2/9*x^18*e*b^3*a + 4/17*x^17*d*b^3*a + 1/4*x^16*c*b^3*a +
2/5*x^15*f*b^2*a^2 + 3/7*x^14*e*b^2*a^2 + 6/13*x^13*d*b^2*a^2 + 1/2*x^12*c*
b^2*a^2 + 4/11*x^11*f*b*a^3 + 2/5*x^10*e*b*a^3 + 4/9*x^9*d*b*a^3 + 1/2*x^8*
c*b*a^3 + 1/7*x^7*f*a^4 + 1/6*x^6*e*a^4 + 1/5*x^5*d*a^4 + 1/4*x^4*c*a^4
```

Sympy [A] time = 0.094753, size = 245, normalized size = 1.24

$$\frac{a^4cx^4}{4} + \frac{a^4dx^5}{5} + \frac{a^4ex^6}{6} + \frac{a^4fx^7}{7} + \frac{a^3bcx^8}{2} + \frac{4a^3bdx^9}{9} + \frac{2a^3bex^{10}}{5} + \frac{4a^3bfx^{11}}{11} + \frac{a^2b^2cx^{12}}{2} + \frac{6a^2b^2dx^{13}}{13} + \frac{3a^2b^2ex^{14}}{7} + \frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

```
[Out] a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x*
*8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**2*
b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2*b*
*2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**18/9
+ 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**22/22
 + b**4*f*x**23/23
```

Giac [A] time = 1.08794, size = 278, normalized size = 1.4

$$\frac{1}{23}b^4fx^{23} + \frac{1}{22}b^4x^{22}e + \frac{1}{21}b^4dx^{21} + \frac{1}{20}b^4cx^{20} + \frac{4}{19}ab^3fx^{19} + \frac{2}{9}ab^3x^{18}e + \frac{4}{17}ab^3dx^{17} + \frac{1}{4}ab^3cx^{16} + \frac{2}{5}a^2b^2fx^{15} + \frac{3}{7}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/23*b^4*f*x^23 + 1/22*b^4*x^22*e + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/1
9*a*b^3*f*x^19 + 2/9*a*b^3*x^18*e + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16 +
2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*x^14*e + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*b^2
*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*x^10*e + 4/9*a^3*b*d*x^9 + 1/2*a^3*
b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*x^6*e + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^4
```

3.485 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx$

Optimal. Leaf size=133

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^
4])/(4*b)
```

Rubi [A] time = 0.124385, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1876, 1167, 205, 208, 1248, 635, 260}

$$\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
 + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^
4])/(4*b)
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx &= \int \left(\frac{c+ex^2}{a-bx^4}+\frac{x\left(d+fx^2\right)}{a-bx^4}\right)dx \\
&= \int \frac{c+ex^2}{a-bx^4}\,dx+\int \frac{x\left(d+fx^2\right)}{a-bx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+fx}{a-bx^2}\,dx,x,x^2\right)+\frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{a}\sqrt{b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{1}{2}d\operatorname{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)+ \\
&= \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right.}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.0655782, size = 214, normalized size = 1.61

$$
-\frac{\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}e+\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d\right)}{4ab^{3/4}}-\frac{\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(-a^{3/4}e-\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d\right)}{4ab^{3/4}}+\frac{\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\tan^{-1}\left(\right.}{2ab^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

```
[Out] ((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4))
 - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/
4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3/4)
*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*x^2]
)/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Maple [A] time = 0.004, size = 177, normalized size = 1.3

$$
\frac{c}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{d}{4}\ln\left(\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt{ab}}-\frac{e}{2b}\arctan
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)
^(1/4)/a*arctan(x/(1/b*a)^(1/4))-1/4*d/(a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/
(-a-x^2*(a*b)^(1/2)))-1/2*e/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4*e/b
/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/4/b*f*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 13.1386, size = 952, normalized size = 7.16

$$-\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3f + t^2\left(96a^3b^2f^2 - 64a^2b^3ce - 32a^2b^3d^2\right) + t\left(-16a^3bf^3 + 32a^2b^2cef + 16a^2b^2d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3*f + _t**2*(96*a**3*b**2*
f**2 - 64*a**2*b**3*c*e - 32*a**2*b**3*d**2) + _t*(-16*a**3*b*f**3 + 32*a**
2*b**2*c*e*f + 16*a**2*b**2*d**2*f - 16*a**2*b**2*d*e**2 - 16*a*b**3*c**2*d
) + a**3*f**4 - 4*a**2*b*c*e*f**2 - 2*a**2*b*d**2*f**2 + 4*a**2*b*d*e**2*f
- a**2*b*e**4 + 4*a*b**2*c**2*d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e
+ a*b**2*d**4 - b**3*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b**3*e**3
- 64*_t**3*a**3*b**4*c**2*e + 128*_t**3*a**3*b**4*c*d**2 + 48*_t**2*a**4*b*
*2*e**3*f + 48*_t**2*a**3*b**3*c**2*e*f - 96*_t**2*a**3*b**3*c*d**2*f + 48*
_t**2*a**3*b**3*c*d*e**2 - 32*_t**2*a**3*b**3*d**3*e - 16*_t**2*a**2*b**4*c
**3*d - 12*_t*a**4*b*e**3*f**2 - 12*_t*a**3*b**2*c**2*e*f**2 + 24*_t*a**3*b
**2*c*d**2*f**2 - 24*_t*a**3*b**2*c*d*e**2*f + 12*_t*a**3*b**2*c*e**4 + 16*
_t*a**3*b**2*d**3*e*f + 12*_t*a**3*b**2*d**2*e**3 + 8*_t*a**2*b**3*c**3*d*f
 + 16*_t*a**2*b**3*c**3*e**2 - 36*_t*a**2*b**3*c**2*d**2*e - 8*_t*a**2*b**3
```

```
*c*d**4 + 4*_t*a*b**4*c**5 + a**4*e**3*f**3 + a**3*b*c**2*e*f**3 - 2*a**3*b
*c*d**2*f**3 + 3*a**3*b*c*d*e**2*f**2 - 3*a**3*b*c*e**4*f - 2*a**3*b*d**3*e
*f**2 - 3*a**3*b*d**2*e**3*f + 3*a**3*b*d*e**5 - a**2*b**2*c**3*d*f**2 - 4*
a**2*b**2*c**3*e**2*f + 9*a**2*b**2*c**2*d**2*e*f + 2*a**2*b**2*c*d**4*f -
5*a**2*b**2*c*d**3*e**2 + 2*a**2*b**2*d**5*e - a*b**3*c**5*f + 5*a*b**3*c**
4*d*e - 5*a*b**3*c**3*d**3)/(a**3*b*e**6 + a**2*b**2*c**2*e**4 - 8*a**2*b**
2*c*d**2*e**3 + 4*a**2*b**2*d**4*e**2 - a*b**3*c**4*e**2 + 8*a*b**3*c**3*d*
*2*e - 4*a*b**3*c**2*d**4 - b**4*c**6))))
```

Giac [B] time = 1.07523, size = 419, normalized size = 3.15

$$-\frac{f\log\left(\left|bx^4-a\right|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}b^2c-\left(-ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2d-\left(-ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*f*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (-a*
b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*d - (
-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-
a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/
8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/(a*b^3)
```

3.486 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}$$

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*
f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]
*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

Rubi [A] time = 0.202951, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1831, 1252, 774, 635, 208, 260, 1280, 1167, 205}

$$\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}f+\sqrt{b}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] -((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*
f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]
*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt[b]
*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a-bx^4}\,dx &= \int \left(\frac{x^3\left(c+ex^2\right)}{a-bx^4}+\frac{x^4\left(d+fx^2\right)}{a-bx^4}\right)dx\\
&= \int \frac{x^3\left(c+ex^2\right)}{a-bx^4}\,dx+\int \frac{x^4\left(d+fx^2\right)}{a-bx^4}\,dx\\
&= -\frac{fx^3}{3b}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+ex)}{a-bx^2}\,dx,x,x^2\right)+\frac{\int \frac{x^2\left(3af+3bdx^2\right)}{a-bx^4}\,dx}{3b}\\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\int \frac{3abd+3abfx^2}{a-bx^4}\,dx}{3b^2}-\frac{\operatorname{Subst}\left(\int \frac{-ae-bcx}{a-bx^2}\,dx,x,x^2\right)}{2b}\\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{x}{a-bx^2}\,dx,x,x^2\right)+\frac{(ae)\operatorname{Subst}\left(\int \frac{1}{a-bx^2}\,dx,x,x^2\right)}{2b}-\ (\\
&= -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+
\end{aligned}
$$

Mathematica [A] time = 0.0842981, size = 221, normalized size = 1.36

$$
\frac{-3\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)\left(a^{3/4}f+\sqrt[4]{a}\sqrt{b}d+\sqrt{a}\sqrt[4]{b}e\right)+3\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\left(a^{3/4}f+\sqrt[4]{a}\sqrt{b}d-\sqrt{a}\sqrt[4]{b}e\right)+6\left(\sqrt[4]{a}\sqrt{b}d-a^{3/4}f\right)\tan}{12b^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

```
[Out] (-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)*Sqrt[b]*d
 - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*Sqrt[b]*d + Sqrt[a]*
b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*x] + 3*(a^(1/4)*Sqrt[b]*d - Sq
rt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/4)*e
*Log[Sqrt[a] + Sqrt[b]*x^2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

Maple [A] time = 0.004, size = 208, normalized size = 1.3

$$-\frac{fx^3}{3b}-\frac{ex^2}{2b}-\frac{dx}{b}+\frac{d}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{d}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)-\frac{ae}{4b}\ln\left(\left(-a+x^2\sqrt{ab}\right)\left(-a-x^2\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

```
[Out] -1/3*f*x^3/b-1/2*e*x^2/b-d*x/b+1/2/b*d*(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/4)
)+1/4/b*d*(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))-1/4/b*a*e/(
a*b)^(1/2)*ln((-a+x^2*(a*b)^(1/2))/(-a-x^2*(a*b)^(1/2)))-1/2/b^2*a*f/(1/b*a
)^(1/4)*arctan(x/(1/b*a)^(1/4))+1/4/b^2*a*f/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/
4))/(x-(1/b*a)^(1/4)))-1/4/b*c*ln(b*x^4-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 12.5476, size = 887, normalized size = 5.48

$$-\operatorname{RootSum}\left(256t^4b^7-256t^3b^6c+t^2\left(-64ab^4df-32ab^4e^2+96b^5c^2\right)+t\left(-16a^2b^2ef^2+32ab^3cdf+16ab^3ce^2-16ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*b**7 - 256*_t**3*b**6*c + _t**2*(-64*a*b**4*d*f - 32*a*b
**4*e**2 + 96*b**5*c**2) + _t*(-16*a**2*b**2*e*f**2 + 32*a*b**3*c*d*f + 16*
a*b**3*c*e**2 - 16*a*b**3*d**2*e - 16*b**4*c**3) - a**3*f**4 + 4*a**2*b*c*e
*f**2 + 2*a**2*b*d**2*f**2 - 4*a**2*b*d*e**2*f + a**2*b*e**4 - 4*a*b**2*c**
2*d*f - 2*a*b**2*c**2*e**2 + 4*a*b**2*c*d**2*e - a*b**2*d**4 + b**3*c**4, L
ambda(_t, _t*log(x + (-64*_t**3*a*b**5*f**3 - 64*_t**3*b**6*d**2*f + 128*_t
**3*b**6*d*e**2 + 48*_t**2*a*b**4*c*f**3 + 48*_t**2*a*b**4*d*e*f**2 - 32*_t
**2*a*b**4*e**3*f + 48*_t**2*b**5*c*d**2*f - 96*_t**2*b**5*c*d*e**2 - 16*_t
**2*b**5*d**3*e + 12*_t*a**2*b**2*d*f**4 + 12*_t*a**2*b**2*e**2*f**3 - 12*_
t*a*b**3*c**2*f**3 - 24*_t*a*b**3*c*d*e*f**2 + 16*_t*a*b**3*c*e**3*f + 16*_
t*a*b**3*d**3*f**2 - 36*_t*a*b**3*d**2*e**2*f - 8*_t*a*b**3*d*e**4 - 12*_t*
b**4*c**2*d**2*f + 24*_t*b**4*c**2*d*e**2 + 8*_t*b**4*c*d**3*e + 4*_t*b**4*
d**5 + 3*a**3*e*f**5 - 3*a**2*b*c*d*f**4 - 3*a**2*b*c*e**2*f**3 - 5*a**2*b*
d*e**3*f**2 + 2*a**2*b*e**5*f + a*b**2*c**3*f**3 + 3*a*b**2*c**2*d*e*f**2 -
 2*a*b**2*c**2*e**3*f - 4*a*b**2*c*d**3*f**2 + 9*a*b**2*c*d**2*e**2*f + 2*a
*b**2*c*d*e**4 + 5*a*b**2*d**4*e*f - 5*a*b**2*d**3*e**3 + b**3*c**3*d**2*f
- 2*b**3*c**3*d*e**2 - b**3*c**2*d**3*e - b**3*c*d**5)/(a**3*f**6 + a**2*b*
d**2*f**4 - 8*a**2*b*d*e**2*f**3 + 4*a**2*b*e**4*f**2 - a*b**2*d**4*f**2 +
8*a*b**2*d**3*e**2*f - 4*a*b**2*d**2*e**4 - b**3*d**6)))) - d*x/b - e*x**2/
(2*b) - f*x**3/(3*b)
```

Giac [B] time = 1.07629, size = 443, normalized size = 2.73

$$-\frac{c\log\left(\left|bx^4-a\right|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-\left(-ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*
b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a*b
^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b
)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(
3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt(2)*((
-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

3.487 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$

Optimal. Leaf size=293

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rubi [A] time = 0.222421, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{b}c\right)\tan^{-1}\left(\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sq
rt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \int \left(\frac{c+ex^2}{a+bx^4}+\frac{x\left(d+fx^2\right)}{a+bx^4}\right)dx$$

$$= \int \frac{c+ex^2}{a+bx^4}\,dx+\int \frac{x\left(d+fx^2\right)}{a+bx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{bc}}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{bc}-\sqrt{ae}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{bc}-\sqrt{ae}\right)\log\left(\sqrt{a}+\right.}{4\sqrt{2}a^{3/4}b}$$

$$= \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{bc}+\sqrt{ae}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{bc}+\sqrt{ae}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-$$

Mathematica [A] time = 0.214556, size = 296, normalized size = 1.01

$$\frac{-\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

```
[Out] (-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(Sqrt[2]
*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^(1/4)
*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 2*a*f*Log[a + b*x^4])/(8*a*b)
```

Maple [A] time = 0.004, size = 294, normalized size = 1.

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
)/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)+1/2*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e/b/
(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(
```

```
1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x-1)+1/4*f*ln(b*x^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 13.2101, size = 950, normalized size = 3.24

$$\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3f + t^2\left(96a^3b^2f^2 + 64a^2b^3ce + 32a^2b^3d^2\right) + t\left(-16a^3bf^3 - 32a^2b^2cef - 16a^2b^2d^2f + 16\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3*f + _t**2*(96*a**3*b**2*f
**2 + 64*a**2*b**3*c*e + 32*a**2*b**3*d**2) + _t*(-16*a**3*b*f**3 - 32*a**2
*b**2*c*e*f - 16*a**2*b**2*d**2*f + 16*a**2*b**2*d*e**2 - 16*a*b**3*c**2*d)
 + a**3*f**4 + 4*a**2*b*c*e*f**2 + 2*a**2*b*d**2*f**2 - 4*a**2*b*d*e**2*f +
 a**2*b*e**4 + 4*a*b**2*c**2*d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e +
 a*b**2*d**4 + b**3*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**4*b**3*e**3 -
64*_t**3*a**3*b**4*c**2*e + 128*_t**3*a**3*b**4*c*d**2 - 48*_t**2*a**4*b**2
*e**3*f + 48*_t**2*a**3*b**3*c**2*e*f - 96*_t**2*a**3*b**3*c*d**2*f + 48*_t
**2*a**3*b**3*c*d*e**2 - 32*_t**2*a**3*b**3*d**3*e + 16*_t**2*a**2*b**4*c**
3*d + 12*_t*a**4*b*e**3*f**2 - 12*_t*a**3*b**2*c**2*e*f**2 + 24*_t*a**3*b**
2*c*d**2*f**2 - 24*_t*a**3*b**2*c*d*e**2*f + 12*_t*a**3*b**2*c*e**4 + 16*_t
*a**3*b**2*d**3*e*f + 12*_t*a**3*b**2*d**2*e**3 - 8*_t*a**2*b**3*c**3*d*f -
 16*_t*a**2*b**3*c**3*e**2 + 36*_t*a**2*b**3*c**2*d**2*e + 8*_t*a**2*b**3*c
*d**4 + 4*_t*a*b**4*c**5 - a**4*e**3*f**3 + a**3*b*c**2*e*f**3 - 2*a**3*b*c
*d**2*f**3 + 3*a**3*b*c*d*e**2*f**2 - 3*a**3*b*c*e**4*f - 2*a**3*b*d**3*e*f
**2 - 3*a**3*b*d**2*e**3*f + 3*a**3*b*d*e**5 + a**2*b**2*c**3*d*f**2 + 4*a*
*2*b**2*c**3*e**2*f - 9*a**2*b**2*c**2*d**2*e*f - 2*a**2*b**2*c*d**4*f + 5*
a**2*b**2*c*d**3*e**2 - 2*a**2*b**2*d**5*e - a*b**3*c**5*f + 5*a*b**3*c**4*
d*e - 5*a*b**3*c**3*d**3)/(a**3*b*e**6 - a**2*b**2*c**2*e**4 + 8*a**2*b**2*
```

```
c*d**2*e**3 - 4*a**2*b**2*d**4*e**2 - a*b**3*c**4*e**2 + 8*a*b**3*c**3*d**2
*e - 4*a*b**3*c**2*d**4 + b**4*c**6))))
```

Giac [A] time = 1.08104, size = 392, normalized size = 1.34

$$\frac{f\log\left(\left|bx^4+a\right|\right)}{4\,b}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^
(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*
e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b
^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^3)
```

3.488 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

Optimal. Leaf size=321

$$\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)}{2\sqrt{}}$$

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/S
qrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(
Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*
x^4])/(4*b)
```

Rubi [A] time = 0.333813, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1831, 1252, 774, 635, 205, 260, 1280, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,f+\sqrt{b}\,d\right)}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/S
qrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/4)*(
Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log[a + b*
x^4])/(4*b)
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a+bx^4}\,dx = \int \left(\frac{x^3\left(c+ex^2\right)}{a+bx^4}+\frac{x^4\left(d+fx^2\right)}{a+bx^4}\right)dx$$

$$= \int \frac{x^3\left(c+ex^2\right)}{a+bx^4}\,dx + \int \frac{x^4\left(d+fx^2\right)}{a+bx^4}\,dx$$

$$= \frac{fx^3}{3b} + \frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+ex)}{a+bx^2}\,dx, x, x^2\right) - \frac{\int \frac{x^2\left(3af-3bdx^2\right)}{a+bx^4}\,dx}{3b}$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} + \frac{\int \frac{-3abd-3abfx^2}{a+bx^4}\,dx}{3b^2} + \frac{\text{Subst}\left(\int \frac{-ae+bcx}{a+bx^2}\,dx, x, x^2\right)}{2b}$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right) - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b} - \cdots$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} + \frac{c\log\left(a+bx^4\right)}{4b} + \frac{\left(\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+\cdots}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}}\cdots}{4\sqrt{2}b^{7/4}}\cdots$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} + \frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}\cdots$$

$$= \frac{dx}{b} + \frac{ex^2}{2b} + \frac{fx^3}{3b} - \frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}} + \frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}} - \frac{\sqrt[4]{a}\left(\sqrt{b}d\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.173454, size = 311, normalized size = 0.97

$$\frac{-3\sqrt{2}\left(a^{3/4}f-\sqrt[4]{a}\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+3\sqrt{2}\left(a^{3/4}f-\sqrt[4]{a}\sqrt{b}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+6b^{3/4}c\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

```
[Out] (24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*S
qrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e + Sqr
t[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(a^(1
/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24*b^(7/4
))
```

Maple [A] time = 0.004, size = 325, normalized size = 1.

$$\frac{fx^3}{3b}+\frac{ex^2}{2b}+\frac{dx}{b}-\frac{d\sqrt{2}}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)-\frac{d\sqrt{2}}{8b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

```
[Out] 1/3*f*x^3/b+1/2*e*x^2/b+d*x/b-1/4/b*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x-1)-1/8/b*d*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/4/b*d*(
1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/2/b*a*e/(a*b)^(1/2
)*arctan(x^2*(b/a)^(1/2))-1/8/b^2*a*f/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)
^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))
)-1/4/b^2*a*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4/b
^2*a*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*c*ln(b*x
^4+a)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 12.1739, size = 886, normalized size = 2.76

$$\text{RootSum}\left(256t^4b^7-256t^3b^6c+t^2\left(64ab^4df+32ab^4e^2+96b^5c^2\right)+t\left(-16a^2b^2ef^2-32ab^3cdf-16ab^3ce^2+16ab^3d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*b**7 - 256*_t**3*b**6*c + _t**2*(64*a*b**4*d*f + 32*a*b**
4*e**2 + 96*b**5*c**2) + _t*(-16*a**2*b**2*e*f**2 - 32*a*b**3*c*d*f - 16*a*
b**3*c*e**2 + 16*a*b**3*d**2*e - 16*b**4*c**3) + a**3*f**4 + 4*a**2*b*c*e*f
**2 + 2*a**2*b*d**2*f**2 - 4*a**2*b*d*e**2*f + a**2*b*e**4 + 4*a*b**2*c**2*
d*f + 2*a*b**2*c**2*e**2 - 4*a*b**2*c*d**2*e + a*b**2*d**4 + b**3*c**4, Lam
bda(_t, _t*log(x + (-64*_t**3*a*b**5*f**3 + 64*_t**3*b**6*d**2*f - 128*_t**
3*b**6*d*e**2 + 48*_t**2*a*b**4*c*f**3 + 48*_t**2*a*b**4*d*e*f**2 - 32*_t**
2*a*b**4*e**3*f - 48*_t**2*b**5*c*d**2*f + 96*_t**2*b**5*c*d*e**2 + 16*_t**
2*b**5*d**3*e - 12*_t*a**2*b**2*d*f**4 - 12*_t*a**2*b**2*e**2*f**3 - 12*_t*
a*b**3*c**2*f**3 - 24*_t*a*b**3*c*d*e*f**2 + 16*_t*a*b**3*c*e**3*f + 16*_t*
a*b**3*d**3*f**2 - 36*_t*a*b**3*d**2*e**2*f - 8*_t*a*b**3*d*e**4 + 12*_t*b*
*4*c**2*d**2*f - 24*_t*b**4*c**2*d*e**2 - 8*_t*b**4*c*d**3*e - 4*_t*b**4*d*
*5 + 3*a**3*e*f**5 + 3*a**2*b*c*d*f**4 + 3*a**2*b*c*e**2*f**3 + 5*a**2*b*d*
e**3*f**2 - 2*a**2*b*e**5*f + a*b**2*c**3*f**3 + 3*a*b**2*c**2*d*e*f**2 - 2
*a*b**2*c**2*e**3*f - 4*a*b**2*c*d**3*f**2 + 9*a*b**2*c*d**2*e**2*f + 2*a*b
**2*c*d*e**4 + 5*a*b**2*d**4*e*f - 5*a*b**2*d**3*e**3 - b**3*c**3*d**2*f +
2*b**3*c**3*d*e**2 + b**3*c**2*d**3*e + b**3*c*d**5)/(a**3*f**6 - a**2*b*d*
*2*f**4 + 8*a**2*b*d*e**2*f**3 - 4*a**2*b*e**4*f**2 - a*b**2*d**4*f**2 + 8*
a*b**2*d**3*e**2*f - 4*a*b**2*d**2*e**4 + b**3*d**6)))) + d*x/b + e*x**2/(2
*b) + f*x**3/(3*b)
```

Giac [A] time = 1.10117, size = 416, normalized size = 1.3

$$\frac{c\log\left(\left|bx^4+a\right|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-\left(ab^3\right)^{\frac{1}{4}}b^2d-\left(ab\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3
)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3)^(1/4
)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/b^4 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^
2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1
/6*(2*b^2*f*x^3 + 3*b^2*x^2*e + 6*b^2*d*x)/b^3
```

3.489 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan}{8\sqrt{2}a^{7/4}}$$

```
[Out] -(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4
)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(
3/4))
```

Rubi [A] time = 0.26987, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{b}c\right)\tan}{8\sqrt{2}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] -(a*f - b*x*(c + d*x + e*x^2))/(4*a*b*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(4*a^(3/2)*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(3/4
)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(
3/4))
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}dx = -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-3c-2dx-ex^2}{a+bx^4}dx}{4a}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} - \frac{\int \left(-\frac{2dx}{a+bx^4}+\frac{-3c-ex^2}{a+bx^4}\right)dx}{4a}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} - \frac{\int \frac{-3c-ex^2}{a+bx^4}dx}{4a} + \frac{d\int \frac{x}{a+bx^4}dx}{2a}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{a+bx^2}dx,x,x^2\right)}{4a} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}dx}{8ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}\right.}{}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16ab} + \frac{\left(\frac{3\sqrt{b}c}{\sqrt{a}}+e\right)\int}{1}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\right.}{}$$

Mathematica [A] time = 0.340096, size = 315, normalized size = 0.99

$$\sqrt{2}\sqrt[4]{b}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}\sqrt[4]{b}\left(3\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*Sq
rt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4
)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
Sqrt[2]*b^(1/4)*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c - a^(
3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^2*b)
```

Maple [A] time = 0.005, size = 362, normalized size = 1.1

$$\frac{cx}{4a\left(bx^4+a\right)}+\frac{3c\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{3c\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] 1/4*c*x/a/(b*x^4+a)+3/32*c/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16*
c/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/16*c/a^2*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*d*x^2/a/(b*x^4+a)
+1/4*d/a/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/4*e*x^3/a/(b*x^4+a)+1/32*e/a
/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^
2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/16*e/a/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x-1)+1/4*f*x^4/a/(b*x^4+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 16.1411, size = 517, normalized size = 1.63

$$\operatorname{RootSum}\left(65536t^4a^7b^3 + t^2\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc^2e^2 - 48abcd^2e +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2*
d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a*b
*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t*lo
g(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304*_t*
*3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*b**2*
d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t*a**4*
b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d**2*e +
 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5 + 120*a*
*2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a*b**2*c**
3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 - 64*a**2*b
*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*a*b**2*c**2
*d**4 + 729*b**3*c**6))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3)/(4*a**2*b
```

```
+ 4*a*b**2*x**4)
```

Giac [A] time = 1.08815, size = 427, normalized size = 1.34

$$\frac{bx^3e + bdx^2 + bcx - af}{4\left(bx^4 + a\right)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d + 3\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3*e + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*s
qrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)
*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sq
rt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)
^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.490 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}-\frac{\left(3\sqrt{a}f+\sqrt{b}d\right)\tan^{-}}{8\sqrt{2}a^{3/4}b}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt
[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d + 3*Sqrt[a]*
f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) - (
(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4))
```

Rubi [A] time = 0.274175, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1823, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}-\frac{\left(3\sqrt{a}f+\sqrt{b}d\right)\tan^{-}}{8\sqrt{2}a^{3/4}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(4*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt
[a]])/(4*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d + 3*Sqrt[a]*
f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*b^(7/4)) - (
(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(3/4)*b^(7/4))
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^2}dx = -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{\int \frac{d+2ex+3fx^2}{a+bx^4}dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{\int \left(\frac{2ex}{a+bx^4}+\frac{d+3fx^2}{a+bx^4}\right)dx}{4b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{\int \frac{d+3fx^2}{a+bx^4}dx}{4b}+\frac{e\int \frac{x}{a+bx^4}dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{e\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}dx,x,x^2\right)}{4b}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}-3f\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}dx}{8b^2}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}+\right.}{}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}+3f\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16b^2}+\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}+3f\right)\int}{16}$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}-$$

$$= -\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}-\frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}}+\frac{\left(\sqrt{b}d+3\right.}{}$$

Mathematica [A] time = 0.269116, size = 294, normalized size = 0.95

$$\frac{-\frac{2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+\sqrt{2}\sqrt{b}d\right)}{a^{3/4}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+\sqrt{2}\sqrt{b}d\right)}{a^{3/4}}+\frac{\sqrt{2}\left(3\sqrt{a}f-\sqrt{b}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}}+}{32b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

```
[Out] ((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*d
 + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[
2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*(
-(Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(3/4) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

Maple [A] time = 0.009, size = 334, normalized size = 1.1

$$\frac{1}{bx^4+a}\left(-\frac{fx^3}{4b}-\frac{ex^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}\right)+\frac{d\sqrt{2}}{32ab}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{d\sqrt{2}}{16ab}\sqrt[4]{\frac{a}{b}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

```
[Out] (-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4*c/b)/(b*x^4+a)+1/32/b*d*(1/b*a)^(1/
4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1
/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/16/b*d*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(1/b*a)^(1/4)*x+1)+1/16/b*d*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x-1)+1/4/b*e/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/32/b^2*f/(1/b*a
)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)
^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+3/16/b^2*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x+1)+3/16/b^2*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 31.1828, size = 508, normalized size = 1.64

$$\mathrm{RootSum}\left(65536t^4a^3b^7 + t^2\left(3072a^2b^4df + 2048a^2b^4e^2\right) + t\left(1152a^2b^2ef^2 - 128ab^3d^2e\right) + 81a^2f^4 + 18abd^2f^2 - 48a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**3*b**7 + _t**2*(3072*a**2*b**4*d*f + 2048*a**2*b**4*
e**2) + _t*(1152*a**2*b**2*e*f**2 - 128*a*b**3*d**2*e) + 81*a**2*f**4 + 18*
a*b*d**2*f**2 - 48*a*b*d*e**2*f + 16*a*b*e**4 + b**2*d**4, Lambda(_t, _t*lo
g(x + (110592*_t**3*a**4*b**5*f**3 - 12288*_t**3*a**3*b**6*d**2*f + 32768*_
t**3*a**3*b**6*d*e**2 + 13824*_t**2*a**3*b**4*d*e*f**2 - 12288*_t**2*a**3*b
**4*e**3*f + 512*_t**2*a**2*b**5*d**3*e + 3888*_t*a**3*b**2*d*f**4 + 5184*_
t*a**3*b**2*e**2*f**3 - 576*_t*a**2*b**3*d**3*f**2 + 1728*_t*a**2*b**3*d**2
*e**2*f + 512*_t*a**2*b**3*d*e**4 + 16*_t*a*b**4*d**5 + 1458*a**3*e*f**5 +
360*a**2*b*d*e**3*f**2 - 192*a**2*b*e**5*f + 30*a*b**2*d**4*e*f - 40*a*b**2
*d**3*e**3)/(729*a**3*f**6 - 81*a**2*b*d**2*f**4 + 864*a**2*b*d*e**2*f**3 -
 576*a**2*b*e**4*f**2 - 9*a*b**2*d**4*f**2 + 96*a*b**2*d**3*e**2*f - 64*a*b
**2*d**2*e**4 + b**3*d**6)))) - (c + d*x + e*x**2 + f*x**3)/(4*a*b + 4*b**2
```

```
*x**4)
```

Giac [A] time = 1.09309, size = 409, normalized size = 1.32

$$-\frac{fx^3+x^2e+dx+c}{4\left(bx^4+a\right)b}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2e+\left(ab^3\right)^{\frac{1}{4}}b^2d+3\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2e+\left(ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(f*x^3 + x^2*e + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*sq
rt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((a*b^3
)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x
^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

3.491 $\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^3} \, dx$

Optimal. Leaf size=351

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}\right.}{64}$$

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*
x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(
3/4))
```

Rubi [A] time = 0.31771, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}-\frac{\left(5\sqrt{a}e+21\sqrt{b}\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] (x*(7*c + 6*d*x + 5*e*x^2))/(32*a^2*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*
x^2))/(8*a*b*(a + b*x^4)^2) + (3*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(5/
2)*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(128*Sqrt[2]*a^(11/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(11/4)*b^(
3/4))
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
```

```
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^3}\,dx = -\frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} - \frac{\int \frac{-7c-6dx-5ex^2}{\left(a+bx^4\right)^2}\,dx}{8a}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{\int \frac{21c+12dx+5ex^2}{a+bx^4}\,dx}{32a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{\int \left(\frac{12dx}{a+bx^4}+\frac{21c+5ex^2}{a+bx^4}\right)dx}{32a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{\int \frac{21c+5ex^2}{a+bx^4}\,dx}{32a^2} + \frac{(3d)\int \frac{x}{a+bx^4}\,dx}{8a^2}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{(3d)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{16a^2} + \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}} - \right.}{}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{2}}}{128\sqrt{2}a^{9/4}b^{3/}}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(\frac{21\sqrt{b}c}{\sqrt{a}}-5e\right)\log\left(\sqrt{a}-\right.}{128\sqrt{2}a}$$

$$= \frac{x\left(7c+6dx+5ex^2\right)}{32a^2\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2} + \frac{3d\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\tan^{-}}{64\sqrt{2}a^{11/4}b}$$

Mathematica [A] time = 0.328364, size = 347, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e-21\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{b}c-5a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} - \frac{32a^2(af-bx(c+x(d+ex)))}{b\left(a+bx^4\right)^2} - \frac{2\sqrt[4]{a}\tan^{-1}\left(1-\right.}{}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

```
[Out] ((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(d
 + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4
)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt
[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*
(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Maple [A] time = 0.005, size = 432, normalized size = 1.2

$$\frac{cx}{8a\left(bx^4+a\right)^2}+\frac{7cx}{32a^2\left(bx^4+a\right)}+\frac{21c\sqrt{2}}{256a^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{21c\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] 1/8*c*x/a/(b*x^4+a)^2+7/32*c/a^2*x/(b*x^4+a)+21/256*c/a^3*(1/b*a)^(1/4)*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
b*a)^(1/4)*x+1)+21/128*c/a^3*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x-1)+1/8*d*x^2/a/(b*x^4+a)^2+3/16*d/a^2*x^2/(b*x^4+a)+3/16*d/a^2/(a*b)
^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8*e*x^3/a/(b*x^4+a)^2+5/32*e/a^2*x^3/(b*x^
4+a)+5/256*e/a^2/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1
/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+5/128*e/a^2/b/(1/
b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+5/128*e/a^2/b/(1/b*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/8*f*x^4/a/(b*x^4+a)^2+1/8
*f/a^2*x^4/(b*x^4+a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 68.5143, size = 578, normalized size = 1.65

$$\mathrm{RootSum}\left(268435456t^4a^{11}b^3+t^2\left(6881280a^6b^2ce+4718592a^6b^2d^2\right)+t\left(153600a^4bde^2-2709504a^3b^2c^2d\right)+625a^2e^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 4718592
*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) + 6
25*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4 +
194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**3 -
4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2 + 30
9657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e + 18207
86688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t*a**5*b
*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b**2*c**2
*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c**5 + 112
500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**5*e + 5834
4300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e**6 - 275625
*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b*d**4*e**2 -
 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 36578304*a*b**2*c
**2*d**4 + 85766121*b**3*c**6)))) + (-4*a**2*f + 11*a*b*c*x + 10*a*b*d*x**2
 + 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x**7)/(32*a**4*b
 + 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

Giac [A] time = 1.1009, size = 478, normalized size = 1.36

$$\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab\right.}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b
^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*
c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)
*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*
(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4
) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*x^7*e + 6*b^2*d*x^6 + 7*b^2*c*x^5 +
9*a*b*x^3*e + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.492 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

Optimal. Leaf size=340

$$-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}-\frac{3\left(\sqrt{a}f+\sqrt{b}d\right)\tan}{64\sqrt{2}a^{7/4}}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2))/
(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(3/2)*b^(3/2
)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(6
4*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt[a
]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a
^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```

Rubi [A] time = 0.328179, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1823, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}-\frac{3\left(\sqrt{a}f+\sqrt{b}d\right)\tan}{64\sqrt{2}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```


```
[Out] -(c + d*x + e*x^2 + f*x^3)/(8*b*(a + b*x^4)^2) + (x*(d + 2*e*x + 3*f*x^2))/
(32*a*b*(a + b*x^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(16*a^(3/2)*b^(3/2
)) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(6
4*Sqrt[2]*a^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*b^(7/4)) - (3*(Sqrt[b]*d - Sqrt[a
]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a
^(7/4)*b^(7/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(128*Sqrt[2]*a^(7/4)*b^(7/4))
```


Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^3}dx = -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{\int \frac{d+2ex+3fx^2}{\left(a+bx^4\right)^2}dx}{8b}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}-\frac{\int \frac{-3d-4ex-3fx^2}{a+bx^4}dx}{32ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}-\frac{\int \left(-\frac{4ex}{a+bx^4}+\frac{-3d-3fx^2}{a+bx^4}\right)dx}{32ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}-\frac{\int \frac{-3d-3fx^2}{a+bx^4}dx}{32ab}+\frac{e\int \frac{x}{a+bx^4}dx}{8ab}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\,\text{Subst}\left(\int \frac{1}{a+bx^2}dx,x,x^2\right)}{16ab}+\frac{\left(3\left(\frac{\sqrt{bd}}{\sqrt{a}}-f\right)\right.}{6}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}+\frac{\left(3\left(\frac{\sqrt{bd}}{\sqrt{a}}+f\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}}{128ab^2}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{bd}-\sqrt{a}f\right)\log\left(\sqrt{a}-\right.}{128\sqrt{2}a^7}$$

$$= -\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}+\frac{x\left(d+2ex+3fx^2\right)}{32ab\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{bd}+\sqrt{a}f\right)\tan^{-1}\left(1\right.}{64\sqrt{2}a^{7/4}b^{7/4}}$$

Mathematica [A] time = 0.349547, size = 329, normalized size = 0.97

$$\frac{-\frac{2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+3\sqrt{2}\sqrt{bd}\right)}{a^{7/4}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f+3\sqrt{2}\sqrt{bd}\right)}{a^{7/4}}+\frac{3\sqrt{2}\left(\sqrt{a}f-\sqrt{bd}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}}{256b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

```
[Out] ((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*(
d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^(1
/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/
4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b]*d)
+ Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4
) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

Maple [A] time = 0.012, size = 373, normalized size = 1.1

$$\frac{1}{\left(bx^4+a\right)^2}\left(\frac{3fx^7}{32a}+\frac{ex^6}{16a}+\frac{dx^5}{32a}-\frac{fx^3}{32b}-\frac{ex^2}{16b}-\frac{3dx}{32b}-\frac{c}{8b}\right)+\frac{3d\sqrt{2}}{256ba^2}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

```
[Out] (3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b-3/32*d*x/
b-1/8*c/b)/(b*x^4+a)^2+3/256/b/a^2*d*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^
(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))
+3/128/b/a^2*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+3/12
8/b/a^2*d*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/16/b/a*
e/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+3/256/b^2/a*f/(1/b*a)^(1/4)*2^(1/2)*l
n((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+
(1/b*a)^(1/2)))+3/128/b^2/a*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^
(1/4)*x+1)+3/128/b^2/a*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)
*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09603, size = 456, normalized size = 1.34

$$\frac{3\,bfx^7 + 2\,bx^6e + bdx^5 - afx^3 - 2\,ax^2e - 3\,adx - 4\,ac}{32\,\left(bx^4 + a\right)^2 ab} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e + 3\,\left(ab^3\right)^{\frac{1}{4}}\,b^2d + 3\,\left(ab^3\right)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{128\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*b*f*x^7 + 2*b*x^6*e + b*d*x^5 - a*f*x^3 - 2*a*x^2*e - 3*a*d*x - 4*a
*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a*b
^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e
+ 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*
d - (a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
 - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

3.493 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx$

Optimal. Leaf size=382

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{ae}+7\right.}{2}$$

```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a
 + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (
(77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```

Rubi [A] time = 0.405867, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}-\frac{\left(15\sqrt{ae}+7\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```


```
[Out] (x*(11*c + 10*d*x + 9*e*x^2))/(96*a^2*(a + b*x^4)^2) + (x*(77*c + 60*d*x +
45*e*x^2))/(384*a^3*(a + b*x^4)) - (a*f - b*x*(c + d*x + e*x^2))/(12*a*b*(a
 + b*x^4)^3) + (5*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(7/2)*Sqrt[b]) - (
(77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(15/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512
*Sqrt[2]*a^(15/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(15/4)*b^(3/4))
```


Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
```

```
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^4}\,dx = -\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-11c-10dx-9ex^2}{\left(a+bx^4\right)^3}\,dx}{12a}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{\int \frac{77c+60dx+45ex^2}{\left(a+bx^4\right)^2}\,dx}{96a^2}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-231c-120d}{a+bx}}{384a}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \left(-\frac{120dx}{a+bx^4} + \right.}{3}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} - \frac{\int \frac{-231c-45ex^2}{a+bx^4}}{384a^3}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{(5d)\,\text{Subst}\left(\right.}{}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-1}\left(\right.}{32a^{7/2}\sqrt{b}}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-1}\left(\right.}{32a^{7/2}\sqrt{b}}$$

$$= \frac{x\left(11c+10dx+9ex^2\right)}{96a^2\left(a+bx^4\right)^2} + \frac{x\left(77c+60dx+45ex^2\right)}{384a^3\left(a+bx^4\right)} - \frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3} + \frac{5d\tan^{-1}\left(\right.}{32a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.408928, size = 379, normalized size = 0.99

$$\frac{\frac{3\sqrt{2}\left(15a^{3/4}e-77\sqrt[4]{a}\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} + \frac{3\sqrt{2}\left(77\sqrt[4]{a}\sqrt{b}c-15a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{3/4}} - \frac{256a^3(af-bx(c+x(d+ex)))}{b\left(a+bx^4\right)^3} + \frac{32a^2x(11c}{(a}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

```
[Out] ((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*
d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x*(d + e*x))))/(b*(a
+ b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*
Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^
(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*
Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
```

```
*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Maple [A] time = 0.014, size = 400, normalized size = 1.1

$$\frac{1}{\left(bx^4+a\right)^3}\left(\frac{15\,b^2ex^{11}}{128\,a^3}+\frac{5\,b^2dx^{10}}{32\,a^3}+\frac{77\,b^2cx^9}{384\,a^3}+\frac{21\,bex^7}{64\,a^2}+\frac{5\,bdx^6}{12\,a^2}+\frac{33\,bcx^5}{64\,a^2}+\frac{113\,ex^3}{384\,a}+\frac{11\,dx^2}{32\,a}+\frac{51\,cx}{128\,a}-\frac{f}{12\,b}\right)+\frac{77\,c}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64/a^2*b
*e*x^7+5/12/a^2*d*b*x^6+33/64/a^2*c*b*x^5+113/384/a*e*x^3+11/32*d/a*x^2+51/
128*c/a*x-1/12*f/b)/(b*x^4+a)^3+77/1024/a^4*c*(1/b*a)^(1/4)*2^(1/2)*ln((x^2
+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a
)^(1/2)))+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x
+1)+77/512/a^4*c*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+5/
32/a^3*d/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+15/1024/a^3*e/b/(1/b*a)^(1/4)*
2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x
*2^(1/2)+(1/b*a)^(1/2)))+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(1/b*a)^(1/4)*x+1)+15/512/a^3*e/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08056, size = 528, normalized size = 1.38

$$\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\left(ab^3\right)^{\frac{1}{4}}b^2c+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\left(ab^3\right)^{\frac{1}{4}}b^2c+15\right.}{512\,a^4b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2
*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/
b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqr
t(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*x^11*e + 60*b^3*d*x^10 + 77*
b^3*c*x^9 + 126*a*b^2*x^7*e + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b
*x^3*e + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.494 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

Optimal. Leaf size=380

$$-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}-\frac{\left(5\sqrt{a}f+7\sqrt{b}d\right)\mathrm{t}}{256\sqrt{2}a}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2))
/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b*x
^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(5/2)*b^(3/2)) - ((7*Sqrt[b]
*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(
11/4)*b^(7/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4
)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rubi [A] time = 0.402178, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1823, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}-\frac{\left(5\sqrt{a}f+7\sqrt{b}d\right)\mathrm{t}}{256\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(12*b*(a + b*x^4)^3) + (x*(d + 2*e*x + 3*f*x^2))
/(96*a*b*(a + b*x^4)^2) + (x*(7*d + 12*e*x + 15*f*x^2))/(384*a^2*b*(a + b*x
^4)) + (e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(32*a^(5/2)*b^(3/2)) - ((7*Sqrt[b]
*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(256*Sqrt[2]*a^(
11/4)*b^(7/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(256*Sqrt[2]*a^(11/4)*b^(7/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4
)*b^(7/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(512*Sqrt[2]*a^(11/4)*b^(7/4))
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^4}dx=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{\int \frac{d+2ex+3fx^2}{\left(a+bx^4\right)^3}dx}{12b}$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}-\frac{\int \frac{-7d-12ex-15fx^2}{\left(a+bx^4\right)^2}dx}{96ab}$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{\int \frac{21d+24ex+15fx^2}{a+bx^4}}{384a^2b}$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{\int\left(\frac{24ex}{a+bx^4}+\frac{21d+1}{a+b}\right.}{384a^2b}$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{\int \frac{21d+15fx^2}{a+bx^4}dx}{384a^2b}+$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\,\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{32a^2b}$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-$$

$$=-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}+\frac{x\left(d+2ex+3fx^2\right)}{96ab\left(a+bx^4\right)^2}+\frac{x\left(7d+12ex+15fx^2\right)}{384a^2b\left(a+bx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}-$$

Mathematica [A] time = 0.421341, size = 366, normalized size = 0.96

$$\frac{\frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2\left(a+bx^4\right)}-\frac{6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f+7\sqrt{2}\sqrt{b}d\right)}{a^{11/4}}+\frac{6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(-16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f+7\sqrt{2}\sqrt{b}d\right)}{a^{11/4}}+\frac{3\sqrt{2}\left(5\sqrt{a}f-7\sqrt{b}d\right.}{}}{3072b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

```
[Out] ((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*d
 + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e +
f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e + 5*
Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*(
7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d + 5*Sqr
t[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4) +
(3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

Maple [A] time = 0.012, size = 403, normalized size = 1.1

$$\frac{1}{\left(bx^4+a\right)^3}\left(\frac{5\,bfx^{11}}{128\,a^2}+\frac{bex^{10}}{32\,a^2}+\frac{7\,bdx^9}{384\,a^2}+\frac{7\,fx^7}{64\,a}+\frac{ex^6}{12\,a}+\frac{3\,dx^5}{64\,a}-\frac{5\,fx^3}{384\,b}-\frac{ex^2}{32\,b}-\frac{7\,dx}{128\,b}-\frac{c}{12\,b}\right)+\frac{7\,d\sqrt{2}}{1024\,ba^3}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

```
[Out] (5/128*f*b/a^2*x^11+1/32/a^2*b*e*x^10+7/384/a^2*d*b*x^9+7/64*f/a*x^7+1/12/a
*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*x/b-1/12*c/b)/(b*x^4
+a)^3+7/1024/b/a^3*d*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+
(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+7/512/b/a^3*d*(
1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+7/512/b/a^3*d*(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/32/b/a^2*e/(a*b)^(1/2)*
arctan(x^2*(b/a)^(1/2))+5/1024/b^2/a^2*f/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b
*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/
2)))+5/512/b^2/a^2*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1
)+5/512/b^2/a^2*f/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09185, size = 513, normalized size = 1.35

$$\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^3b^4}+\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)}{512a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

```
[Out] 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^3
)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3
*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d +
5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(2)*(7*
(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*x^10*e + 7*b^2*d*x^9 +
 42*a*b*f*x^7 + 32*a*b*x^6*e + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*x^2*e -
21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

3.495 $\int x^4 \left(c+dx+ex^2+fx^3\right) \sqrt{a+bx^4}\, dx$

Optimal. Leaf size=418

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}-\frac{a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+}}{15b^{3/2}\left(\sqrt{a}+\right.}$$

```
[Out] (2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*Sqrt[a + b*x^4])/63 + (f*x^4*(
a + b*x^4)^(3/2))/(10*b) - ((8*a*f - 15*b*d*x^2)*(a + b*x^4)^(3/2))/(120*b^
2) - (a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/
4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^4]) -
 (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.381155, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 833, 780, 195, 217, 206}

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}-\frac{a^2d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```


```
[Out] (2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*Sqrt[a + b*x^4])/63 + (f*x^4*(
a + b*x^4)^(3/2))/(10*b) - ((8*a*f - 15*b*d*x^2)*(a + b*x^4)^(3/2))/(120*b^
2) - (a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/
4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^4]) -
 (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(105*b^(7/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
```

```
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\left(x^4\left(c+ex^2\right)\sqrt{a+bx^4}+x^5\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$=\int x^4\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^5\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}+\frac{1}{2}\,\mathrm{Subst}\left(\int x^2(d+fx)\sqrt{a+bx^2}\,dx,x,x^2\right)+\ldots$$

$$=\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}+\frac{fx^4\left(a+bx^4\right)^{3/2}}{10b}+\frac{\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$=\frac{2acx\sqrt{a+bx^4}}{21b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+bx^4}+\frac{fx^4\left(a+b\ldots\right.}{10b}$$

$$=\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9c+7ex^2\right)\sqrt{a+\ldots}$$

$$=\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\ldots$$

$$=\frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\ldots$$

Mathematica [C] time = 0.63576, size = 202, normalized size = 0.48

$$\frac{\sqrt{a+bx^4}\left(-\frac{315a^{3/2}\sqrt{b}d\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{\frac{bx^4}{a}+1}}-\frac{720abcx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}+720bcx\left(a+bx^4\right)+315bdx^2\left(a+2bx^4\right)-\frac{560abex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\ldots\right)}{\sqrt{\frac{bx^4}{a}+1}}\ldots\right.}{5040b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(720*b*c*x*(a + b*x^4) + 560*b*e*x^3*(a + b*x^4) + 315*b*d
*x^2*(a + 2*b*x^4) + 168*f*(a + b*x^4)*(-2*a + 3*b*x^4) - (315*a^(3/2)*Sqrt
[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a] - (720*a*b*c*x*Hy
pergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (560*a
*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/
a]))/(5040*b^2)
```

Maple [C] time = 0.032, size = 390, normalized size = 0.9

$$-\frac{f\left(-3\,bx^4+2\,a\right)}{30\,b^2}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{ex^7}{9}\sqrt{bx^4+a}+\frac{2\,aex^3}{45\,b}\sqrt{bx^4+a}-\frac{2\,i}{15}ea^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] -1/30*f*(b*x^4+a)^(3/2)*(-3*b*x^4+2*a)/b^2+1/9*e*x^7*(b*x^4+a)^(1/2)+2/45*a
*e*x^3*(b*x^4+a)^(1/2)/b-2/15*I*e/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+2/15*I*e/b^(3/2)*a^(5/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*
x^2*(b*x^4+a)^(3/2)/b-1/16*a*d*x^2*(b*x^4+a)^(1/2)/b-1/16*d/b^(3/2)*a^2*ln(
x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/7*c*x^5*(b*x^4+a)^(1/2)+2/21*a*c*x*(b*x^4+a)
^(1/2)/b-2/21*c/b*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^7+ex^6+dx^5+cx^4\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 7.09639, size = 252, normalized size = 0.6

$$\frac{a^{\frac{3}{2}}dx^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}dx^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*d*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*
d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*e*x**7*gamma(7/4)*hyper((-1/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*d*asinh(sqrt(b
)*x**2/sqrt(a))/(16*b**(3/2)) + f*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**
2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)),
(sqrt(a)*x**8/8, True)) + b*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.496 $\int x^3\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=394

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\right.}$$

```
[Out] (2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*Sqrt[a + b*x^4])/63 + ((4*c +
3*e*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(15*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.334251, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1833, 1252, 780, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(Sq
rt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*Sqrt[a + b*x^4])/63 + ((4*c +
3*e*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(15*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^3\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\left(x^3\left(c+ex^2\right)\sqrt{a+bx^4}+x^4\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$=\int x^3\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^4\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+bx^4}+\frac{1}{2}\,\text{Subst}\left(\int x(c+ex)\sqrt{a+bx^2}\,dx,x,x^2\right)+\frac{1}{63}(2$$

$$=\frac{2afx^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+bx^4}+\frac{\left(4c+3ex^2\right)\left(a+bx^4\right)^{3/2}}{24b}-\frac{(2a}{}$$

$$=\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+b}$$

$$=\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}+\frac{1}{63}x^5\left(9d+7fx^2\right)\sqrt{a+b}$$

$$=\frac{2adx\sqrt{a+bx^4}}{21b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{63}$$

Mathematica [C] time = 0.630121, size = 215, normalized size = 0.55

$$\frac{\sqrt{a+bx^4}\left(63e\left(\sqrt{b}x^2\left(a+2bx^4\right)-\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{\frac{bx^4}{a}+1}}\right)+168\sqrt{b}c\left(a+bx^4\right)-\frac{144a\sqrt{b}dx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}+144\sqrt{b}dx\left(a+bx^4\right)-\right.}{1008b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(168*Sqrt[b]*c*(a + b*x^4) + 144*Sqrt[b]*d*x*(a + b*x^4) +
 112*Sqrt[b]*f*x^3*(a + b*x^4) + 63*e*(Sqrt[b]*x^2*(a + 2*b*x^4) - (a^(3/2)
*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (144*a*Sqrt[b]*d*x*
Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (112
*a*Sqrt[b]*f*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 +
(b*x^4)/a]))/(1008*b^(3/2))
```

Maple [C] time = 0.008, size = 380, normalized size = 1.

$$\frac{fx^7}{9}\sqrt{bx^4+a}+\frac{2x^3af}{45b}\sqrt{bx^4+a}-\frac{2i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/9*f*x^7*(b*x^4+a)^(1/2)+2/45*a*f*x^3*(b*x^4+a)^(1/2)/b-2/15*I*f/b^(3/2)*a
^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)+2/15*I*f/b^(3/2)*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)+1/8*e*x^2*(b*x^4+a)^(3/2)/b-1/16*a*e*x^2*(b*x^4+a
)^(1/2)/b-1/16*e/b^(3/2)*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/7*d*x^5*(b*x
^4+a)^(1/2)+2/21*a*d*x*(b*x^4+a)^(1/2)/b-2/21*d/b*a^2/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4
+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*c/b*(b*x^4+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bx^4+a\right)^{\frac{3}{2}}c}{6\,b}+\int\left(fx^6+ex^5+dx^4\right)\sqrt{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^4 + a)^(3/2)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)*sqrt(b*x^4 +
a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(fx^6+ex^5+dx^4+cx^3\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 6.29747, size = 212, normalized size = 0.54

$$\frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}ex^6}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{a^2e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*
e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x**7*gamma(7/4)*hyper((-1/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*e*asinh(sqrt(b
```

```
)*x**2/sqrt(a))/(16*b**(3/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
 + b*x**4)**(3/2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

3.497 $\int x^2 \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=369

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\ldots\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*c + 5*e*
x^2)*Sqrt[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^
2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) - (2*a^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)
*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*
b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.297924, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 780, 195, 217, 206}

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\ldots\right.\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a*
c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*c + 5*e*
x^2)*Sqrt[a + b*x^4])/35 + ((4*d + 3*f*x^2)*(a + b*x^4)^(3/2))/(24*b) - (a^
2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) - (2*a^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)
*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*
b^(5/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
```

```
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\left(x^2\left(c+ex^2\right)\sqrt{a+bx^4}+x^3\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$=\int x^2\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^3\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{1}{2}\operatorname{Subst}\left(\int x(d+fx)\sqrt{a+bx^2}\,dx,x,x^2\right)+\frac{1}{3\ldots}$$

$$=\frac{2aex\sqrt{a+bx^4}}{21b}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}}{24b}-\ldots$$

$$=\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{\left(4d+3fx^2\ldots\right.}{2\ldots}$$

$$=\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{\ldots}$$

$$=\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{\ldots}$$

Mathematica [C] time = 0.72146, size = 182, normalized size = 0.49

$$\frac{1}{336}\sqrt{a+bx^4}\left(-\frac{21a^{3/2}f\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}\sqrt{\frac{bx^4}{a}+1}}+\frac{112cx^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}+\frac{56d\left(a+bx^4\right)}{b}-\frac{48aex\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{b\sqrt{\frac{bx^4}{a}+1}}+\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*((56*d*(a + b*x^4))/b + (48*e*x*(a + b*x^4))/b + (21*f*x^2
*(a + 2*b*x^4))/b - (21*a^(3/2)*f*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(3/2)*
Sqrt[1 + (b*x^4)/a]) - (48*a*e*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4
)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (112*c*x^3*Hypergeometric2F1[-1/2, 3/4, 7/
4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/336
```

Maple [C] time = 0.01, size = 361, normalized size = 1.

$$\frac{fx^2}{8b}\left(bx^4+a\right)^{\frac{3}{2}}-\frac{afx^2}{16b}\sqrt{bx^4+a}-\frac{fa^2}{16}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{ex^5}{7}\sqrt{bx^4+a}+\frac{2aex}{21b}\sqrt{bx^4+a}-\frac{2a^2e}{21b}\sqrt{1-ix^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/8*f*x^2*(b*x^4+a)^(3/2)/b-1/16*a*f*x^2*(b*x^4+a)^(1/2)/b-1/16*f/b^(3/2)*a
^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/7*e*x^5*(b*x^4+a)^(1/2)+2/21*a*e*x*(b*
x^4+a)^(1/2)/b-2/21*e/b*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)+1/6*d/b*(b*x^4+a)^(3/2)+1/5*c*x^3*(b*x^4+a)^(1/2)+
2/5*I*c*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)-2/5*I*c*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)
*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^5+ex^4+dx^3+cx^2\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 6.00464, size = 212, normalized size = 0.57

$$\frac{a^{\frac{3}{2}}fx^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}fx^6}{16\sqrt{1+\frac{bx^4}{a}}}-\frac{a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+d\Bigg(\Bigg\{$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(3/4)*hyper
((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*e*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqrt(1 + b*x**4/a)) - a**2*f*asinh(sqrt(b)*
x**2/sqrt(a))/(16*b**(3/2)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a +
```

```
b*x**4)**(3/2)/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.498 $\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=354

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2*a*d*x*Sqr
t[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*d + 5*f*x^2)*Sq
rt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*b) + (a*c*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*Sqrt[a
]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.273206, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1833, 1248, 641, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2*a*d*x*Sqr
t[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*d + 5*f*x^2)*Sq
rt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*b) + (a*c*ArcTanh[(Sqrt[b]*x^2
)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*Sqrt[a
]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

```
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\left(x\left(c+ex^2\right)\sqrt{a+bx^4}+x^2\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$=\int x\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x^2\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{1}{2}\operatorname{Subst}\left(\int(c+ex)\sqrt{a+bx^2}\,dx,x,x^2\right)+\frac{1}{35}(2a)$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}-\frac{(2a)\int\frac{5af-21bdx}{\sqrt{a+bx^4}}}{105b}$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+bx^4}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+}$$

$$=\frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{35}x^3\left(7d+5fx^2\right)\sqrt{a+}$$

Mathematica [C] time = 0.184122, size = 211, normalized size = 0.6

$$\frac{\sqrt{a+bx^4}\left(21bcx^2\sqrt{\frac{bx^4}{a}+1}+21\sqrt{a}\sqrt{b}c\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+28bdx^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)+14bex^4\sqrt{\frac{bx^4}{a}+1}+14ae\sqrt{\frac{bx^4}{a}+1}-\right.}{84b\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(14*a*e*Sqrt[1 + (b*x^4)/a] + 12*a*f*x*Sqrt[1 + (b*x^4)/a]
 + 21*b*c*x^2*Sqrt[1 + (b*x^4)/a] + 14*b*e*x^4*Sqrt[1 + (b*x^4)/a] + 12*b*f
*x^5*Sqrt[1 + (b*x^4)/a] + 21*Sqrt[a]*Sqrt[b]*c*ArcSinh[(Sqrt[b]*x^2)/Sqrt[
a]] - 12*a*f*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 28*b*d*x^3
*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(84*b*Sqrt[1 + (b*x^4)/a
])
```

Maple [C] time = 0.013, size = 337, normalized size = 1.

$$\frac{fx^5}{7}\sqrt{bx^4+a}+\frac{2afx}{21b}\sqrt{bx^4+a}-\frac{2fa^2}{21b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/7*f*x^5*(b*x^4+a)^(1/2)+2/21*a*f*x*(b*x^4+a)^(1/2)/b-2/21*f/b*a^2/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*e*(b*x
^4+a)^(3/2)/b+1/5*x^3*d*(b*x^4+a)^(1/2)+2/5*I*d*a^(3/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*d*a^(3/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)+1/4*c*x^2*(b*x^4+a)^(1/2)+1/4*c*a/b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^4+a}\left(fx^4+ex^3+dx^2+cx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2 + c*x), x)
```

Sympy [A] time = 4.28085, size = 158, normalized size = 0.45

$$\frac{\sqrt{a}cx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+e\left(\begin{cases}\frac{\sqrt{a}x^4}{4}\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*f*x**5*g
amma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4
)) + a*c*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + e*Piecewise((sqrt(a)*x**
4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

3.499 $\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$

Optimal. Leaf size=331

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] +
 Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^(3
/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*
a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4
]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.194189, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {1885, 1177, 1198, 220, 1196, 1248, 641, 195, 217, 206}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] +
 Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^(3
/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*
a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4
]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\left(\left(c+ex^2\right)\sqrt{a+bx^4}+x\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$
$$=\int\left(c+ex^2\right)\sqrt{a+bx^4}\,dx+\int x\left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}\int\frac{10ac+6aex^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}\operatorname{Subst}\left(\int(d+fx)\sqrt{a}\right.$$
$$=\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}+\frac{1}{2}d\operatorname{Subst}\left(\int\sqrt{a+bx^2}\,dx,x,x^2\right)$$
$$=\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)}{6b}$$
$$=\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)}{6b}$$
$$=\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x\left(5c+3ex^2\right)\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)}{6b}$$

Mathematica [C] time = 0.128262, size = 171, normalized size = 0.52

$$\frac{\sqrt{a+bx^4}\left(12bcx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+3bdx^2\sqrt{\frac{bx^4}{a}+1}+3\sqrt{a}\sqrt{b}d\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+4bex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)+2bfx^4\right.}{12b\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(2*a*f*Sqrt[1 + (b*x^4)/a] + 3*b*d*x^2*Sqrt[1 + (b*x^4)/a]
 + 2*b*f*x^4*Sqrt[1 + (b*x^4)/a] + 3*Sqrt[a]*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2
)/Sqrt[a]] + 12*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*b
*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(12*b*Sqrt[1 + (b*
x^4)/a])
```

Maple [C] time = 0.007, size = 313, normalized size = 1.

$$\frac{f}{6b}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{x^3e}{5}\sqrt{bx^4+a}+\frac{2i}{5}ea^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

```
[Out] 1/6*f*(b*x^4+a)^(3/2)/b+1/5*x^3*e*(b*x^4+a)^(1/2)+2/5*I*e*a^(3/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*
```

```
I*e*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)+1/4*d*x^2*(b*x^4+a)^(1/2)+1/4*d*a/b^(1/2)*ln(x^2*b^(1/2)+(
b*x^4+a)^(1/2))+1/3*x*c*(b*x^4+a)^(1/2)+2/3*c*a/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Sympy [A] time = 4.14321, size = 156, normalized size = 0.47

$$\frac{\sqrt{a}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}}+f\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{oth}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*e*x**3*gamm
a(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))
+ a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + f*Piecewise((sqrt(a)*x**4/4
, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.500 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$

Optimal. Leaf size=345

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x
^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.255159, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1833, 1252, 815, 844, 217, 206, 266, 63, 208, 1177, 1198, 220, 1196}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{5b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```


```
[Out] (2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x
^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x}\,dx = \int \left(\frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x}+\left(d+fx^2\right)\sqrt{a+bx^4}\right)dx$$

$$= \int \frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x}\,dx+\int \left(d+fx^2\right)\sqrt{a+bx^4}\,dx$$

$$= \frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}+\frac{1}{15}\int \frac{10ad+6afx^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(c+ex)\sqrt{a+}}{x}\right.$$

$$= \frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}+\frac{\mathrm{Subst}\left(\int \frac{2abc+abex}{x\sqrt{a+bx^2}}\,dx,x,x^2\right.}{4b}$$

$$= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}-\frac{2a^{5/4}}{}$$

$$= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}-\frac{2a^{5/4}}{}$$

$$= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}+\frac{ae\tan}{}$$

$$= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2c+ex^2\right)\sqrt{a+bx^4}+\frac{1}{15}x\left(5d+3fx^2\right)\sqrt{a+bx^4}+\frac{ae\tan}{}$$

Mathematica [C] time = 0.357081, size = 208, normalized size = 0.6

$$\frac{3a^{3/2}e\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3\sqrt{b}\left(\left(a+bx^4\right)\left(2c+ex^2\right)-2\sqrt{a}c\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+12a\sqrt{b}dx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\right.}{12\sqrt{b}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

```
[Out] (3*a^(3/2)*e*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*Sqrt[b]
*((2*c + e*x^2)*(a + b*x^4) - 2*Sqrt[a]*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a +
b*x^4]/Sqrt[a]]) + 12*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-
1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*x^4)/a]*Hyperg
eometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/(12*Sqrt[b]*Sqrt[a + b*x^4])
```

Maple [C] time = 0.015, size = 339, normalized size = 1.

$$\frac{fx^3}{5}\sqrt{bx^4+a}+\frac{2i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}-\frac{2i}{5}fa^{\frac{3}{2}}\sqrt{1-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x)
```

```
[Out] 1/5*f*x^3*(b*x^4+a)^(1/2)+2/5*I*f*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^
(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*f*a^(3/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*e*x
^2*(b*x^4+a)^(1/2)+1/4*e*a/b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*x*d*
(b*x^4+a)^(1/2)+2/3*d*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)+1/2*c*(b*x^4+a)^(1/2)-1/2*c*a^(1/2)*ln((2*a+2*a^(1/2)*
(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Sympy [C] time = 8.39242, size = 204, normalized size = 0.59

$$-\frac{\sqrt{a}c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{\sqrt{a}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ac}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*d*x*gamma(1/4)*hyper((
-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*e*x*
*2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*c/(2*sqrt(b)*x**2*sqrt(a/(
b*x**4) + 1)) + a*e*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.501 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$

Optimal. Leaf size=341

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\left(3c-ex^2\right)}{3x}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-$$

```
[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*Sq
rt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.263187, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1833, 1272, 1198, 220, 1196, 1252, 815, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^4}\left(3c-ex^2\right)}{3x}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{2\sqrt[4]{a}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```


```
[Out] (2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*Sq
rt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^2}\,dx = \int \left(\frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x^2}+\frac{\left(d+fx^2\right)\sqrt{a+bx^4}}{x}\right)dx$$

$$= \int \frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x^2}\,dx + \int \frac{\left(d+fx^2\right)\sqrt{a+bx^4}}{x}\,dx$$

$$= -\frac{\left(3c-ex^2\right)\sqrt{a+bx^4}}{3x} + \frac{1}{2}\,\text{Subst}\left(\int \frac{(d+fx)\sqrt{a+bx^2}}{x}\,dx, x, x^2\right) - \frac{2}{3}\int \frac{-ae \cdots}{\sqrt{a \cdots}}$$

$$= -\frac{\left(3c-ex^2\right)\sqrt{a+bx^4}}{3x} + \frac{1}{4}\left(2d+fx^2\right)\sqrt{a+bx^4} + \frac{\text{Subst}\left(\int \frac{2abd+abfx}{x\sqrt{a+bx^2}}\,dx, x, x^2 \cdots\right.}{4b}$$

$$= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{\left(3c-ex^2\right)\sqrt{a+bx^4}}{3x} + \frac{1}{4}\left(2d+fx^2\right)\sqrt{a+bx^4} - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{\left(3c-ex^2\right)\sqrt{a+bx^4}}{3x} + \frac{1}{4}\left(2d+fx^2\right)\sqrt{a+bx^4} - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{\left(3c-ex^2\right)\sqrt{a+bx^4}}{3x} + \frac{1}{4}\left(2d+fx^2\right)\sqrt{a+bx^4} + \frac{af\tanh^{-1}\cdots}{4\cdots}$$

$$= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{\left(3c-ex^2\right)\sqrt{a+bx^4}}{3x} + \frac{1}{4}\left(2d+fx^2\right)\sqrt{a+bx^4} + \frac{af\tanh^{-1}\cdots}{4\cdots}$$

Mathematica [C] time = 0.391284, size = 208, normalized size = 0.61

$$\frac{x\left(\sqrt{b}\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a+bx^4}\left(2d+fx^2\right)-2\sqrt{a}d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+4\sqrt{b}ex\sqrt{a+bx^4}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+\sqrt{a}f\sqrt{a+bx\cdots}\right.}{4\sqrt{b}x\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

```
[Out] (-4*Sqrt[b]*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)/
a)] + x*(Sqrt[a]*f*Sqrt[a + b*x^4]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]
*Sqrt[1 + (b*x^4)/a]*((2*d + f*x^2)*Sqrt[a + b*x^4] - 2*Sqrt[a]*d*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]]) + 4*Sqrt[b]*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[
-1/2, 1/4, 5/4, -((b*x^4)/a)]))/(4*Sqrt[b]*x*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.012, size = 339, normalized size = 1.

$$\frac{fx^2}{4}\sqrt{bx^4+a}+\frac{af}{4}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+\frac{ex}{3}\sqrt{bx^4+a}+\frac{2\,ae}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x)
```

```
[Out] 1/4*x^2*f*(b*x^4+a)^(1/2)+1/4*f*a/b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1
/3*e*x*(b*x^4+a)^(1/2)+2/3*e*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d*(b*x^4+a)^(1/2)-1/2*d*a^(1/2)*ln((2*a+2*a
^(1/2)*(b*x^4+a)^(1/2))/x^2)-c/x*(b*x^4+a)^(1/2)+2*I*c*b^(1/2)*a^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2*I*c*b
^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Sympy [C] time = 5.70388, size = 206, normalized size = 0.6

$$\frac{\sqrt{a}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{ad}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*e*x
*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5
/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a/(b*
x**4) + 1)) + a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*x**2/
(2*sqrt(a/(b*x**4) + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.502 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

Optimal. Leaf size=342

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\left(c-ex^2\right)}{2x^2}+\frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+}}\right.$$

```
[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqrt
[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.262424, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1833, 1252, 813, 844, 217, 206, 266, 63, 208, 1272, 1198, 220, 1196}

$$-\frac{\sqrt{a+bx^4}\left(c-ex^2\right)}{2x^2}+\frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{\sqrt{a+bx^4}\left(3d-fx^2\right)}{3x}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqrt
[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c*A
rcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
```

```
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^3}\,dx = \int \left(\frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x^3}+\frac{\left(d+fx^2\right)\sqrt{a+bx^4}}{x^2}\right)dx$$

$$= \int \frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x^3}\,dx + \int \frac{\left(d+fx^2\right)\sqrt{a+bx^4}}{x^2}\,dx$$

$$= -\frac{\left(3d-fx^2\right)\sqrt{a+bx^4}}{3x}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+ex)\sqrt{a+bx^2}}{x^2}\,dx,x,x^2\right)-\frac{2}{3}\int \frac{-af-3}{\sqrt{a+}}$$

$$= -\frac{\left(c-ex^2\right)\sqrt{a+bx^4}}{2x^2}-\frac{\left(3d-fx^2\right)\sqrt{a+bx^4}}{3x}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{-2ae-2bcx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right.$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-ex^2\right)\sqrt{a+bx^4}}{2x^2}-\frac{\left(3d-fx^2\right)\sqrt{a+bx^4}}{3x}-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\right.}{}$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-ex^2\right)\sqrt{a+bx^4}}{2x^2}-\frac{\left(3d-fx^2\right)\sqrt{a+bx^4}}{3x}-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\right.}{}$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-ex^2\right)\sqrt{a+bx^4}}{2x^2}-\frac{\left(3d-fx^2\right)\sqrt{a+bx^4}}{3x}+\frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\right.$$

$$= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-ex^2\right)\sqrt{a+bx^4}}{2x^2}-\frac{\left(3d-fx^2\right)\sqrt{a+bx^4}}{3x}+\frac{1}{2}\sqrt{b}c\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.233381, size = 204, normalized size = 0.6

$$\frac{\sqrt{a}\sqrt{b}cx^2\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-2adx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)-\sqrt{a}ex^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+2afx^3\sqrt{\frac{bx^4}{a}}}{2x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

```
[Out] (-(a*c) + a*e*x^2 - b*c*x^4 + b*e*x^6 + Sqrt[a]*Sqrt[b]*c*x^2*Sqrt[1 + (b*x
^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*ArcTa
nh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[-1/2, -1/4, 3/4, -((b*x^4)/a)] + 2*a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(2*x^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.016, size = 360, normalized size = 1.1

$$\frac{fx}{3}\sqrt{bx^4+a}+\frac{2af}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{2}\sqrt{bx^4+a}-\frac{e}{2}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x)
```

```
[Out] 1/3*x*f*(b*x^4+a)^(1/2)+2/3*f*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e*(b*x^4+a)^(1/2)-1/2*e*a^(1/2)*ln((2*a+2*
a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/2*c/a/x^2*(b*x^4+a)^(3/2)+1/2*c*b/a*x^2*(b*
x^4+a)^(1/2)+1/2*c*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-d/x*(b*x^4+a)^(1
/2)+2*I*d*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(
1/2)*b^(1/2))^(1/2),I)-2*I*d*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Sympy [C] time = 5.27143, size = 230, normalized size = 0.67

$$-\frac{\sqrt{a}c}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{ae}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hyper((-1/2,
 -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*e*asin
h(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/
4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a*e/(2*sqrt(b)*x**2*sqrt(a/
(b*x**4) + 1)) + sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x**2/(
2*sqrt(a/(b*x**4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.503 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

Optimal. Leaf size=357

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\left(c-3ex^2\right)}{3x^3}-\frac{\sqrt{a+bx^4}\left(d-fx\right.}{2x^2}$$

```
[Out] (-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[
b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d - f*x^2)*Sqrt[a + b
*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sq
rt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c + 3*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4
])
```

Rubi [A] time = 0.295318, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1833, 1272, 1282, 1198, 220, 1196, 1252, 813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^4}\left(c-3ex^2\right)}{3x^3}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}\left(d-fx^2\right)}{2x^2}+\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```


```
[Out] (-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[
b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d - f*x^2)*Sqrt[a + b
*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sq
rt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c + 3*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4
])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
```

```
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^4}\,dx = \int \left(\frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x^4}+\frac{\left(d+fx^2\right)\sqrt{a+bx^4}}{x^3}\right)dx$$

$$= \int \frac{\left(c+ex^2\right)\sqrt{a+bx^4}}{x^4}\,dx+\int \frac{\left(d+fx^2\right)\sqrt{a+bx^4}}{x^3}\,dx$$

$$= -\frac{\left(c-3ex^2\right)\sqrt{a+bx^4}}{3x^3}+\frac{1}{2}\,\text{Subst}\left(\int \frac{(d+fx)\sqrt{a+bx^2}}{x^2}\,dx,x,x^2\right)-\frac{2}{3}\int \frac{-3ae}{x^2\sqrt{}}$$

$$= -\frac{2e\sqrt{a+bx^4}}{x}-\frac{\left(c-3ex^2\right)\sqrt{a+bx^4}}{3x^3}-\frac{\left(d-fx^2\right)\sqrt{a+bx^4}}{2x^2}-\frac{1}{4}\,\text{Subst}\left(\int \frac{-2}{x}\right.$$

$$= -\frac{2e\sqrt{a+bx^4}}{x}-\frac{\left(c-3ex^2\right)\sqrt{a+bx^4}}{3x^3}-\frac{\left(d-fx^2\right)\sqrt{a+bx^4}}{2x^2}+\frac{1}{2}(bd)\,\text{Subst}\left(\int\right.$$

$$= -\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-3ex^2\right)\sqrt{a+bx^4}}{3x^3}-\frac{\left(d-fx^2\right)\sqrt{a+bx^4}}{2x^2}$$

$$= -\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-3ex^2\right)\sqrt{a+bx^4}}{3x^3}-\frac{\left(d-fx^2\right)\sqrt{a+bx^4}}{2x^2}$$

$$= -\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{\left(c-3ex^2\right)\sqrt{a+bx^4}}{3x^3}-\frac{\left(d-fx^2\right)\sqrt{a+bx^4}}{2x^2}$$

Mathematica [C] time = 0.301453, size = 205, normalized size = 0.57

$$\frac{3x\left(\sqrt{a}\sqrt{b}dx^2\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-2aex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)-\sqrt{a}fx^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-ad+af}{6x^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

```
[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a)
] + 3*x*(-(a*d) + a*f*x^2 - b*d*x^4 + b*f*x^6 + Sqrt[a]*Sqrt[b]*d*x^2*Sqrt[
1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*f*x^2*Sqrt[a + b*x^
4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.013, size = 362, normalized size = 1.

$$\frac{f}{2}\sqrt{bx^4+a}-\frac{f}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)-\frac{c}{3x^3}\sqrt{bx^4+a}+\frac{2bc}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x)
```

```
[Out] 1/2*f*(b*x^4+a)^(1/2)-1/2*f*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
-1/3*c/x^3*(b*x^4+a)^(1/2)+2/3*c*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/a/x^2*(b*x^4+a)^(3/2)+1/2*d*b/a*x^2*(
b*x^4+a)^(1/2)+1/2*d*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-e*(b*x^4+a)^(1
/2)/x+2*I*e*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)-2*I*e*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Sympy [C] time = 5.41421, size = 235, normalized size = 0.66

$$\frac{\sqrt{a}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{af}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*e*ga
mma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2*sqr
t(a/(b*x**4) + 1)) + sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*f*x*
*2/(2*sqrt(a/(b*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.504 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

Optimal. Leaf size=329

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)-\frac{bc\tanh}{}$$

```
[Out] -((((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4])/12 + (2*S
qrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (Sqrt[b]*e*ArcTanh[(S
qrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4
*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sq
rt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.280873, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1825, 1832, 266, 63, 208, 1885, 275, 217, 206, 1198, 220, 1196}

$$-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\right.\right.}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```


```
[Out] -((((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4])/12 + (2*S
qrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (Sqrt[b]*e*ArcTanh[(S
qrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4
*Sqrt[a]) - (2*a^(1/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sq
rt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^5}\,dx = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{4}-\frac{dx}{3}-\frac{ex^2}{2}-fx^3}{x\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{d}{3}-\frac{ex}{2}-fx^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{2}(bc)\int \frac{}{x\sqrt{a}}$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int\left(-\frac{ex}{2\sqrt{a+bx^4}}+\frac{-\frac{d}{3}-fx^2}{\sqrt{a+bx^4}}\right)dx+$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{d}{3}-fx^2}{\sqrt{a+bx^4}}\,dx+\frac{1}{4}c\,\mathrm{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{1}{2}(be)\,\mathrm{Subst}\left(\int\right.$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}-$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{1}{2}\sqrt{b}e\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.$$

Mathematica [C] time = 0.233866, size = 175, normalized size = 0.53

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\left(3ac\sqrt{\frac{bx^4}{a}+1}+3bcx^4\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+4adx\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^4}{a}\right)+6aex^2\sqrt{\frac{bx^4}{a}+1}-6\sqrt{a}\sqrt{b}ex^4\sinh^{-1}\right.}{12x^4\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

```
[Out] -(Sqrt[1 + (b*x^4)/a]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*x^
4)/a] - 6*Sqrt[a]*Sqrt[b]*e*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*c*x^4*
ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -
((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)])
)/(12*x^4*Sqrt[a + b*x^4])
```

Maple [C] time = 0.016, size = 385, normalized size = 1.2

$$-\frac{d}{3\,x^{3}}\sqrt{bx^{4}+a}+\frac{2\,bd}{3}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^{4}+a}}-\frac{c}{4\,ax^{4}}\left(bx^{4}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x)
```

```
[Out] -1/3*d/x^3*(b*x^4+a)^(1/2)+2/3*d*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*c/a/x^4*(b*x^4+a)^(3/2)-1/4*c*b/a^(1/2)
*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*c*b/a*(b*x^4+a)^(1/2)-1/2*e/a/
x^2*(b*x^4+a)^(3/2)+1/2*e*b/a*x^2*(b*x^4+a)^(1/2)+1/2*e*b^(1/2)*ln(x^2*b^(1
/2)+(b*x^4+a)^(1/2))-f/x*(b*x^4+a)^(1/2)+2*I*f*b^(1/2)*a^(1/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2*I*f*b^(1/2)*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^{4}+a}\left(fx^{3}+ex^{2}+dx+c\right)}{x^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Sympy [C] time = 5.73538, size = 211, normalized size = 0.64

$$\frac{\sqrt{a}d\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x^{3}\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}e}{2x^{2}\sqrt{1+\frac{bx^{4}}{a}}}+\frac{\sqrt{a}f\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^{4}}+1}}{4x^{2}}+\frac{\sqrt{b}e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{2}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*f*ga
mma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(
3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(b)*x
**2/sqrt(a))/2 - b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x**2/(
2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.505 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

Optimal. Leaf size=360

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{5a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4])/60 -
 (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a*(Sq
rt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(
3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3
/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.323537, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 844, 217, 206, 266, 63, 208}

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{5a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```


```
[Out] -(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4])/60 -
 (2*b*c*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a*(Sq
rt[a] + Sqrt[b]*x^2)) + (Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
2 - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(
3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3
/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}-\frac{fx^3}{2}}{x^2\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-(2b)\int \left(\frac{-\frac{c}{5}-\frac{ex^2}{3}}{x^2\sqrt{a+bx^4}}+\frac{-\frac{d}{4}-\frac{fx^2}{2}}{x\sqrt{a+bx^4}}\right.$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{5}-\frac{ex^2}{3}}{x^2\sqrt{a+bx^4}}\,dx-(2b)\int -$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}-b\,\mathrm{Subst}\left(\int \frac{-\frac{d}{4}}{x\sqrt{a}}\right.$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}-\frac{\left(2b^{3/2}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}}{5\sqrt{a}}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Mathematica [C] time = 0.233774, size = 179, normalized size = 0.5

$$-\frac{\sqrt{a+bx^4}\left(12ac\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^4}{a}\right)+5x\left(3ad\sqrt{\frac{bx^4}{a}+1}+3bdx^4\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+4aex\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^4}{a}\right)+6\right.\right.}{60ax^5\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)]
 + 5*x*(3*a*d*Sqrt[1 + (b*x^4)/a] + 6*a*f*x^2*Sqrt[1 + (b*x^4)/a] - 6*Sqrt[
a]*Sqrt[b]*f*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*d*x^4*ArcTanh[Sqrt[1
+ (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a)])))
/(60*a*x^5*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.015, size = 404, normalized size = 1.1

$$-\frac{e}{3\,x^3}\sqrt{bx^4+a}+\frac{2\,be}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{c}{5\,x^5}\sqrt{bx^4+a}-\frac{2\,b}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x)
```

```
[Out] -1/3*e/x^3*(b*x^4+a)^(1/2)+2/3*e*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*c/x^5*(b*x^4+a)^(1/2)-2/5*b*c*(b*x^4+a)
^(1/2)/a/x+2/5*I*c/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*c/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*d/a/x^4*(b*x^4+a)^
(3/2)-1/4*d*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*d*b/a*(b*
x^4+a)^(1/2)-1/2*f/a/x^2*(b*x^4+a)^(3/2)+1/2*f*b/a*x^2*(b*x^4+a)^(1/2)+1/2*
f*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Sympy [C] time = 6.0442, size = 216, normalized size = 0.6

$$\frac{\sqrt{a}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}f}{2x^2\sqrt{1+\frac{bx^4}{a}}}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2}+\frac{\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*f/(2*x**2*sqrt(1 +
b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*f*asinh(sqrt
(b)*x**2/sqrt(a))/2 - b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*f*x
**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.506 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

Optimal. Leaf size=352

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{5a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4])/60 -
 (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(
3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqrt[
a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.316338, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {14, 1825, 1833, 1252, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{5a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] -(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4])/60 -
 (b*c*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(
3/2)*d*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*ArcTanh[Sqrt
[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) - (2*b^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d + 5*Sqrt[
a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}-\frac{fx^3}{3}}{x^3\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{6}-\frac{ex^2}{4}}{x^3\sqrt{a+bx^4}}+\frac{-\frac{d}{5}-\frac{fx^2}{3}}{x^2\sqrt{a+bx^4}}\right)$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{6}-\frac{ex^2}{4}}{x^3\sqrt{a+bx^4}}\,dx-(2b)\int \frac{-\frac{d}{5}}{x^2\sqrt{}}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{2bd\sqrt{a+bx^4}}{5ax}-b\,\mathrm{Subst}\left(\int \frac{-\frac{c}{6}-}{x^2\sqrt{a+}}\right.$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}-\frac{\left(2b^{3/2}\right.}{}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}d}{5a\left(\sqrt{\right.}}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}d}{5a\left(\sqrt{\right.}}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2}-\frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}d}{5a\left(\sqrt{\right.}}$$

Mathematica [C] time = 0.243303, size = 145, normalized size = 0.41

$$-\frac{\sqrt{a+bx^4}\left(5\left(\sqrt{\frac{bx^4}{a}+1}\left(2ac+3aex^2+2bcx^4\right)+3bex^6\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+4afx^3\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^4}{a}\right)\right)+12adx\,{}_2F_1\left(\right.\right.}{60ax^6\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(12*a*d*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a
)] + 5*(Sqrt[1 + (b*x^4)/a]*(2*a*c + 3*a*e*x^2 + 2*b*c*x^4) + 3*b*e*x^6*Arc
Tanh[Sqrt[1 + (b*x^4)/a]] + 4*a*f*x^3*Hypergeometric2F1[-3/4, -1/2, 1/4, -(
```

```
(b*x^4)/a)]]))/(60*a*x^6*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.017, size = 361, normalized size = 1.

$$-\frac{f}{3x^3}\sqrt{bx^4+a}+\frac{2fb}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{d}{5x^5}\sqrt{bx^4+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x)
```

```
[Out] -1/3*f/x^3*(b*x^4+a)^(1/2)+2/3*f*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*d/x^5*(b*x^4+a)^(1/2)-2/5*b*d*(b*x^4+a)
^(1/2)/a/x+2/5*I*d/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2/5*I*d/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*e/a/x^4*(b*x^4+a)^
(3/2)-1/4*e*b/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*e*b/a*(b*
x^4+a)^(1/2)-1/6*c*(b*x^4+a)^(3/2)/x^6/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bx^4+a\right)^{\frac{3}{2}}c}{6\,ax^6}+\int\frac{\sqrt{bx^4+a}\left(fx^2+ex+d\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(b*x^4 + a)^(3/2)*c/(a*x^6) + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x +
 d)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

Sympy [C] time = 5.70158, size = 189, normalized size = 0.54

$$\frac{\sqrt{a}d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6a}-\frac{be\,\mathrm{as}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

```
[Out] sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,),
b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(b)*c*sqrt(a/(b*x**4) +
 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c*sqrt(a/
(b*x**4) + 1)/(6*a) - b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

3.507 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$

Optimal. Leaf size=375

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\right.}{5a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^4])/420
 - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b*x^4])/(6*a*x^2) - (
2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*a*(Sqrt
[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) -
(2*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*
x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.361593, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 807, 266, 63, 208}

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```


```
[Out] -(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^4])/420
 - (2*b*c*Sqrt[a + b*x^4])/(21*a*x^3) - (b*d*Sqrt[a + b*x^4])/(6*a*x^2) - (
2*b*e*Sqrt[a + b*x^4])/(5*a*x) + (2*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*a*(Sqrt
[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*Sqrt[a]) -
(2*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*
x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(105*a^(5/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{7}-\frac{dx}{6}-\frac{ex^2}{5}-\frac{fx^3}{4}}{x^4\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{7}-\frac{ex^2}{5}}{x^4\sqrt{a+bx^4}}+\frac{-\frac{d}{6}-\cdots}{x^3\sqrt{a+\cdots}}\right.$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-(2b)\int\frac{-\frac{c}{7}-\frac{ex^2}{5}}{x^4\sqrt{a+bx^4}}\,dx-(2b)\int\cdots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-b\,\mathrm{Subst}\left(\int\frac{\cdots}{x^2\cdots}\right.$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\cdots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\cdots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\cdots$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3}-\frac{bd\sqrt{a+bx^4}}{6ax^2}-\cdots$$

Mathematica [C] time = 0.240489, size = 145, normalized size = 0.39

$$-\frac{\sqrt{a+bx^4}\left(60ac\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{bx^4}{a}\right)+35x\left(\sqrt{\frac{bx^4}{a}+1}\left(2ad+3afx^2+2bdx^4\right)+3bfx^6\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)\right)+84ae\cdots\right.}{420ax^7\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(35*x*(Sqrt[1 + (b*x^4)/a]*(2*a*d + 3*a*f*x^2 + 2*b*d*x^4
) + 3*b*f*x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 60*a*c*Hypergeometric2F1[-7/4
, -1/2, -3/4, -((b*x^4)/a)] + 84*a*e*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4
```

```
, -((b*x^4)/a)]))/(420*a*x^7*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.018, size = 385, normalized size = 1.

$$-\frac{e}{5x^5}\sqrt{bx^4+a}-\frac{2be}{5ax}\sqrt{bx^4+a}+\frac{2i}{5}eb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x)
```

```
[Out] -1/5*e/x^5*(b*x^4+a)^(1/2)-2/5*b*e*(b*x^4+a)^(1/2)/a/x+2/5*I*e/a^(1/2)*b^(3
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
-2/5*I*e/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)-1/4*f/a/x^4*(b*x^4+a)^(3/2)-1/4*f*b/a^(1/2)*ln((2*a+2
*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/4*f*b/a*(b*x^4+a)^(1/2)-1/6*d*(b*x^4+a)^(3
/2)/x^6/a-1/7*c/x^7*(b*x^4+a)^(1/2)-2/21*b*c*(b*x^4+a)^(1/2)/a/x^3-2/21*c/a
*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Sympy [C] time = 6.31268, size = 192, normalized size = 0.51

$$\frac{\sqrt{a}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2}-\frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - sqrt(b)*d*sqrt(a/(b*x**4)
 + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d*sqrt(
a/(b*x**4) + 1)/(6*a) - b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.508 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

Optimal. Leaf size=400

$$-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}+\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{\right.}}{}$$

```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b*x^4])/
840 - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a + b*x^4])/(21*a*x^3)
 - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*Sqrt[a + b*x^4])/(5*a*x) + (2*b
^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d - 21
*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b
*x^4])
```

Rubi [A] time = 0.386205, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1825, 1833, 1252, 835, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{}{(\sqrt{a}}}}{5a^{3/4}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```


```
[Out] -(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b*x^4])/
840 - (b*c*Sqrt[a + b*x^4])/(16*a*x^4) - (2*b*d*Sqrt[a + b*x^4])/(21*a*x^3)
 - (b*e*Sqrt[a + b*x^4])/(6*a*x^2) - (2*b*f*Sqrt[a + b*x^4])/(5*a*x) + (2*b
^(3/2)*f*x*Sqrt[a + b*x^4])/(5*a*(Sqrt[a] + Sqrt[b]*x^2)) + (b^2*c*ArcTanh[
Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3/2)) - (2*b^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*d - 21
*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*a^(5/4)*Sqrt[a + b
*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
```

```
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{8}-\frac{dx}{7}-\frac{ex^2}{6}-\frac{fx^3}{5}}{x^5\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{8}-\frac{ex^2}{6}}{x^5\sqrt{a+bx^4}}+\frac{-\frac{d}{7}-\cdots}{x^4\sqrt{a+\cdots}}\right.$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{8}-\frac{ex^2}{6}}{x^5\sqrt{a+bx^4}}\,dx-(2b)\int$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-b\,\text{Subst}\left(\int \frac{\cdots}{x^3\cdots}\right.$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-$$

$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-$$

Mathematica [C] time = 0.165423, size = 146, normalized size = 0.36

$$-\frac{\sqrt{a+bx^4}\left(7x\left(5\left(a+bx^4\right)\sqrt{\frac{bx^4}{a}+1}\left(a^2e+b^2cx^6\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^4}{a}+1\right)\right)+6a^3fx\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^4}{a}\right)\right)+30a^3d\,{}_2F_1\right.}{210a^3x^7\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(30*a^3*d*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^4)/a
)] + 7*x*(6*a^3*f*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a)] + 5*(
a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*e + b^2*c*x^6*Hypergeometric2F1[3/2, 3,
 5/2, 1 + (b*x^4)/a]))))/(210*a^3*x^7*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.019, size = 408, normalized size = 1.

$$-\frac{f}{5x^5}\sqrt{bx^4+a}-\frac{2fb}{5ax}\sqrt{bx^4+a}+\frac{2i}{5}fb^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x)
```

```
[Out] -1/5*f/x^5*(b*x^4+a)^(1/2)-2/5*b*f*(b*x^4+a)^(1/2)/a/x+2/5*I*f/a^(1/2)*b^(3
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
-2/5*I*f/a^(1/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)-1/8*c/a/x^8*(b*x^4+a)^(3/2)+1/16*c*b/a^2/x^4*(b*x^4+a
)^(3/2)+1/16*c*b^2/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*c*b
^2/a^2*(b*x^4+a)^(1/2)-1/6*e*(b*x^4+a)^(3/2)/x^6/a-1/7*d/x^7*(b*x^4+a)^(1/2
)-2/21*b*d*(b*x^4+a)^(1/2)/a/x^3-2/21*d/a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

Sympy [C] time = 7.90392, size = 246, normalized size = 0.62

$$\frac{\sqrt{a}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}-\frac{ac}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{3\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a*c/(8*sqrt(b)*x**10*sqrt
(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*e*
sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4) + 1))
 - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

3.509 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}} dx$

Optimal. Leaf size=425

$$-\frac{b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}+$$

```
[Out] -(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4])/504
- (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*x^4])/(16*a*x^4) - (
2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt[a + b*x^4])/(6*a*x^2) + (2*b^
2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2*(
Sqrt[a] + Sqrt[b]*x^2)) + (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3
/2)) + (2*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4)*Sqr
t[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.433128, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 835, 807, 266, 63, 208}

$$-\frac{b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+7\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}+\frac{2b^{9/4}c\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```


```
[Out] -(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4])/504
- (2*b*c*Sqrt[a + b*x^4])/(45*a*x^5) - (b*d*Sqrt[a + b*x^4])/(16*a*x^4) - (
2*b*e*Sqrt[a + b*x^4])/(21*a*x^3) - (b*f*Sqrt[a + b*x^4])/(6*a*x^2) + (2*b^
2*c*Sqrt[a + b*x^4])/(15*a^2*x) - (2*b^(5/2)*c*x*Sqrt[a + b*x^4])/(15*a^2*(
Sqrt[a] + Sqrt[b]*x^2)) + (b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*a^(3
/2)) + (2*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(7/4)*Sqr
t[a + b*x^4]) - (b^(7/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(105*a^(7/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}}{x^{10}}\,dx = -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{9}-\frac{dx}{8}-\frac{ex^2}{7}-\frac{fx^3}{6}}{x^6\sqrt{a+bx^4}}\,dx$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int\left(\frac{-\frac{c}{9}-\frac{ex^2}{7}}{x^6\sqrt{a+bx^4}}+\frac{-\frac{d}{8}-\frac{f}{6}}{x^5\sqrt{a+b}}\right.$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-(2b)\int \frac{-\frac{c}{9}-\frac{ex^2}{7}}{x^6\sqrt{a+bx^4}}\,dx-(2b)\int$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-b\,\text{Subst}\left(\int \frac{-}{x^3\sqrt{}}\right.$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-2$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-2$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-2$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-2$$

$$= -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}-2$$

Mathematica [C] time = 0.16672, size = 148, normalized size = 0.35

$$-\frac{\sqrt{a+bx^4}\left(3x^2\left(7x\left(a+bx^4\right)\sqrt{\frac{bx^4}{a}+1}\left(a^2f+b^2dx^6\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^4}{a}+1\right)\right)+6a^3e\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{bx^4}{a}\right)\right)+14a^3c\,{}_2F_1\left(-\right.\right.}{126a^3x^9\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(14*a^3*c*Hypergeometric2F1[-9/4, -1/2, -5/4, -((b*x^4)/a
)] + 3*x^2*(6*a^3*e*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^4)/a)] + 7*x
*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*f + b^2*d*x^6*Hypergeometric2F1[3/2,
3, 5/2, 1 + (b*x^4)/a]))))/(126*a^3*x^9*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.018, size = 429, normalized size = 1.

$$-\frac{d}{8\,ax^8}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{bd}{16\,a^2x^4}\left(bx^4+a\right)^{\frac{3}{2}}+\frac{b^2d}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}-\frac{b^2d}{16\,a^2}\sqrt{bx^4+a}-\frac{f}{6\,x^6a}\left(bx^4+a\right)^{\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x)
```

```
[Out] -1/8*d/a/x^8*(b*x^4+a)^(3/2)+1/16*d*b/a^2/x^4*(b*x^4+a)^(3/2)+1/16*d*b^2/a^
(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/16*d*b^2/a^2*(b*x^4+a)^(1/2
)-1/6*f*(b*x^4+a)^(3/2)/x^6/a-1/7*e/x^7*(b*x^4+a)^(1/2)-2/21*b*e*(b*x^4+a)^
(1/2)/a/x^3-2/21*e/a*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)-1/9*c/x^9*(b*x^4+a)^(1/2)-2/45*b*c*(b*x^4+a)^(1/2)/a/
x^5+2/15*b^2*c*(b*x^4+a)^(1/2)/a^2/x-2/15*I*c/a^(3/2)*b^(5/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+2/15*I*c/a^(3/2)
*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Sympy [C] time = 8.46443, size = 246, normalized size = 0.58

$$\frac{\sqrt{a}c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}-\frac{ad}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}}-\frac{3\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)
```

```
[Out] sqrt(a)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**9*gamma(-5/4)) + sqrt(a)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a*d/(8*sqrt(b)*x**10*sqrt
(a/(b*x**4) + 1)) - 3*sqrt(b)*d/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b)*f*
sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d/(16*a*x**2*sqrt(a/(b*x**4) + 1))
 - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x*
*2))/(16*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.510 $\int x^4 \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=476

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}e+65\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}-\frac{a^3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3ex\sqrt{a}\cdots}{65b^{3/2}\left(\sqrt{a}+\cdots\right.}$$

```
[Out] (4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*c + 77*e*x^2)*Sqrt[a + b*x^4])
/3003 - (a*d*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*c + 11*e*x^2)*(a + b*
x^4)^(3/2))/143 + (f*x^4*(a + b*x^4)^(5/2))/(14*b) - ((12*a*f - 35*b*d*x^2)
*(a + b*x^4)^(5/2))/(420*b^2) - (a^3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6
5*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[b]*c + 77*Sqrt[a]*e)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.439691, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 833, 780, 195, 217, 206}

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}e+65\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}-\frac{a^3d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*c + 77*e*x^2)*Sqrt[a + b*x^4])
/3003 - (a*d*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*c + 11*e*x^2)*(a + b*
x^4)^(3/2))/143 + (f*x^4*(a + b*x^4)^(5/2))/(14*b) - ((12*a*f - 35*b*d*x^2)
*(a + b*x^4)^(5/2))/(420*b^2) - (a^3*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6
5*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[b]*c + 77*Sqrt[a]*e)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int\left(x^4\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^5\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$

$$=\int x^4\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x^5\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx$$

$$=\frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x^2(d+fx)\left(a+bx^2\right)^{3/2}\,dx,\right.$$

$$=\frac{2ax^5\left(117c+77ex^2\right)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{fx^4\left(a\right.}{1}$$

$$=\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5\left(117c+77ex^2\right)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5\left(13c+11ex^2\right)\left(a\right.$$

$$=\frac{4a^2cx\sqrt{a+bx^4}}{77b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5\left(117c+77ex^2\right)\sqrt{a+bx^4}}{3003}-\frac{adx^2}{}$$

$$=\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5\left(117c+77ex^2\right)}{3003}$$

$$=\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$=\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Mathematica [C] time = 0.864564, size = 225, normalized size = 0.47

$$\frac{\sqrt{a+bx^4}\left(5005bdx^2\left(3a^2+14abx^4+8b^2x^8\right)-\frac{43680a^2bcx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}-\frac{15015a^{5/2}\sqrt{b}d\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{\frac{bx^4}{a}+1}}-\frac{36960a^2bex^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{b}{}\right.}{\sqrt{\frac{bx^4}{a}+1}}\right.}{480480b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*(43680*b*c*x*(a + b*x^4)^2 + 36960*b*e*x^3*(a + b*x^4)^2 +
 6864*f*(a + b*x^4)^2*(-2*a + 5*b*x^4) + 5005*b*d*x^2*(3*a^2 + 14*a*b*x^4 +
 8*b^2*x^8) - (15015*a^(5/2)*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt
[1 + (b*x^4)/a] - (43680*a^2*b*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*
^4)/a)])/Sqrt[1 + (b*x^4)/a] - (36960*a^2*b*e*x^3*Hypergeometric2F1[-3/2, 3
/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(480480*b^2)
```

Maple [C] time = 0.039, size = 462, normalized size = 1.

$$-\frac{f\left(-5\,bx^{4}+2\,a\right)\left(b^{2}x^{8}+2\,abx^{4}+a^{2}\right)}{70\,b^{2}}\sqrt{bx^{4}+a}+\frac{bex^{11}}{13}\sqrt{bx^{4}+a}+\frac{5\,aex^{7}}{39}\sqrt{bx^{4}+a}+\frac{4\,a^{2}ex^{3}}{195\,b}\sqrt{bx^{4}+a}-\frac{4\,i}{65}ea^{\frac{7}{2}}\sqrt{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] -1/70*f*(b*x^4+a)^(1/2)*(-5*b*x^4+2*a)*(b^2*x^8+2*a*b*x^4+a^2)/b^2+1/13*e*b
*x^11*(b*x^4+a)^(1/2)+5/39*e*a*x^7*(b*x^4+a)^(1/2)+4/195*a^2*e*x^3*(b*x^4+a
)^(1/2)/b-4/65*I*e/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4/65*I*e/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/12*d*b*x^10*(b*x^4+
a)^(1/2)+7/48*d*a*x^6*(b*x^4+a)^(1/2)+1/32*a^2*d*x^2*(b*x^4+a)^(1/2)/b-1/32
*d/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/11*c*b*x^9*(b*x^4+a)^(1/2)
+13/77*c*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*c*x*(b*x^4+a)^(1/2)/b-4/77*c/b*a^3/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{4}+a\right)^{\frac{3}{2}}\left(fx^{3}+ex^{2}+dx+c\right)x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^{11}+bex^{10}+bdx^{9}+bcx^{8}+afx^{7}+aex^{6}+adx^{5}+acx^{4}\right)\sqrt{bx^{4}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^11 + b*e*x^10 + b*d*x^9 + b*c*x^8 + a*f*x^7 + a*e*x^6 + a*d
*x^5 + a*c*x^4)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 18.0418, size = 462, normalized size = 0.97

$$\frac{a^{\frac{5}{2}}dx^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}dx^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bcx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\right.}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*d*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/
2)*d*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*c*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(13/4)) + 11*sqrt(a)*b*d*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**11*
gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(15/4)) - a**3*d*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*f*Piecewise(
(-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*s
qrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b*f*Piecewise((4*a
**3*sqrt(a + b*x**4)/(105*b**3) - 2*a**2*x**4*sqrt(a + b*x**4)/(105*b**2) +
 a*x**8*sqrt(a + b*x**4)/(70*b) + x**12*sqrt(a + b*x**4)/14, Ne(b, 0)), (sq
rt(a)*x**12/12, True)) + b**2*d*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.511 $\int x^3 \left(c + dx + ex^2 + f x^3\right) \left(a + bx^4\right)^{3/2} dx$

Optimal. Leaf size=452

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx}{65b^{3/2}\left(\sqrt{\right.}\right.}$$

```
[Out] (4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*d + 77*f*x^2)*Sqrt[a + b*x^4])
/3003 - (a*e*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*d + 11*f*x^2)*(a + b*
x^4)^(3/2))/143 + ((6*c + 5*e*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*e*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*f*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65
*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^
(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.40782, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1833, 1252, 780, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[a + b*x^4])/(65*b^(3
/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*d + 77*f*x^2)*Sqrt[a + b*x^4])
/3003 - (a*e*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*d + 11*f*x^2)*(a + b*
x^4)^(3/2))/143 + ((6*c + 5*e*x^2)*(a + b*x^4)^(5/2))/(60*b) - (a^3*e*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*f*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65
*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^
(7/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int\left(x^3\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^4\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$

$$=\int x^3\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx+\int x^4\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx$$

$$=\frac{1}{143}x^5\left(13d+11fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x(c+ex)\left(a+bx^2\right)^{3/2}\,d\ldots\right.$$

$$=\frac{2ax^5\left(117d+77fx^2\right)\sqrt{a+bx^4}}{3003}+\frac{1}{143}x^5\left(13d+11fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{(6\ldots}{\ldots}$$

$$=\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5\left(117d+77fx^2\right)\sqrt{a+bx^4}}{3003}-\frac{aex^2\left(a+bx^4\right)^{3/2}}{48b}+\ldots$$

$$=\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5\left(117d+77\ldots\right.}{300\ldots}$$

$$=\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{2ax^5\left(117d+77\ldots\right.}{300\ldots}$$

$$=\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}-\frac{4a^3fx\sqrt{a+bx\ldots}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b\ldots}\right.}$$

Mathematica [C] time = 0.746668, size = 238, normalized size = 0.53

$$\frac{\sqrt{a+bx^4}\left(715e\left(\sqrt{b}x^2\left(3a^2+14abx^4+8b^2x^8\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{\frac{bx^4}{a}+1}}\right)-\frac{6240a^2\sqrt{b}dx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}-\frac{5280a^2\sqrt{b}fx^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\ldots\right)}{\sqrt{\frac{bx^4}{a}+1}}\ldots\right.}{68640b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*(6864*Sqrt[b]*c*(a + b*x^4)^2 + 6240*Sqrt[b]*d*x*(a + b*x^
4)^2 + 5280*Sqrt[b]*f*x^3*(a + b*x^4)^2 + 715*e*(Sqrt[b]*x^2*(3*a^2 + 14*a*
b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b
*x^4)/a]) - (6240*a^2*Sqrt[b]*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^
4)/a)])/Sqrt[1 + (b*x^4)/a] - (5280*a^2*Sqrt[b]*f*x^3*Hypergeometric2F1[-3/
2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(68640*b^(3/2))
```

Maple [C] time = 0.007, size = 434, normalized size = 1.

$$\frac{bfx^{11}}{13}\sqrt{bx^4+a}+\frac{5afx^7}{39}\sqrt{bx^4+a}+\frac{4a^2fx^3}{195b}\sqrt{bx^4+a}-\frac{4i}{65}fa^{\frac{7}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/13*f*b*x^11*(b*x^4+a)^(1/2)+5/39*f*a*x^7*(b*x^4+a)^(1/2)+4/195*a^2*f*x^3*
(b*x^4+a)^(1/2)/b-4/65*I*f/b^(3/2)*a^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4/65*I*f/b^(3/2)*a^(7/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/12*e*b*x^10
*(b*x^4+a)^(1/2)+7/48*e*a*x^6*(b*x^4+a)^(1/2)+1/32*a^2*e*x^2*(b*x^4+a)^(1/2
)/b-1/32*e/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/11*d*b*x^9*(b*x^4+
a)^(1/2)+13/77*d*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*d*x*(b*x^4+a)^(1/2)/b-4/77*
d/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)+1/10*c/b*(b*x^4+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bx^4+a\right)^{\frac{5}{2}}c}{10\,b}+\int\left(bfx^{10}+bex^9+bdx^8+afx^6+aex^5+adx^4\right)\sqrt{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^4 + a)^(5/2)*c/b + integrate((b*f*x^10 + b*e*x^9 + b*d*x^8 + a*f*
x^6 + a*e*x^5 + a*d*x^4)*sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bfx^{10}+bex^9+bdx^8+bcx^7+afx^6+aex^5+adx^4+acx^3\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^10 + b*e*x^9 + b*d*x^8 + b*c*x^7 + a*f*x^6 + a*e*x^5 + a*d*
x^4 + a*c*x^3)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 15.0307, size = 398, normalized size = 0.88

$$\frac{a^{\frac{5}{2}}ex^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}ex^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bdx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right.}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(3/
2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**7*gamma(7/4)*hyper((-1/2,
 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(13/4)) + 11*sqrt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*x**11*
gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*c*Piecewise(
(sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*c*Piecew
ise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x*
*8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*e*x**14/(
12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

3.512 $\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=427

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\mathrm{t}\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*
(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*f*x^2*(a + b*x^4)^(3/2))/(48*b
) + (x^3*(11*c + 9*e*x^2)*(a + b*x^4)^(3/2))/99 + ((6*d + 5*f*x^2)*(a + b*x
^4)^(5/2))/(60*b) - (a^3*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3
/2)) - (4*a^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqr
t[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.352719, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1833, 1274, 1280, 1198, 220, 1196, 1252, 780, 195, 217, 206}

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/(32*b) + (
4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*
(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*f*x^2*(a + b*x^4)^(3/2))/(48*b
) + (x^3*(11*c + 9*e*x^2)*(a + b*x^4)^(3/2))/99 + ((6*d + 5*f*x^2)*(a + b*x
^4)^(5/2))/(60*b) - (a^3*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3
/2)) - (4*a^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqr
t[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
```

```
+ m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}dx=\int\left(x^2\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^3\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$

$$=\int x^2\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}dx+\int x^3\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}dx$$

$$=\frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int x(d+fx)\left(a+bx^2\right)^{3/2}dx,x,x$$

$$=\frac{2ax^3\left(77c+45ex^2\right)\sqrt{a+bx^4}}{1155}+\frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{\left(6d+5fx\right.}{}$$

$$=\frac{4a^2ex\sqrt{a+bx^4}}{77b}+\frac{2ax^3\left(77c+45ex^2\right)\sqrt{a+bx^4}}{1155}-\frac{afx^2\left(a+bx^4\right)^{3/2}}{48b}+\frac{1}{99}x^3$$

$$=\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{2ax^3\left(77c+45ex^2\right)\sqrt{a+bx^4}}{1155}-\frac{afx^2\left(a\right.}{}$$

$$=\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3\left(77c+45ex\right.}{1155}$$

$$=\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3\left(77c+45ex\right.}{1155}$$

Mathematica [C] time = 0.863065, size = 205, normalized size = 0.48

$$\frac{\sqrt{a+bx^4}\left(\frac{55f\left(\sqrt{b}x^2\left(3a^2+14abx^4+8b^2x^8\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{\frac{bx^4}{a}+1}}\right)}{b^{3/2}}-\frac{480a^2ex\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{b\sqrt{\frac{bx^4}{a}+1}}+\frac{1760acx^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}+\frac{528d\left(a+bx^4\right)^2}{b}+\frac{480e}{}\right.}{5280}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((528*d*(a + b*x^4)^2)/b + (480*e*x*(a + b*x^4)^2)/b + (55
*f*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt
[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]))/b^(3/2) - (480*a^2*e*x*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (1760*a*c*x
^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/5
280
```

Maple [C] time = 0.01, size = 413, normalized size = 1.

$$\frac{bfx^{10}}{12}\sqrt{bx^4+a}+\frac{7afx^6}{48}\sqrt{bx^4+a}+\frac{a^2fx^2}{32b}\sqrt{bx^4+a}-\frac{a^3f}{32}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{bex^9}{11}\sqrt{bx^4+a}+\frac{13aex^5}{77}\sqrt{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/12*f*b*x^10*(b*x^4+a)^(1/2)+7/48*f*a*x^6*(b*x^4+a)^(1/2)+1/32*a^2*f*x^2*(
b*x^4+a)^(1/2)/b-1/32*f/b^(3/2)*a^3*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/11*e*
b*x^9*(b*x^4+a)^(1/2)+13/77*e*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*e*x*(b*x^4+a)^
(1/2)/b-4/77*e/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)+1/10*d/b*(b*x^4+a)^(5/2)+1/9*c*b*x^7*(b*x^4+a)^(1/2)+11/4
5*c*a*x^3*(b*x^4+a)^(1/2)+4/15*I*c*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*c*a^(5/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bfx^9+bex^8+bdx^7+bcx^6+afx^5+aex^4+adx^3+acx^2\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^9 + b*e*x^8 + b*d*x^7 + b*c*x^6 + a*f*x^5 + a*e*x^4 + a*d*x
^3 + a*c*x^2)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 14.209, size = 398, normalized size = 0.93

$$\frac{a^{\frac{5}{2}}fx^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}fx^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bcx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\\ \frac{11}{4}\end{matrix}\right.}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma(3/4)*hype
r((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*
e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(9/4)) + 17*a**(3/2)*f*x**6/(96*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x**7*
gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(1
1/4)) + sqrt(a)*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b*x**4/a
)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*Piecewise((sqrt
(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*d*Piecewise((
-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sq
rt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*f*x**14/(12*sq
rt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.513 $\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=409

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E}{15b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a^
2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(77*
d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3*(1
1*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^
2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9
/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
1155*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.324278, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1833, 1248, 641, 195, 217, 206, 1274, 1280, 1198, 220, 1196}

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```


```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a^
2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(77*
d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3*(1
1*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (3*a^
2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*d*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9
/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
1155*b^(5/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}dx=\int\left(x\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x^2\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$

$$=\int x\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}dx+\int x^2\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}dx$$

$$=\frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int(c+ex)\left(a+bx^2\right)^{3/2}dx,x,x$$

$$=\frac{2ax^3\left(77d+45fx^2\right)\sqrt{a+bx^4}}{1155}+\frac{1}{99}x^3\left(11d+9fx^2\right)\left(a+bx^4\right)^{3/2}+\frac{e\left(a+b\right.}{10}$$

$$=\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{2ax^3\left(77d+45fx^2\right)\sqrt{a+bx^4}}{1155}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{1}{99}$$

$$=\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{2ax^3\left(77d+45fx^2\right)\sqrt{a+bx^4}}{1155}+\frac{1}{8}c$$

$$=\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3\left(77d+\right.}{1}$$

$$=\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3\left(77d+\right.}{1}$$

Mathematica [C] time = 0.701316, size = 196, normalized size = 0.48

$$\frac{\sqrt{a+bx^4}\left(165c\left(\frac{3a^{5/2}\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\left(a+bx^4\right)}+5ax^2+2bx^6\right)-\frac{240a^2fx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{b\sqrt{\frac{bx^4}{a}+1}}+\frac{880adx^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}+\frac{264e\left(a+bx^4\right)^2}{b}\right.}{2640}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((264*e*(a + b*x^4)^2)/b + (240*f*x*(a + b*x^4)^2)/b + 165
*c*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2
)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) - (240*a^2*f*x*Hypergeometric2F1[-3/2, 1
/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (880*a*d*x^3*Hypergeometr
ic2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/2640
```

Maple [C] time = 0.015, size = 392, normalized size = 1.

$$\frac{bfx^9}{11}\sqrt{bx^4+a}+\frac{13afx^5}{77}\sqrt{bx^4+a}+\frac{4a^2fx}{77b}\sqrt{bx^4+a}-\frac{4a^3f}{77b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/11*f*b*x^9*(b*x^4+a)^(1/2)+13/77*f*a*x^5*(b*x^4+a)^(1/2)+4/77*a^2*f*x*(b*
x^4+a)^(1/2)/b-4/77*f/b*a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)+1/10*e*(b*x^4+a)^(5/2)/b+1/9*d*b*x^7*(b*x^4+a)^(1/
2)+11/45*d*a*x^3*(b*x^4+a)^(1/2)+4/15*I*d*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*d*a^(5/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)+1/8*c*b*x^6*(b*x^4+a)^(1/2)+5/16*a*c*x^2*(b*x^4+a)^(1/2)+3/16*c*a^2*ln(x^
2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bfx^8 + bex^7 + bdx^6 + bcx^5 + afx^4 + aex^3 + adx^2 + acx\right)\sqrt{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^8 + b*e*x^7 + b*d*x^6 + b*c*x^5 + a*f*x^4 + a*e*x^3 + a*d*x
^2 + a*c*x)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 10.1743, size = 396, normalized size = 0.97

$$\frac{a^{\frac{3}{2}}cx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}cx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bcx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bdx^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1 + b*x**4/
a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (
```

```
9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(16*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9/4)*hy
per((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 3*a**
2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise((sqrt(a)*x**4/
4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*Piecewise((-a**2*sqr
t(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*
x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*c*x**10/(8*sqrt(a)*sqrt
(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

3.514 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx$

Optimal. Leaf size=382

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(S
qrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (d*
x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f*(a
 + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(
3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.254981, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {1885, 1177, 1198, 220, 1196, 1248, 641, 195, 217, 206}

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(S
qrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (d*
x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f*(a
 + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(
3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \int \left(\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}+x\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$
$$= \int \left(c+ex^2\right)\left(a+bx^4\right)^{3/2}\,dx + \int x\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx$$
$$= \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{1}{21}\int \left(18ac+14aex^2\right)\sqrt{a+bx^4}\,dx + \frac{1}{2}\text{Subst}\Big($$
$$= \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{f\left(a+bx^4\right)^{5/2}}{10b}$$
$$= \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{8}dx^2\left(a+bx^4\right)^{3/2} + \frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)$$
$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{8}dx$$
$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{8}dx$$
$$= \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4} + \frac{1}{8}dx$$

Mathematica [C] time = 0.552603, size = 175, normalized size = 0.46

$$\frac{1}{240}\sqrt{a+bx^4}\left(15d\left(\frac{3a^{5/2}\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\left(a+bx^4\right)}+5ax^2+2bx^6\right)+\frac{240acx\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{\frac{bx^4}{a}+1}}+\frac{80aex^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\right.}{\sqrt{\frac{bx^4}{a}+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^4]*((24*f*(a + b*x^4)^2)/b + 15*d*(5*a*x^2 + 2*b*x^6 + (3*a^(
5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^
4))) + (240*a*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 +
 (b*x^4)/a] + (80*a*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/
Sqrt[1 + (b*x^4)/a]))/240
```

Maple [C] time = 0.006, size = 368, normalized size = 1.

$$\frac{f}{10b}\left(bx^4+a\right)^{\frac{5}{2}}+\frac{bex^7}{9}\sqrt{bx^4+a}+\frac{11\,aex^3}{45}\sqrt{bx^4+a}+\frac{4\,i}{15}ea^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

```
[Out] 1/10*f*(b*x^4+a)^(5/2)/b+1/9*e*b*x^7*(b*x^4+a)^(1/2)+11/45*e*a*x^3*(b*x^4+a
)^(1/2)+4/15*I*e*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
```

```
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*e*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*b*x^6*(b*x^4+a)^(
1/2)+5/16*a*d*x^2*(b*x^4+a)^(1/2)+3/16*d*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)
)/b^(1/2)+1/7*c*b*x^5*(b*x^4+a)^(1/2)+3/7*c*a*x*(b*x^4+a)^(1/2)+4/7*c*a^2/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a), x)
```

Sympy [A] time = 9.64242, size = 394, normalized size = 1.03

$$\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}dx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}dx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(-\right.}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*d*x**2/(
16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*c*x**5*gamma(5/4)*
hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqr
t(a)*b*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*hyper((
-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a**2*d*a
sinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqrt(a)*x**4/4, Eq
(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*Piecewise((-a**2*sqrt(a +
```

```
 b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)
/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*d*x**10/(8*sqrt(a)*sqrt(1 +
b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.515 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=403

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (a*(8*c
+ 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2)*Sqrt[a + b*x^4])/1
05 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 + (x*(9*d + 7*f*x^2)*(a + b*x^4
)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b])
 - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15
*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3
/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.35252, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1833, 1252, 815, 844, 217, 206, 266, 63, 208, 1177, 1198, 220, 1196}

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{15b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (a*(8*c
+ 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2)*Sqrt[a + b*x^4])/1
05 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 + (x*(9*d + 7*f*x^2)*(a + b*x^4
)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b])
 - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15
*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3
/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x}\,dx = \int \left(\frac{\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}}{x}+\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\right)dx$$

$$= \int \frac{\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}}{x}\,dx + \int \left(d+fx^2\right)\left(a+bx^4\right)^{3/2}\,dx$$

$$= \frac{1}{63}x\left(9d+7fx^2\right)\left(a+bx^4\right)^{3/2} + \frac{1}{21}\int \left(18ad+14afx^2\right)\sqrt{a+bx^4}\,dx + \frac{1}{2}\,\mathrm{Su}$$

$$= \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a+bx^4} + \frac{1}{24}\left(4c+3ex^2\right)\left(a+bx^4\right)^{3/2} + \frac{1}{63}x\left(9d+7\right.$$

$$= \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a+bx^4} + \frac{1}{24}\left(4c+3ex^2\right.$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a}$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a}$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a}$$

$$= \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\left(8c+3ex^2\right)\sqrt{a+bx^4} + \frac{2}{105}ax\left(15d+7fx^2\right)\sqrt{a}$$

Mathematica [C] time = 0.512375, size = 224, normalized size = 0.56

$$\frac{1}{6}c\left(\sqrt{a+bx^4}\left(4a+bx^4\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+\frac{1}{16}e\sqrt{a+bx^4}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^4}{a}+1}}+5ax^2+2bx^6\right)+\frac{adx\sqrt{a+bx}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

```
[Out] (e*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sq
rt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (c*(Sqrt[a + b*x^4]*(4*a + b*x
^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 + (a*d*x*Sqrt[a + b*x^
4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] + (
a*f*x^3*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/(3
*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.019, size = 411, normalized size = 1.

$$\frac{bfx^7}{9}\sqrt{bx^4+a}+\frac{11\,x^3af}{45}\sqrt{bx^4+a}+\frac{4\,i}{15}fa^{\frac{5}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x)
```

```
[Out] 1/9*f*b*x^7*(b*x^4+a)^(1/2)+11/45*f*a*x^3*(b*x^4+a)^(1/2)+4/15*I*f*a^(5/2)/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-4/15*I*f*a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+1/8*e*b*x^6*(b*x^4+a)^(1/2)+5/16*e*a*x^2*(b*x^4+
a)^(1/2)+3/16*e*a^2*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+1/7*d*b*x^5*(b*
x^4+a)^(1/2)+3/7*d*a*x*(b*x^4+a)^(1/2)+4/7*d*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*c*b*x^4*(b*x^4+a)^(1/2)+2
/3*c*a*(b*x^4+a)^(1/2)-1/2*c*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x, x)
```

Sympy [A] time = 26.8634, size = 405, normalized size = 1.

$$-\frac{a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

```
[Out] -a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(1/4)*hyper
((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)*e
*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/(16*sqrt(1 + b*x**4/a)) + a**(
3/2)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(7/4)) + sqrt(a)*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*sqrt(a/(b*x**4)
+ 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*c*x**
2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a
+ b*x**4)**(3/2)/(6*b), True)) + b**2*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.516 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2} dx$

Optimal. Leaf size=404

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(\right.}{5\sqrt{a+bx^4}}$$

```
[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a*
e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])/1
6 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^4)^
(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) -
 (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*S
qrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)
*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.348052, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1833, 1272, 1177, 1198, 220, 1196, 1252, 815, 844, 217, 206, 266, 63, 208}

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```


```
[Out] (12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a*
e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])/1
6 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^4)^
(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) -
 (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*S
qrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)
*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
```

```
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \left(\frac{(c+ex^2)(a+bx^4)^{3/2}}{x^2} + \frac{(d+fx^2)(a+bx^4)^{3/2}}{x}\right)dx$$

$$= \int \frac{(c+ex^2)(a+bx^4)^{3/2}}{x^2}\,dx + \int \frac{(d+fx^2)(a+bx^4)^{3/2}}{x}\,dx$$

$$= -\frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{2}\text{Subst}\left(\int \frac{(d+fx)(a+bx^2)^{3/2}}{x}\,dx, x, x^2\right) - \frac{6}{7}$$

$$= \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} - \frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{24}(4d+3fx^2)($$

$$= \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+bx^4} - \frac{(7c-ex^2)(a}{7x}$$

$$= \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+}$$

$$= \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+}$$

$$= \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+}$$

$$= \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+}$$

Mathematica [C] time = 0.523102, size = 222, normalized size = 0.55

$$\frac{1}{6}d\left(\sqrt{a+bx^4}(4a+bx^4) - 3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) + \frac{1}{16}f\sqrt{a+bx^4}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^4}{a}+1}} + 5ax^2 + 2bx^6\right) - \frac{ac\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

```
[Out] (f*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/Sq
rt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (d*(Sqrt[a + b*x^4]*(4*a + b*x
^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 - (a*c*Sqrt[a + b*x^4]
*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)])/(x*Sqrt[1 + (b*x^4)/a])
+ (a*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/S
qrt[1 + (b*x^4)/a]
```

Maple [C] time = 0.013, size = 411, normalized size = 1.

$$\frac{bfx^6}{8}\sqrt{bx^4+a} + \frac{5afx^2}{16}\sqrt{bx^4+a} + \frac{3fa^2}{16}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}} + \frac{bex^5}{7}\sqrt{bx^4+a} + \frac{3aex}{7}\sqrt{bx^4+a} + \frac{4a^2e}{7}\sqrt{1-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x)
```

```
[Out] 1/8*f*b*x^6*(b*x^4+a)^(1/2)+5/16*f*a*x^2*(b*x^4+a)^(1/2)+3/16*f*a^2*ln(x^2*
b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+1/7*e*b*x^5*(b*x^4+a)^(1/2)+3/7*e*a*x*(b*x
^4+a)^(1/2)+4/7*e*a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+1/6*d*b*x^4*(b*x^4+a)^(1/2)+2/3*d*a*(b*x^4+a)^(1/2)-1/2*
d*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-c*a*(b*x^4+a)^(1/2)/x+1/5
*c*b*x^3*(b*x^4+a)^(1/2)+12/5*I*c*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*c*a^(3/2)*b^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^2, x)
```

Sympy [A] time = 11.26, size = 406, normalized size = 1.

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}fx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1+\frac{bx^4}{a}}}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*
e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(5/4)) + a**(3/2)*f*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sqrt(1
 + b*x**4/a)) + sqrt(a)*b*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*f
*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1)
) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*d*x**2/(2
*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*
x**4)**(3/2)/(6*b), True)) + b**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.517 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3} dx$

Optimal. Leaf size=406

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E}{5\sqrt{a+bx^4}}$$

```
[Out] (12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*a*e +
3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*Sqrt[a + b*x^4])/
35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2) - ((7*d - f*x^2)*(a + b*x^4)
^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 -
(a^(3/2)*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*Sq
rt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)*
Sqrt[a + b*x^4])
```

Rubi [A] time = 0.343212, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1833, 1252, 813, 815, 844, 217, 206, 266, 63, 208, 1272, 1177, 1198, 220, 1196}

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\right.}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```



```
[Out] (12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*a*e +
3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*Sqrt[a + b*x^4])/
35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2) - ((7*d - f*x^2)*(a + b*x^4)
^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 -
(a^(3/2)*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)*(21*Sq
rt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*b^(1/4)*
Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}dx = \int \left(\frac{(c+ex^2)(a+bx^4)^{3/2}}{x^3} + \frac{(d+fx^2)(a+bx^4)^{3/2}}{x^2}\right)dx$$

$$= \int \frac{(c+ex^2)(a+bx^4)^{3/2}}{x^3}dx + \int \frac{(d+fx^2)(a+bx^4)^{3/2}}{x^2}dx$$

$$= -\frac{(7d-fx^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{2}\text{Subst}\left(\int \frac{(c+ex)(a+bx^2)^{3/2}}{x^2}dx, x, x^2\right) - \frac{6}{7}$$

$$= \frac{2}{35}x(5af+21bdx^2)\sqrt{a+bx^4} - \frac{(3c-ex^2)(a+bx^4)^{3/2}}{6x^2} - \frac{(7d-fx^2)(a+b}{7x}$$

$$= \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a+bx^4} - \frac{(3c-ex^2)(a}{6x^2}$$

$$= \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a}$$

$$= \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a}$$

$$= \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a}$$

$$= \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)} + \frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4} + \frac{2}{35}x(5af+21bdx^2)\sqrt{a}$$

Mathematica [C] time = 0.361783, size = 194, normalized size = 0.48

$$\frac{x\left(ex\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a+bx^4}(4a+bx^4) - 3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) - 6ad\sqrt{a+bx^4}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right) + 6afx^2\sqrt{a+bx^4}\,{}_2}{6x^2\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

```
[Out] (-3*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] +
x*(e*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*ArcTa
nh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*d*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2
, -1/4, 3/4, -((b*x^4)/a)] + 6*a*f*x^2*Sqrt[a + b*x^4]*Hypergeometric2F1[-3
/2, 1/4, 5/4, -((b*x^4)/a)]))/(6*x^2*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.02, size = 409, normalized size = 1.

$$\frac{bfx^5}{7}\sqrt{bx^4+a} + \frac{3afx}{7}\sqrt{bx^4+a} + \frac{4fa^2}{7}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x)
```

```
[Out] 1/7*f*b*x^5*(b*x^4+a)^(1/2)+3/7*f*a*x*(b*x^4+a)^(1/2)+4/7*f*a^2/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/6*e*b*x^4*(b
*x^4+a)^(1/2)+2/3*e*a*(b*x^4+a)^(1/2)-1/2*e*a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^
4+a)^(1/2))/x^2)+1/4*c*b*x^2*(b*x^4+a)^(1/2)+3/4*c*a*b^(1/2)*ln(x^2*b^(1/2)
+(b*x^4+a)^(1/2))-1/2*c*a/x^2*(b*x^4+a)^(1/2)-d*a*(b*x^4+a)^(1/2)/x+1/5*d*b
*x^3*(b*x^4+a)^(1/2)+12/5*I*d*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*d*a^(3/2)*b^(1/2)/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^3, x)
```

Sympy [A] time = 9.06234, size = 377, normalized size = 0.93

$$-\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}bcx^2\sqrt{1+\frac{bx^4}{a}}}{4}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

```
[Out] -a**(3/2)*c/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*d*gamma(-1/4)*hyper((-1/
2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*e*a
sinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*f*x*gamma(1/4)*hyper((-1/2, 1/4),
 (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*c*x**2*sqrt(1
 + b*x**4/a)/4 - sqrt(a)*b*c*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**3
*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7
/4)) + sqrt(a)*b*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(9/4)) + a**2*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1))
+ 3*a*sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*e*x**2/(2*sqrt(a/
(b*x**4) + 1)) + b*e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(
3/2)/(6*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.518 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=408

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E}{5\sqrt{a+bx^4}}$$

```
[Out] (12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*(9*a*e
- 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*Sqrt[a + b*x^4]
)/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15*x^3) - ((3*d - f*x^2)*(a + b*
x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/4 - (a^(3/2)*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*
e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*
b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(15*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.335922, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1833, 1272, 1198, 220, 1196, 1252, 813, 815, 844, 217, 206, 266, 63, 208}

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-}\right.}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```


```
[Out] (12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*(9*a*e
- 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*Sqrt[a + b*x^4]
)/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15*x^3) - ((3*d - f*x^2)*(a + b*
x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/4 - (a^(3/2)*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*
e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*
b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(15*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
```

```
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
```

```
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^4}dx = \int\left(\frac{\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}}{x^4}+\frac{\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}}{x^3}\right)dx$$

$$= \int \frac{\left(c+ex^2\right)\left(a+bx^4\right)^{3/2}}{x^4}dx + \int \frac{\left(d+fx^2\right)\left(a+bx^4\right)^{3/2}}{x^3}dx$$

$$= -\frac{\left(5c-3ex^2\right)\left(a+bx^4\right)^{3/2}}{15x^3} - \frac{2}{5}\int \frac{\left(-3ae-5bcx^2\right)\sqrt{a+bx^4}}{x^2}dx + \frac{1}{2}\text{Subst}\Big($$

$$= -\frac{2\left(9ae-5bcx^2\right)\sqrt{a+bx^4}}{15x} - \frac{\left(5c-3ex^2\right)\left(a+bx^4\right)^{3/2}}{15x^3} - \frac{\left(3d-fx^2\right)\left(a+bx\right.}{6x^2}$$

$$= -\frac{2\left(9ae-5bcx^2\right)\sqrt{a+bx^4}}{15x} + \frac{1}{4}\left(2af+3bdx^2\right)\sqrt{a+bx^4} - \frac{\left(5c-3ex^2\right)\left(a+\right.}{15x^3}$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2\left(9ae-5bcx^2\right)\sqrt{a+bx^4}}{15x} + \frac{1}{4}\left(2af+3bdx^2\right)\sqrt{a+bx}$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2\left(9ae-5bcx^2\right)\sqrt{a+bx^4}}{15x} + \frac{1}{4}\left(2af+3bdx^2\right)\sqrt{a+bx}$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2\left(9ae-5bcx^2\right)\sqrt{a+bx^4}}{15x} + \frac{1}{4}\left(2af+3bdx^2\right)\sqrt{a+bx}$$

$$= \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2\left(9ae-5bcx^2\right)\sqrt{a+bx^4}}{15x} + \frac{1}{4}\left(2af+3bdx^2\right)\sqrt{a+bx}$$

Mathematica [C] time = 0.357646, size = 194, normalized size = 0.48

$$\frac{x^2\left(fx\sqrt{\frac{bx^4}{a}+1}\left(\sqrt{a+bx^4}\left(4a+bx^4\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)-6ae\sqrt{a+bx^4}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)\right)-2ac\sqrt{a+bx^4}\,{}_2}{6x^3\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

```
[Out] (-2*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] -
3*a*d*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] +
x^2*(f*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*e*Sqrt[a + b*x^4]*Hypergeometric2F1[-3
/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.014, size = 408, normalized size = 1.

$$\frac{bfx^4}{6}\sqrt{bx^4+a}+\frac{2af}{3}\sqrt{bx^4+a}-\frac{f}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)-\frac{ac}{3x^3}\sqrt{bx^4+a}+\frac{bcx}{3}\sqrt{bx^4+a}+\frac{4abc}{3}\sqrt{1-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x)
```

```
[Out] 1/6*f*b*x^4*(b*x^4+a)^(1/2)+2/3*f*a*(b*x^4+a)^(1/2)-1/2*f*a^(3/2)*ln((2*a+2
*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/3*c*a*(b*x^4+a)^(1/2)/x^3+1/3*c*b*x*(b*x^4
+a)^(1/2)+4/3*c*a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)+1/4*d*b*x^2*(b*x^4+a)^(1/2)+3/4*d*a*b^(1/2)*ln(x^2*b^(1/2)
+(b*x^4+a)^(1/2))-1/2*d*a/x^2*(b*x^4+a)^(1/2)-e*a*(b*x^4+a)^(1/2)/x+1/5*e*b
*x^3*(b*x^4+a)^(1/2)+12/5*I*e*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*e*a^(3/2)*b^(1/2)/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac)\sqrt{bx^4+a}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^4, x)
```

Sympy [A] time = 9.12796, size = 381, normalized size = 0.93

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}bcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\cdots\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
```

```
[Out] a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*e
*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) +
 sqrt(a)*b*d*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 + b*x**
4/a)) + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**4) + 1))
 + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*f*x**2/(2*sqrt(a
/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**
(3/2)/(6*b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.519 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=386

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}}{5\sqrt{a+bx^4}}$$

```
[Out] (12*a*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (3*b*(c +
e*x^2)*Sqrt[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*Sqrt[a + b*x^4])/15 - ((
(3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^(3/2))/12 + (3*a*
Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(5/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*b^(1/4)*(5*Sqrt[b]*d +
9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.346981, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {14, 1825, 1833, 1252, 815, 844, 217, 206, 266, 63, 208, 1177, 1198, 220, 1196}

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```


```
[Out] (12*a*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (3*b*(c +
e*x^2)*Sqrt[a + b*x^4])/4 + (2*b*x*(5*d + 9*f*x^2)*Sqrt[a + b*x^4])/15 - ((
(3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^(3/2))/12 + (3*a*
Sqrt[b]*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*c*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(5/4)*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(3/4)*b^(1/4)*(5*Sqrt[b]*d +
9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{4}-\frac{dx}{3}-\frac{ex^2}{2}-fx^3\right)\sqrt{}}{x}$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-(6b)\int\left(\frac{\left(-\frac{c}{4}-\frac{ex^2}{2}\right)\sqrt{a+bx^4}}{x}+\right.$$

$$= -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-(6b)\int \frac{\left(-\frac{c}{4}-\frac{ex^2}{2}\right)\sqrt{a+bx^4}}{x}dx$$

$$= \frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}-\frac{1}{5}(2$$

$$= \frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\right.$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}$$

$$= \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5(\sqrt{a}+\sqrt{b}x^2)}+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}$$

Mathematica [C] time = 0.218816, size = 163, normalized size = 0.42

$$\frac{\sqrt{a+bx^4}\left(3x\left(-5a^3e\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^4}{a}\right)-10a^3fx\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)+bcx^2(a+bx^4)^2\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^4}{a}\right.\right.\right.}{30a^2x^3\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-10*a^3*d*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)
] + 3*x*(-5*a^3*e*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] - 10*a^3
*f*x*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)] + b*c*x^2*(a + b*x^4)
^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a])))/(30
*a^2*x^3*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.021, size = 409, normalized size = 1.1

$$-\frac{ad}{3x^3}\sqrt{bx^4+a}+\frac{bdx}{3}\sqrt{bx^4+a}+\frac{4bda}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x)
```

```
[Out] -1/3*d*a*(b*x^4+a)^(1/2)/x^3+1/3*d*b*x*(b*x^4+a)^(1/2)+4/3*d*a*b/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c*b*(b*x^
4+a)^(1/2)-3/4*c*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*c*a/
x^4*(b*x^4+a)^(1/2)+1/4*e*b*x^2*(b*x^4+a)^(1/2)+3/4*e*a*b^(1/2)*ln(x^2*b^(1
/2)+(b*x^4+a)^(1/2))-1/2*e*a/x^2*(b*x^4+a)^(1/2)-f*a*(b*x^4+a)^(1/2)/x+1/5*
f*b*x^3*(b*x^4+a)^(1/2)+12/5*I*f*a^(3/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*f*a^(3/2)*b^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac\right)\sqrt{bx^4 + a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^5, x)
```

Sympy [C] time = 10.7074, size = 379, normalized size = 0.98

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}} e}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bc \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bdx\Gamma\left(\frac{1}{4}\right) {}_{2}F}{4\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

```
[Out] a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*f
```

```
*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*x*ga
mma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)
) + sqrt(a)*b*e*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(1 + b*
x**4/a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2)
 + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*e*asinh(sqrt(b)*
x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.520 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=387

$$\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\right.}{}$$

```
[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(9*c -
5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqrt[a + b*x^4])/4 - ((
(12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/2))/60 +
(3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqrt[b]
*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^
4])
```

Rubi [A] time = 0.349002, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {14, 1825, 1833, 1272, 1198, 220, 1196, 1252, 815, 844, 217, 206, 266, 63, 208}

$$\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (12*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*b*(9*c -
5*e*x^2)*Sqrt[a + b*x^4])/(15*x) + (3*b*(d + f*x^2)*Sqrt[a + b*x^4])/4 - ((
(12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/2))/60 +
(3*a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/4 - (3*Sqrt[a]*b*d*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*c*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqrt[b]
*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a + b*x^
4])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
```

```
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{5}-\frac{dx}{4}-\frac{ex^2}{3}-\frac{fx^3}{2}\right.}{x^2}$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \left(\frac{\left(-\frac{c}{5}-\frac{ex^2}{3}\right)\sqrt{a+bx^4}}{x^2}\right.$$

$$= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{5}-\frac{ex^2}{3}\right)\sqrt{a+bx^4}}{x^2}$$

$$= -\frac{2b\left(9c-5ex^2\right)\sqrt{a+bx^4}}{15x} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\left(a+bx^4\right)^{3/2} - (3$$

$$= -\frac{2b\left(9c-5ex^2\right)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b\left(d+fx^2\right)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}\right.$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(9c-5ex^2\right)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b\left(d+fx^2\right)\sqrt{a+bx^4} - \frac{1}{60}$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(9c-5ex^2\right)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b\left(d+fx^2\right)\sqrt{a+bx^4} - \frac{1}{60}$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(9c-5ex^2\right)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b\left(d+fx^2\right)\sqrt{a+bx^4} - \frac{1}{60}$$

$$= \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(9c-5ex^2\right)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b\left(d+fx^2\right)\sqrt{a+bx^4} - \frac{1}{60}$$

Mathematica [C] time = 0.223091, size = 165, normalized size = 0.43

$$\frac{\sqrt{a+bx^4}\left(-6a^3c\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)-10a^3ex^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^4}{a}\right)-15a^3fx^3\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^4}{a}\right)+3bdx^5\left(a+\right.\right.}{30a^2x^5\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-6*a^3*c*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)
] - 10*a^3*e*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] - 15*a^3*
f*x^3*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + 3*b*d*x^5*(a + b*x
^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(
30*a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.015, size = 409, normalized size = 1.1

$$-\frac{ae}{3x^3}\sqrt{bx^4+a}+\frac{bxe}{3}\sqrt{bx^4+a}+\frac{4aeb}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x)
```

```
[Out] -1/3*e*a*(b*x^4+a)^(1/2)/x^3+1/3*e*b*x*(b*x^4+a)^(1/2)+4/3*e*a*b/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*c*a*(b*x^
4+a)^(1/2)/x^5-7/5*c*b*(b*x^4+a)^(1/2)/x+12/5*I*c*b^(3/2)*a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*c*b^(
3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)+1/2*d*b*(b*x^4+a)^(1/2)-3/4*d*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+
a)^(1/2))/x^2)-1/4*d*a/x^4*(b*x^4+a)^(1/2)+1/4*f*b*x^2*(b*x^4+a)^(1/2)+3/4*
f*a*b^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*f*a/x^2*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^6, x)
```

Sympy [C] time = 11.0208, size = 386, normalized size = 1.

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{a^{\frac{3}{2}}f}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bc\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{3\sqrt{a}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

```
[Out] a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*f/(2*x**2*sqrt(1
 + b*x**4/a)) + sqrt(a)*b*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*
x**2))/4 + sqrt(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4 - sqrt
(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x
**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*f*asinh(sqrt
(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.521 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=392

$$\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}d\ldots}{5\left(\sqrt{a}\ldots\right.}$$

```
[Out] (12*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (b*(2*c - 3*
e*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*Sqrt[a + b*x^4])/(15
*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^(3/2
))/60 + (b^(3/2)*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b
*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqr
t[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a +
b*x^4])
```

Rubi [A] time = 0.342193, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {14, 1825, 1833, 1252, 813, 844, 217, 206, 266, 63, 208, 1272, 1198, 220, 1196}

$$\frac{1}{2}b^{3/2}c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{2\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15\sqrt{a+bx^4}}+\frac{12b^{3/2}dx\sqrt{a+b\ldots}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```


```
[Out] (12*b^(3/2)*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (b*(2*c - 3*
e*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (2*b*(9*d - 5*f*x^2)*Sqrt[a + b*x^4])/(15
*x) - (((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^(3/2
))/60 + (b^(3/2)*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b
*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/4 - (12*a^(1/4)*b^(5/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(1/4)*b^(3/4)*(9*Sqr
t[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*Sqrt[a +
b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
```

```
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^7}\,dx = -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{6}-\frac{dx}{5}-\frac{ex^2}{4}-\frac{fx^3}{3}\right.}{x^3}$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2}-(6b)\int\left(\frac{\left(-\frac{c}{6}-\frac{ex^2}{4}\right)\sqrt{a+bx^4}}{x^3}\right.$$

$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{6}-\frac{ex^2}{4}\right)\sqrt{a+bx^4}}{x^3}$$

$$= -\frac{2b\left(9d-5fx^2\right)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\left(a+bx^4\right)^{3/2}-($$

$$= -\frac{b\left(2c-3ex^2\right)\sqrt{a+bx^4}}{4x^2}-\frac{2b\left(9d-5fx^2\right)\sqrt{a+bx^4}}{15x}-\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15}{x}\right.$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b\left(2c-3ex^2\right)\sqrt{a+bx^4}}{4x^2}-\frac{2b\left(9d-5fx^2\right)\sqrt{a+bx^4}}{15x}-$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b\left(2c-3ex^2\right)\sqrt{a+bx^4}}{4x^2}-\frac{2b\left(9d-5fx^2\right)\sqrt{a+bx^4}}{15x}-$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b\left(2c-3ex^2\right)\sqrt{a+bx^4}}{4x^2}-\frac{2b\left(9d-5fx^2\right)\sqrt{a+bx^4}}{15x}-$$

$$= \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{b\left(2c-3ex^2\right)\sqrt{a+bx^4}}{4x^2}-\frac{2b\left(9d-5fx^2\right)\sqrt{a+bx^4}}{15x}-$$

Mathematica [C] time = 0.201954, size = 163, normalized size = 0.42

$$\frac{\sqrt{a+bx^4}\left(-5a^3c\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^4}{a}\right)-6a^3dx\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)-10a^3fx^3\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^4}{a}\right)+3bex^6\left(a+\right.\right.}{30a^2x^6\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-5*a^3*c*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)
] - 6*a^3*d*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] - 10*a^3*f*
x^3*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] + 3*b*e*x^6*(a + b*x^4
)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))/(30
*a^2*x^6*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.02, size = 408, normalized size = 1.

$$-\frac{af}{3x^3}\sqrt{bx^4+a}+\frac{fbx}{3}\sqrt{bx^4+a}+\frac{4abf}{3}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x)
```

```
[Out] -1/3*f*a*(b*x^4+a)^(1/2)/x^3+1/3*f*b*x*(b*x^4+a)^(1/2)+4/3*f*a*b/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*d*a*(b*x^
4+a)^(1/2)/x^5-7/5*d*b*(b*x^4+a)^(1/2)/x+12/5*I*d*b^(3/2)*a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-12/5*I*d*b^(
3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)+1/2*e*b*(b*x^4+a)^(1/2)-3/4*e*a^(1/2)*b*ln((2*a+2*a^(1/2)*(b*x^4+
a)^(1/2))/x^2)-1/4*e*a/x^4*(b*x^4+a)^(1/2)+1/2*c*b^(3/2)*ln(x^2*b^(1/2)+(b*
x^4+a)^(1/2))-1/6*c*a/x^6*(b*x^4+a)^(1/2)-2/3*c*b/x^2*(b*x^4+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac\right)\sqrt{bx^4 + a}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^7, x)
```

Sympy [C] time = 9.89324, size = 406, normalized size = 1.04

$$\frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bc}{2x^2\sqrt{1 + \frac{bx^4}{a}}} + \frac{\sqrt{a}bd\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

```
[Out] a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
```

```
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*c/(2*x**2*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4
*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)
*x**2))/4 + sqrt(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_
polar(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*x**4) -
 a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*x**2*sqrt(a/(b*
x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*c*asinh(sqrt(b)*
x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*c*x**2/(2
*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

3.522 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=412

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}ex\sqrt{}}{5\left(\sqrt{a}+\right.}$$

```
[Out] (-12*b*e*Sqrt[a + b*x^4])/(5*x) + (12*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*(Sqrt
[a] + Sqrt[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*Sqrt[a + b*x^4])/(35*x^3) - (b*
(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (((60*c)/x^7 + (70*d)/x^6 + (84*
e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(3/2))/420 + (b^(3/2)*d*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/4 - (12*a^(1/4)*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[
a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.393938, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {14, 1825, 1833, 1272, 1282, 1198, 220, 1196, 1252, 813, 844, 217, 206, 266, 63, 208}

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+\frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```


```
[Out] (-12*b*e*Sqrt[a + b*x^4])/(5*x) + (12*b^(3/2)*e*x*Sqrt[a + b*x^4])/(5*(Sqrt
[a] + Sqrt[b]*x^2)) - (2*b*(5*c - 21*e*x^2)*Sqrt[a + b*x^4])/(35*x^3) - (b*
(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(4*x^2) - (((60*c)/x^7 + (70*d)/x^6 + (84*
e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(3/2))/420 + (b^(3/2)*d*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/2 - (3*Sqrt[a]*b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])
/4 - (12*a^(1/4)*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[
a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
```

```
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{7}-\frac{dx}{6}-\frac{ex^2}{5}-\frac{f}{}\right.}{x^4}$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} - (6b)\int\left(\frac{\left(-\frac{c}{7}-\frac{ex^2}{5}\right)\sqrt{a+}}{x^4}\right.$$

$$= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2} - (6b)\int \frac{\left(-\frac{c}{7}-\frac{ex^2}{5}\right)\sqrt{a+b}}{x^4}$$

$$= -\frac{2b\left(5c-21ex^2\right)\sqrt{a+bx^4}}{35x^3} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\left(a+bx^4\right)^{3/2}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} - \frac{2b\left(5c-21ex^2\right)\sqrt{a+bx^4}}{35x^3} - \frac{b\left(2d-3fx^2\right)\sqrt{a+bx^4}}{4x^2} - \frac{1}{42}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} - \frac{2b\left(5c-21ex^2\right)\sqrt{a+bx^4}}{35x^3} - \frac{b\left(2d-3fx^2\right)\sqrt{a+bx^4}}{4x^2} - \frac{1}{42}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(5c-21ex^2\right)\sqrt{a+bx^4}}{35x^3} - \frac{b\left(2d-\right.}{}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(5c-21ex^2\right)\sqrt{a+bx^4}}{35x^3} - \frac{b\left(2d-\right.}{}$$

$$= -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b\left(5c-21ex^2\right)\sqrt{a+bx^4}}{35x^3} - \frac{b\left(2d-\right.}{}$$

Mathematica [C] time = 0.221303, size = 164, normalized size = 0.4

$$\frac{\sqrt{a+bx^4}\left(7x\left(-5a^3d\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^4}{a}\right)-6a^3ex\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)+3bfx^6\left(a+bx^4\right)^2\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\right.\right.\right.}{210a^2x^7\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^4]*(-30*a^3*c*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a
)] + 7*x*(-5*a^3*d*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)] - 6*a^
3*e*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] + 3*b*f*x^6*(a + b*
x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a])))
/(210*a^2*x^7*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.015, size = 411, normalized size = 1.

$$-\frac{ae}{5x^5}\sqrt{bx^4+a}-\frac{7be}{5x}\sqrt{bx^4+a}+\frac{12i}{5}eb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x)
```

```
[Out] -1/5*e*a*(b*x^4+a)^(1/2)/x^5-7/5*b*e*(b*x^4+a)^(1/2)/x+12/5*I*e*b^(3/2)*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-12/5*I*e*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)+1/2*f*b*(b*x^4+a)^(1/2)-3/4*f*a^(1/2)*b*ln((2*a+2*a
^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/4*f*a/x^4*(b*x^4+a)^(1/2)+1/2*d*b^(3/2)*ln(x
^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*d*a/x^6*(b*x^4+a)^(1/2)-2/3*d*b/x^2*(b*x^4+
a)^(1/2)-1/7*c*a*(b*x^4+a)^(1/2)/x^7-3/7*c*b*(b*x^4+a)^(1/2)/x^3+4/7*c*b^2/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^8, x)
```

Sympy [C] time = 11.4562, size = 415, normalized size = 1.01

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}bc\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}bd}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{ab}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

```
[Out] a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*c*gamma(-3/4)
*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
- sqrt(a)*b*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hyper((
-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)
*b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(6*
x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*f/(2*x**2*sqr
t(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*d*asinh(s
qrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2*d*
x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.523 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=377

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{1}{2}b^{3/2}e\tanh^{-1}$$

```
[Out] -(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*Sqrt[a + b*x^4])/
560 + (12*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (((105
*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4)^(3/2))/840 +
 (b^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[Sq
rt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (12*a^(1/4)*b^(5/4)*f*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*d + 2
1*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a + b
*x^4])
```

Rubi [A] time = 0.370944, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1825, 1832, 266, 63, 208, 1885, 275, 217, 206, 1198, 220, 1196}

$$-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{35\sqrt[4]{a}\sqrt{a+bx^4}}+\frac{1}{2}b^{3/2}e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```


```
[Out] -(b*((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)*Sqrt[a + b*x^4])/
560 + (12*b^(3/2)*f*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (((105
*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4)^(3/2))/840 +
 (b^(3/2)*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (3*b^2*c*ArcTanh[Sq
rt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (12*a^(1/4)*b^(5/4)*f*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*b^(5/4)*(5*Sqrt[b]*d + 2
1*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35*a^(1/4)*Sqrt[a + b
*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^9}\,dx = -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{8}-\frac{dx}{7}-\frac{ex^2}{6}-f\right.}{x^5}$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{840}\left(\right.$$

$$= -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4}+\frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{1}{840}\left(\right.$$

Mathematica [C] time = 0.293568, size = 174, normalized size = 0.46

$$-\frac{\sqrt{a+bx^4}\left(7\left(40a^2ex^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^4}{a}\right)+48a^2fx^3\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)+15c\left(3b^2x^8\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+a\left(2a\right.\right.\right.\right.}{1680ax^8\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(240*a^2*d*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4
)/a)] + 7*(15*c*(a*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 3*b^2*x^8*ArcTanh[
```

```
Sqrt[1 + (b*x^4)/a]]) + 40*a^2*e*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(
(b*x^4)/a)] + 48*a^2*f*x^3*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)
]))/(1680*a*x^8*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.023, size = 416, normalized size = 1.1

$$-\frac{af}{5x^5}\sqrt{bx^4+a}-\frac{7fb}{5x}\sqrt{bx^4+a}+\frac{12i}{5}fb^{\frac{3}{2}}\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x)
```

```
[Out] -1/5*f*a*(b*x^4+a)^(1/2)/x^5-7/5*f*b*(b*x^4+a)^(1/2)/x+12/5*I*f*b^(3/2)*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-12/5*I*f*b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)-3/16*c*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2
))/x^2)-1/8*c*a/x^8*(b*x^4+a)^(1/2)-5/16*c*b/x^4*(b*x^4+a)^(1/2)+1/2*e*b^(3
/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/6*e*a/x^6*(b*x^4+a)^(1/2)-2/3*e*b/x^2
*(b*x^4+a)^(1/2)-1/7*d*a*(b*x^4+a)^(1/2)/x^7-3/7*d*b*(b*x^4+a)^(1/2)/x^3+4/
7*d*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^9, x)
```

Sympy [C] time = 12.9674, size = 444, normalized size = 1.18

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}bd\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}be}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

```
[Out] a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*d*gamma(-3/4)
*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4))
- sqrt(a)*b*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hyper((
-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**2*c/(8
*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**
4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c*sqrt(a/(b
*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)
*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*asinh(sqrt(b)*x**2/sqrt(a))/2 - 3*b*
*2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*e*x**2/(2*sqrt(a)*sq
rt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

3.524 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=405

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*Sqrt[a + b*x^4]
)/1680 - (4*b^2*c*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*c*x*Sqrt[a + b*x^4
])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7
+ (84*f)/x^6)*(a + b*x^4)^(3/2))/504 + (b^(3/2)*f*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a + b*x^4]])/2 - (3*b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a])
- (4*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a +
 b*x^4]) + (2*b^(7/4)*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.425, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 844, 217, 206, 266, 63, 208}

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```


```
[Out] -(b*((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*Sqrt[a + b*x^4]
)/1680 - (4*b^2*c*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*c*x*Sqrt[a + b*x^4
])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7
+ (84*f)/x^6)*(a + b*x^4)^(3/2))/504 + (b^(3/2)*f*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a + b*x^4]])/2 - (3*b^2*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a])
- (4*b^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a +
 b*x^4]) + (2*b^(7/4)*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
```

```
x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx = -\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2}-(6b)\int \frac{\left(-\frac{c}{9}-\frac{dx}{8}-\frac{ex^2}{7}-\frac{fx}{6}\right.}{x^6}$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}\right.$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}\right.$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$= -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

Mathematica [C] time = 0.314219, size = 174, normalized size = 0.43

$$-\frac{\sqrt{a+bx^4}\left(3x\left(7\left(8a^2fx^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^4}{a}\right)+9b^2dx^8\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+3ad\left(2a+5bx^4\right)\sqrt{\frac{bx^4}{a}+1}\right)+48a^2ex\,{}_2F_1\right.\right.}{1008ax^9\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(112*a^2*c*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/
a)] + 3*x*(48*a^2*e*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)] + 7
*(3*a*d*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 9*b^2*d*x^8*ArcTanh[Sqrt[1 +
(b*x^4)/a]] + 8*a^2*f*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)]
))))/(1008*a*x^9*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.018, size = 437, normalized size = 1.1

$$-\frac{3\,b^2 d}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{ad}{8\,x^8}\sqrt{bx^4+a}-\frac{5\,bd}{16\,x^4}\sqrt{bx^4+a}+\frac{f}{2}b^{\frac{3}{2}}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)-\frac{af}{6\,x^6}\sqrt{bx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x)
```

```
[Out] -3/16*d*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/8*d*a/x^8*(b*
x^4+a)^(1/2)-5/16*d*b/x^4*(b*x^4+a)^(1/2)+1/2*f*b^(3/2)*ln(x^2*b^(1/2)+(b*x
^4+a)^(1/2))-1/6*f*a/x^6*(b*x^4+a)^(1/2)-2/3*f*b/x^2*(b*x^4+a)^(1/2)-1/7*e*
a*(b*x^4+a)^(1/2)/x^7-3/7*e*b*(b*x^4+a)^(1/2)/x^3+4/7*e*b^2/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/9*c*a*(b*x^4+a)^
(1/2)/x^9-11/45*c*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2*c*(b*x^4+a)^(1/2)/a/x+4/15
*I*c/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-4/15*I*c/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^7 + bex^6 + bdx^5 + bcx^4 + afx^3 + aex^2 + adx + ac)\sqrt{bx^4 + a}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^10, x)
```

Sympy [C] time = 13.5714, size = 449, normalized size = 1.11

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4}, -\frac{1}{2}\\ -\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4}, -\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4}, -\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{3}{4}\right){}_2F_1}{4x^3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

```
[Out] a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*c*gamma(-5/4)
*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)
) + sqrt(a)*b*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a)) - a**2
*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*sqrt(a/(
b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d*sqrt
(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sqrt(a/(b*x**4) + 1)) - b**
(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*f*asinh(sqrt(b)*x**2/sqrt(a))/2 -
 3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*f*x**2/(2*sqrt(
a)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.525 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=399

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*Sqrt[a + b*x^4]
)/1680 - (b^2*c*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*d*Sqrt[a + b*x^4])/(15
*a*x) + (4*b^(5/2)*d*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - ((
(252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x^4)^(3/2))/
2520 - (3*b^2*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)
*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) + (
2*b^(7/4)*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.420172, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {14, 1825, 1833, 1252, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```


```
[Out] -(b*((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/x^3)*Sqrt[a + b*x^4]
)/1680 - (b^2*c*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*d*Sqrt[a + b*x^4])/(15
*a*x) + (4*b^(5/2)*d*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - ((
(252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x^4)^(3/2))/
2520 - (3*b^2*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)
*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) + (
2*b^(7/4)*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(105*a^(3/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
```

```
0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx = -\frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right)^{3/2}}{2520} - (6b)\int \frac{\left(-\frac{c}{10}-\frac{dx}{9}-\frac{ex^2}{8}-\frac{fx^3}{7}\right)\sqrt{a+}}{x^7}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right.}{2520}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right.}{2520}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)\left(a+bx^4\right.}{2520}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}\right.}{2}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} - $$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b}{15}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b}{15}$$

$$= -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b}{15}$$

Mathematica [C] time = 0.326002, size = 171, normalized size = 0.43

$$-\frac{\sqrt{a+bx^4}\left(63\sqrt{\frac{bx^4}{a}+1}\left(2a^2\left(4c+5ex^2\right)+abx^4\left(16c+25ex^2\right)+8b^2cx^8\right)+560a^2dx\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^4}{a}\right)+720a^2fx^3\,{}_2\right.}{5040ax^{10}\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(63*Sqrt[1 + (b*x^4)/a]*(8*b^2*c*x^8 + 2*a^2*(4*c + 5*e*x
^2) + a*b*x^4*(16*c + 25*e*x^2)) + 945*b^2*e*x^10*ArcTanh[Sqrt[1 + (b*x^4)/
a]] + 560*a^2*d*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)] + 720*a
^2*f*x^3*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)]))/(5040*a*x^10*S
qrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.019, size = 417, normalized size = 1.1

$$-\frac{c\left(b^2x^8+2\,abx^4+a^2\right)}{10\,x^{10}a}\sqrt{bx^4+a}-\frac{3\,b^2e}{16}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{ae}{8\,x^8}\sqrt{bx^4+a}-\frac{5\,be}{16\,x^4}\sqrt{bx^4+a}-\frac{af}{7\,x^7}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x)
```

```
[Out] -1/10*c*(b*x^4+a)^(1/2)/x^10/a*(b^2*x^8+2*a*b*x^4+a^2)-3/16*e*b^2/a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/8*e*a/x^8*(b*x^4+a)^(1/2)-5/16*e*b
/x^4*(b*x^4+a)^(1/2)-1/7*f*a*(b*x^4+a)^(1/2)/x^7-3/7*f*b*(b*x^4+a)^(1/2)/x^
3+4/7*f*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-1/9*d*a*(b*x^4+a)^(1/2)/x^9-11/45*d*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2
*d*(b*x^4+a)^(1/2)/a/x+4/15*I*d/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*d/a^(1/2)*b^(5/2)/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bx^4+a\right)^{\frac{5}{2}}c}{10\,ax^{10}}+\int\frac{\left(bfx^6+bex^5+bdx^4+afx^2+aex+ad\right)\sqrt{bx^4+a}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(b*x^4 + a)^(5/2)*c/(a*x^10) + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4
 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^10, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^{11}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^11, x)
```

Sympy [C] time = 15.602, size = 398, normalized size = 1.

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}bd\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\sqrt{a}bf\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(-\right.}{4x^3\Gamma\left(\frac{1}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

```
[Out] a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*d*gamma(-5/4)
*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)
) + sqrt(a)*b*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*
pi)/a)/(4*x**3*gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1))
- a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**6*sqrt(
a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)*e*sq
rt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*sqrt(a/(b*x**4) + 1)) - b
**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*e*asinh(sqrt(a)/(sqrt(b)*x**
2))/(16*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11, x)
```

3.526 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}} dx$

Optimal. Leaf size=424

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E}{15a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*Sqrt[a + b*
x^4])/18480 - (4*b^2*c*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*d*Sqrt[a + b*x^4]
)/(10*a*x^2) - (4*b^2*e*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*e*x*Sqrt[a +
 b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((360*c)/x^11 + (396*d)/x^10 + (
440*e)/x^9 + (495*f)/x^8)*(a + b*x^4)^(3/2))/3960 - (3*b^2*f*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*c - 77*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*
x^4])
```

Rubi [A] time = 0.461727, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 807, 266, 63, 208}

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```


```
[Out] -(b*((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (3465*f)/x^4)*Sqrt[a + b*
x^4])/18480 - (4*b^2*c*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*d*Sqrt[a + b*x^4]
)/(10*a*x^2) - (4*b^2*e*Sqrt[a + b*x^4])/(15*a*x) + (4*b^(5/2)*e*x*Sqrt[a +
 b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)) - (((360*c)/x^11 + (396*d)/x^10 + (
440*e)/x^9 + (495*f)/x^8)*(a + b*x^4)^(3/2))/3960 - (3*b^2*f*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/(16*Sqrt[a]) - (4*b^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4]) - (2*b^(9/4)*(15*Sqrt[b]*c - 77*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*
x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^{12}}\,dx = -\frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\left(a+bx^4\right)^{3/2}}{3960} - (6b)\int \frac{\left(-\frac{c}{11}-\frac{dx}{10}-\frac{ex^2}{9}-\frac{fx^3}{8}\right)\cdots}{x^8}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\cdots}{3960}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\cdots}{3960}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)\cdots}{3960}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}\cdots\right.}{\cdots}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx\cdots}}{10ax^2}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx\cdots}}{10ax^2}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx\cdots}}{10ax^2}$$

$$= -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2c\sqrt{a+bx^4}}{77ax^3} - \frac{b^2d\sqrt{a+bx\cdots}}{10ax^2}$$

Mathematica [C] time = 0.38134, size = 172, normalized size = 0.41

$$-\frac{\sqrt{a+bx^4}\left(11x\left(9\sqrt{\frac{bx^4}{a}+1}\left(2a^2\left(4d+5fx^2\right)+abx^4\left(16d+25fx^2\right)+8b^2dx^8\right)+80a^2ex\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^4}{a}\right)+135\cdots\right.\right.}{7920ax^{11}\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(720*a^2*c*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^4)
/a)] + 11*x*(9*Sqrt[1 + (b*x^4)/a]*(8*b^2*d*x^8 + 2*a^2*(4*d + 5*f*x^2) + a
*b*x^4*(16*d + 25*f*x^2)) + 135*b^2*f*x^10*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 8
0*a^2*e*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)])))/(7920*a*x^11
*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.02, size = 441, normalized size = 1.

$$-\frac{ac}{11\,x^{11}}\sqrt{bx^4+a}-\frac{13\,bc}{77\,x^7}\sqrt{bx^4+a}-\frac{4\,b^2c}{77\,ax^3}\sqrt{bx^4+a}-\frac{4\,b^3c}{77\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x)
```

```
[Out] -1/11*c*a*(b*x^4+a)^(1/2)/x^11-13/77*c*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2*c*(b*
x^4+a)^(1/2)/a/x^3-4/77*c/a*b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)-3/16*f*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)
^(1/2))/x^2)-1/8*f*a/x^8*(b*x^4+a)^(1/2)-5/16*f*b/x^4*(b*x^4+a)^(1/2)-1/9*e
*a*(b*x^4+a)^(1/2)/x^9-11/45*e*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2*e*(b*x^4+a)^(
1/2)/a/x+4/15*I*e/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*e/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/10*d*(b*x^4+a)^(1/2)
/x^10/a*(b^2*x^8+2*a*b*x^4+a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^{12}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^12, x)
```

Sympy [C] time = 15.4933, size = 401, normalized size = 0.95

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}bc\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}be\Gamma\left(-\frac{5}{4}\right){}_2F}{4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

```
[Out] a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyper((-9/4, -1/2), (-5
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*c*gamma(-7
/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) +
 1)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(16*x**6*
sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3/2)
*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*x**2*sqrt(a/(b*x**4) + 1)
) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*f*asinh(sqrt(a)/(sqrt(b
)*x**2))/(16*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

3.527 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}} dx$

Optimal. Leaf size=449

$$-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}+\frac{b^3c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{4b^{9/4}f\left(\sqrt{a}+\right.}{}$$

```
[Out] -(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*Sqrt[a + b*
x^4])/18480 - (b^2*c*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b^2*d*Sqrt[a + b*x^4]
)/(77*a*x^3) - (b^2*e*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*f*Sqrt[a + b*x^4
])/(15*a*x) + (4*b^(5/2)*f*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)
) - (((165*c)/x^12 + (180*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b*x^4)
^(3/2))/1980 + (b^3*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) - (4*b
^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4
]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.490132, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1825, 1833, 1252, 835, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{b^3c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}-\frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{}}{15a^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```


```
[Out] -(b*((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2464*f)/x^5)*Sqrt[a + b*
x^4])/18480 - (b^2*c*Sqrt[a + b*x^4])/(32*a*x^4) - (4*b^2*d*Sqrt[a + b*x^4]
)/(77*a*x^3) - (b^2*e*Sqrt[a + b*x^4])/(10*a*x^2) - (4*b^2*f*Sqrt[a + b*x^4
])/(15*a*x) + (4*b^(5/2)*f*x*Sqrt[a + b*x^4])/(15*a*(Sqrt[a] + Sqrt[b]*x^2)
) - (((165*c)/x^12 + (180*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b*x^4)
^(3/2))/1980 + (b^3*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) - (4*b
^(9/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a + b*x^4
]) - (2*b^(9/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(1155*a^(5/4)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
```

```
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
```

```
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = -\frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(a+bx^4)^{3/2}}{1980} - (6b)\int \frac{\left(-\frac{c}{12}-\frac{dx}{11}-\frac{ex^2}{10}-\frac{fx^3}{9}\right)\sqrt{}}{x^9}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(}{1980}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(}{1980}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(}{1980}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}\right.}{}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx}}{77ax^3}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx}}{77ax^3}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx}}{77ax^3}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx}}{77ax^3}$$

$$= -\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx}}{77ax^3}$$

Mathematica [C] time = 0.191164, size = 149, normalized size = 0.33

$$-\frac{\sqrt{a+bx^4}\left(11x\left(9\left(a+bx^4\right)^2\sqrt{\frac{bx^4}{a}+1}\left(a^3e-b^3cx^{10}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^4}{a}+1\right)\right)+10a^5fx\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^4}{a}\right)\right)+90a^5d\right.}{990a^4x^{11}\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(90*a^5*d*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^4)/
a)] + 11*x*(10*a^5*f*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)] +
9*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*e - b^3*c*x^10*Hypergeometric2F1[5
/2, 4, 7/2, 1 + (b*x^4)/a]))))/(990*a^4*x^11*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.023, size = 462, normalized size = 1.

$$-\frac{ad}{11\,x^{11}}\sqrt{bx^4+a}-\frac{13\,bd}{77\,x^7}\sqrt{bx^4+a}-\frac{4\,b^2d}{77\,ax^3}\sqrt{bx^4+a}-\frac{4\,b^3d}{77\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x)
```

```
[Out] -1/11*d*a*(b*x^4+a)^(1/2)/x^11-13/77*d*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2*d*(b*
x^4+a)^(1/2)/a/x^3-4/77*d/a*b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)-1/10*e*(b*x^4+a)^(1/2)/x^10/a*(b^2*x^8+2*a*b*x
^4+a^2)-7/48*c*b/x^8*(b*x^4+a)^(1/2)-1/32*b^2*c*(b*x^4+a)^(1/2)/a/x^4+1/32*
c/a^(3/2)*b^3*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/12*c*a/x^12*(b*x^4+
a)^(1/2)-1/9*f*a*(b*x^4+a)^(1/2)/x^9-11/45*f*b*(b*x^4+a)^(1/2)/x^5-4/15*b^2
*f*(b*x^4+a)^(1/2)/a/x+4/15*I*f/a^(1/2)*b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-4/15*I*f/a^(1/2)*b^(5/2)/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^{13}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^13, x)
```

Sympy [C] time = 20.1769, size = 403, normalized size = 0.9

$$\frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}bd\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{\sqrt{a}bf\Gamma\left(-\frac{5}{4}\right){}_2F_1}{4x^5\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

```
[Out] a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyper((-9/4, -1/2), (-5
/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*d*gamma(-7
/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3
/4)) + sqrt(a)*b*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*x**4)
+ 1)) - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a
/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*x**4) + 1)) - b
**(3/2)*e*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*c/(32*a*x**2*sqrt(a/(b*x
**4) + 1)) - b**(5/2)*e*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*c*asinh(sqrt(a)/
(sqrt(b)*x**2))/(32*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13, x)
```

3.528 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} dx$

Optimal. Leaf size=474

$$-\frac{2b^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}-\frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}$$

```
[Out] -(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^6)*Sqrt[a
+ b*x^4])/240240 - (4*b^2*c*Sqrt[a + b*x^4])/(195*a*x^5) - (b^2*d*Sqrt[a +
b*x^4])/(32*a*x^4) - (4*b^2*e*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*f*Sqrt[a +
 b*x^4])/(10*a*x^2) + (4*b^3*c*Sqrt[a + b*x^4])/(65*a^2*x) - (4*b^(7/2)*c*x
*Sqrt[a + b*x^4])/(65*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (((660*c)/x^13 + (715*
d)/x^12 + (780*e)/x^11 + (858*f)/x^10)*(a + b*x^4)^(3/2))/8580 + (b^3*d*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^4]) - (2*b^(11/4)*(77*Sqr
t[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7/4
)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.549361, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {14, 1825, 1833, 1282, 1198, 220, 1196, 1252, 835, 807, 266, 63, 208}

$$-\frac{2b^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}-\frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4b^3c\sqrt{a+bx^4}}{65a^2x}+\frac{4b^{13/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```


```
[Out] -(b*((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7 + (24024*f)/x^6)*Sqrt[a
+ b*x^4])/240240 - (4*b^2*c*Sqrt[a + b*x^4])/(195*a*x^5) - (b^2*d*Sqrt[a +
b*x^4])/(32*a*x^4) - (4*b^2*e*Sqrt[a + b*x^4])/(77*a*x^3) - (b^2*f*Sqrt[a +
 b*x^4])/(10*a*x^2) + (4*b^3*c*Sqrt[a + b*x^4])/(65*a^2*x) - (4*b^(7/2)*c*x
*Sqrt[a + b*x^4])/(65*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (((660*c)/x^13 + (715*
d)/x^12 + (780*e)/x^11 + (858*f)/x^10)*(a + b*x^4)^(3/2))/8580 + (b^3*d*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]])/(32*a^(3/2)) + (4*b^(13/4)*c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(65*a^(7/4)*Sqrt[a + b*x^4]) - (2*b^(11/4)*(77*Sqr
t[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*a^(7/4
)*Sqrt[a + b*x^4])
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = -\frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} - (6b)\int \frac{\left(-\frac{c}{13}-\frac{dx}{12}-\frac{ex^2}{11}-\frac{fx^3}{10}\right)}{x^{10}}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858}{x^{10}}\right.}{8580}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858}{x^{10}}\right.}{8580}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858}{x^{10}}\right.}{8580}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \left(\frac{660c}{x^{13}}+\right.$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+}}{32ax}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+}}{32ax}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+}}{32ax}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+}}{32ax}$$

$$= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+}}{32ax}$$

Mathematica [C] time = 0.183965, size = 151, normalized size = 0.32

$$-\frac{\sqrt{a+bx^4}\left(13x^2\left(11x\left(a+bx^4\right)^2\sqrt{\frac{bx^4}{a}+1}\left(a^3f-b^3dx^{10}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^4}{a}+1\right)\right)+10a^5e\,{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{bx^4}{a}\right)\right)+110\right.}{1430a^4x^{13}\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

```
[Out] -(Sqrt[a + b*x^4]*(110*a^5*c*Hypergeometric2F1[-13/4, -3/2, -9/4, -((b*x^4)
/a)] + 13*x^2*(10*a^5*e*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^4)/a)]
+ 11*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*f - b^3*d*x^10*Hypergeometric
2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(1430*a^4*x^13*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.02, size = 483, normalized size = 1.

$$-\frac{ae}{11\,x^{11}}\sqrt{bx^4+a}-\frac{13\,be}{77\,x^7}\sqrt{bx^4+a}-\frac{4\,b^2e}{77\,ax^3}\sqrt{bx^4+a}-\frac{4\,eb^3}{77\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x)
```

```
[Out] -1/11*e*a*(b*x^4+a)^(1/2)/x^11-13/77*e*b*(b*x^4+a)^(1/2)/x^7-4/77*b^2*e*(b*
x^4+a)^(1/2)/a/x^3-4/77*e/a*b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)-1/13*c*a*(b*x^4+a)^(1/2)/x^13-5/39*c*b*(b*x^4+
a)^(1/2)/x^9-4/195*b^2*c*(b*x^4+a)^(1/2)/a/x^5+4/65*b^3*c*(b*x^4+a)^(1/2)/a
^2/x-4/65*I*c/a^(3/2)*b^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)+4/65*I*c/a^(3/2)*b^(7/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-7/48*d*b/x^8*(b*x^4+a)^(1/
2)-1/32*b^2*d*(b*x^4+a)^(1/2)/a/x^4+1/32*d/a^(3/2)*b^3*ln((2*a+2*a^(1/2)*(b
*x^4+a)^(1/2))/x^2)-1/12*d*a/x^12*(b*x^4+a)^(1/2)-1/10*f*(b*x^4+a)^(1/2)/x^
10/a*(b^2*x^8+2*a*b*x^4+a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bfx^7+bex^6+bdx^5+bcx^4+afx^3+aex^2+adx+ac\right)\sqrt{bx^4+a}}{x^{14}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*x
 + a*c)*sqrt(b*x^4 + a)/x^14, x)
```

Sympy [C] time = 21.7902, size = 403, normalized size = 0.85

$$\frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4},-\frac{1}{2}\\-\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)}+\frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}+\frac{\sqrt{a}bc\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)}+\frac{\sqrt{a}be\Gamma\left(-\frac{7}{4}\right){}_2F_1}{4x^7\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

```
[Out] a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hyper((-11/4, -1/2), (
-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + sqrt(a)*b*c*gamma
(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma
(-5/4)) + sqrt(a)*b*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/(b*x**
4) + 1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqr
t(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/(b*x**4) + 1))
- b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*d/(32*a*x**2*sqrt(a/(
b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*d*asinh(sqrt(
a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

3.529 $\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=361

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan\ldots\right.}{5b^{7/4}\sqrt{a+bx^4}}$$

```
[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a
 + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*
x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])/(12*b^2) - (a*d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*
Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x
^4])
```

Rubi [A] time = 0.30039, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1833, 1280, 1198, 220, 1196, 1252, 833, 780, 217, 206}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{5b^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```


```
[Out] (c*x*Sqrt[a + b*x^4])/(3*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a
 + b*x^4])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*
x^2)) - ((4*a*f - 3*b*d*x^2)*Sqrt[a + b*x^4])/(12*b^2) - (a*d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*c + 9*
Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x
^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
```

```
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4 \left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{x^4 \left(c+ex^2\right)}{\sqrt{a+bx^4}}+\frac{x^5 \left(d+fx^2\right)}{\sqrt{a+bx^4}}\right)dx$$
$$= \int \frac{x^4 \left(c+ex^2\right)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^5 \left(d+fx^2\right)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+fx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{x^2\left(3ae-5bcx^2\right)}{\sqrt{a+bx^4}}\,dx}{5b}$$
$$= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}+\frac{\int \frac{-5abc-9abex^2}{\sqrt{a+bx^4}}\,dx}{15b^2}+\frac{\text{Subst}\left(\int \frac{x(-2af+3bdx)}{\sqrt{a+bx^2}}\right.}{6b}$$
$$= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{\left(4af-3bdx^2\right)\sqrt{a+bx^4}}{12b^2}-\frac{(ad)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(4af-3bdx^2\right)\sqrt{a+}}{12b^2}$$
$$= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(4af-3bdx^2\right)\sqrt{a+}}{12b^2}$$

Mathematica [C] time = 0.130861, size = 212, normalized size = 0.59

$$\frac{-20a^2f-20abcx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+20abcx+15abdx^2-15a\sqrt{b}d\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-12abex^3\sqrt{\frac{bx^4}{a}}+}{60b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (-20*a^2*f + 20*a*b*c*x + 15*a*b*d*x^2 + 12*a*b*e*x^3 - 10*a*b*f*x^4 + 20*b
^2*c*x^5 + 15*b^2*d*x^6 + 12*b^2*e*x^7 + 10*b^2*f*x^8 - 15*a*Sqrt[b]*d*Sqrt
[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*b*c*x*Sqrt[1 + (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*b*e*x^3*Sqrt
[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^2*Sqr
t[a + b*x^4])
```

Maple [C] time = 0.018, size = 335, normalized size = 0.9

$$-\frac{f\left(-bx^4+2a\right)}{6b^2}\sqrt{bx^4+a}+\frac{ex^3}{5b}\sqrt{bx^4+a}-\frac{3i}{5}ea^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/6*f*(b*x^4+a)^(1/2)*(-b*x^4+2*a)/b^2+1/5*e*x^3*(b*x^4+a)^(1/2)/b-3/5*I*e
/b^(3/2)*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*
```

```
(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)+3/5*I*e/b^(3/2)*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipt
icE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*d*x^2/b*(b*x^4+a)^(1/2)-1/4*d*a/b^(3
/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*c*x*(b*x^4+a)^(1/2)/b-1/3*c/b*a/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^7 + ex^6 + dx^5 + cx^4}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 6.13838, size = 177, normalized size = 0.49

$$\frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases} -\frac{a\sqrt{a+bx^4}}{3b^2} + \frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right)}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b*x
**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), True)) + c*x**5*gamma(5/4)*hyper(
(1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e*x*
*7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{\sqrt{bx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

3.530 $\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=336

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(\cdots\right.}{5b^{7/4}\sqrt{a+bx^4}}$$

```
[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt
[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a +
b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) +
(3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*
x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.265053, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1833, 1252, 780, 217, 206, 1280, 1198, 220, 1196}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\cdots\right.\right.}{5b^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (d*x*Sqrt[a + b*x^4])/(3*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt
[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*x^2)*Sqrt[a +
b*x^4])/(4*b) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) +
(3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*
x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{x^3\left(c+ex^2\right)}{\sqrt{a+bx^4}}+\frac{x^4\left(d+fx^2\right)}{\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{x^3\left(c+ex^2\right)}{\sqrt{a+bx^4}}\,dx + \int \frac{x^4\left(d+fx^2\right)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+ex)}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\int \frac{x^2\left(3af-5bdx^2\right)}{\sqrt{a+bx^4}}\,dx}{5b}$$

$$= \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{\left(2c+ex^2\right)\sqrt{a+bx^4}}{4b} + \frac{\int \frac{-5abd-9abfx^2}{\sqrt{a+bx^4}}\,dx}{15b^2} - \frac{(ae)\,\text{Subst}}{}$$

$$= \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} + \frac{\left(2c+ex^2\right)\sqrt{a+bx^4}}{4b} - \frac{(ae)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+b}}\right.}{4b}$$

$$= \frac{dx\sqrt{a+bx^4}}{3b} + \frac{fx^3\sqrt{a+bx^4}}{5b} - \frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{\left(2c+ex^2\right)\sqrt{a+bx^4}}{4b} - \frac{ae\tanh^{-1}}{4}$$

Mathematica [C] time = 0.145449, size = 212, normalized size = 0.63

$$\frac{30\sqrt{b}c\left(a+bx^4\right) - 20a\sqrt{b}dx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right) + 20\sqrt{b}dx\left(a+bx^4\right) + 15\sqrt{b}ex^2\left(a+bx^4\right) - 15ae\sqrt{a+bx^4}\,\text{tan}}{60b^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (30*Sqrt[b]*c*(a + b*x^4) + 20*Sqrt[b]*d*x*(a + b*x^4) + 15*Sqrt[b]*e*x^2*(
a + b*x^4) + 12*Sqrt[b]*f*x^3*(a + b*x^4) - 15*a*e*Sqrt[a + b*x^4]*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a
+ b*x^4])
```

Maple [C] time = 0.007, size = 325, normalized size = 1.

$$\frac{fx^3}{5b}\sqrt{bx^4+a} - \frac{3i}{5}fa^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + \frac{3i}{5}fa^{\frac{3}{2}}\sqrt{1-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/5*f*x^3*(b*x^4+a)^(1/2)/b-3/5*I*f/b^(3/2)*a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*f/b^(3/2)*a^(3/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*e
*x^2/b*(b*x^4+a)^(1/2)-1/4*e*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/3*
d*x*(b*x^4+a)^(1/2)/b-1/3*d/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
```

```
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c/b*(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{bx^4+a}c}{2b}+\int\frac{fx^6+ex^5+dx^4}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)/sqrt(b*x^4 + a)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^6+ex^5+dx^4+cx^3}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 5.02286, size = 156, normalized size = 0.46

$$\frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*e*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + c*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar
(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/
4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^3+ex^2+dx+c\right)x^3}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

3.531 $\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=308

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\cdots\right.\right.}{b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqr
t[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*f*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.221998, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1833, 1280, 1198, 220, 1196, 1252, 780, 217, 206}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```


```
[Out] (e*x*Sqrt[a + b*x^4])/(3*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqr
t[b]*x^2)) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/(4*b) - (a*f*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*c - Sqrt[a]*e)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx^4}}\,dx &= \int \left(\frac{x^2\left(c+ex^2\right)}{\sqrt{a+bx^4}}+\frac{x^3\left(d+fx^2\right)}{\sqrt{a+bx^4}}\right)dx \\
&= \int \frac{x^2\left(c+ex^2\right)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^3\left(d+fx^2\right)}{\sqrt{a+bx^4}}\,dx \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(d+fx)}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{ae-3bcx^2}{\sqrt{a+bx^4}}\,dx}{3b} \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{\left(2d+fx^2\right)\sqrt{a+bx^4}}{4b}-\frac{\left(\sqrt{a}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\frac{\left(\sqrt{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}}{3b} \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(2d+fx^2\right)\sqrt{a+bx^4}}{4b}-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)}}}{b^{3/4}\sqrt{a+b}} \\
&= \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{\left(2d+fx^2\right)\sqrt{a+bx^4}}{4b}-\frac{af\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{\sqrt[4]{a}c\left(\sqrt{}\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.168889, size = 193, normalized size = 0.63

$$
\frac{4b^{3/2}cx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)+6\sqrt{b}d\left(a+bx^4\right)-4a\sqrt{b}ex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+4\sqrt{b}ex\left(a+bx^4\right)+3\sqrt{b}fx}{12b^{3/2}\sqrt{a+bx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (6*Sqrt[b]*d*(a + b*x^4) + 4*Sqrt[b]*e*x*(a + b*x^4) + 3*Sqrt[b]*f*x^2*(a +
 b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 4*
a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4
)/a)] + 4*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4
, -((b*x^4)/a)])/(12*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.006, size = 248, normalized size = 0.8

$$
\frac{fx^2}{4b}\sqrt{bx^4+a}-\frac{af}{4}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}+\frac{ex}{3b}\sqrt{bx^4+a}-\frac{ae}{3b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/4*f*x^2/b*(b*x^4+a)^(1/2)-1/4*f*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
+1/3*e*x*(b*x^4+a)^(1/2)/b-1/3*e/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/b*(b*x^4+a)^(1/2)+I*c*a^(1/2)/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

```
-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{\sqrt{bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^5 + ex^4 + dx^3 + cx^2}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 4.7238, size = 156, normalized size = 0.51

$$\frac{\sqrt{a}fx^2\sqrt{1 + \frac{bx^4}{a}}}{4b} - \frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + d\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for}\ b = 0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/sqrt(a))/(
4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(
2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar
(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4
,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{\sqrt{bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

3.532 $\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*x
^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.198289, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1833, 1248, 641, 217, 206, 1280, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```


```
[Out] (e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*x
^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{x\left(c+ex^2\right)}{\sqrt{a+bx^4}}+\frac{x^2\left(d+fx^2\right)}{\sqrt{a+bx^4}}\right)dx$$
$$= \int \frac{x\left(c+ex^2\right)}{\sqrt{a+bx^4}}\,dx+\int \frac{x^2\left(d+fx^2\right)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{fx\sqrt{a+bx^4}}{3b}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{af-3bdx^2}{\sqrt{a+bx^4}}\,dx}{3b}$$
$$= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\frac{(\text{\textbackslash}}{}$$
$$= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-}\right.}{b^{3/4}\sqrt{a+bx^4}}$$
$$= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)}{}$$

Mathematica [C] time = 0.094539, size = 160, normalized size = 0.54

$$\frac{3\sqrt{b}c\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)+2bdx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)-2afx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+3ae+2afx+3}{6b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

```
[Out] (3*a*e + 2*a*f*x + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 2*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hyperg
eometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(6*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.005, size = 229, normalized size = 0.8

$$\frac{fx}{3b}\sqrt{bx^4+a}-\frac{af}{3b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{2b}\sqrt{bx^4+a}+id\sqrt{a}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/3*f*x*(b*x^4+a)^(1/2)/b-1/3*f/b*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e*(b*x^4+a)^(1/2)/b+I*d*a^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-
EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1
/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^4 + ex^3 + dx^2 + cx}{\sqrt{bx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^4 + e*x^3 + d*x^2 + c*x)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 3.5603, size = 129, normalized size = 0.43

$$e\left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for } b = 0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{cases}\right) + \frac{c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) +
 c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gamma(3/4)*hyper((1/2,
3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + f*x**5*gam
ma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x}{\sqrt{bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a), x)
```

3.533 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[
b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4
)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^
(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.141569, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[
b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4
)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^
(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
b^(3/4)*Sqrt[a + b*x^4])
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \int \left(\frac{c+ex^2}{\sqrt{a+bx^4}}+\frac{x\left(d+fx^2\right)}{\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{c+ex^2}{\sqrt{a+bx^4}}\,dx + \int \frac{x\left(d+fx^2\right)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+fx}{\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}} + \left(c+\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \cdots$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \cdots$$

$$= \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(\cdots\right.}{b^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.0940698, size = 150, normalized size = 0.54

$$\frac{cx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

```
[Out] (f*Sqrt[a + b*x^4])/(2*b) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*S
qrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^
4)/a)])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.004, size = 208, normalized size = 0.8

$$\frac{f}{2b}\sqrt{bx^4+a}+ie\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*f*(b*x^4+a)^(1/2)/b+I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*
(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+c/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Sympy [A] time = 2.62666, size = 128, normalized size = 0.46

$$f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True)) +
 d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma(
3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7
/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.534 $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=285

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)
*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.179943, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1832, 266, 63, 208, 1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```


```
[Out] (f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)
*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```


Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = c\int \frac{1}{x\sqrt{a+bx^4}}\,dx+\int \frac{d+ex+fx^2}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{1}{4}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^4\right)+\int \left(\frac{ex}{\sqrt{a+bx^4}}+\frac{d+fx^2}{\sqrt{a+bx^4}}\right)dx$$
$$= \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^4}\right)}{2b}+e\int \frac{x}{\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{\sqrt{a+bx^4}}\,dx$$
$$= -\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\left(\sqrt{a}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}+\left(d+\frac{\sqrt{a}f}{\sqrt{b}}\right)$$
$$= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}$$
$$= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}}{b^{3/4}\sqrt{a+b}}$$

Mathematica [C] time = 0.227222, size = 159, normalized size = 0.56

$$-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{dx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a
+ b*x^4]/Sqrt[a]])/(2*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (f*x^3*Sqrt[1 + (b*x^4)/a]
*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.013, size = 222, normalized size = 0.8

$$if\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}}+\frac{e}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x)
```

```
[Out] I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*e*ln(x^2*b^
(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/
```

```
2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{bx^5 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^5 + a*x), x)
```

Sympy [C] time = 4.06959, size = 126, normalized size = 0.44

$$\frac{e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
```

```
[Out] e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt(b)*x**2))
/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*
pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

3.535 $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=309

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d
*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*
e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^
4])
```

Rubi [A] time = 0.224348, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1833, 1282, 1198, 220, 1196, 1252, 844, 217, 206, 266, 63, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{c\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] -((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d
*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqrt[a]*
e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^
4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \int \left(\frac{c+ex^2}{x^2\sqrt{a+bx^4}}+\frac{d+fx^2}{x\sqrt{a+bx^4}}\right)dx$$
$$= \int \frac{c+ex^2}{x^2\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x\sqrt{a+bx^4}}\,dx$$
$$= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{1}{2}\,\text{Subst}\left(\int \frac{d+fx}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-ae-bcx^2}{\sqrt{a+bx^4}}\,dx}{a}$$
$$= -\frac{c\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}+\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)+\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{a+}}$$
$$= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}+\frac{\left(\sqrt{bc}\right.}{}$$
$$= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\text{ta}\right.}{a^{3/4}\sqrt{a+bx^4}}$$
$$= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right.}{}$$

Mathematica [C] time = 0.227375, size = 157, normalized size = 0.51

$$-\frac{c\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{ex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a
+ b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-
1/4, 1/2, 3/4, -((b*x^4)/a)])/(x*Sqrt[a + b*x^4]) + (e*x*Sqrt[1 + (b*x^4)/a
]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] time = 0.009, size = 299, normalized size = 1.

$$\frac{f}{2}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{b}}+e\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{d}{2}\ln\left(\frac{1}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)+e/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/a^(1/2)*ln((2*a+2*a^(1/2
)*(b*x^4+a)^(1/2))/x^2)-c*(b*x^4+a)^(1/2)/a/x+I*c*b^(1/2)/a^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*c*b^(1/2)/
a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{bx^6 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^6 + a*x^2), x)
```

Sympy [C] time = 3.3097, size = 128, normalized size = 0.41

$$\frac{f \operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{c\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{d \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
```

```
[Out] f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((-1/4, 1/2)
, (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - d*asinh(sqrt
(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,),
b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

3.536 $\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=300

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(2*a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*d*x*S
qrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a
^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.215868, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1833, 1252, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(2*a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*d*x*S
qrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4]/Sq
rt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a
^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \int \left(\frac{c+ex^2}{x^3\sqrt{a+bx^4}} + \frac{d+fx^2}{x^2\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{c+ex^2}{x^3\sqrt{a+bx^4}}\,dx + \int \frac{d+fx^2}{x^2\sqrt{a+bx^4}}\,dx$$

$$= -\frac{d\sqrt{a+bx^4}}{ax} + \frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{x^2\sqrt{a+bx^2}}\,dx, x, x^2\right) - \frac{\int \frac{-af-bdx^2}{\sqrt{a+bx^4}}\,dx}{a}$$

$$= -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} - \frac{\left(\sqrt{b}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, x^2\right) + \Bigg(\ldots$$

$$= -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\ldots\right.\right.}{a^{3/4}\sqrt{a+bx^4}}\ldots$$

$$= -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\ldots\right.\right.}{a^{3/4}\sqrt{a+bx^4}}\ldots$$

$$= -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\ldots}}{a^{3\ldots}}$$

Mathematica [C] time = 0.139661, size = 148, normalized size = 0.49

$$-\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{d\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}} - \frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{fx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(2*a*x^2) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sq
rt[a]) - (d*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^4)
/a)])/(x*Sqrt[a + b*x^4]) + (f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4,
 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Maple [C] time = 0.015, size = 293, normalized size = 1.

$$f\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{e}{2}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}} \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x)
```

```
[Out] f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1
/2*e/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/2*c*(b*x^4+a)^(1/2)/
```

```
a/x^2-d*(b*x^4+a)^(1/2)/a/x+I*d*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*d*b^(1/2)/a^(1/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{bx^7 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^7 + a*x^3), x)
```

Sympy [C] time = 3.09551, size = 126, normalized size = 0.42

$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4} + 1}}{2a} + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{e \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4, 1/2), (3
/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - e*asinh(sqrt(a)/
(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

3.537 $\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=323

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{a^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(3*a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) - (e*Sqrt[a
+ b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2))
 - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*e*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]
*c - 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a
 + b*x^4])
```

Rubi [A] time = 0.258399, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1833, 1282, 1198, 220, 1196, 1252, 807, 266, 63, 208}

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(3*a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) - (e*Sqrt[a
+ b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2))
 - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*e*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]
*c - 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a
 + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = \int \left(\frac{c+ex^2}{x^4\sqrt{a+bx^4}}+\frac{d+fx^2}{x^3\sqrt{a+bx^4}}\right)dx$$
$$= \int \frac{c+ex^2}{x^4\sqrt{a+bx^4}}\,dx+\int \frac{d+fx^2}{x^3\sqrt{a+bx^4}}\,dx$$
$$= -\frac{c\sqrt{a+bx^4}}{3ax^3}+\frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-3ae+bcx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a}$$
$$= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\int \frac{-abc+3abex^2}{\sqrt{a+bx^4}}\,dx}{3a^2}+\frac{1}{2}f\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,\right.$$
$$= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}-\frac{\left(\sqrt{b}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}-\frac{\left(\sqrt{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^4}}}{3a}$$
$$= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E}{a^{3/4}\sqrt{a+bx^4}}$$
$$= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\right.}{}$$

Mathematica [C] time = 0.157824, size = 149, normalized size = 0.46

$$\frac{-2ac\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^4}{a}\right)-3x\left(2aex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^4}{a}\right)+\sqrt{a}fx^2\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+ad+bd\right.}{6ax^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)]
 - 3*x*(a*d + b*d*x^4 + Sqrt[a]*f*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^
4]/Sqrt[a]] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -((b*x^4)/a)]))/(6*a*x^3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.01, size = 316, normalized size = 1.

$$-\frac{f}{2}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{c}{3ax^3}\sqrt{bx^4+a}-\frac{bc}{3a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/2*f/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/3*c*(b*x^4+a)^(1/2
)/a/x^3-1/3*c*b/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-1/2*d*(b*x^4+a)^(1/2)/a/x^2-e*(b*x^4+a)^(1/2)/a/x+I*e*b^(1/2
```

```
)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)-I*e*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{bx^8 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^8 + a*x^4), x)
```

Sympy [C] time = 3.44096, size = 131, normalized size = 0.41

$$-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{f\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4, 1/2), (1
/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + e*gamma(-1/4)
*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/
4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

3.538 $\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=346

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x\right.}{}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(4*a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) - (e*Sqrt[a
+ b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*f*x*Sqrt[a + b*x
^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(
4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sq
rt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.2813, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1833, 1252, 835, 807, 266, 63, 208, 1282, 1198, 220, 1196}

$$\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+b}{(\sqrt{a}+}}}{a^{3/4}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(4*a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) - (e*Sqrt[a
+ b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*f*x*Sqrt[a + b*x
^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(
4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sq
rt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
```

```
Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = \int\left(\frac{c+ex^2}{x^5\sqrt{a+bx^4}}+\frac{d+fx^2}{x^4\sqrt{a+bx^4}}\right)dx$$
$$= \int \frac{c+ex^2}{x^5\sqrt{a+bx^4}}\,dx + \int \frac{d+fx^2}{x^4\sqrt{a+bx^4}}\,dx$$
$$= -\frac{d\sqrt{a+bx^4}}{3ax^3}+\frac{1}{2}\text{Subst}\left(\int \frac{c+ex}{x^3\sqrt{a+bx^2}}\,dx,x,x^2\right)-\frac{\int \frac{-3af+bdx^2}{x^2\sqrt{a+bx^4}}\,dx}{3a}$$
$$= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\int \frac{-abd+3abfx^2}{\sqrt{a+bx^4}}\,dx}{3a^2}-\frac{\text{Subst}\left(\int \frac{-2ae+bcx}{x^2\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4a}$$
$$= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx,x,x^2\right)}{4a}-$$
$$= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}\right.}{}$$
$$= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}\right.}{}$$
$$= -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.}{4a^{3/2}}$$

Mathematica [C] time = 0.147496, size = 147, normalized size = 0.42

$$-\frac{\sqrt{a+bx^4}\left(3ac\sqrt{\frac{bx^4}{a}+1}-3bcx^4\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+4adx\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^4}{a}\right)+6aex^2\sqrt{\frac{bx^4}{a}+1}+12afx^3\,{}_2F_1\left(-\frac{1}{4},\right.\right.}{12a^2x^4\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^4]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*x^4)/a
] - 3*b*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4
, 1/2, 1/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/4, 1/2, 3/4, -(
(b*x^4)/a)]))/(12*a^2*x^4*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.017, size = 335, normalized size = 1.

$$-\frac{d}{3\,ax^3}\sqrt{bx^4+a}-\frac{bd}{3\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{c}{4\,ax^4}\sqrt{bx^4+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/3*d*(b*x^4+a)^(1/2)/a/x^3-1/3*d*b/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*c*(b*x^4+a)^(1/2)/a/x^4+1/4*c*b/a^(
3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/2*e*(b*x^4+a)^(1/2)/a/x^2-f*
(b*x^4+a)^(1/2)/a/x+I*f*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-I*f*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{bx^9+ax^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^9 + a*x^5), x)
```

Sympy [C] time = 4.53686, size = 158, normalized size = 0.46

$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4ax^2}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{2a}+\frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}+\frac{f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)
/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x
**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + b*c*asinh(sqrt(a)/(sqrt(b
```

```
)*x**2))/(4*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

3.539 $\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=377

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)}{5}$$

```
[Out] -(c*Sqrt[a + b*x^4])/(5*a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) - (e*Sqrt[a
+ b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (3*b*c*Sqrt[a + b*x^4
])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[b]*x^
2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (b^(3/4)
*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*a^
(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.326546, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1833, 1282, 1198, 220, 1196, 1252, 835, 807, 266, 63, 208}

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}}{5a^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -(c*Sqrt[a + b*x^4])/(5*a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) - (e*Sqrt[a
+ b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (3*b*c*Sqrt[a + b*x^4
])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[b]*x^
2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)*c*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (b^(3/4)
*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*a^
(7/4)*Sqrt[a + b*x^4])
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
```

```
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}dx = \int \left(\frac{c+ex^2}{x^6\sqrt{a+bx^4}}+\frac{d+fx^2}{x^5\sqrt{a+bx^4}}\right)dx$$

$$= \int \frac{c+ex^2}{x^6\sqrt{a+bx^4}}dx + \int \frac{d+fx^2}{x^5\sqrt{a+bx^4}}dx$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} + \frac{1}{2}\text{Subst}\left(\int \frac{d+fx}{x^3\sqrt{a+bx^2}}dx, x, x^2\right) - \frac{\int \frac{-5ae+3bcx^2}{x^4\sqrt{a+bx^4}}dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} + \frac{\int \frac{-9abc-5abex^2}{x^2\sqrt{a+bx^4}}dx}{15a^2} - \frac{\text{Subst}\left(\int \frac{-2af+bdx}{x^2\sqrt{a+bx^2}}dx, x, x^2\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} - \frac{f\sqrt{a+bx^4}}{2ax^2} + \frac{3bc\sqrt{a+bx^4}}{5a^2x} - \frac{\int \frac{5a^2be+9ab^2cx^2}{\sqrt{a+bx^4}}dx}{15a^3} - \ldots$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} - \frac{f\sqrt{a+bx^4}}{2ax^2} + \frac{3bc\sqrt{a+bx^4}}{5a^2x} + \frac{\left(3b^{3/2}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}dx}{5a^{3/2}} \ldots$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} - \frac{f\sqrt{a+bx^4}}{2ax^2} + \frac{3bc\sqrt{a+bx^4}}{5a^2x} - \frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \ldots$$

$$= -\frac{c\sqrt{a+bx^4}}{5ax^5} - \frac{d\sqrt{a+bx^4}}{4ax^4} - \frac{e\sqrt{a+bx^4}}{3ax^3} - \frac{f\sqrt{a+bx^4}}{2ax^2} + \frac{3bc\sqrt{a+bx^4}}{5a^2x} - \frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \ldots$$

Mathematica [C] time = 0.208193, size = 134, normalized size = 0.36

$$-\frac{\sqrt{a+bx^4}\left(12ac\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^4}{a}\right)+5x\left(3a\sqrt{\frac{bx^4}{a}+1}\left(d+2fx^2\right)-3bdx^4\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)+4aex\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\ldots\right.\right.\right.}{60a^2x^5\sqrt{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4)/a)]
+ 5*x*(3*a*(d + 2*f*x^2)*Sqrt[1 + (b*x^4)/a] - 3*b*d*x^4*ArcTanh[Sqrt[1 + (
b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)])))/(60
*a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

Maple [C] time = 0.01, size = 354, normalized size = 0.9

$$-\frac{e}{3\,ax^3}\sqrt{bx^4+a} - \frac{be}{3\,a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{c}{5\,ax^5}\sqrt{bx^4+a} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/3*e*(b*x^4+a)^(1/2)/a/x^3-1/3*e*b/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/5*c*(b*x^4+a)^(1/2)/a/x^5+3/5*b*c*(b*
x^4+a)^(1/2)/a^2/x-3/5*I*c/a^(3/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/5*I*c/a^(3/2)*b^(3/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/4*d*(b*x^4+a
)^(1/2)/a/x^4+1/4*d*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/2*f
*(b*x^4+a)^(1/2)/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{bx^{10} + ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^10 + a*x^6), x)
```

Sympy [C] time = 4.88255, size = 163, normalized size = 0.43

$$-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)
/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a
```

```
)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + b*d*asinh(sqrt(a)/(s
qrt(b)*x**2))/(4*a**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + a}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

3.540 $\int \frac{x^6\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=365

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{3cx\sqrt{a}}{2b^{3/2}\left(\sqrt{a}\right.}$$

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*Sqrt[a + b*x^4])/(4*b
^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (3*a*f*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*S
qrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)
*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.522866, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {1828, 1885, 1888, 1198, 220, 1196, 1819, 1815, 641, 217, 206}

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/b^2 + (e*x*Sqrt[a + b*x^4])/(3*b^2) + (f*x^2*Sqrt[a + b*x^4])/(4*b
^2) + (3*c*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (3*a*f*
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2)) - (3*a^(1/4)*c*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*S
qrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)
*Sqrt[a + b*x^4])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1819

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
```

```
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2be+2a^2bfx-3ab^2cx^2-4ab^2dx^3-2ab^2ex^4-2ab^2fx^5}{\sqrt{a+bx^4}}\,dx}{2ab^3}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{a^2be-3ab^2cx^2-2ab^2ex^4}{\sqrt{a+bx^4}} + \frac{x\left(2a^2bf-4ab^2dx^2-2ab^2fx^4\right)}{\sqrt{a+bx^4}}\right)dx}{2ab^3}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2be-3ab^2cx^2-2ab^2ex^4}{\sqrt{a+bx^4}}\,dx}{2ab^3} - \frac{\int \frac{x\left(2a^2bf-4ab^2dx^2-2ab^2fx^4\right)}{\sqrt{a+bx^4}}\,dx}{2ab^3}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{ex\sqrt{a+bx^4}}{3b^2} - \frac{\int \frac{5a^2b^2e-9ab^3cx^2}{\sqrt{a+bx^4}}\,dx}{6ab^4} - \frac{\text{Subst}\left(\int \frac{2a^2bf-4ab^2}{\sqrt{a}}\right.}{4a}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx^4}}{4b^2} - \frac{\text{Subst}\left(\int \frac{6a^2b^2f-8ab^3dx}{\sqrt{a+bx^2}}\right.}{8ab^4}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{b^2} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx^4}}{4b^2} + \frac{3cx\sqrt{a}}{2b^{3/2}\left(\sqrt{a}\right.}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{b^2} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx^4}}{4b^2} + \frac{3cx\sqrt{a}}{2b^{3/2}\left(\sqrt{a}\right.}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{b^2} + \frac{ex\sqrt{a+bx^4}}{3b^2} + \frac{fx^2\sqrt{a+bx^4}}{4b^2} + \frac{3cx\sqrt{a}}{2b^{3/2}\left(\sqrt{a}\right.}$$

Mathematica [C] time = 0.173399, size = 220, normalized size = 0.6

$$\frac{-9a^{3/2}f\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-12b^{3/2}cx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)+12a\sqrt{b}d-10a\sqrt{b}ex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\right.}{12b^{5/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (12*a*Sqrt[b]*d + 10*a*Sqrt[b]*e*x + 9*a*Sqrt[b]*f*x^2 + 12*b^(3/2)*c*x^3 +
 6*b^(3/2)*d*x^4 + 4*b^(3/2)*e*x^5 + 3*b^(3/2)*f*x^6 - 9*a^(3/2)*f*Sqrt[1 +
 (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 10*a*Sqrt[b]*e*x*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*b^(3/2)*c*x^3*Sq
rt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(12*b^(5/
2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.027, size = 378, normalized size = 1.

$$\frac{f x^6}{4 b}\frac{1}{\sqrt{b x^4+a}}+\frac{3 a f x^2}{4 b^2}\frac{1}{\sqrt{b x^4+a}}-\frac{3 a f}{4}\ln\left(x^2\sqrt{b}+\sqrt{b x^4+a}\right) b^{-\frac{5}{2}}+\frac{a e x}{2 b^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right) b}}+\frac{e x}{3 b^2}\sqrt{b x^4+a}-\frac{5 a e}{6 b^2}\sqrt{1-i x^2\surd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/4*f*x^6/b/(b*x^4+a)^(1/2)+3/4*f/b^2*a*x^2/(b*x^4+a)^(1/2)-3/4*f/b^(5/2)*a
*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+1/2*e/b^2*x*a/((x^4+1/b*a)*b)^(1/2)+1/3*e*
x*(b*x^4+a)^(1/2)/b^2-5/6*e/b^2*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2-1/2*c/
b*x^3/((x^4+1/b*a)*b)^(1/2)+3/2*I*c/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*I*c/b^(3/2)*a^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(f x^3+e x^2+d x+c\right) x^6}{\left(b x^4+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(f x^9+e x^8+d x^7+c x^6\right)\sqrt{b x^4+a}}{b^2 x^8+2 a b x^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^9 + e*x^8 + d*x^7 + c*x^6)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x
^4 + a^2), x)
```

Sympy [A] time = 34.6687, size = 202, normalized size = 0.55

$$d\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+f\left(\frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^6}{4\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+\frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b,
 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x**
4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b*sq
rt(1 + b*x**4/a))) + c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3/2, 9
/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x^6}{\left(bx^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

3.541 $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{c\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2b^{3/2}}$$

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*Sqrt[a + b*x^4])/(2*b
^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(
7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.367229, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1828, 1885, 1248, 641, 217, 206, 1888, 1198, 220, 1196}

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```


```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/b^2 + (f*x*Sqrt[a + b*x^4])/(3*b^2) + (3*d*x*Sqrt[a + b*x^4])/(2*b
^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]
)/(2*b^(3/2)) - (3*a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(
7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
```

```
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^5\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{a^2f-2abcx-3abdx^2-4abex^3-2abfx^4}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{x\left(-2abc-4abex^2\right)}{\sqrt{a+bx^4}}+\frac{a^2f-3abdx^2-2abfx^4}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} - \frac{\int \frac{x\left(-2abc-4abex^2\right)}{\sqrt{a+bx^4}}dx}{2ab^2} - \frac{\int \frac{a^2f-3abdx^2-2abfx^4}{\sqrt{a+bx^4}}dx}{2ab^2}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{fx\sqrt{a+bx^4}}{3b^2} - \frac{\int \frac{5a^2bf-9ab^2dx^2}{\sqrt{a+bx^4}}dx}{6ab^3} - \frac{\text{Subst}\left(\int \frac{-2abc-4abex}{\sqrt{a+bx^2}}d\right.}{4ab^2}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}dx,x,x^2\right)}{2b}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}d\,(}{}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{b^2} + \frac{fx\sqrt{a+bx^4}}{3b^2} + \frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\tanh}{2}$$

Mathematica [C] time = 0.151856, size = 176, normalized size = 0.51

$$\frac{3\sqrt{a}\sqrt{b}c\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-6bdx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)-5afx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+6ae+5afx-3}{6b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (6*a*e + 5*a*f*x - 3*b*c*x^2 + 6*b*d*x^3 + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[a
]*Sqrt[b]*c*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 5*a*f*x*Sq
rt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 6*b*d*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*b^
2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.016, size = 358, normalized size = 1.

$$\frac{afx}{2b^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} + \frac{fx}{3b^2}\sqrt{bx^4+a} - \frac{5af}{6b^2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f/b^2*x*a/((x^4+1/b*a)*b)^(1/2)+1/3*f*x*(b*x^4+a)^(1/2)/b^2-5/6*f/b^2*a
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
```

```
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/
2*e*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2-1/2*d/b*x^3/((x^4+1/b*a)*b)^(1/2)+3/2*I
*d/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^
(1/2))^(1/2),I)-3/2*I*d/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c*x^2/b/(b*x^4+a)^(1/2)+1/2*c/b^(3
/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx^8+ex^7+dx^6+cx^5\right)\sqrt{bx^4+a}}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^8 + e*x^7 + d*x^6 + c*x^5)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x
^4 + a^2), x)
```

Sympy [A] time = 25.7538, size = 172, normalized size = 0.5

$$c\left(\frac{\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+e\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{fx^9\Gamma\left(\frac{9}{4}\right)2}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*
x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x
**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + d*x**7*gamma(7/4)*hyper((3/2
, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f*x**
9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^5}{(bx^4 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)
```

3.542 $\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=314

$$\frac{(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}(3\sqrt{a}e+\sqrt{b}c)\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a}+\sqrt{b}x^2)}-$$

```
[Out] -(x*(c + d*x + e*x^2 + f*x^3))/(2*b*Sqrt[a + b*x^4]) + (f*Sqrt[a + b*x^4])/
b^2 + (3*e*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*e*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + 3*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a
 + b*x^4])
```

Rubi [A] time = 0.26789, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1828, 1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$\frac{(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}(3\sqrt{a}e+\sqrt{b}c)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}(\sqrt{a}+\sqrt{b}x^2)}-\frac{3\sqrt[4]{a}e(\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -(x*(c + d*x + e*x^2 + f*x^3))/(2*b*Sqrt[a + b*x^4]) + (f*Sqrt[a + b*x^4])/
b^2 + (3*e*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcT
anh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) - (3*a^(1/4)*e*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + 3*S
qrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a
 + b*x^4])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
```

```
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4 \left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-2abdx-3abex^2-4abfx^3}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abc-3abex^2}{\sqrt{a+bx^4}} + \frac{x\left(-2abd-4abfx^2\right)}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} - \frac{\int \frac{-abc-3abex^2}{\sqrt{a+bx^4}}\,dx}{2ab^2} - \frac{\int \frac{x\left(-2abd-4abfx^2\right)}{\sqrt{a+bx^4}}\,dx}{2ab^2}$$

$$= -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{-2abd-4abfx}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4ab^2} - \frac{\left(3\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}} + \cdots$$

$$= -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\cdots}}{2b^{\cdots}}$$

$$= -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\cdots}}{2b^{\cdots}}$$

$$= -\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{b^2} + \frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \cdots$$

Mathematica [C] time = 0.145698, size = 166, normalized size = 0.53

$$\frac{bcx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right) + \sqrt{a}\sqrt{b}d\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) - 2bex^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right) + 2af - bcx - b\cdots}{2b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a*f - b*c*x - b*d*x^2 + 2*b*e*x^3 + b*f*x^4 + Sqrt[a]*Sqrt[b]*d*Sqrt[1 +
 (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + b*c*x*Sqrt[1 + (b*x^4)/a]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.008, size = 340, normalized size = 1.1

$$\frac{f\left(bx^4+2a\right)}{2b^2}\frac{1}{\sqrt{bx^4+a}} - \frac{ex^3}{2b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} + \frac{3i}{2}e\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\cdots}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2-1/2*e/b*x^3/((x^4+1/b*a)*b)^(1/2)+3/2
*I*e/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
```

```
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-3/2*I*e/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d*x^2/b/(b*x^4+a)^(1/2)+1/2*d/b^
(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*c/b*x/((x^4+1/b*a)*b)^(1/2)+1/2*c
/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3+ex^2+dx+c\right)x^4}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(fx^7+ex^6+dx^5+cx^4\right)\sqrt{bx^4+a}}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^7 + e*x^6 + d*x^5 + c*x^4)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x
^4 + a^2), x)
```

Sympy [A] time = 19.9864, size = 172, normalized size = 0.55

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+f\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1}{4a^{\frac{3}{2}}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*
x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x
**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + c*x**5*gamma(5/4)*hyper((5/4
, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**7*
gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)
*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^4}{(bx^4 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

3.543 $\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{}}{}$$

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(2*b*Sqrt[a + b*x^4]) + (3*f*x*Sqrt[a + b*x^4])/
(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x
^4]])/(2*b^(3/2)) - (3*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2
*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.199901, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1823, 1885, 275, 217, 206, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -(c + d*x + e*x^2 + f*x^3)/(2*b*Sqrt[a + b*x^4]) + (3*f*x*Sqrt[a + b*x^4])/
(2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x
^4]])/(2*b^(3/2)) - (3*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2
*b^(7/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 1823

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Pq*(
a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[D[Pq, x]*
(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && Eq
Q[m - n + 1, 0] && LtQ[p, -1]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \frac{d+2ex+3fx^2}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \left(\frac{2ex}{\sqrt{a+bx^4}}+\frac{d+3fx^2}{\sqrt{a+bx^4}}\right)dx}{2b}$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{\int \frac{d+3fx^2}{\sqrt{a+bx^4}}\,dx}{2b}+\frac{e\int \frac{x}{\sqrt{a+bx^4}}\,dx}{b}$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{e\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^2\right)}{2b}-\frac{\left(3\sqrt{a}f\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}}+$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-$$

$$= -\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}-$$

Mathematica [C] time = 0.128945, size = 181, normalized size = 0.61

$$\frac{\sqrt{b}dx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+\sqrt{a}e\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-2\sqrt{b}fx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)-\sqrt{b}c-\sqrt{b}dx-\sqrt{}}{2b^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*c) - Sqrt[b]*d*x - Sqrt[b]*e*x^2 + 2*Sqrt[b]*f*x^3 + Sqrt[a]*e*S
qrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]*d*x*Sqrt[1 + (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*Sqrt[b]*f*x^3*S
qrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^(3/
2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.007, size = 331, normalized size = 1.1

$$-\frac{fx^3}{2b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i}{2}f\sqrt{a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)b^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{3i}{2}f\sqrt{a}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*f/b*x^3/((x^4+1/b*a)*b)^(1/2)+3/2*I*f/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*I*f/b^(3/2)*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-1/2*e*x^2/b/(b*x^4+a)^(1/2)+1/2*e/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
-1/2*d/b*x/((x^4+1/b*a)*b)^(1/2)+1/2*d/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*c/b/(b*x^4+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c}{2\sqrt{bx^4+a}b}+\int\frac{fx^6+ex^5+dx^4}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*c/(sqrt(b*x^4 + a)*b) + integrate((f*x^6 + e*x^5 + d*x^4)/(b*x^4 + a)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx^6+ex^5+dx^4+cx^3\right)\sqrt{bx^4+a}}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x
^4 + a^2), x)
```

Sympy [A] time = 17.2315, size = 156, normalized size = 0.53

$$c\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)+e\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right)+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right| \ldots\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1
 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((3/2, 7/4), (
11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3+ex^2+dx+c\right)x^3}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

3.544 $\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a*b*Sqrt[a + b*x^4]) - (d*Sqrt[a
+ b*x^4])/(2*a*b) - (c*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) + (c*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((Sqr
t[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)*b^(5
/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.250941, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1828, 1885, 1198, 220, 1196, 1248, 641, 217, 206}

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```


```
[Out] -(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a*b*Sqrt[a + b*x^4]) - (d*Sqrt[a
+ b*x^4])/(2*a*b) - (c*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2)) + (c*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) - ((Sqr
t[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)*b^(5
/4)*Sqrt[a + b*x^4])
```


Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
```

```
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}dx = -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-abe-2abfx+b^2cx^2+2b^2dx^3}{\sqrt{a+bx^4}}dx}{2ab^2}$$

$$= -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{\int \left(\frac{-abe+b^2cx^2}{\sqrt{a+bx^4}} + \frac{x\left(-2abf+2b^2dx^2\right)}{\sqrt{a+bx^4}}\right)dx}{2ab^2}$$

$$= -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{\int \frac{-abe+b^2cx^2}{\sqrt{a+bx^4}}dx}{2ab^2} - \frac{\int \frac{x\left(-2abf+2b^2dx^2\right)}{\sqrt{a+bx^4}}dx}{2ab^2}$$

$$= -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{-2abf+2b^2dx}{\sqrt{a+bx^2}}dx, x, x^2\right)}{4ab^2} + \frac{c\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}dx}{2\sqrt{a}\sqrt{b}} - \Bigg(\frac{\sqrt{b}c}{\sqrt{a}} -$$

$$= -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{}{(\sqrt{a}}}}{2a^{3/4}}$$

$$= -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{}{(\sqrt{a}}}}{2a^{3/4}}$$

$$= -\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} +$$

Mathematica [C] time = 0.196435, size = 165, normalized size = 0.5

$$\frac{3a^{3/2}f\sqrt{\frac{bx^4}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+2b^{3/2}cx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)-3a\sqrt{b}(d+x(e+fx))+3a\sqrt{b}ex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\right.}{6ab^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*Sqrt[b]*(d + x*(e + f*x)) + 3*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*ArcSinh[(
Sqrt[b]*x^2)/Sqrt[a]] + 3*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b^(3/2)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.009, size = 331, normalized size = 1.

$$-\frac{fx^2}{2b}\frac{1}{\sqrt{bx^4+a}}+\frac{f}{2}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)b^{-\frac{3}{2}}-\frac{ex}{2b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e}{2b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*f*x^2/b/(b*x^4+a)^(1/2)+1/2*f/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-
1/2*e/b*x/((x^4+1/b*a)*b)^(1/2)+1/2*e/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
```

```
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*d/b/(b*x^4+a)^(1/2)+1/2*c*x^3/a/((
x^4+1/b*a)*b)^(1/2)-1/2*I*c/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*I*c/a^(1/2)/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^3 + ex^2 + dx + c\right)x^2}{\left(bx^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx^5 + ex^4 + dx^3 + cx^2\right)\sqrt{bx^4 + a}}{b^2x^8 + 2\,abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((f*x^5 + e*x^4 + d*x^3 + c*x^2)*sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x
^4 + a^2), x)
```

Sympy [A] time = 15.565, size = 156, normalized size = 0.47

$$d\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\ b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle| b\right.}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1
 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((5/4, 3/2), (
9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3 + ex^2 + dx + c)x^2}{(bx^4 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

3.545 $\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a*b*Sqrt[a + b*x^4]) - (e*Sqrt[a
+ b*x^4])/(2*a*b) - (d*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
^2)) + (d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a
 + b*x^4]) - ((Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(3/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.192557, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1828, 1885, 261, 1198, 220, 1196}

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] -(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a*b*Sqrt[a + b*x^4]) - (e*Sqrt[a
+ b*x^4])/(2*a*b) - (d*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
^2)) + (d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a
 + b*x^4]) - ((Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(3/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 1828

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a
 + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x],
x], x] - Simp[(x*R*(a + b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] +
1)), x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] &&
 LtQ[p, -1] && IGtQ[m, 0]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx &= -\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{\int \frac{-af+bdx^2+2bex^3}{\sqrt{a+bx^4}}\,dx}{2ab}\\
&= -\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{\int \left(\frac{2bex^3}{\sqrt{a+bx^4}}+\frac{-af+bdx^2}{\sqrt{a+bx^4}}\right)dx}{2ab}\\
&= -\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{\int \frac{-af+bdx^2}{\sqrt{a+bx^4}}\,dx}{2ab}-\frac{e\int \frac{x^3}{\sqrt{a+bx^4}}\,dx}{a}\\
&= -\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{e\sqrt{a+bx^4}}{2ab}+\frac{d\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}}-\frac{\left(\frac{\sqrt{b}d}{\sqrt{a}}-f\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b}\\
&= -\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}-\frac{e\sqrt{a+bx^4}}{2ab}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a\cdots}{(\sqrt{a}+\cdots}}}{2a^{3/4}b^{\cdots}}\end{aligned}$$

Mathematica [C] time = 0.0769455, size = 116, normalized size = 0.38

$$\frac{2bdx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)+3afx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-3ae-3afx+3bcx^2}{6ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*e - 3*a*f*x + 3*b*c*x^2 + 3*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 250, normalized size = 0.8

$$f\left(-\frac{x}{2\,b}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{1}{2\,b}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\right)-\frac{e}{2\,b}\frac{1}{\sqrt{bx^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] f*(-1/2/b*x/((x^4+1/b*a)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*e/b/(b*x^4+a)^(1/2)+d*(1/2*x^3/a/
((x^4+1/b*a)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*
(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)))+1/2*c/(b*x^4+a)^(1/2)/a*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^2}{2\sqrt{bx^4+a}a}+\int\frac{fx^4+ex^3+dx^2}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/(sqrt(b*x^4 + a)*a) + integrate((f*x^4 + e*x^3 + d*x^2)/(b*x^4 +
a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^4+ex^3+dx^2+cx\right)}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2 + c*x)/(b^2*x^8 + 2*a*b*x^4
 + a^2), x)
```

Sympy [A] time = 14.6385, size = 133, normalized size = 0.44

$$e\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*gamma(3/4)*hyper((3/4,
 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + f*x**5*g
amma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*g
amma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^3+ex^2+dx+c)x}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2), x)
```

3.546 $\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] -(e*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a*f - b*x*(
c + d*x + e*x^2))/(2*a*b*Sqrt[a + b*x^4]) + (e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c - Sqrt[a]*e)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.117198, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1854, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{af}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] -(e*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a*f - b*x*(
c + d*x + e*x^2))/(2*a*b*Sqrt[a + b*x^4]) + (e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c - Sqrt[a]*e)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{af-bx\left(c+dx+ex^2\right)}{2ab\sqrt{a+bx^4}}-\frac{\int \frac{-c+ex^2}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= -\frac{af-bx\left(c+dx+ex^2\right)}{2ab\sqrt{a+bx^4}}+\frac{e\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\sqrt{b}}+\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= -\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{af-bx\left(c+dx+ex^2\right)}{2ab\sqrt{a+bx^4}}+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.0540039, size = 116, normalized size = 0.42

$$\frac{3bcx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+2bex^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)-3af+3bcx+3bdx^2}{6ab\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

```
[Out] (-3*a*f + 3*b*c*x + 3*b*d*x^2 + 3*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Maple [C] time = 0.006, size = 250, normalized size = 0.9

$$-\frac{f}{2\,b}\frac{1}{\sqrt{bx^4+a}}+e\left(\frac{x^3}{2\,a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i}{2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*f/b/(b*x^4+a)^(1/2)+e*(1/2*x^3/a/((x^4+1/b*a)*b)^(1/2)-1/2*I/a^(1/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))+1/2*d/(b*x^4+a)^(1/2)/a*x^2
```

```
+c*(1/2*x/a/((x^4+1/b*a)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\left(bx^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{b^2x^8 + 2\,abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^8 + 2*a*b*x^4 + a
^2), x)
```

Sympy [A] time = 13.9834, size = 131, normalized size = 0.48

$$f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for}\: b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| {\frac{bx^4e^{i\pi}}{a}} \right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1 + \frac{bx^4}{a}}} + \frac{ex^3\Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| {\frac{bx^4e^{i\pi}}{a}} \right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

```
[Out] f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + e*x**3*gamm
a(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\left(bx^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

3.547 $\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

```
[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (c*Sqrt[a +
 b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^
2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) + (f*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d
- Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqr
t[a + b*x^4])
```

Rubi [A] time = 0.291992, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1829, 1832, 266, 63, 208, 1885, 261, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}+\frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (c*Sqrt[a +
 b*x^4])/(2*a^2) - (f*x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^
2)) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) + (f*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d
- Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(3/4)*Sqr
t[a + b*x^4])
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1832

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
```

```
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}dx = \frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-bdx+bfx^3-\frac{2b^2cx^4}{a}}{x\sqrt{a+bx^4}}dx}{2ab}$$

$$= \frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-bd+bfx^2-\frac{2b^2cx^3}{a}}{\sqrt{a+bx^4}}dx}{2ab} + \frac{c\int \frac{1}{x\sqrt{a+bx^4}}dx}{a}$$

$$= \frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(-\frac{2b^2cx^3}{a\sqrt{a+bx^4}}+\frac{-bd+bfx^2}{\sqrt{a+bx^4}}\right)dx}{2ab} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx,x,x^4\right)}{4a}$$

$$= \frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-bd+bfx^2}{\sqrt{a+bx^4}}dx}{2ab} + \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{a+bx^4}\right)}{2ab} + \frac{(bc)\ \ldots}{\ldots}$$

$$= \frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{f\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}dx}{2\sqrt{a}\sqrt{b}} + \frac{\left(d-\ldots\right)}{\ldots}$$

$$= \frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{f\ \ldots}{\ldots}$$

Mathematica [C] time = 0.126972, size = 125, normalized size = 0.39

$$\frac{3c\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^4}{a}+1\right)+x\left(3d\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+2fx^2\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)+3d+3ex\right)}{6a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] (3*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] + x*(3*d + 3*e*x + 3*d*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*f*x^
2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(6*a
*Sqrt[a + b*x^4])
```

Maple [C] time = 0.017, size = 336, normalized size = 1.

$$\frac{fx^3}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} - \frac{i}{2}f\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}\frac{1}{\sqrt{b}} + \frac{i}{2}f\sqrt{1-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f*x^3/a/((x^4+1/b*a)*b)^(1/2)-1/2*I*f/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)/b^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*I*f/a^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

```
+1/2*e/(b*x^4+a)^(1/2)/a*x^2+1/2*d*x/a/((x^4+1/b*a)*b)^(1/2)+1/2*d/a/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^
2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*c/a/(
b*x^4+a)^(1/2)-1/2*c/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\left(bx^4 + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + a}\left(fx^3 + ex^2 + dx + c\right)}{b^2x^9 + 2\,abx^5 + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^9 + 2*a*b*x^5 + a
^2*x), x)
```

Sympy [C] time = 24.4472, size = 289, normalized size = 0.89

$$c\left(\frac{2a^3\sqrt{1 + \frac{bx^4}{a}}}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1 + \frac{bx^4}{a}} + 1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4} - \frac{2a^2bx^4\log\left(\sqrt{1 + \frac{bx^4}{a}} + 1\right)}{4a^{\frac{9}{2}} + 4a^{\frac{7}{2}}bx^4}\right) + \frac{dx\Gamma\left(\frac{1}{4}\right)2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*
x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/
2) + 4*a**(7/2)*b*x**4)) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*
x**4/a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*a**(3/2)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\left(bx^4 + a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

3.548 $\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{}}{2a^2}$$

```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*c*x*Sqrt[a + b*
x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(
7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.382575, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1829, 1833, 1835, 1584, 1198, 220, 1196, 21, 266, 50, 63, 208}

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}cx\sqrt{a+}}{2a^2\left(\sqrt{a}+\sqrt{}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```


```
[Out] (x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (d*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*c*x*Sqrt[a + b*
x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/(2*a^(3/2)) - (3*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(
7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a + b*x^4])
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
```

```
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-bex^2-\frac{b^2cx^4}{a}-\frac{2b^2dx^5}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bc-bex^2-\frac{b^2cx^4}{a}}{x^2\sqrt{a+bx^4}}+\frac{-2bd-\frac{2b^2dx^4}{a}}{x\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-bex^2-\frac{b^2cx^4}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bd-\frac{2b^2dx^4}{a}}{x\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{2abex+6b^2cx^3}{x\sqrt{a+bx^4}}\,dx}{4a^2b} + \frac{d\int \frac{\sqrt{a+bx^4}}{x}\,dx}{a^2}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{2abe+6b^2cx^2}{\sqrt{a+bx^4}}\,dx}{4a^2b} + \frac{d\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^4\right)}{4a^2}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}c\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}} + \frac{d\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{a+b\ldots}}{\sqrt{a}}\right.}{2a^{3/2}}$$

Mathematica [C] time = 0.114255, size = 123, normalized size = 0.36

$$\frac{-2c\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^4}{a}\right)+dx\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^4}{a}+1\right)+x^2\left(e\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+e+fx\right)}{2ax\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] (d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] - 2*c*Sqrt[1 + (b*x^4)/
a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)] + x^2*(e + f*x + e*Sqrt[
1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(2*a*x*Sqrt
[a + b*x^4])
```

Maple [C] time = 0.01, size = 355, normalized size = 1.

$$\frac{fx^2}{2a}\frac{1}{\sqrt{bx^4+a}}+\frac{ex}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e}{2a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f/(b*x^4+a)^(1/2)/a*x^2+1/2*e*x/a/((x^4+1/b*a)*b)^(1/2)+1/2*e/a/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/a/(b
*x^4+a)^(1/2)-1/2*d/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/2*c*b
*x^3/a^2/((x^4+1/b*a)*b)^(1/2)-c*(b*x^4+a)^(1/2)/a^2/x+3/2*I*c*b^(1/2)/a^(3
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
-3/2*I*c*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{b^2x^{10}+2\,abx^6+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^10 + 2*a*b*x^6 +
a^2*x^2), x)
```

Sympy [C] time = 34.1444, size = 291, normalized size = 0.85

$$d\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{c\Gamma\left(-\frac{1}{4}\right)\dots}{4\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*
x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/
2) + 4*a**(7/2)*b*x**4)) + c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((1/4, 3
/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x**2/(2*
a**(3/2)*sqrt(1 + b*x**4/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\left(bx^4 + a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

3.549 $\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2a^2x^2}-$$

```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d*Sqrt[a + b*x^4])/(a
^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (
e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + S
qrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a
 + b*x^4])
```

Rubi [A] time = 0.477778, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1829, 1833, 1835, 1584, 1198, 220, 1196, 21, 266, 50, 63, 208}

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{2a^2x^2}+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{d\sqrt{a+b}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```


```
[Out] (x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (e*Sqrt[a +
 b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(2*a^2*x^2) - (d*Sqrt[a + b*x^4])/(a
^2*x) + (3*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (
e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a^(3/2)) - (3*b^(1/4)*d*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + ((3*Sqrt[b]*d + S
qrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*b^(1/4)*Sqrt[a
 + b*x^4])
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx = \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-2bex^2-bfx^3-\frac{b^2dx^5}{a}-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bd-bfx^2-\frac{b^2dx^4}{a}}{x^2\sqrt{a+bx^4}}+\frac{-2bc-2bex^2-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bd-bfx^2-\frac{b^2dx^4}{a}}{x^2\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bc-2bex^2-\frac{2b^2ex^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{8abex+8b^2ex^5}{x^2\sqrt{a+bx^4}}\,dx}{8a^2b} + \frac{\int \frac{2abfx+6b^2dx^{3}}{x\sqrt{a+bx^4}}}{4a^2b}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{\int \frac{8abe+8b^2ex^4}{x\sqrt{a+bx^4}}\,dx}{8a^2b} + \frac{\int \frac{2abf+6b^2dx^2}{\sqrt{a+bx^4}}}{4a^2b}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} - \frac{\left(3\sqrt{b}d\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}} + \frac{e\int \frac{\sqrt{a+bx^4}}{x}}{a^2}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\right.}{}$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} -$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} -$$

$$= \frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{2a^2x^2} - \frac{d\sqrt{a+bx^4}}{a^2x} + \frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} -$$

Mathematica [C] time = 0.106415, size = 140, normalized size = 0.38

$$\frac{-2adx\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^4}{a}\right)+aex^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^4}{a}+1\right)+afx^3\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-ac+afx^3-2bcx^4}{2a^2x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-(a*c) + a*f*x^3 - 2*b*c*x^4 + a*e*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 +
 (b*x^4)/a] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4,
 -((b*x^4)/a)] + a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((b*x^4)/a)])/(2*a^2*x^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.016, size = 363, normalized size = 1.

$$\frac{fx}{2a}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f}{2a}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\frac{e}{2a}\frac{1}{\sqrt{bx^4+a}}-\frac{e}{2}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f*x/a/((x^4+1/b*a)*b)^(1/2)+1/2*f/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*e/a/(b*x^4+a)^(1/2)-1/2*e/a^(3/2)*
ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-1/2*c/x^2*(2*b*x^4+a)/(b*x^4+a)^(1/
2)/a^2-1/2*d*b*x^3/a^2/((x^4+1/b*a)*b)^(1/2)-d*(b*x^4+a)^(1/2)/a^2/x+3/2*I*
d*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-3/2*I*d*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellip
ticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{b^2x^{11}+2\,abx^7+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^11 + 2*a*b*x^7 +
a^2*x^3), x)
```

Sympy [C] time = 42.4373, size = 316, normalized size = 0.86

$$c\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right)+e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

```
[Out] c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**
4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 +
b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)
/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2), (3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma(1/4)*h
yper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

3.550 $\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-9\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3a^2x^3}$$

```
[Out] -(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (f*Sqrt[a
+ b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(3*a^2*x^3) - (d*Sqrt[a + b*x^4])/(
2*a^2*x^2) - (e*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*e*x*Sqrt[a + b*x^4])/
(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a
^(3/2)) - (3*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*S
qrt[a + b*x^4]) - (b^(1/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.606259, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1829, 1833, 1835, 1585, 1584, 1198, 220, 1196, 21, 266, 50, 63, 208}

$$-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-9\sqrt{a}e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```


```
[Out] -(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(2*a^2*Sqrt[a + b*x^4]) + (f*Sqrt[a
+ b*x^4])/(2*a^2) - (c*Sqrt[a + b*x^4])/(3*a^2*x^3) - (d*Sqrt[a + b*x^4])/(
2*a^2*x^2) - (e*Sqrt[a + b*x^4])/(a^2*x) + (3*Sqrt[b]*e*x*Sqrt[a + b*x^4])/
(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*a
^(3/2)) - (3*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*S
qrt[a + b*x^4]) - (b^(1/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```


Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
```

```
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bdx-2bex^2-2bfx^3+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}-\frac{2b^2fx^7}{a}}{x^4\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \left(\frac{-2bc-2bex^2+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}}{x^4\sqrt{a+bx^4}}+\frac{-2bd-2bfx^2-\frac{2b^2fx^6}{a}}{x^3\sqrt{a+bx^4}}\right)dx}{2ab}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{\int \frac{-2bc-2bex^2+\frac{b^2cx^4}{a}-\frac{b^2ex^6}{a}}{x^4\sqrt{a+bx^4}}\,dx}{2ab} - \frac{\int \frac{-2bd-2bfx^2-\frac{2b^2fx^6}{a}}{x^3\sqrt{a+bx^4}}\,dx}{2ab}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} + \frac{\int \frac{12abex-10b^2cx^3+6b^2ex^5}{x^3\sqrt{a+bx^4}}\,dx}{12a^2b} + \frac{\int \frac{8}{}}{}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} + \frac{\int \frac{12abe-10b^2cx^2+6b^2ex^4}{x^2\sqrt{a+bx^4}}\,dx}{12a^2b} + \frac{\int \frac{8a}{}}{}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \frac{\int \frac{20ab^2cx-36ab^2ex^3}{x\sqrt{a+bx^4}}}{24a^3b}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \frac{\int \frac{20ab^2c-36ab^2ex^2}{\sqrt{a+bx^4}}\,d}{24a^3b}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} - \frac{(3}{}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} + \frac{3}{2a^2}$$

$$= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}} + \frac{f\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{3a^2x^3} - \frac{d\sqrt{a+bx^4}}{2a^2x^2} - \frac{e\sqrt{a+bx^4}}{a^2x} + \frac{3}{2a^2}$$

Mathematica [C] time = 0.119275, size = 136, normalized size = 0.35

$$\frac{-2ac\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^4}{a}\right)-3x\left(2aex\sqrt{\frac{bx^4}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^4}{a}\right)-afx^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^4}{a}+1\right)+ad+2bdx^4\right.}{6a^2x^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] (-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^4)/a)]
 - 3*x*(a*d + 2*b*d*x^4 - a*f*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^
4)/a] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*
x^4)/a)]))/(6*a^2*x^3*Sqrt[a + b*x^4])
```

Maple [C] time = 0.011, size = 383, normalized size = 1.

$$\frac{f}{2a}\frac{1}{\sqrt{bx^4+a}}-\frac{f}{2}\ln\left(\frac{1}{x^2}\left(2a+2\sqrt{a}\sqrt{bx^4+a}\right)\right)a^{-\frac{3}{2}}-\frac{c}{3x^3a^2}\sqrt{bx^4+a}-\frac{bcx}{2a^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5bc}{6a^2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2*f/a/(b*x^4+a)^(1/2)-1/2*f/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^
2)-1/3*c*(b*x^4+a)^(1/2)/x^3/a^2-1/2*c*b*x/a^2/((x^4+1/b*a)*b)^(1/2)-5/6*c*
b/a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-1/2*d/x^2*(2*b*x^4+a)/(b*x^4+a)^(1/2)/a^2-1/2*e*b*x^3/a^2/((x^4+1/b*a)*b
)^(1/2)-e*(b*x^4+a)^(1/2)/a^2/x+3/2*I*e*b^(1/2)/a^(3/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*I*e*b^(1/2)/a^(3/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)}{b^2x^{12}+2\,abx^8+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^12 + 2*a*b*x^8 +
a^2*x^4), x)
```

Sympy [C] time = 41.3974, size = 321, normalized size = 0.83

$$d\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right)+f\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] d*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**
4) + 1))) + f*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 +
b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)
/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) +
1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-3/4)*hyper((-3/4, 3/2), (1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4)) + e*gamma(-1/4)
*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3
/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^3 + ex^2 + dx + c}{\left(bx^4 + a\right)^{\frac{3}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

3.551 $\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=269

$$\frac{c(gx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{g(m+1)}+\frac{d(gx)^{m+2}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{g^2(m+2)}+$$

```
[Out] (c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
-((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)^(2 + m)*(a + b*x^4)
^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(g^2*(2 + m)*
(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 + m)*(a + b*x^4)^p*Hypergeometric2F1[(3 +
m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) + (f*(g
*x)^(4 + m)*(a + b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((b*
x^4)/a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.256017, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1833, 1336, 365, 364}

$$\frac{c(gx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{g(m+1)}+\frac{d(gx)^{m+2}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{g^2(m+2)}+$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```


```
[Out] (c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
-((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)^(2 + m)*(a + b*x^4)
^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(g^2*(2 + m)*
(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 + m)*(a + b*x^4)^p*Hypergeometric2F1[(3 +
m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) + (f*(g
*x)^(4 + m)*(a + b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((b*
x^4)/a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```


Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx = \int \left((gx)^m\left(c+ex^2\right)\left(a+bx^4\right)^p + \frac{(gx)^{1+m}\left(d+fx^2\right)\left(a+bx^4\right)^p}{g}\right)dx$$

$$= \frac{\int (gx)^{1+m}\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx}{g} + \int (gx)^m\left(c+ex^2\right)\left(a+bx^4\right)^p\,dx$$

$$= \frac{\int \left(d(gx)^{1+m}\left(a+bx^4\right)^p + \frac{f(gx)^{3+m}\left(a+bx^4\right)^p}{g^2}\right)dx}{g} + \int \left(c(gx)^m\left(a+bx^4\right)^p + \right.$$

$$= c\int (gx)^m\left(a+bx^4\right)^p\,dx + \frac{f\int (gx)^{3+m}\left(a+bx^4\right)^p\,dx}{g^3} + \frac{e\int (gx)^{2+m}\left(a+\right.}{g^2}$$

$$= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{bx^4}{a}\right)^p\,dx + \frac{\left(f\left(a+bx^4\right)^p\left(1+\right.\right.}{}$$

$$= \frac{c(gx)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{4},-p;\frac{5+m}{4};-\frac{bx^4}{a}\right)}{g(1+m)} + \frac{d(gx)^{2+m}\left(\right.}{}$$

Mathematica [A] time = 0.214322, size = 174, normalized size = 0.65

$$x(gx)^m\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(\frac{c\,{}_2F_1\left(\frac{m+1}{4},-p;\frac{m+5}{4};-\frac{bx^4}{a}\right)}{m+1}+x\left(\frac{d\,{}_2F_1\left(\frac{m+2}{4},-p;\frac{m+6}{4};-\frac{bx^4}{a}\right)}{m+2}+x\left(\frac{e\,{}_2F_1\left(\frac{m+3}{4},-p;\frac{m+7}{4};-\right.}{m+3}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (x*(g*x)^m*(a + b*x^4)^p*((c*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -(
(b*x^4)/a)])/(1 + m) + x*((d*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -(
(b*x^4)/a)])/(2 + m) + x*((e*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -(
(b*x^4)/a)])/(3 + m) + (f*x*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4, -((
b*x^4)/a)])/(4 + m)))))/(1 + (b*x^4)/a)^p
```

Maple [F] time = 0.314, size = 0, normalized size = 0.

$$\int (gx)^m\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((fx^3 + ex^2 + dx + c)(bx^4 + a)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx^3 + ex^2 + dx + c)(bx^4 + a)^p (gx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

3.552 $\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=143

$$\frac{cx\left(a+bx^4\right)^{p+1}{}_2F_1\left(1,p+\frac{5}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{a}+\frac{dx^2\left(a+bx^4\right)^{p+1}{}_2F_1\left(1,p+\frac{3}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{2a}+\frac{ex^3\left(a+bx^4\right)^{p+1}{}_2F_1\left(1,p+\frac{7}{4};\frac{7}{4};-\right.}{3a}$$

```
[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^(1 + p)*Hypergeome
tric2F1[1, 5/4 + p, 5/4, -((b*x^4)/a)])/a + (d*x^2*(a + b*x^4)^(1 + p)*Hype
rgeometric2F1[1, 3/2 + p, 3/2, -((b*x^4)/a)])/(2*a) + (e*x^3*(a + b*x^4)^(1
 + p)*Hypergeometric2F1[1, 7/4 + p, 7/4, -((b*x^4)/a)])/(3*a)
```

Rubi [A] time = 0.131025, antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1885, 1204, 246, 245, 365, 364, 1248, 641}

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hypergeometric2F
1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (d*x^2*(a + b*x^4)^p*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(2*(1 + (b*x^4)/a)^p) + (e*x^3*
(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4
)/a)^p)
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx &= \int \left(\left(c+ex^2\right)\left(a+bx^4\right)^p + x\left(d+fx^2\right)\left(a+bx^4\right)^p\right)dx\\ &= \int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx + \int x\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int (d+fx)\left(a+bx^2\right)^p\,dx, x, x^2\right) + \int \left(c\left(a+bx^4\right)^p + ex^2\left(a+bx^4\right)^p\right) d\\ &= \frac{f\left(a+bx^4\right)^{1+p}}{4b(1+p)} + c\int \left(a+bx^4\right)^p\,dx + \frac{1}{2}d\,\mathrm{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, x^2\right) + e\int\\ &= \frac{f\left(a+bx^4\right)^{1+p}}{4b(1+p)} + \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int\left(1+\frac{bx^4}{a}\right)^p\,dx + \frac{1}{2}\left(d\left(a+bx^4\right)^p\right.\\ &= \frac{f\left(a+bx^4\right)^{1+p}}{4b(1+p)} + cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + \frac{1}{2}dx^2\left(a+b\right.\end{aligned}$$

Mathematica [A] time = 0.111298, size = 147, normalized size = 1.03

$$\frac{1}{12}\left(a+bx^4\right)^p\left(12cx\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+6dx^2\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^4}{a}\right)+4ex^3\left(\frac{bx^4}{a}+1\right)^{-p}\,{}_2F_1\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*((3*f*(a + b*x^4))/(b*(1 + p)) + (12*c*x*Hypergeometric2F1[1
/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (6*d*x^2*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (4*e*x^3*Hypergeometric2F1
[3/4, -p, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p))/12
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \left(f x^{3}+e x^{2}+d x +c \right) \left(b \,x^{4}+a \right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(f x^{3}+e x^{2}+d x +c\right)\left(b x^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(f x^{3}+e x^{2}+d x+c\right)\left(b x^{4}+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Sympy [A] time = 70.5717, size = 141, normalized size = 0.99

$$\frac{a^{p} c x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{a^{p} d x^{2} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}{2} + \frac{a^{p} e x^{3} \Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}{4 \Gamma\left(\frac{7}{4}\right)} + f \left(\begin{cases} \frac{a^{p} x^{4}}{4} \\ \frac{\begin{cases} \frac{\left(a + b x^{4}\right)^{p + 1}}{p + 1} & \text{for}\: p \neq \\ \log{\left(a + b x^{4} \right)} & \text{otherw} \end{cases}}{4 b} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/a)
/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4)) + f*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise(((a +
b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b), True)
```

)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(f x^3 + e x^2 + d x + c\right)\left(b x^4 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

3.553 $\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=175

$$\frac{c\left(a+bx^4\right)^{p+1}}{4b(p+1)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^4}{a}\right)$$

```
[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometric
2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^4)
^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)])/(6*(1 + (b*x^4)/a)^p) + (
f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 +
 (b*x^4)/a)^p)
```

Rubi [A] time = 0.18225, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1833, 1252, 764, 261, 365, 364, 1336}

$$\frac{c\left(a+bx^4\right)^{p+1}}{4b(p+1)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] (c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometric
2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^4)
^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)])/(6*(1 + (b*x^4)/a)^p) + (
f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 +
 (b*x^4)/a)^p)
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx &= \int\left(x^3\left(c+ex^2\right)\left(a+bx^4\right)^p+x^4\left(d+fx^2\right)\left(a+bx^4\right)^p\right)dx\\
&= \int x^3\left(c+ex^2\right)\left(a+bx^4\right)^p\,dx+\int x^4\left(d+fx^2\right)\left(a+bx^4\right)^p\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x(c+ex)\left(a+bx^2\right)^p\,dx,x,x^2\right)+\int\left(dx^4\left(a+bx^4\right)^p+fx^6\left(a+bx\right.\right.\\
&= \frac{1}{2}c\operatorname{Subst}\left(\int x\left(a+bx^2\right)^p\,dx,x,x^2\right)+d\int x^4\left(a+bx^4\right)^p\,dx+\frac{1}{2}e\operatorname{Subst}\left(\int x\right.\\
&= \frac{c\left(a+bx^4\right)^{1+p}}{4b(1+p)}+\left(d\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^4\left(1+\frac{bx^4}{a}\right)^p\,dx+\frac{1}{2}\left(e\left(a+b\right.\right.\\
&= \frac{c\left(a+bx^4\right)^{1+p}}{4b(1+p)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6\left(a\right.
\end{aligned}$$

Mathematica [A] time = 0.114865, size = 145, normalized size = 0.83

$$\frac{\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(105c\left(a+bx^4\right)\left(\frac{bx^4}{a}+1\right)^p+84bd(p+1)x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+70be(p+1)x^6\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^4}{a}\right)\right.}{420b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^p*(105*c*(a + b*x^4)*(1 + (b*x^4)/a)^p + 84*b*d*(1 + p)*x^5*Hy
pergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeome
tric2F1[3/2, -p, 5/2, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2F1[
7/4, -p, 11/4, -((b*x^4)/a)]))/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int x^3\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

```
[Out] int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(fx^6+ex^5+dx^4+cx^3\right)\left(bx^4+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

3.554 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$

Optimal. Leaf size=8

$$-\log(1-x)$$

```
[Out] -Log[1 - x]
```

Rubi [A] time = 0.0098654, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586, 31}

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2+x^3+x^4}{1-x^5} \, dx &= \int \frac{1}{1-x} \, dx \\ &= -\log(1-x) \end{aligned}$$

Mathematica [A] time = 0.0010466, size = 8, normalized size = 1.

$$-\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

```
[Out] -Log[1 - x]
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3+x^2+x+1)/(-x^5+1),x)
```

```
[Out] -ln(-1+x)
```

Maxima [A] time = 0.91599, size = 8, normalized size = 1.

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="maxima")
```

```
[Out] -log(x - 1)
```

Fricas [A] time = 1.34621, size = 18, normalized size = 2.25

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="fricas")
```

```
[Out] -log(x - 1)
```

Sympy [A] time = 0.059698, size = 5, normalized size = 0.62

$$-\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
```

```
[Out] -log(x - 1)
```

Giac [A] time = 1.0536, size = 9, normalized size = 1.12

$$-\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1))
```

3.555 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx$

Optimal. Leaf size=10

$$\frac{1}{2}\log(2x+3)$$

[Out] Log[3 + 2*x]/2

Rubi [A] time = 0.0146238, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1586, 31}

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

```
[Out] Log[3 + 2*x]/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx &= \int \frac{1}{3+2x} dx \\ &= \frac{1}{2}\log(3+2x) \end{aligned}$$

Mathematica [A] time = 0.0010186, size = 10, normalized size = 1.

$$\frac{1}{2}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
,x]
```

```
[Out] Log[3 + 2*x]/2
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$\frac{\ln(3+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x)
```

```
[Out] 1/2*ln(3+2*x)
```

Maxima [A] time = 0.928414, size = 11, normalized size = 1.1

$$\frac{1}{2}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="maxima")
```

```
[Out] 1/2*log(2*x + 3)
```

Fricas [A] time = 1.34647, size = 24, normalized size = 2.4

$$\frac{1}{2}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="fricas")
```

```
[Out] 1/2*log(2*x + 3)
```

Sympy [A] time = 0.070157, size = 7, normalized size = 0.7

$$\frac{\log(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

```
[Out] log(2*x + 3)/2
```

Giac [A] time = 1.05907, size = 12, normalized size = 1.2

$$\frac{1}{2}\log(|2x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algori
thm="giac")
```

```
[Out] 1/2*log(abs(2*x + 3))
```

3.556 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\log(3-2x)$$

```
[Out] -Log[3 - 2*x]/2
```

Rubi [A] time = 0.0133504, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1586, 31}

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]

[Out] -Log[3 - 2*x]/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} dx &= \int \frac{1}{3-2x} dx\\ &= -\frac{1}{2}\log(3-2x)\end{aligned}$$

Mathematica [A] time = 0.0010264, size = 10, normalized size = 1.

$$-\frac{1}{2}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
,x]

[Out] -Log[3 - 2*x]/2
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$-\frac{\ln \left(-3+2 x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x)
```

```
[Out] -1/2*ln(-3+2*x)
```

Maxima [A] time = 0.920725, size = 11, normalized size = 1.1

$$-\frac{1}{2} \log \left(2 x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="maxima")
```

```
[Out] -1/2*log(2*x - 3)
```

Fricas [A] time = 1.35855, size = 26, normalized size = 2.6

$$-\frac{1}{2} \log \left(2 x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="fricas")
```

```
[Out] -1/2*log(2*x - 3)
```

Sympy [A] time = 0.068114, size = 8, normalized size = 0.8

$$-\frac{\log \left(2x-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

```
[Out] -log(2*x - 3)/2
```

Giac [A] time = 1.05218, size = 12, normalized size = 1.2

$$-\frac{1}{2} \log \left(|2 x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algorit
hm="giac")
```

```
[Out] -1/2*log(abs(2*x - 3))
```

3.557 $\int \frac{81+36x^2+16x^4}{729-64x^6} \, dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] ArcTanh[(2*x)/3]/6
```

Rubi [A] time = 0.009933, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586, 206}

$$\frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] ArcTanh[(2*x)/3]/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{81+36x^2+16x^4}{729-64x^6} \, dx &= \int \frac{1}{9-4x^2} \, dx\\ &= \frac{1}{6}\tanh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [B] time = 0.0026819, size = 21, normalized size = 2.1

$$\frac{1}{12}\log(2x+3)-\frac{1}{12}\log(3-2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -Log[3 - 2*x]/12 + Log[3 + 2*x]/12
```

Maple [B] time = 0.006, size = 18, normalized size = 1.8

$$\frac{\ln \left(3+2 x \right)}{12}-\frac{\ln \left(-3+2 x \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729),x)
```

```
[Out] 1/12*ln(3+2*x)-1/12*ln(-3+2*x)
```

Maxima [B] time = 0.895089, size = 23, normalized size = 2.3

$$\frac{1}{12} \log \left(2 x+3\right)-\frac{1}{12} \log \left(2 x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Fricas [B] time = 1.35319, size = 53, normalized size = 5.3

$$\frac{1}{12} \log \left(2 x+3\right)-\frac{1}{12} \log \left(2 x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Sympy [B] time = 0.0933, size = 15, normalized size = 1.5

$$-\frac{\log \left(x-\frac{3}{2}\right)}{12}+\frac{\log \left(x+\frac{3}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/12 + log(x + 3/2)/12
```

Giac [B] time = 1.04777, size = 20, normalized size = 2.

$$\frac{1}{12} \log \left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{12} \log \left(\left|x-\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

3.558 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6} \, dx$

Optimal. Leaf size=24

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Rubi [A] time = 0.0237435, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1586, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{81+54x-24x^3-16x^4}{729-64x^6} \, dx &= \int \frac{1}{9-6x+4x^2} \, dx \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-108-x^2} \, dx, x, -6+8x\right)\right) \\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0063597, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Maple [A] time = 0.003, size = 17, normalized size = 0.7

$$\frac{\sqrt{3}}{9}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x)
```

```
[Out] 1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.37219, size = 22, normalized size = 0.92

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Fricas [A] time = 1.35482, size = 58, normalized size = 2.42

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Sympy [A] time = 0.114066, size = 24, normalized size = 1.

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
```

```
[Out] sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

Giac [A] time = 1.05202, size = 22, normalized size = 0.92

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

3.559 $\int \frac{3-2x}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Rubi [A] time = 0.049007, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1586, 2058, 628, 618, 204}

$$-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{3-2x}{729-64x^6}\,dx = \int \frac{1}{243+162x+108x^2+72x^3+48x^4+32x^5}\,dx$$
$$= \int \left(\frac{1}{243(3+2x)} + \frac{3-4x}{486\left(9-6x+4x^2\right)} + \frac{1}{54\left(9+6x+4x^2\right)}\right) dx$$
$$= \frac{1}{486}\log(3+2x) + \frac{1}{486}\int \frac{3-4x}{9-6x+4x^2}\,dx + \frac{1}{54}\int \frac{1}{9+6x+4x^2}\,dx$$
$$= \frac{1}{486}\log(3+2x) - \frac{1}{972}\log\left(9-6x+4x^2\right) - \frac{1}{27}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, 6+8x\right)$$
$$= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}} + \frac{1}{486}\log(3+2x) - \frac{1}{972}\log\left(9-6x+4x^2\right)$$

Mathematica [A] time = 0.0169689, size = 50, normalized size = 1.

$$-\frac{1}{972}\log\left(4x^2-6x+9\right) + \frac{1}{486}\log(2x+3) + \frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6*
x + 4*x^2]/972
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$\frac{\ln(3+2x)}{486} + \frac{\sqrt{3}}{486}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right) - \frac{\ln\left(4x^2-6x+9\right)}{972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)/(-64*x^6+729),x)
```

```
[Out] 1/486*ln(3+2*x)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/972*ln(4*x^2-6
*x+9)
```

Maxima [A] time = 1.38798, size = 51, normalized size = 1.02

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{972}\log\left(4x^2-6x+9\right) + \frac{1}{486}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

Fricas [A] time = 1.43269, size = 128, normalized size = 2.56

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(2*x + 3)
```

Sympy [A] time = 0.162269, size = 46, normalized size = 0.92

$$\frac{\log\left(x+\frac{3}{2}\right)}{486}-\frac{\log\left(4x^2-6x+9\right)}{972}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9 +
 sqrt(3)/3)/486
```

Giac [A] time = 1.08276, size = 53, normalized size = 1.06

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log\left(|2x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
1/486*log(abs(2*x + 3))
```

3.560 $\int \frac{3+2x}{729-64x^6} \, dx$

Optimal. Leaf size=50

$$\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 + Log[9 + 6
*x + 4*x^2]/972
```

Rubi [A] time = 0.0481794, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1586, 2058, 618, 204, 628}

$$\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(729 - 64*x^6),x]
```


```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) - Log[3 - 2*x]/486 + Log[9 + 6
*x + 4*x^2]/972
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3+2x}{729-64x^6}\,dx &= \int \frac{1}{243-162x+108x^2-72x^3+48x^4-32x^5}\,dx\\
&= \int \left(-\frac{1}{243(-3+2x)}+\frac{1}{54\left(9-6x+4x^2\right)}+\frac{3+4x}{486\left(9+6x+4x^2\right)}\right)dx\\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{486}\int \frac{3+4x}{9+6x+4x^2}\,dx+\frac{1}{54}\int \frac{1}{9-6x+4x^2}\,dx\\
&= -\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)-\frac{1}{27}\,\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{486}\log(3-2x)+\frac{1}{972}\log\left(9+6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0134169, size = 46, normalized size = 0.92

$$\frac{1}{972}\left(\log\left(4x^2+6x+9\right)-2\log(3-2x)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x +
4*x^2])/972
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$-\frac{\ln(-3+2x)}{486}+\frac{\ln\left(4x^2+6x+9\right)}{972}+\frac{\sqrt{3}}{486}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(-64*x^6+729),x)
```

```
[Out] -1/486*ln(-3+2*x)+1/972*ln(4*x^2+6*x+9)+1/486*3^(1/2)*arctan(1/18*(8*x-6)*3
^(1/2))
```

Maxima [A] time = 1.3875, size = 51, normalized size = 1.02

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

Fricas [A] time = 1.40525, size = 128, normalized size = 2.56

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log\left(2x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(2*x - 3)
```

Sympy [A] time = 0.16509, size = 46, normalized size = 0.92

$$-\frac{\log\left(x-\frac{3}{2}\right)}{486}+\frac{\log\left(4x^2+6x+9\right)}{972}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/486
```

Giac [A] time = 1.04905, size = 53, normalized size = 1.06

$$\frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log\left(|2x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
1/486*log(abs(2*x - 3))
```

3.561 $\int \frac{9-6x+4x^2}{729-64x^6}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x
]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rubi [A] time = 0.0512375, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1586, 2058, 634, 618, 204, 628}

$$-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*x
]/108 - Log[9 + 6*x + 4*x^2]/324
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{9-6x+4x^2}{729-64x^6}\,dx &= \int \frac{1}{81+54x-24x^3-16x^4}\,dx \\
&= \int \left(-\frac{1}{162(-3+2x)}+\frac{1}{54(3+2x)}+\frac{3-2x}{81\left(9+6x+4x^2\right)}\right) dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)+\frac{1}{81}\int \frac{3-2x}{9+6x+4x^2}\,dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\int \frac{6+8x}{9+6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9+6x+4x^2}\,dx \\
&= -\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)-\frac{1}{9}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,6 \right. \\
&= \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0126314, size = 56, normalized size = 0.93

$$
\frac{1}{324}\left(-\log\left(4x^2+6x+9\right)-\log(3-2x)+3\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3 + 2*x] -
Log[9 + 6*x + 4*x^2])/324
```

Maple [A] time = 0.007, size = 47, normalized size = 0.8

$$
\frac{\ln(3+2x)}{108}-\frac{\ln(-3+2x)}{324}-\frac{\ln\left(4x^2+6x+9\right)}{324}+\frac{\sqrt{3}}{162}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729),x)
```

```
[Out] 1/108*ln(3+2*x)-1/324*ln(-3+2*x)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arctan
(1/18*(8*x+6)*3^(1/2))
```

Maxima [A] time = 1.37365, size = 62, normalized size = 1.03

$$
\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(2x+3)-\frac{1}{324}\log(2x-3)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Fricas [A] time = 1.43479, size = 157, normalized size = 2.62

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(2x+3)-\frac{1}{324}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Sympy [A] time = 0.181276, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{324}+\frac{\log\left(x+\frac{3}{2}\right)}{108}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

Giac [A] time = 1.04567, size = 65, normalized size = 1.08

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log(|2x+3|)-\frac{1}{324}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
1/108*log(abs(2*x + 3)) - 1/324*log(abs(2*x - 3))
```

3.562 $\int \frac{9+6x+4x^2}{729-64x^6} dx$

Optimal. Leaf size=60

$$\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 + Log[3 + 2*
x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rubi [A] time = 0.0485636, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1586, 2058, 634, 618, 204, 628}

$$\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/108 + Log[3 + 2*
x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{9+6x+4x^2}{729-64x^6}\,dx &= \int \frac{1}{81-54x+24x^3-16x^4}\,dx\\
&= \int \left(-\frac{1}{54(-3+2x)}+\frac{1}{162(3+2x)}+\frac{3+2x}{81\left(9-6x+4x^2\right)}\right)dx\\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{81}\int \frac{3+2x}{9-6x+4x^2}\,dx\\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{18}\int \frac{1}{9-6x+4x^2}\,dx\\
&= -\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)-\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6\right.\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(3+2x)+\frac{1}{324}\log\left(9-6x+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0108217, size = 52, normalized size = 0.87

$$\frac{1}{324}\left(\log\left(4x^2-6x+9\right)-3\log(3-2x)+\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x] +
 Log[9 - 6*x + 4*x^2])/324
```

Maple [A] time = 0.006, size = 47, normalized size = 0.8

$$\frac{\ln(3+2x)}{324}-\frac{\ln(-3+2x)}{108}+\frac{\ln\left(4x^2-6x+9\right)}{324}+\frac{\sqrt{3}}{162}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729),x)
```

```
[Out] 1/324*ln(3+2*x)-1/108*ln(-3+2*x)+1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan
(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.38407, size = 62, normalized size = 1.03

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(2x+3)-\frac{1}{108}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Fricas [A] time = 1.43353, size = 157, normalized size = 2.62

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(2x+3)-\frac{1}{108}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Sympy [A] time = 0.179838, size = 56, normalized size = 0.93

$$-\frac{\log\left(x-\frac{3}{2}\right)}{108}+\frac{\log\left(x+\frac{3}{2}\right)}{324}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(3
)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

Giac [A] time = 1.072, size = 65, normalized size = 1.08

$$\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log(|2x+3|)-\frac{1}{108}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))
```

3.563 $\int \frac{27-8x^3}{729-64x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Rubi [A] time = 0.0262205, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {26, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{27-8x^3}{729-64x^6}\,dx &= \int \frac{1}{27+8x^3}\,dx\\
&= \frac{1}{27}\int \frac{1}{3+2x}\,dx + \frac{1}{27}\int \frac{6-2x}{9-6x+4x^2}\,dx\\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\int \frac{-6+8x}{9-6x+4x^2}\,dx + \frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx\\
&= \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)\\
&= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0056041, size = 50, normalized size = 1.

$$
-\frac{1}{108}\log\left(4x^2-6x+9\right) + \frac{1}{54}\log(2x+3) + \frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*x
 + 4*x^2]/108
```

Maple [A] time = 0.005, size = 39, normalized size = 0.8

$$
\frac{\ln(3+2x)}{54} - \frac{\ln\left(4x^2-6x+9\right)}{108} + \frac{\sqrt{3}}{54}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-8*x^3+27)/(-64*x^6+729),x)
```

```
[Out] 1/54*ln(3+2*x)-1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/
2))
```

Maxima [A] time = 1.37319, size = 51, normalized size = 1.02

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

Fricas [A] time = 1.3748, size = 126, normalized size = 2.52

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\log\left(4x^2-6x+9\right)+\frac{1}{54}\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) + 1
/54*log(2*x + 3)
```

Sympy [A] time = 0.138369, size = 48, normalized size = 0.96

$$\frac{\log\left(x+\frac{3}{2}\right)}{54}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{108}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x**3+27)/(-64*x**6+729),x)
```

```
[Out] log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

Giac [A] time = 1.04442, size = 47, normalized size = 0.94

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{108}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)+\frac{1}{54}\log\left(\left|x+\frac{3}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4) +
 1/54*log(abs(x + 3/2))
```

3.564 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6} dx$

Optimal. Leaf size=50

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Rubi [A] time = 0.0428354, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1586, 2058, 634, 618, 204, 628}

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] -ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx &= \int \frac{1}{27-36x+24x^2-8x^3}\,dx\\ &= \int \left(-\frac{1}{9(-3+2x)}+\frac{2x}{9\left(9-6x+4x^2\right)}\right)dx\\ &= -\frac{1}{18}\log(3-2x)+\frac{2}{9}\int \frac{x}{9-6x+4x^2}\,dx\\ &= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\int \frac{-6+8x}{9-6x+4x^2}\,dx+\frac{1}{6}\int \frac{1}{9-6x+4x^2}\,dx\\ &= -\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx,x,-6+8x\right)\\ &= -\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{18}\log(3-2x)+\frac{1}{36}\log\left(9-6x+4x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0097146, size = 50, normalized size = 1.

$$\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)+\frac{\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

```
[Out] ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*x
 + 4*x^2]/36
```

Maple [A] time = 0.004, size = 39, normalized size = 0.8

$$-\frac{\ln(-3+2x)}{18}+\frac{\ln\left(4x^2-6x+9\right)}{36}+\frac{\sqrt{3}}{54}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x)
```

```
[Out] -1/18*ln(-3+2*x)+1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1
/2))
```

Maxima [A] time = 1.36732, size = 51, normalized size = 1.02

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

Fricas [A] time = 1.41497, size = 124, normalized size = 2.48

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(2*x - 3)
```

Sympy [A] time = 0.145883, size = 48, normalized size = 0.96

$$-\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9
- sqrt(3)/3)/54
```

Giac [A] time = 1.06169, size = 53, normalized size = 1.06

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(|2x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] 1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1/
18*log(abs(2*x - 3))
```

3.565 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\log(4x^2-6x+9)}{17496}-\frac{\log(4x^2+6x+9)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

```
[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*
x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```

Rubi [A] time = 0.118137, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1586, 2074, 634, 618, 204, 628}

$$-\frac{\log(4x^2-6x+9)}{17496}-\frac{\log(4x^2+6x+9)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```



```
[Out] -1/(2916*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 + 2*
x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3+2x)^2\left(243-162x+108x^2-72x^3+48x^4-32x^5\right)}\,dx$$

$$= \int \left(-\frac{1}{8748(-3+2x)}+\frac{1}{1458(3+2x)^2}+\frac{5}{8748(3+2x)}+\frac{3}{4374\left(9\right.}\right.$$

$$= -\frac{1}{2916(3+2x)}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}+\frac{\int\frac{3-2x}{9-6x+4x^2}\,dx}{4374}+$$

$$= -\frac{1}{2916(3+2x)}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\int\frac{-6+8x}{9-6x+4x^2}\,dx}{17496}-$$

$$= -\frac{1}{2916(3+2x)}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\log\left(9-6x+4x^2\right.}{17496}$$

$$= -\frac{1}{2916(3+2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(3-2x)}{17496}+\frac{5}{}$$

Mathematica [A] time = 0.0872429, size = 100, normalized size = 0.91

$$\frac{-3\log\left(4x^2-6x+9\right)-3\log\left(4x^2+6x+9\right)-\frac{18}{2x+3}-3\log(3-2x)+15\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+6\sqrt{3}\tan^{-}}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcTa
n[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6*x
 + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

Maple [A] time = 0.01, size = 85, normalized size = 0.8

$$-\frac{1}{8748+5832\,x}+\frac{5\,\ln(3+2\,x)}{17496}-\frac{\ln(-3+2\,x)}{17496}-\frac{\ln\left(4\,x^2+6\,x+9\right)}{17496}+\frac{\sqrt{3}}{8748}\arctan\left(\frac{(8\,x+6)\sqrt{3}}{18}\right)-\frac{\ln\left(4\,x^2-6\,x\right.}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x)
```

```
[Out] -1/2916/(3+2*x)+5/17496*ln(3+2*x)-1/17496*ln(-3+2*x)-1/17496*ln(4*x^2+6*x+9
)+1/8748*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/17496*ln(4*x^2-6*x+9)+1/262
44*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.40531, size = 113, normalized size = 1.03

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right)-\frac{1}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="maxima")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3) - 1/17496*log(2*x - 3)
```

Fricas [A] time = 1.49653, size = 342, normalized size = 3.11

$$\frac{6\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3(2x+3)\log\left(4x^2+6x+9\right)-3(2x+\cdots}{52488(2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="fricas")
```

```
[Out] 1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*x
 + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) - 3*
(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)*lo
g(2*x - 3) - 18)/(2*x + 3)
```

Sympy [A] time = 0.358786, size = 105, normalized size = 0.95

$$-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{5\log\left(x+\frac{3}{2}\right)}{17496}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{26244}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{8748}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,x
)
```

```
[Out] -log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496
- log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/2
6244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)
```

Giac [A] time = 1.07194, size = 116, normalized size = 1.05

$$\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right)-\frac{1}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, algo
rithm="giac")
```

```
[Out] 1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/174
96*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*x -
 3))
```

3.566 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$

Optimal. Leaf size=110

$$\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[
(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*
x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

Rubi [A] time = 0.117263, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1586, 2074, 634, 618, 204, 628}

$$\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan[
(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 + 2*
x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{\left(729-64x^6\right)^2}dx &= \int \frac{1}{(3-2x)^2\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)}dx\\ &= \int \left(\frac{1}{1458(-3+2x)^2}-\frac{5}{8748(-3+2x)}+\frac{1}{8748(3+2x)}+\frac{3}{4374\,(9\right.\\ &= \frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\int \frac{3+2x}{9-6x+4x^2}dx}{4374}+\int\\ &= \frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\int \frac{-6+8x}{9-6x+4x^2}dx}{17496}+\int\\ &= \frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(3+2x)}{17496}+\frac{\log\left(9-6x+4x^2\right)}{17496}\\ &= \frac{1}{2916(3-2x)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}-\frac{5\log(3-2x)}{17496}+\frac{l}{}\end{aligned}$$

Mathematica [A] time = 0.076873, size = 97, normalized size = 0.88

$$\frac{3\left(\log\left(4x^2-6x+9\right)+\log\left(4x^2+6x+9\right)+\frac{6}{3-2x}-5\log(3-2x)+\log(2x+3)\right)+6\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{4}{3}\right.}{52488}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)
^2,x]
```

```
[Out] (6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*S
qrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x + 4
*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

Maple [A] time = 0.009, size = 85, normalized size = 0.8

$$\frac{\ln(3+2x)}{17496}-\frac{1}{-8748+5832x}-\frac{5\ln(-3+2x)}{17496}+\frac{\ln\left(4x^2+6x+9\right)}{17496}+\frac{\sqrt{3}}{26244}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)+\frac{\ln\left(4x^2-6\right.}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x)
```

```
[Out] 1/17496*ln(3+2*x)-1/2916/(-3+2*x)-5/17496*ln(-3+2*x)+1/17496*ln(4*x^2+6*x+9
)+1/26244*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/17496*ln(4*x^2-6*x+9)+1/87
48*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.38071, size = 113, normalized size = 1.03

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="maxima")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3) - 5/17496*log(2*x - 3)
```

Fricas [A] time = 1.43763, size = 342, normalized size = 3.11

$$\frac{2\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+6\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(2x-3)\log\left(4x^2+6x+9\right)+3(2x-}{52488(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="fricas")
```

```
[Out] 1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*x
 - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) + 3*
(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)*lo
g(2*x - 3) - 18)/(2*x - 3)
```

Sympy [A] time = 0.351426, size = 105, normalized size = 0.95

$$-\frac{5\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{8748}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{26244}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x)
```

```
[Out] -5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496
+ log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/8
748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)
```

Giac [A] time = 1.07353, size = 116, normalized size = 1.05

$$\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algor
ithm="giac")
```

```
[Out] 1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*s
qrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/174
96*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*x -
 3))
```

3.567 $\int \frac{81+36x^2+16x^4}{(729-64x^6)^2} dx$

Optimal. Leaf size=81

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(
13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[(2
*x)/3]/8748
```

Rubi [A] time = 0.0692554, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1586, 1170, 207, 618, 204}

$$\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] 1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(
13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[(2
*x)/3]/8748
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \int \frac{1}{(9-4x^2)^2(81+36x^2+16x^4)}\,dx$$
$$= \int \left(\frac{1}{8748(-3+2x)^2} + \frac{1}{8748(3+2x)^2} - \frac{1}{1458(-9+4x^2)} + \frac{1}{4374(9-6x+4x^2)} + \frac{}{4374(9}\right.$$
$$= \frac{1}{17496(3-2x)} - \frac{1}{17496(3+2x)} + \frac{\int \frac{1}{9-6x+4x^2}\,dx}{4374} + \frac{\int \frac{1}{9+6x+4x^2}\,dx}{4374} - \frac{\int \frac{1}{-9+4x^2}\,dx}{1458}$$
$$= \frac{1}{17496(3-2x)} - \frac{1}{17496(3+2x)} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748} - \frac{\text{Subst}\left(\int \frac{1}{-108-x^2}\,dx, x, -6+8x\right)}{2187} - \text{Subs}$$
$$= \frac{1}{17496(3-2x)} - \frac{1}{17496(3+2x)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2x}{3}\right)}{8748}$$

Mathematica [C] time = 0.430905, size = 122, normalized size = 1.51

$$\frac{\frac{36x}{9-4x^2} - 9\log(3-2x) + 9\log(2x+3) + 3\sqrt{3}\tan^{-1}\left(\frac{1}{3}\left(\sqrt{3}-i\right)x\right) + 4i\sqrt{3}\tanh^{-1}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right) + \left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\text{t}}{157464}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4*I)*Sqrt[3
]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTanh
[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464
```

Maple [A] time = 0.01, size = 68, normalized size = 0.8

$$-\frac{1}{52488+34992x} + \frac{\ln(3+2x)}{17496} - \frac{1}{-52488+34992x} - \frac{\ln(-3+2x)}{17496} + \frac{\sqrt{3}}{39366}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right) + \frac{\sqrt{3}}{39366}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x)
```

```
[Out] -1/17496/(3+2*x)+1/17496*ln(3+2*x)-1/17496/(-3+2*x)-1/17496*ln(-3+2*x)+1/39
366*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/39366*3^(1/2)*arctan(1/18*(8*x-6
)*3^(1/2))
```

Maxima [A] time = 1.37462, size = 82, normalized size = 1.01

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374(4x^2-9)} + \frac{1}{17496}\log(2x+3) - \frac{}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/17496*
log(2*x - 3)
```

Fricas [A] time = 1.52705, size = 259, normalized size = 3.2

$$\frac{4\sqrt{3}\left(4x^2-9\right)\arctan\left(\frac{4}{81}\sqrt{3}\left(2x^3+9x\right)\right)+4\sqrt{3}\left(4x^2-9\right)\arctan\left(\frac{2}{9}\sqrt{3}x\right)+9\left(4x^2-9\right)\log\left(2x+3\right)-9\left(4x^2-9\right)\log}{157464\left(4x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqrt
(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4*x
^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)
```

Sympy [A] time = 0.201229, size = 70, normalized size = 0.86

$$-\frac{x}{17496x^2-39366}+\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{2\sqrt{3}x}{9}\right)+2\operatorname{atan}\left(\frac{8\sqrt{3}x^3}{81}+\frac{4\sqrt{3}x}{9}\right)\right)}{78732}-\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3)
*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/17496
```

Giac [A] time = 1.06105, size = 85, normalized size = 1.05

$$\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2-9\right)}+\frac{1}{17496}\log\left(|2x+3|\right)-\frac{1}{1749}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1/1
7496*log(abs(2*x - 3))
```

3.568 $\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=92

$$\frac{x}{4374\left(4x^2-6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) -
 Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Lo
g[9 + 6*x + 4*x^2]/52488
```

Rubi [A] time = 0.116453, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{x}{4374\left(4x^2-6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3]) -
 Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 + Lo
g[9 + 6*x + 4*x^2]/52488
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9-6x+4x^2\right)^2\left(81+54x-24x^3-16x^4\right)}\,dx$$

$$= \int \left(-\frac{1}{13122(-3+2x)} + \frac{1}{39366(3+2x)} + \frac{3-x}{729\left(9-6x+4x^2\right)^2} + \frac{39-4x}{78732\left(9-6x+4x^2\right)} \right.$$

$$= -\frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} + \frac{\int \frac{39-4x}{9-6x+4x^2}\,dx}{78732} + \frac{\int \frac{3+4x}{9+6x+4x^2}\,dx}{26244} + \frac{1}{729}\int \frac{3-x}{\left(9-6x+4x^2\right)^2}$$

$$= \frac{x}{4374\left(9-6x+4x^2\right)} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} + \frac{\log\left(9+6x+4x^2\right)}{52488} - \frac{\int \frac{-6+8x}{9-6x+4x^2}\,dx}{157464}$$

$$= \frac{x}{4374\left(9-6x+4x^2\right)} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log\left(9-6x+4x^2\right)}{157464} + \frac{\log\left(9+6x+\right.}{52488}$$

$$= \frac{x}{4374\left(9-6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log\left(9-6x+4x^2\right)}{157464}$$

Mathematica [A] time = 0.029823, size = 84, normalized size = 0.91

$$\frac{\frac{36x}{4x^2-6x+9} - \log\left(4x^2-6x+9\right) + 3\log\left(4x^2+6x+9\right) - 6\log(3-2x) + 2\log(2x+3) + 12\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)}{157464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

```
[Out] ((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*L
og[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2
])/157464
```

Maple [A] time = 0.01, size = 73, normalized size = 0.8

$$\frac{\ln(3+2x)}{78732} - \frac{\ln(-3+2x)}{26244} + \frac{\ln\left(4x^2+6x+9\right)}{52488} + \frac{x}{17496}\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)^{-1} - \frac{\ln\left(4x^2-6x+9\right)}{157464} + \frac{\sqrt{3}}{13122}\arctan\left(\frac{(8x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x)
```

```
[Out] 1/78732*ln(3+2*x)-1/26244*ln(-3+2*x)+1/52488*ln(4*x^2+6*x+9)+1/17496*x/(x^2
-3/2*x+9/4)-1/157464*ln(4*x^2-6*x+9)+1/13122*3^(1/2)*arctan(1/18*(8*x-6)*3^
(1/2))
```

Maxima [A] time = 1.38606, size = 100, normalized size = 1.09

$$\frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(4x^2-6x+9\right)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
+ 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*lo
g(2*x + 3) - 1/26244*log(2*x - 3)
```

Fricas [A] time = 1.65431, size = 338, normalized size = 3.67

$$\frac{12\sqrt{3}\left(4x^2-6x+9\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\left(4x^2-6x+9\right)\log\left(4x^2+6x+9\right)-\left(4x^2-6x+9\right)\log\left(4x^2-6x\right.}{157464\left(4x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/157464*(12*sqrt(3)*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(4
*x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9) - (4*x^2 - 6*x + 9)*log(4*x^2 - 6*x +
9) + 2*(4*x^2 - 6*x + 9)*log(2*x + 3) - 6*(4*x^2 - 6*x + 9)*log(2*x - 3) +
36*x)/(4*x^2 - 6*x + 9)
```

Sympy [A] time = 0.284686, size = 82, normalized size = 0.89

$$\frac{x}{17496x^2-26244x+39366}-\frac{\log\left(x-\frac{3}{2}\right)}{26244}+\frac{\log\left(x+\frac{3}{2}\right)}{78732}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464}+\frac{\log\left(4x^2+6x+9\right)}{52488}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}\right.}{13122}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
```

```
[Out] x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732
- log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*at
an(4*sqrt(3)*x/9 - sqrt(3)/3)/13122
```

Giac [A] time = 1.05603, size = 103, normalized size = 1.12

$$\frac{1}{13122}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(4x^2-6x+9\right)}+\frac{1}{52488}\log\left(4x^2+6x+9\right)-\frac{1}{157464}\log\left(4x^2-6x+9\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
+ 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*lo
g(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))
```

3.569 $\int \frac{3-2x}{(729-64x^6)^2} dx$

Optimal. Leaf size=148

$$\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(}{42}$$

```
[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x
]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rubi [A] time = 0.171598, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{3-x}{708588\left(4x^2-6x+9\right)}+\frac{x}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(}{42}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] -1/(708588*(3 + 2*x)) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 + 2*x
]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{3-2x}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3-2x)\left(243+162x+108x^2+72x^3+48x^4+32x^5\right)^2}\,dx$$

$$= \int \left(-\frac{1}{2125764(-3+2x)}+\frac{1}{354294(3+2x)^2}+\frac{1}{236196(3+2x)}-\frac{x}{39366\left(9-6x+4x^2\right)^2}+\frac{}{70858}\right.$$

$$= -\frac{1}{708588(3+2x)}-\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}+\frac{\int\frac{33+2x}{9+6x+4x^2}\,dx}{2125764}+\frac{\int\frac{7-6x}{9-6x+4x^2}\,dx}{708588}-\frac{\int\frac{x}{\left(9-6x+4x^2\right)^2}\,d}{39366}$$

$$= -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)}-\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}$$

$$= -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)}-\frac{\log(3-2x)}{4251528}+\frac{\log(3+2x)}{472392}$$

$$= -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)}-\frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\tan^{-1}\left(\frac{3}{3}\right)}{157464}$$

Mathematica [A] time = 0.0569492, size = 119, normalized size = 0.8

$$\frac{\frac{1944x}{32x^5+48x^4+72x^3+108x^2+162x+243}-9\log\left(4x^2-6x+9\right)+\log\left(4x^2+6x+9\right)-2\log(3-2x)+18\log(2x+3)+2\sqrt{3}\tan^{-1}\left(\frac{4}{3}\right.}{8503056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*Sqrt[3]*Ar
cTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 2
*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x + 4*
x^2])/8503056
```

Maple [A] time = 0.016, size = 115, normalized size = 0.8

$$-\frac{1}{2125764+1417176x}+\frac{\ln(3+2x)}{472392}-\frac{\ln(-3+2x)}{4251528}+\frac{x}{944784}\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)^{-1}+\frac{\ln(4x^2+6x+9)}{8503056}+\frac{\sqrt{3}}{472392}\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)/(-64*x^6+729)^2,x)
```

```
[Out] -1/708588/(3+2*x)+1/472392*ln(3+2*x)-1/4251528*ln(-3+2*x)+1/944784*x/(x^2+3
/2*x+9/4)+1/8503056*ln(4*x^2+6*x+9)+1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^
(1/2))-1/708588*(1/4*x-3/4)/(x^2-3/2*x+9/4)-1/944784*ln(4*x^2-6*x+9)+1/4251
528*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.37397, size = 142, normalized size = 0.96

$$\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(32x^5+48x^4+72x^3+108x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*
x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) +
 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)
```

Fricas [B] time = 1.68333, size = 736, normalized size = 4.97

$$18\sqrt{3}\left(32x^5+48x^4+72x^3+108x^2+162x+243\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}\left(32x^5+48x^4+72x^3+108x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*ar
ctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2
 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^3 +
 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*x^3
+ 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 + 72*x^
3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3 + 108
*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3 + 108*
x^2 + 162*x + 243)
```

Sympy [A] time = 0.502706, size = 124, normalized size = 0.84

$$\frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882}-\frac{\log\left(x-\frac{3}{2}\right)}{4251528}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}-\frac{\log\left(x^2-\frac{3x}{2}+\right.}{944784}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x**6+729)**2,x)
```

```
[Out] x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 10628
82) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4)/
944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqr
t(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Giac [A] time = 1.07046, size = 150, normalized size = 1.01

$$\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(4x^2+6x+9\right)\left(4x^2-6x+9\right)(2x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
+ 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/
472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))
```

3.570 $\int \frac{3+2x}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=146

$$\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(}{47}$$

```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3
+ 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/
4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```

Rubi [A] time = 0.172511, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(}{47}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```


```
[Out] 1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(3
+ 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*x]/
4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{3+2x}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{(3+2x)\left(243-162x+108x^2-72x^3+48x^4-32x^5\right)^2}\,dx$$

$$= \int \left(\frac{1}{354294(-3+2x)^2} - \frac{1}{236196(-3+2x)} + \frac{1}{2125764(3+2x)} + \frac{3-x}{39366\left(9-6x+4x^2\right)^2} + \frac{1}{21257\ldots}\right.$$

$$= \frac{1}{708588(3-2x)} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528} + \frac{\int \frac{33-2x}{9-6x+4x^2}\,dx}{2125764} + \frac{\int \frac{7+6x}{9+6x+4x^2}\,dx}{708588} + \frac{\int \frac{3-x}{\left(9-6x+4x^2\right)^2}\,dx}{39366}\ldots$$

$$= \frac{1}{708588(3-2x)} + \frac{x}{236196\left(9-6x+4x^2\right)} - \frac{3+x}{708588\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528}\ldots$$

$$= \frac{1}{708588(3-2x)} + \frac{x}{236196\left(9-6x+4x^2\right)} - \frac{3+x}{708588\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528}\ldots$$

$$= \frac{1}{708588(3-2x)} + \frac{x}{236196\left(9-6x+4x^2\right)} - \frac{3+x}{708588\left(9+6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4\ldots}{3\sqrt{3}}\right)}{1417176\sqrt{\ldots}}$$

Mathematica [A] time = 0.0817094, size = 121, normalized size = 0.83

$$\frac{\frac{1944x}{-32x^5+48x^4-72x^3+108x^2-162x+243} - \log\left(4x^2-6x+9\right) + 9\log\left(4x^2+6x+9\right) - 18\log(3-2x) + 2\log(2x+3) + 18\sqrt{3}\tan^{-1}\ldots}{8503056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

```
[Out] ((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*Sqrt[3]*A
rcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 1
8*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x + 4*
x^2])/8503056
```

Maple [A] time = 0.013, size = 115, normalized size = 0.8

$$\frac{\ln (3+2x)}{4251528}-\frac{1}{-2125764+1417176x}-\frac{\ln (-3+2x)}{472392}+\frac{1}{708588}\left(-\frac{x}{4}-\frac{3}{4}\right)\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)^{-1}+\frac{\ln \left(4x^2+6x+9\right)}{944784}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(-64*x^6+729)^2,x)
```

```
[Out] 1/4251528*ln(3+2*x)-1/708588/(-3+2*x)-1/472392*ln(-3+2*x)+1/708588*(-1/4*x-
3/4)/(x^2+3/2*x+9/4)+1/944784*ln(4*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(1/18
*(8*x+6)*3^(1/2))+1/944784*x/(x^2-3/2*x+9/4)-1/8503056*ln(4*x^2-6*x+9)+1/47
2392*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.38129, size = 142, normalized size = 0.97

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(32x^5-48x^4+72x^3-108x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*
x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) +
 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)
```

Fricas [B] time = 1.50641, size = 736, normalized size = 5.04

$$\frac{2\sqrt{3}\left(32x^5-48x^4+72x^3-108x^2+162x-243\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+18\sqrt{3}\left(32x^5-48x^4+72x^3-108x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*arc
tan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2
 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*x^3
- 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3 - 108
*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3 - 108*
x^2 + 162*x - 243)
```

Sympy [A] time = 0.501011, size = 124, normalized size = 0.85

$$-\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882}-\frac{\log\left(x-\frac{3}{2}\right)}{472392}+\frac{\log\left(x+\frac{3}{2}\right)}{4251528}-\frac{\log\left(x^2-\frac{3x}{2}+\right.}{8503056}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 1062
882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/4)
/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sq
rt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528
```

Giac [A] time = 1.06199, size = 150, normalized size = 1.03

$$\frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2+6x+9\right)\left(4x^2-6x+9\right)(2x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*x
- 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) + 1/
4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))
```

3.571 $\int \frac{9-6x+4x^2}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=142

$$\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2}{354294}$$

```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 -
 Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```

Rubi [A] time = 0.148439, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1586, 2074, 634, 618, 204, 628, 614}

$$\frac{4x+3}{236196\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2}{354294}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```


```
[Out] 1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*x)
/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/118098 -
 Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{9-6x+4x^2}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9-6x+4x^2\right)\left(81+54x-24x^3-16x^4\right)^2}\,dx$$

$$= \int \left(\frac{1}{236196(-3+2x)^2} - \frac{1}{177147(-3+2x)} + \frac{1}{78732(3+2x)^2} + \frac{1}{59049(3+2x)} + \frac{3-2x}{236196\left(9-6x\right.}\right.$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} + \frac{\int \frac{21-10x}{9+6x+4x^2}\,dx}{708588} + \frac{\int \frac{3-2x}{9-6x+4x^2}\,dx}{236196}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} - \frac{\int \frac{-}{9-}}{94}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\left(9+6x+4x^2\right)} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} - \frac{\log\left(\right.}{}$$

$$= \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196\left(9+6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\mathrm{lo}}{}$$

Mathematica [A] time = 0.0630649, size = 111, normalized size = 0.78

$$\frac{\frac{648x}{-16x^4-24x^3+54x+81} - 3\log\left(4x^2-6x+9\right) - 5\log\left(4x^2+6x+9\right) - 8\log(3-2x) + 24\log(2x+3) + 2\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 18}{2834352}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*Log
[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352
```

Maple [A] time = 0.013, size = 111, normalized size = 0.8

$$-\frac{1}{472392+314928\,x}+\frac{\ln\left(3+2\,x\right)}{118098}-\frac{1}{-1417176+944784\,x}-\frac{\ln\left(-3+2\,x\right)}{354294}-\frac{1}{708588}\left(-3\,x-\frac{9}{4}\right)\left(x^{2}+\frac{3\,x}{2}+\frac{9}{4}\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-6*x+9)/(-64*x^6+729)^2,x)
```

```
[Out] -1/157464/(3+2*x)+1/118098*ln(3+2*x)-1/472392/(-3+2*x)-1/354294*ln(-3+2*x)-
1/708588*(-3*x-9/4)/(x^2+3/2*x+9/4)-5/2834352*ln(4*x^2+6*x+9)+1/157464*3^(1
/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/944784*ln(4*x^2-6*x+9)+1/1417176*3^(1/2)
*arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.37607, size = 128, normalized size = 0.9

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)-\frac{x}{4374\left(16\,x^{4}+24\,x^{3}-54\,x-81\right)}-\frac{5}{2834\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/2834352*
log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x + 3
) - 1/354294*log(2*x - 3)
```

Fricas [A] time = 1.46151, size = 539, normalized size = 3.8

$$18\sqrt{3}\left(16\,x^{4}+24\,x^{3}-54\,x-81\right)\arctan\left(\tfrac{1}{9}\sqrt{3}(4\,x+3)\right)+2\sqrt{3}\left(16\,x^{4}+24\,x^{3}-54\,x-81\right)\arctan\left(\tfrac{1}{9}\sqrt{3}(4\,x-3)\right)-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
 + 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 + 24
*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 81)*l
og(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 + 24*x^3 - 54*x - 81)
```

Sympy [A] time = 0.474707, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^{4}+104976x^{3}-236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{354294}+\frac{\log\left(x+\frac{3}{2}\right)}{118098}-\frac{\log\left(x^{2}-\frac{3x}{2}+\frac{9}{4}\right)}{944784}-\frac{5\log\left(x^{2}+\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 + l
og(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2 +
9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)*at
an(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

Giac [A] time = 1.06484, size = 143, normalized size = 1.01

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2+6x+9\right)(2x+3)(2x-3)}-\frac{}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3)) -
5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*l
og(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))
```

3.572 $\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2} dx$

Optimal. Leaf size=142

$$-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-}{11809}$$

```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/
(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 +
 (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```

Rubi [A] time = 0.144815, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1586, 2074, 614, 618, 204, 634, 628}

$$-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-}{11809}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```


```
[Out] 1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x)/
(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/354294 +
 (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(9+6x+4x^2\right)\left(81-54x+24x^3-16x^4\right)^2}\,dx$$

$$= \int \left(\frac{1}{78732(-3+2x)^2} - \frac{1}{59049(-3+2x)} + \frac{1}{236196(3+2x)^2} + \frac{1}{177147(3+2x)} + \frac{1}{4374\left(9-6x+\right.}\right.$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{\log(3-2x)}{118098} + \frac{\log(3+2x)}{354294} + \frac{\int \frac{21+10x}{9-6x+4x^2}\,dx}{708588} + \frac{\int \frac{3+2x}{9+6x+4x^2}\,dx}{236196}$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\left(9-6x+4x^2\right)} - \frac{\log(3-2x)}{118098} + \frac{\log(3+2x)}{354294} + \int$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\left(9-6x+4x^2\right)} - \frac{\log(3-2x)}{118098} + \frac{\log(3+2x)}{354294} + 5\log$$

$$= \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\left(9-6x+4x^2\right)} - \frac{\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} - \text{lo}$$

Mathematica [A] time = 0.0619316, size = 111, normalized size = 0.78

$$\frac{\frac{648x}{-16x^4+24x^3-54x+81} + 5\log\left(4x^2-6x+9\right) + 3\log\left(4x^2+6x+9\right) - 24\log(3-2x) + 8\log(2x+3) + 18\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 2}{2834352}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

```
[Out] ((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*Log
[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352
```

Maple [A] time = 0.013, size = 111, normalized size = 0.8

$$-\frac{1}{1417176+944784\,x}+\frac{\ln\left(3+2\,x\right)}{354294}-\frac{1}{-472392+314928\,x}-\frac{\ln\left(-3+2\,x\right)}{118098}+\frac{\ln\left(4\,x^{2}+6\,x+9\right)}{944784}+\frac{\sqrt{3}}{1417176}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+6*x+9)/(-64*x^6+729)^2,x)
```

```
[Out] -1/472392/(3+2*x)+1/354294*ln(3+2*x)-1/157464/(-3+2*x)-1/118098*ln(-3+2*x)+
1/944784*ln(4*x^2+6*x+9)+1/1417176*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/7
08588*(3*x-9/4)/(x^2-3/2*x+9/4)+5/2834352*ln(4*x^2-6*x+9)+1/157464*3^(1/2)*
arctan(1/18*(8*x-6)*3^(1/2))
```

Maxima [A] time = 1.37685, size = 128, normalized size = 0.9

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)-\frac{x}{4374\left(16\,x^{4}-24\,x^{3}+54\,x-81\right)}+\frac{1}{9447}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784*l
og(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x + 3
) - 1/118098*log(2*x - 3)
```

Fricas [A] time = 1.4611, size = 539, normalized size = 3.8

$$2\sqrt{3}\left(16\,x^{4}-24\,x^{3}+54\,x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+18\sqrt{3}\left(16\,x^{4}-24\,x^{3}+54\,x-81\right)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
+ 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x -
3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 - 24
*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81)*lo
g(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(16*x^4
 - 24*x^3 + 54*x - 81)
```

Sympy [A] time = 0.46865, size = 116, normalized size = 0.82

$$-\frac{x}{69984x^{4}-104976x^{3}+236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{118098}+\frac{\log\left(x+\frac{3}{2}\right)}{354294}+\frac{5\log\left(x^{2}-\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\log\left(x^{2}+\frac{3x}{2}+\frac{9}{4}\right)}{944784}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 + l
og(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2 +
 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*ata
n(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176
```

Giac [A] time = 1.06027, size = 143, normalized size = 1.01

$$\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2-6x+9\right)(2x+3)(2x-3)}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3)) +
1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*l
og(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))
```

3.573 $\int \frac{27-8x^3}{\left(729-64x^6\right)^2} \, dx$

Optimal. Leaf size=113

$$\frac{x}{4374\left(8x^3+27\right)}-\frac{7\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}}{52}$$

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3])
+ ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*
Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^
2]/314928
```

Rubi [A] time = 0.0818849, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1404, 414, 522, 200, 31, 634, 618, 204, 628}

$$\frac{x}{4374\left(8x^3+27\right)}-\frac{7\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}-\frac{\log(3-2x)}{157464}+\frac{7\log(2x+3)}{472392}-\frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\tan^{-1}}{52}$$

Antiderivative was successfully verified.

```
[In] Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] x/(4374*(27 + 8*x^3)) - (7*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3])
+ ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/157464 + (7*
Log[3 + 2*x])/472392 - (7*Log[9 - 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^
2]/314928
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(27-8x^3\right)\left(27+8x^3\right)^2}\,dx$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{\int \frac{-1080+128x^3}{(27-8x^3)(27+8x^3)}\,dx}{34992}$$

$$= \frac{x}{4374\left(27+8x^3\right)} + \frac{\int \frac{1}{27-8x^3}\,dx}{2916} + \frac{7\int \frac{1}{27+8x^3}\,dx}{8748}$$

$$= \frac{x}{4374\left(27+8x^3\right)} + \frac{\int \frac{1}{3-2x}\,dx}{78732} + \frac{\int \frac{6+2x}{9+6x+4x^2}\,dx}{78732} + \frac{7\int \frac{1}{3+2x}\,dx}{236196} + \frac{7\int \frac{6-2x}{9-6x+4x^2}\,dx}{236196}$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} + \frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{314928} - \frac{7\int \frac{-6+8x}{9-6x+4x^2}\,dx}{944784} + \frac{\int \frac{1}{9+6x+4}}{1749}$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log\left(9-6x+4x^2\right)}{944784} + \frac{\log\left(9+6x+4x^2\right)}{314928} -$$

$$= \frac{x}{4374\left(27+8x^3\right)} - \frac{7\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log(9-}{944}$$

Mathematica [A] time = 0.0501018, size = 103, normalized size = 0.91

$$\frac{\frac{216x}{8x^3+27} - 7\log\left(4x^2-6x+9\right) + 3\log\left(4x^2+6x+9\right) - 6\log(3-2x) + 14\log(2x+3) + 14\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) + 6\sqrt{3}\tan}{944784}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[
3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*Log
[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Maple [A] time = 0.013, size = 102, normalized size = 0.9

$$-\frac{1}{236196+157464\,x} + \frac{7\,\ln(3+2\,x)}{472392} - \frac{\ln(-3+2\,x)}{157464} + \frac{\ln\left(4\,x^2+6\,x+9\right)}{314928} + \frac{\sqrt{3}}{157464}\arctan\left(\frac{(8\,x+6)\sqrt{3}}{18}\right) - \frac{1}{11809}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-8*x^3+27)/(-64*x^6+729)^2,x)
```

```
[Out] -1/78732/(3+2*x)+7/472392*ln(3+2*x)-1/157464*ln(-3+2*x)+1/314928*ln(4*x^2+6
*x+9)+1/157464*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))-1/118098*(-3/4*x-9/8)/(
x^2-3/2*x+9/4)-7/944784*ln(4*x^2-6*x+9)+7/472392*3^(1/2)*arctan(1/18*(8*x-6
)*3^(1/2))
```

Maxima [A] time = 1.38086, size = 117, normalized size = 1.04

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\left(8\,x^3+27\right)} + \frac{1}{314928}\log\left(4\,x^2+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log(2*
x - 3)
```

Fricas [A] time = 1.42664, size = 377, normalized size = 3.34

$$\frac{6\sqrt{3}\left(8x^3+27\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+14\sqrt{3}\left(8x^3+27\right)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\left(8x^3+27\right)\log\left(4x^2+6x+9\right)-}{944784\left(8x^3+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] 1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3)
*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 + 6*
x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x + 3)
 - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)
```

Sympy [A] time = 0.3835, size = 110, normalized size = 0.97

$$\frac{x}{34992x^3+118098}-\frac{\log\left(x-\frac{3}{2}\right)}{157464}+\frac{7\log\left(x+\frac{3}{2}\right)}{472392}-\frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

```
[Out] x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*l
og(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3)*
atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(
3)/3)/157464
```

Giac [A] time = 1.05941, size = 120, normalized size = 1.06

$$\frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{x}{4374\left(8x^3+27\right)}+\frac{1}{314928}\log\left(4x^2+6x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] 1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1/
9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x + 9
) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/157464*l
og(abs(2*x - 3))
```

3.574 $\int \frac{27+36x+24x^2+8x^3}{\left(729-64x^6\right)^2} \, dx$

Optimal. Leaf size=131

$$-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}$$

```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 -
 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(15746
4*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*
x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Rubi [A] time = 0.147723, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1586, 2074, 638, 618, 204, 634, 628}

$$-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}$$

Antiderivative was successfully verified.

```
[In] Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```


```
[Out] 1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3 -
 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(15746
4*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 - 6*
x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{27+36x+24x^2+8x^3}{\left(729-64x^6\right)^2}\,dx = \int \frac{1}{\left(27-36x+24x^2-8x^3\right)^2\left(27+36x+24x^2+8x^3\right)}\,dx$$

$$= \int \left(\frac{1}{13122(-3+2x)^2} - \frac{7}{78732(-3+2x)} + \frac{1}{236196(3+2x)} + \frac{3+2x}{4374\left(9-6x+4x^2\right)^2} + \frac{}{11}\right.$$

$$= \frac{1}{26244(3-2x)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{3+17x}{9-6x+4x^2}\,dx}{118098} + \frac{\int \frac{x}{9+6x+4x^2}\,dx}{39366} + \frac{\int \frac{3}{(9-6}}{4}$$

$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244\left(9-6x+4x^2\right)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{\int \frac{6+8x}{9+6x+4x^2}\,dx}{314928} +$$

$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244\left(9-6x+4x^2\right)} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{17\log\left(9-6x+\right.}{944784}$$

$$= \frac{1}{26244(3-2x)} - \frac{3-2x}{26244\left(9-6x+4x^2\right)} - \frac{11\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{7\log(3-2x}{157464}$$

Mathematica [A] time = 0.0637757, size = 111, normalized size = 0.85

$$\frac{\frac{216x}{-8x^3+24x^2-36x+27} + 17\log\left(4x^2-6x+9\right) + 3\log\left(4x^2+6x+9\right) - 42\log(3-2x) + 2\log(2x+3) + 22\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right) - 2}{944784}$$

Antiderivative was successfully verified.

```
[In] Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

```
[Out] ((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqr
t[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Log[
3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Maple [A] time = 0.013, size = 102, normalized size = 0.8

$$\frac{\ln(3+2x)}{472392}-\frac{1}{-78732+52488x}-\frac{7\ln(-3+2x)}{157464}+\frac{\ln(4x^2+6x+9)}{314928}-\frac{\sqrt{3}}{472392}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)+\frac{1}{118098}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x)
```

```
[Out] 1/472392*ln(3+2*x)-1/26244/(-3+2*x)-7/157464*ln(-3+2*x)+1/314928*ln(4*x^2+6
*x+9)-1/472392*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/118098*(9/4*x-27/8)/(
x^2-3/2*x+9/4)+17/944784*ln(4*x^2-6*x+9)+11/472392*3^(1/2)*arctan(1/18*(8*x
-6)*3^(1/2))
```

Maxima [A] time = 1.3948, size = 128, normalized size = 0.98

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374(8x^3-24x^2+36x-27)}+\frac{1}{3149}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928*l
og(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x + 3
) - 7/157464*log(2*x - 3)
```

Fricas [A] time = 1.44256, size = 531, normalized size = 4.05

$$-\frac{2\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-22\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

```
[Out] -1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x +
 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x - 3)
) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24*x^
2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*
x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3 - 24*
x^2 + 36*x - 27)
```

Sympy [A] time = 0.46846, size = 119, normalized size = 0.91

$$-\frac{x}{34992x^3-104976x^2+157464x-118098}-\frac{7\log\left(x-\frac{3}{2}\right)}{157464}+\frac{\log\left(x+\frac{3}{2}\right)}{472392}+\frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right.}{314928}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

```
[Out] -x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464 +
 log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2
 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqrt(3
)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Giac [A] time = 1.04995, size = 134, normalized size = 1.02

$$-\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-\frac{x}{4374\left(4x^2-6x+9\right)(2x-3)}+\frac{1}{314928}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

```
[Out] -1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/314928*
log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs(2*
x + 3)) - 7/157464*log(abs(2*x - 3))
```

3.575 $\int \frac{x(27-2x^3)}{729-64x^6} dx$

Optimal. Leaf size=99

$$\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[
3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/288 + (5*Log[9 - 6*x
 + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rubi [A] time = 0.0627053, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1511, 292, 31, 634, 618, 204, 628}

$$\frac{5}{576}\log\left(4x^2-6x+9\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(2x+3)-\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{4x+3}{3\sqrt{3}}\right)}{32\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (-5*ArcTan[(3 - 4*x)/(3*Sqrt[3])])/(96*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[
3])]/(32*Sqrt[3]) - Log[3 - 2*x]/96 - (5*Log[3 + 2*x])/288 + (5*Log[9 - 6*x
 + 4*x^2])/576 + Log[9 + 6*x + 4*x^2]/192
```

Rule 1511

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)
^m/(q - c*x^n), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^n), x
], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(27-2x^3)}{729-64x^6}\,dx &= 3\int \frac{x}{216-64x^3}\,dx+5\int \frac{x}{216+64x^3}\,dx\\ &= \frac{1}{24}\int \frac{1}{6-4x}\,dx-\frac{1}{24}\int \frac{6-4x}{36+24x+16x^2}\,dx-\frac{5}{72}\int \frac{1}{6+4x}\,dx+\frac{5}{72}\int \frac{6+4x}{36-24x+16x^2}\,dx\\ &= -\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{1}{192}\int \frac{24+32x}{36+24x+16x^2}\,dx+\frac{5}{576}\int \frac{-24+32x}{36-24x+16x^2}\,dx-\frac{3}{8}\\ &= -\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(9-6x+4x^2\right)+\frac{1}{192}\log\left(9+6x+4x^2\right)+\frac{3}{4}\operatorname{Subst}\left(\right.\\ &= -\frac{5\tan^{-1}\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}}-\frac{\tan^{-1}\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}}-\frac{1}{96}\log(3-2x)-\frac{5}{288}\log(3+2x)+\frac{5}{576}\log\left(9-6x+4x^2\right)+\frac{1}{19}\end{aligned}$$

Mathematica [A] time = 0.0139777, size = 91, normalized size = 0.92

$$\frac{1}{576}\left(5\log\left(4x^2-6x+9\right)+3\log\left(4x^2+6x+9\right)-6\log(3-2x)-10\log(2x+3)+10\sqrt{3}\tan^{-1}\left(\frac{4x-3}{3\sqrt{3}}\right)-6\sqrt{3}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3*
Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*L
og[9 + 6*x + 4*x^2])/576
```

Maple [A] time = 0.007, size = 76, normalized size = 0.8

$$-\frac{5\ln(3+2x)}{288}-\frac{\ln(-3+2x)}{96}+\frac{\ln\left(4x^2+6x+9\right)}{192}-\frac{\sqrt{3}}{96}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)+\frac{5\ln\left(4x^2-6x+9\right)}{576}+\frac{5\sqrt{3}}{288}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*x^3+27)/(-64*x^6+729),x)
```

```
[Out] -5/288*ln(3+2*x)-1/96*ln(-3+2*x)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/2)*arctan(
1/18*(8*x+6)*3^(1/2))+5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-
```

```
6)*3^(1/2))
```

Maxima [A] time = 1.37515, size = 101, normalized size = 1.02

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Fricas [A] time = 1.44422, size = 257, normalized size = 2.6

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9) -
5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

Sympy [A] time = 0.327847, size = 102, normalized size = 1.03

$$-\frac{\log\left(x-\frac{3}{2}\right)}{96}-\frac{5\log\left(x+\frac{3}{2}\right)}{288}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{576}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{192}+\frac{5\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\right.}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

```
[Out] -log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + log
(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288 -
sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

Giac [A] time = 1.06298, size = 93, normalized size = 0.94

$$-\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)+\frac{5}{576}\log\left(x^2-\frac{3}{2}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqrt
(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9/4
) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))
```

3.576 $\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx$

Optimal. Leaf size=162

$$\frac{(cx)^{m+1}\left(a^2bf+a^3(-g)-ab^2e+b^3d\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ab^3c(m+1)}+\frac{(cx)^{m+1}\left(a^2g-abf+b^2e\right)}{b^3c(m+1)}+\frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+$$

```
[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m)
/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 + m)
) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))
```

Rubi [A] time = 0.167104, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1844, 20, 30, 364}

$$\frac{(cx)^{m+1}\left(a^2bf+a^3(-g)-ab^2e+b^3d\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ab^3c(m+1)}+\frac{(cx)^{m+1}\left(a^2g-abf+b^2e\right)}{b^3c(m+1)}+\frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] ((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m)
/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 + m)
) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m \left(d + ex^n + fx^{2n} + gx^{3n}\right)}{a + bx^n} dx = \int \left(\frac{\left(b^2e - abf + a^2g\right)(cx)^m}{b^3} + \frac{(bf - ag)x^n(cx)^m}{b^2} + \frac{gx^{2n}(cx)^m}{b} + \frac{\left(b^3d - ab^2e + a^2b\right.}{b^3 (a +}\right.$$

$$= \frac{\left(b^2e - abf + a^2g\right)(cx)^{1+m}}{b^3c(1+m)} + \frac{g \int x^{2n}(cx)^m \, dx}{b} + \frac{(bf - ag) \int x^n(cx)^m \, dx}{b^2} + \frac{\left(b^3d - \right.}{}$$

$$= \frac{\left(b^2e - abf + a^2g\right)(cx)^{1+m}}{b^3c(1+m)} + \frac{\left(b^3d - ab^2e + a^2bf - a^3g\right)(cx)^{1+m} {}_2F_1\left(1, \frac{1+m}{n}; \frac{1+m+n}{n}\right.}{ab^3c(1+m)}$$

$$= \frac{(bf - ag)x^{1+n}(cx)^m}{b^2(1+m+n)} + \frac{gx^{1+2n}(cx)^m}{b(1+m+2n)} + \frac{\left(b^2e - abf + a^2g\right)(cx)^{1+m}}{b^3c(1+m)} + \frac{\left(b^3d - ab^2e\right.}{}$$

Mathematica [A] time = 0.341273, size = 130, normalized size = 0.8

$$\frac{x(cx)^m \left(\frac{\left(a^2bf + a^3(-g) - ab^2e + b^3d\right) {}_2F_1\left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{a(m+1)} + \frac{a^2g - abf + b^2e}{m+1} + \frac{bx^n(bf - ag)}{m+n+1} + \frac{b^2gx^{2n}}{m+2n+1} \right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^m*((b^2*e - a*b*f + a^2*g)/(1 + m) + (b*(b*f - a*g)*x^n)/(1 + m +
n) + (b^2*g*x^(2*n))/(1 + m + 2*n) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*H
ypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m))))/
b^3
```

Maple [F] time = 0.384, size = 0, normalized size = 0.

$$\int \frac{(cx)^m \left(d + ex^n + fx^{2n} + gx^{3n}\right)}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(b^3c^md - ab^2c^me + a^2bc^mf - a^3c^mg\right) \int \frac{x^m}{b^4x^n + ab^3} dx + \frac{\left(m^2 + m(n+2) + n + 1\right)b^2c^mgxe^{(m\log(x) + 2n\log(x))} + \left(\left(m^2 + m(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="max
ima")
```

```
[Out] (b^3*c^m*d - a*b^2*c^m*e + a^2*b*c^m*f - a^3*c^m*g)*integrate(x^m/(b^4*x^n
+ a*b^3), x) + ((m^2 + m*(n + 2) + n + 1)*b^2*c^m*g*x*e^(m*log(x) + 2*n*log
```

```
(x)) + ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^m*e - (m^2 + m*(3*n + 2
) + 2*n^2 + 3*n + 1)*a*b*c^m*f + (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^2*
c^m*g)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c^m*f - (m^2 + 2*m*(n + 1
) + 2*n + 1)*a*b*c^m*g)*x*e^(m*log(x) + n*log(x)))/((m^3 + 3*m^2*(n + 1) +
(2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(gx^{3n} + fx^{2n} + ex^{n} + d\right)(cx)^{m}}{bx^{n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="fri
cas")
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3n} + fx^{2n} + ex^{n} + d\right)(cx)^{m}}{bx^{n} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="gia
c")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

3.577 $\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx$

Optimal. Leaf size=84

$$\frac{3a^2bcx^{n+1}}{n+1}+a^3cx+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))/(1 + 2*n)
 + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4*b*n)
```

Rubi [A] time = 0.0557268, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 244, 261}

$$\frac{3a^2bcx^{n+1}}{n+1}+a^3cx+\frac{3ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{4bn}+\frac{b^3cx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] a^3*c*x + (3*a^2*b*c*x^(1 + n))/(1 + n) + (3*a*b^2*c*x^(1 + 2*n))/(1 + 2*n)
 + (b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x^n)^4)/(4*b*n)
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx &= c\int (a+bx^n)^3\,dx+d\int x^{-1+n}(a+bx^n)^3\,dx\\ &= \frac{d(a+bx^n)^4}{4bn}+c\int \left(a^3+3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}\right)dx\\ &= a^3cx+\frac{3a^2bcx^{1+n}}{1+n}+\frac{3ab^2cx^{1+2n}}{1+2n}+\frac{b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{4bn}\end{aligned}$$

Mathematica [A] time = 0.163315, size = 108, normalized size = 1.29

$$\frac{x\left(c+dx^{n-1}\right)\left(\frac{12a^2bcx^{n+1}}{n+1}+4a^3cx+\frac{12ab^2cx^{2n+1}}{2n+1}+\frac{d(a+bx^n)^4}{bn}+\frac{4b^3cx^{3n+1}}{3n+1}\right)}{4(cx+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

```
[Out] (x*(c + d*x^(-1 + n))*(4*a^3*c*x + (12*a^2*b*c*x^(1 + n))/(1 + n) + (12*a*b
^2*c*x^(1 + 2*n))/(1 + 2*n) + (4*b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b*x
^n)^4)/(b*n)))/(4*(c*x + d*x^n))
```

Maple [A] time = 0.015, size = 130, normalized size = 1.6

$$a^3cx + \frac{a^3de^{n\ln(x)}}{n} + \frac{ab^2d\left(e^{n\ln(x)}\right)^3}{n} + \frac{b^3cx\left(e^{n\ln(x)}\right)^3}{1+3n} + \frac{b^3d\left(e^{n\ln(x)}\right)^4}{4n} + \frac{3a^2bd\left(e^{n\ln(x)}\right)^2}{2n} + 3\frac{acb^2x\left(e^{n\ln(x)}\right)^2}{1+2n} + 3\frac{a^2bc}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))*(a+b*x^n)^3,x)
```

```
[Out] a^3*c*x+a^3*d/n*exp(n*ln(x))+a*b^2*d/n*exp(n*ln(x))^3+b^3*c/(1+3*n)*x*exp(n
*ln(x))^3+1/4*b^3*d/n*exp(n*ln(x))^4+3/2*a^2*b*d/n*exp(n*ln(x))^2+3*a*c*b^2
/(1+2*n)*x*exp(n*ln(x))^2+3*b*a^2*c/(1+n)*x*exp(n*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.54533, size = 660, normalized size = 7.86

$$\frac{4\left(6a^3cn^4 + 11a^3cn^3 + 6a^3cn^2 + a^3cn\right)x + \left(6b^3dn^3 + 11b^3dn^2 + 6b^3dn + b^3d\right)x^{4n} + 4\left(6ab^2dn^3 + 11ab^2dn^2 + 6ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (6*b^3*d*n^
3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*b^2*d*n^3 + 11*a*b^2
*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3 + 3*b^3*c*n^2 + b^3*c*n)*x)*x
^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*(3*a
*b^2*c*n^3 + 4*a*b^2*c*n^2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 + 11*a^
3*d*n^2 + 6*a^3*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*b*c*n)
*x)*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Sympy [A] time = 6.76931, size = 1251, normalized size = 14.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) -
3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -1)
), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*a*b
**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2), Eq
(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*a**2*
b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x) - 3*b
**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n, 0)),
(24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n**3*x/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44*n**3 + 2
4*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*
d*n**3*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*d*n**2*x**n/(24*n
**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x**n/(24*n**4 + 44*n**3 + 24*n
**2 + 4*n) + 4*a**3*d*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*
c*n**3*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n**2*x*x**n
/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 12*a**2*b*c*n*x*x**n/(24*n**4 + 44*n
**3 + 24*n**2 + 4*n) + 36*a**2*b*d*n**3*x**(2*n)/(24*n**4 + 44*n**3 + 24*n*
*2 + 4*n) + 66*a**2*b*d*n**2*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) +
 36*a**2*b*d*n*x**(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d*x*
*(2*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n**3*x*x**(2*n)/(2
4*n**4 + 44*n**3 + 24*n**2 + 4*n) + 48*a*b**2*c*n**2*x*x**(2*n)/(24*n**4 +
44*n**3 + 24*n**2 + 4*n) + 12*a*b**2*c*n*x*x**(2*n)/(24*n**4 + 44*n**3 + 24
*n**2 + 4*n) + 24*a*b**2*d*n**3*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n
) + 44*a*b**2*d*n**2*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a*b*
*2*d*n*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a*b**2*d*x**(3*n)/(
24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 8*b**3*c*n**3*x*x**(3*n)/(24*n**4 + 44
*n**3 + 24*n**2 + 4*n) + 12*b**3*c*n**2*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*
n**2 + 4*n) + 4*b**3*c*n*x*x**(3*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6
*b**3*d*n**3*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 11*b**3*d*n**2*
x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*b**3*d*n*x**(4*n)/(24*n**4
 + 44*n**3 + 24*n**2 + 4*n) + b**3*d*x**(4*n)/(24*n**4 + 44*n**3 + 24*n**2
+ 4*n), True))
```

Giac [B] time = 1.06869, size = 529, normalized size = 6.3

$$\frac{24\,a^3cn^4x + 8\,b^3cn^3xx^{3\,n} + 36\,ab^2cn^3xx^{2\,n} + 72\,a^2bcn^3xx^n + 44\,a^3cn^3x + 6\,b^3dn^3x^{4\,n} + 24\,ab^2dn^3x^{3\,n} + 12\,b^3cn^2xx^{3\,n} + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 72
*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^3*
x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n^2*
x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x + 11*b
^3*d*n^2*x^(4*n) + 44*a*b^2*d*n^2*x^(3*n) + 4*b^3*c*n*x*x^(3*n) + 66*a^2*b*
d*n^2*x^(2*n) + 12*a*b^2*c*n*x*x^(2*n) + 44*a^3*d*n^2*x^n + 12*a^2*b*c*n*x*
x^n + 4*a^3*c*n*x + 6*b^3*d*n*x^(4*n) + 24*a*b^2*d*n*x^(3*n) + 36*a^2*b*d*n
```

```
*x^(2*n) + 24*a^3*d*n*x^n + b^3*d*x^(4*n) + 4*a*b^2*d*x^(3*n) + 6*a^2*b*d*x
^(2*n) + 4*a^3*d*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

3.578 $\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx$

Optimal. Leaf size=61

$$a^2cx+\frac{2abcx^{n+1}}{n+1}+\frac{d(a+bx^n)^3}{3bn}+\frac{b^2cx^{2n+1}}{2n+1}$$

```
[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 + 2*n) + (d*
(a + b*x^n)^3)/(3*b*n)
```

Rubi [A] time = 0.0402319, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 244, 261}

$$a^2cx+\frac{2abcx^{n+1}}{n+1}+\frac{d(a+bx^n)^3}{3bn}+\frac{b^2cx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] a^2*c*x + (2*a*b*c*x^(1 + n))/(1 + n) + (b^2*c*x^(1 + 2*n))/(1 + 2*n) + (d*
(a + b*x^n)^3)/(3*b*n)
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx&=c\int (a+bx^n)^2\,dx+d\int x^{-1+n}(a+bx^n)^2\,dx\\&=\frac{d(a+bx^n)^3}{3bn}+c\int \left(a^2+2abx^n+b^2x^{2n}\right)dx\\&=a^2cx+\frac{2abcx^{1+n}}{1+n}+\frac{b^2cx^{1+2n}}{1+2n}+\frac{d(a+bx^n)^3}{3bn}\end{aligned}$$

Mathematica [A] time = 0.112459, size = 120, normalized size = 1.97

$$\frac{3a^2b\left(2n^2+3n+1\right)(cnx+dx^n)+a^3d\left(2n^2+3n+1\right)+3ab^2(2n+1)x^n(2cnx+d(n+1)x^n)+b^3(n+1)x^{2n}(3cnx+d(2n}{3bn(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

```
[Out] (a^3*d*(1 + 3*n + 2*n^2) + 3*a^2*b*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a*
b^2*(1 + 2*n)*x^n*(2*c*n*x + d*(1 + n)*x^n) + b^3*(1 + n)*x^(2*n)*(3*c*n*x
+ d*(1 + 2*n)*x^n))/(3*b*n*(1 + n)*(1 + 2*n))
```

Maple [A] time = 0.012, size = 87, normalized size = 1.4

$$a^{2}cx+\frac{a^{2}d\mathrm{e}^{n\ln(x)}}{n}+\frac{bda\left(\mathrm{e}^{n\ln(x)}\right)^{2}}{n}+\frac{b^{2}cx\left(\mathrm{e}^{n\ln(x)}\right)^{2}}{1+2n}+\frac{b^{2}d\left(\mathrm{e}^{n\ln(x)}\right)^{3}}{3n}+2\frac{abcx\mathrm{e}^{n\ln(x)}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))*(a+b*x^n)^2,x)
```

```
[Out] a^2*c*x+a^2*d/n*exp(n*ln(x))+b*d*a/n*exp(n*ln(x))^2+b^2*c/(1+2*n)*x*exp(n*l
n(x))^2+1/3*b^2*d/n*exp(n*ln(x))^3+2*a*b*c/(1+n)*x*exp(n*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.53706, size = 347, normalized size = 5.69

$$\frac{3\left(2a^{2}cn^{3}+3a^{2}cn^{2}+a^{2}cn\right)x+\left(2b^{2}dn^{2}+3b^{2}dn+b^{2}d\right)x^{3n}+3\left(2abdn^{2}+3abdn+abd+\left(b^{2}cn^{2}+b^{2}cn\right)x\right)x^{2n}+3}{3\left(2n^{3}+3n^{2}+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3*b^2*d*n +
 b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d + (b^2*c*n^2 + b^2*c*n
)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a^2*d + 2*(2*a*b*c*n^2 + a*b*c*
n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

Sympy [A] time = 3.25191, size = 552, normalized size = 9.05

$$\begin{cases} a^{2}cx-\frac{a^{2}d}{x}+2abc\log(x)-\frac{abd}{x^{2}}-\frac{b^{2}c}{x}-\frac{b^{2}d}{3x^{3}} \\ a^{2}cx-\frac{2a^{2}d}{\sqrt{x}}+4abc\sqrt{x}-\frac{2abd}{x}+b^{2}c\log(x)-\frac{2b^{2}d}{3x^{\frac{3}{2}}} \\ (a+b)^{2}(cx+d\log(x)) \\ \frac{6a^{2}cn^{3}x}{6n^{3}+9n^{2}+3n}+\frac{9a^{2}cn^{2}x}{6n^{3}+9n^{2}+3n}+\frac{3a^{2}cnx}{6n^{3}+9n^{2}+3n}+\frac{6a^{2}dn^{2}x^{n}}{6n^{3}+9n^{2}+3n}+\frac{9a^{2}dnx^{n}}{6n^{3}+9n^{2}+3n}+\frac{3a^{2}dx^{n}}{6n^{3}+9n^{2}+3n}+\frac{12abcn^{2}xx^{n}}{6n^{3}+9n^{2}+3n}+\frac{6abcnxx^{n}}{6n^{3}+9n^{2}+3n}+\frac{6abdn^{2}x^{2n}}{6n^{3}+9n^{2}+3n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x - b
**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x)
- 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a + b)
**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*n) +
 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n**2 +
3*n) + 6*a**2*d*n**2*x**n/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*n*x**n/(6*n**3
 + 9*n**2 + 3*n) + 3*a**2*d*x**n/(6*n**3 + 9*n**2 + 3*n) + 12*a*b*c*n**2*x*
x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6
*a*b*d*n**2*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 9*a*b*d*n*x**(2*n)/(6*n**3 +
 9*n**2 + 3*n) + 3*a*b*d*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x
*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2
+ 3*n) + 2*b**2*d*n**2*x**(3*n)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x**(3*
n)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x**(3*n)/(6*n**3 + 9*n**2 + 3*n), True)
)
```

Giac [B] time = 1.07394, size = 265, normalized size = 4.34

$$\frac{6\,a^{2}cn^{3}x + 3\,b^{2}cn^{2}xx^{2\,n} + 12\,abcn^{2}xx^{n} + 9\,a^{2}cn^{2}x + 2\,b^{2}dn^{2}x^{3\,n} + 6\,abdn^{2}x^{2\,n} + 3\,b^{2}cnxx^{2\,n} + 6\,a^{2}dn^{2}x^{n} + 6\,abcnxx^{n} + 3}{3\left(2\,n^{3} + 3\,n^{2} + n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*n
^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) + 6*
a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*b*d
*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*x^n)
/(2*n^3 + 3*n^2 + n)
```

3.579 $\int (c+dx^{-1+n})(a+bx^n)\,dx$

Optimal. Leaf size=41

$$acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)
```

Rubi [A] time = 0.0227337, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1891, 14}

$$acx+\frac{adx^n}{n}+\frac{bcx^{n+1}}{n+1}+\frac{bdx^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x^(1 + n))/(1 + n)
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (c+dx^{-1+n})(a+bx^n)\,dx &= c\int (a+bx^n)\,dx+d\int x^{-1+n}(a+bx^n)\,dx\\ &= acx+\frac{bcx^{1+n}}{1+n}+d\int \left(ax^{-1+n}+bx^{-1+2n}\right)dx\\ &= acx+\frac{adx^n}{n}+\frac{bdx^{2n}}{2n}+\frac{bcx^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0962162, size = 42, normalized size = 1.02

$$\frac{2a(cnx+dx^n)+bx^n\left(\frac{2cnx}{n+1}+dx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

```
[Out] (2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

Maple [A] time = 0.011, size = 45, normalized size = 1.1

$$acx + \frac{ad\mathrm{e}^{n\ln(x)}}{n} + \frac{bcx\mathrm{e}^{n\ln(x)}}{1+n} + \frac{bd\left(\mathrm{e}^{n\ln(x)}\right)^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))*(a+b*x^n),x)
```

```
[Out] a*c*x+a*d/n*exp(n*ln(x))+b*c/(1+n)*x*exp(n*ln(x))+1/2*b*d/n*exp(n*ln(x))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48533, size = 128, normalized size = 3.12

$$\frac{2\left(acn^2 + acn\right)x + (bdn + bd)x^{2n} + 2(bcnx + adn + ad)x^n}{2\left(n^2 + n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n + a
*d)*x^n)/(n^2 + n)
```

Sympy [A] time = 1.50908, size = 163, normalized size = 3.98

$$\begin{cases} acx - \frac{ad}{x} + bc\log(x) - \frac{bd}{2x^2} & \text{for } n = -1 \\ (a+b)(cx + d\log(x)) & \text{for } n = 0 \\ \frac{2acn^2x}{2n^2+2n} + \frac{2acnx}{2n^2+2n} + \frac{2adnx^n}{2n^2+2n} + \frac{2adx^n}{2n^2+2n} + \frac{2bcnxx^n}{2n^2+2n} + \frac{bdnx^{2n}}{2n^2+2n} + \frac{bdx^{2n}}{2n^2+2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

```
[Out] Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)*
(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*n*
*2 + 2*n) + 2*a*d*n*x**n/(2*n**2 + 2*n) + 2*a*d*x**n/(2*n**2 + 2*n) + 2*b*c
*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x**(2*n)/(2*n**2 + 2*n) + b*d*x**(2*n)/(2*
```

```
n**2 + 2*n), True))
```

Giac [A] time = 1.05632, size = 88, normalized size = 2.15

$$\frac{2\,acn^2x + 2\,bcnxx^n + 2\,acnx + bdnx^{2\,n} + 2\,adnx^n + bdx^{2\,n} + 2\,adx^n}{2\left(n^2 + n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n
+ b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

3.580 $\int \left(c + dx^{-1+n}\right) dx$

Optimal. Leaf size=12

$$cx + \frac{dx^n}{n}$$

[Out] c*x + (d*x^n)/n

Rubi [A] time = 0.0028014, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$cx + \frac{dx^n}{n}$$

Antiderivative was successfully verified.

[In] Int[c + d*x^(-1 + n),x]

[Out] c*x + (d*x^n)/n

Rubi steps

$$\int \left(c + dx^{-1+n}\right) dx = cx + \frac{dx^n}{n}$$

Mathematica [A] time = 0.0013776, size = 12, normalized size = 1.

$$cx + \frac{dx^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[c + d*x^(-1 + n),x]

[Out] c*x + (d*x^n)/n

Maple [A] time = 0.041, size = 13, normalized size = 1.1

$$cx + \frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c+d*x^(-1+n),x)

[Out] c*x+d*x^n/n

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50071, size = 36, normalized size = 3.

$$\frac{cnx + dxx^{n-1}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="fricas")
```

```
[Out] (c*n*x + d*x*x^(n - 1))/n
```

Sympy [A] time = 0.057868, size = 15, normalized size = 1.25

$$cx + d\left(\begin{cases} \frac{x^n}{n} & \text{for}\ n - 1 \neq -1 \\ \log(x) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x**(-1+n),x)
```

```
[Out] c*x + d*Piecewise((x**n/n, Ne(n - 1, -1)), (log(x), True))
```

Giac [A] time = 1.04143, size = 16, normalized size = 1.33

$$cx + \frac{dx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c+d*x^(-1+n),x, algorithm="giac")
```

```
[Out] c*x + d*x^n/n
```

3.581 $\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx$

Optimal. Leaf size=42

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a +
 b*x^n])/(b*n)
```

Rubi [A] time = 0.0304338, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 245, 260}

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a +
 b*x^n])/(b*n)
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx &= c\int \frac{1}{a+bx^n}\,dx+d\int \frac{x^{-1+n}}{a+bx^n}\,dx\\ &= \frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0561254, size = 42, normalized size = 1.

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

```
[Out] (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a +
 b*x^n])/(b*n)
```

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int \frac{c + dx^{-1+n}}{a + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n),x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\log(x)}{b} + \int \frac{bcx - ad}{b^2xx^n + abx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^{n-1} + c}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Sympy [A] time = 9.52058, size = 65, normalized size = 1.55

$$d\left(\begin{cases} \frac{\log(x)}{a} & \text{for}\: b = 0 \wedge n = 0 \\ \frac{x^n}{an} & \text{for}\: b = 0 \\ \frac{\log(x)}{a+b} & \text{for}\: n = 0 \\ \frac{\log\left(\frac{a}{b} + x^n\right)}{bn} & \text{otherwise} \end{cases}\right) + \frac{cx\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n),x)
```

```
[Out] d*Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x**n/(a*n), Eq(b, 0)), (log(x
)/(a + b), Eq(n, 0)), (log(a/b + x**n)/(b*n), True)) + c*x*lerchphi(b*x**n*
exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

3.582 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} \, dx$

Optimal. Leaf size=44

$$\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

```
[Out] -(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b
*x^n)/a)])/a^2
```

Rubi [A] time = 0.0288111, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 245, 261}

$$\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

```
[Out] -(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b
*x^n)/a)])/a^2
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} \, dx &= c\int \frac{1}{(a+bx^n)^2} \, dx+d\int \frac{x^{-1+n}}{(a+bx^n)^2} \, dx\\ &= -\frac{d}{bn\,(a+bx^n)}+\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0754271, size = 44, normalized size = 1.

$$\frac{cx\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{abn+b^2nx^n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

```
[Out] -(d/(a*b*n + b^2*n*x^n)) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(
(b*x^n)/a)])/a^2
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$c(n-1)\int \frac{1}{abnx^n+a^2n}\,dx+\frac{bcx-ad}{ab^2nx^n+a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*b^2*n*x^n
+ a^2*b*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{dx^{n-1}+c}{b^2x^{2n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] time = 22.7527, size = 313, normalized size = 7.11

$$c\left(\frac{nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+ \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

```
[Out] c*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gam
ma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamm
a(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 +
1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/
n) + b*n**3*x**n*gamma(1 + 1/n)))) + d*Piecewise((zoo*log(x), Eq(a, 0) & Eq
(b, 0) & Eq(n, 0)), (-x**(-n)/(b**2*n), Eq(a, 0)), (zoo*x**n/n, Eq(b, -a*x*
*(-n))), (log(x)/(a + b)**2, Eq(n, 0)), (x**n/(a**2*n + a*b*n*x**n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1}+c}{(bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

3.583 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=46

$$\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\,(a+bx^n)^2}$$

```
[Out] -d/(2*b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(
(b*x^n)/a)])/a^3
```

Rubi [A] time = 0.0295255, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1891, 245, 261}

$$\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] -d/(2*b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(
(b*x^n)/a)])/a^3
```

Rule 1891

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[A, Int[(a + b*x^n)^p, x], x] + Dist[B, Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx &= c\int \frac{1}{(a+bx^n)^3}\,dx+d\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx\\ &= -\frac{d}{2bn\,(a+bx^n)^2}+\frac{cx\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0744282, size = 63, normalized size = 1.37

$$\frac{2bcnx\left(a+bx^n\right)^2 {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)-a^3d}{2a^3bn\left(a+bx^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

```
[Out] (-(a^3*d) + 2*b*c*n*x*(a + b*x^n)^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1)
, -((b*x^n)/a)])/(2*a^3*b*n*(a + b*x^n)^2)
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{c+dx^{-1+n}}{\left(a+bx^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,n^2-3\,n+1\right)c\int \frac{1}{2\left(a^2bn^2x^n+a^3n^2\right)}\,dx+\frac{b^2c(2\,n-1)xx^n+abc(3\,n-1)x-a^2dn}{2\left(a^2b^3n^2x^{2\,n}+2\,a^3b^2n^2x^n+a^4bn^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b^2*
c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b^3*n^2*x^(2*n) + 2*a
^3*b^2*n^2*x^n + a^4*b*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^{n-1}+c}{b^3x^{3\,n}+3\,ab^2x^{2\,n}+3\,a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{n-1} + c}{(bx^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

3.584 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=305

$$\frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}}}{(m}$$

```
[Out] (d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 +
 m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) + (e*x^(1 + n)*(c*x)^
m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (1 + m + 2*n)/n
, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -
((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^m*Sqrt
[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -(
(b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```

Rubi [A] time = 0.231849, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1844, 365, 364, 20}

$$\frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}}}{(m}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```


```
[Out] (d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 +
 m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) + (e*x^(1 + n)*(c*x)^
m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (1 + m + 2*n)/n
, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -
((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^m*Sqrt
[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -(
(b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```


Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rubi steps

$$\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{\sqrt{a+bx^n}}\,dx=\int\left(\frac{d(cx)^m}{\sqrt{a+bx^n}}+\frac{ex^n(cx)^m}{\sqrt{a+bx^n}}+\frac{fx^{2n}(cx)^m}{\sqrt{a+bx^n}}+\frac{gx^{3n}(cx)^m}{\sqrt{a+bx^n}}\right)dx$$

$$=d\int\frac{(cx)^m}{\sqrt{a+bx^n}}\,dx+e\int\frac{x^n(cx)^m}{\sqrt{a+bx^n}}\,dx+f\int\frac{x^{2n}(cx)^m}{\sqrt{a+bx^n}}\,dx+g\int\frac{x^{3n}(cx)^m}{\sqrt{a+bx^n}}\,dx$$

$$=\left(ex^{-m}(cx)^m\right)\int\frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx+\left(fx^{-m}(cx)^m\right)\int\frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx+\left(gx^{-m}(cx)^m\right)\int\frac{x}{\sqrt{a}}$$

$$=\frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}+\frac{\left(ex^{-m}(cx)^m\sqrt{1+\frac{bx^n}{a}}\right)\int\frac{x^{m+n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}$$

$$=\frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}+\frac{ex^{1+n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{n}\right.}{(1+m+n)\sqrt{a+bx}}$$

Mathematica [A] time = 0.373558, size = 206, normalized size = 0.68

$$\frac{x(cx)^m\sqrt{\frac{bx^n}{a}+1}\left(\frac{d\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{m+1}+x^n\left(\frac{e\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+x^n\left(\frac{f\,{}_2F_1\left(\frac{1}{2},\frac{m+2n+1}{n};\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{gx^n\,{}_2F_1\left(\frac{1}{2},\frac{m+3n+1}{n};\frac{m+}{}\right.}{m+3n+1}\right.\right.\right.}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

```
[Out] (x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*((d*Hypergeometric2F1[1/2, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[1/2, (1 + m + n
)/n, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometric2F
1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) + (g*
x^n*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -((b*x^n)/a)])
/(1 + m + 3*n)))))/Sqrt[a + b*x^n]
```

Maple [F] time = 0.464, size = 0, normalized size = 0.

$$\int (cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)\frac{1}{\sqrt{a+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3n} + fx^{2n} + ex^n + d\right)(cx)^m}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(gx^{3n} + fx^{2n} + ex^n + d\right)(cx)^m}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)
```

3.585 $\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rubi [A] time = 0.394729, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {6741, 1816}

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!= u]
```

Rule 1816

```
Int[((x_)^(m_.)*((e_) + (h_.)*(x_)^(n_.) + (f_.)*(x_)^(q_.) + (g_.)*(x_)^(r_.)))/((a_) + (c_.)*(x_)^(n_.))^(3/2), x_Symbol] :> -Simp[(2*a*g + 4*a*h*x^(n/4) - 2*c*f*x^(n/2))/(a*c*n*Sqrt[a + c*x^n]), x] /; FreeQ[{a, c, e, f, g, h, m, n}, x] && EqQ[q, n/4] && EqQ[r, (3*n)/4] && EqQ[4*m - n + 4, 0] && EqQ[c*e + a*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx &= \int \frac{x^{-1+\frac{n}{4}}\left(-ah+bfx^{n/4}+bgx^{3n/4}+bhx^n\right)}{(a+bx^n)^{3/2}}\,dx\\ &= -\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.21546, size = 45, normalized size = 1.

$$\frac{2bfx^{n/2}-2a\left(g+2hx^{n/4}\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^
(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

```
[Out] (2*b*f*x^(n/2) - 2*a*(g + 2*h*x^(n/4)))/(a*n*Sqrt[a + b*x^n])
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int \left(-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}\right)(a+bx^n)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b
*x^n)^(3/2),x)
```

```
[Out] int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+b
*x^n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bhx^{\frac{5}{4}n-1}+bgx^{n-1}+bfx^{\frac{1}{2}n-1}-ahx^{\frac{1}{4}n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

Fricas [A] time = 1.895, size = 153, normalized size = 3.4

$$\frac{2\sqrt{bx^4x^{n-4}+a}\left(bfx^2x^{\frac{1}{2}n-2}-2\,ahxx^{\frac{1}{4}n-1}-ag\right)}{abnx^4x^{n-4}+a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1/4*n - 1)
- a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-1+5/
4*n))/(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n)
)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(1
/4*n - 1))/(b*x^n + a)^(3/2), x)
```

3.586 $\int (cx)^m \left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx$

Optimal. Leaf size=273

$$\frac{e(cx)^{m+2}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{n},-p;\frac{m+n+2}{n};-\frac{bx^n}{a}\right)}{c^2(m+2)}+\frac{f(cx)^{m+3}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3}{n},-p;\frac{m+n+3}{n};-\right.}{c^3(m+3)}$$

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(c*x)^(2 + m)*(a + b*
x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(c^2*
(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*x)^(3 + m)*(a + b*x^n)^p*Hypergeometric2
F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x^n)/a
)^p) + (g*(c*x)^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p, (4 +
 m + n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.18508, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1844, 365, 364}

$$\frac{e(cx)^{m+2}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{n},-p;\frac{m+n+2}{n};-\frac{bx^n}{a}\right)}{c^2(m+2)}+\frac{f(cx)^{m+3}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3}{n},-p;\frac{m+n+3}{n};-\right.}{c^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(c*x)^(2 + m)*(a + b*
x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(c^2*
(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*x)^(3 + m)*(a + b*x^n)^p*Hypergeometric2
F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x^n)/a
)^p) + (g*(c*x)^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p, (4 +
 m + n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx = \int \left(d(cx)^m(a+bx^n)^p + \frac{e(cx)^{1+m}(a+bx^n)^p}{c} + \frac{f(cx)^{2+m}(a+bx^n)^p}{c^2} + \frac{g(cx)}{}\right.$$

$$= d\int (cx)^m(a+bx^n)^p\,dx + \frac{e\int (cx)^{1+m}(a+bx^n)^p\,dx}{c} + \frac{f\int (cx)^{2+m}(a+bx^n}{c^2}$$

$$= \left(d(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^m\left(1+\frac{bx^n}{a}\right)^p dx + \frac{\left(e(a+bx^n)^p\left(1+\frac{bx^n}{a}\right.\right.}{}$$

$$= \frac{d(cx)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)} + \frac{e(cx)^{2+m}(a}{}$$

Mathematica [A] time = 0.24311, size = 178, normalized size = 0.65

$$x(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{d\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{m+1}+x\left(\frac{e\,{}_2F_1\left(\frac{m+2}{n},-p;\frac{m+n+2}{n};-\frac{bx^n}{a}\right)}{m+2}+x\left(\frac{f\,{}_2F_1\left(\frac{m+3}{n},-p;\frac{m+n+}{n}\right.}{m+3}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(1 + m) + x*((e*Hypergeometric2F1[(2 + m)/n, -p, (2 + m +
n)/n, -((b*x^n)/a)])/(2 + m) + x*((f*Hypergeometric2F1[(3 + m)/n, -p, (3 +
m + n)/n, -((b*x^n)/a)])/(3 + m) + (g*x*Hypergeometric2F1[(4 + m)/n, -p, (4
 + m + n)/n, -((b*x^n)/a)])/(4 + m)))))/(1 + (b*x^n)/a)^p
```

Maple [F] time = 0.437, size = 0, normalized size = 0.

$$\int (cx)^m\left(gx^3+fx^2+ex+d\right)(a+bx^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

```
[Out] int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^3+fx^2+ex+d\right)(bx^n+a)^p\,(cx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p\,(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.587 $\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$

Optimal. Leaf size=297

$$\frac{d(cx)^{m+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};\right.}{m+n+1}$$

```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x^(1 + n)*(c*x)^m*(a
+ b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/
a)])/((1 + m + n)*(1 + (b*x^n)/a)^p) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^n)^p
*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1
 + m + 2*n)*(1 + (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((1 + m +
 3*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.208185, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1844, 365, 364, 20}

$$\frac{d(cx)^{m+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};\right.}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```



```
[Out] (d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)
/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x^(1 + n)*(c*x)^m*(a
+ b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/
a)])/((1 + m + n)*(1 + (b*x^n)/a)^p) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^n)^p
*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1
 + m + 2*n)*(1 + (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((1 + m +
 3*n)*(1 + (b*x^n)/a)^p)
```


Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rubi steps

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx = \int \left(d(cx)^m (a+bx^n)^p + ex^n(cx)^m (a+bx^n)^p + fx^{2n}(cx)^m (a+bx^n)^p +\right.$$

$$= d\int (cx)^m (a+bx^n)^p \, dx + e\int x^n(cx)^m (a+bx^n)^p \, dx + f\int x^{2n}(cx)^m$$

$$= \left(ex^{-m}(cx)^m\right)\int x^{m+n} (a+bx^n)^p \, dx + \left(fx^{-m}(cx)^m\right)\int x^{m+2n} (a+bx^n)$$

$$= \frac{d(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)} + \left(ex^{-m}\right.$$

$$= \frac{d(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n},-p;\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{c(1+m)} + \frac{ex^{1+n}(}{}$$

Mathematica [A] time = 0.273287, size = 204, normalized size = 0.69

$$x(cx)^m (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(\frac{d\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{m+1} + x^n\left(\frac{e\,{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1} + x^n\left(\frac{f\,{}_2F_1\left(\frac{m+2n}{n}\right.}{n}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[(1 + m + n)/n, -p, (1
+ m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometric2F1[(1 + m
 + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) + (g*x^n*Hyper
geometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/(1 + m +
3*n)))))/(1 + (b*x^n)/a)^p
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

```
[Out] int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(gx^{3n}+fx^{2n}+ex^n+d\right)(bx^n+a)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="m
axima")
```

```
[Out] integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(gx^{3\,n}+fx^{2\,n}+ex^{n}+d\right)\left(bx^{n}+a\right)^{p}\left(cx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="f
ricas")
```

```
[Out] integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.588 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx$

Optimal. Leaf size=162

$$\frac{x(ae-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}-\frac{x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(n+2)}+\frac{x\left(x^{n/2}(ba\right.}{ab}$$

```
[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d*(2 - n) -
 a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2,
-((b*x^n)/a)])/(a^2*b*n*(2 + n)) + ((a*e - b*c*(1 - n))*x*Hypergeometric2F1
[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rubi [A] time = 0.124185, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1892, 1418, 245, 364}

$$\frac{x(ae-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}-\frac{x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a^2bn(n+2)}+\frac{x\left(x^{n/2}(ba\right.}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```


```
[Out] (x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) - ((b*d*(2 - n) -
 a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2,
-((b*x^n)/a)])/(a^2*b*n*(2 + n)) + ((a*e - b*c*(1 - n))*x*Hypergeometric2F1
[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```


Rule 1892

```
Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{A = Coeff[P3, x
^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff
[P3, x^(n/2), 3]}, -Simp[(x*(b*A - a*C + (b*B - a*D)*x^(n/2)))*(a + b*x^n)^(
p + 1))/(a*b*n*(p + 1)), x] - Dist[1/(2*a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3) +
 2))*x^(n/2), x], x], x]] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3] &
& ILtQ[p, -1]
```

Rule 1418

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Dist[d
, Int[1/(a + c*x^(2*n)), x], x] + Dist[e, Int[x^n/(a + c*x^(2*n)), x], x] /
; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (Po
sQ[a*c] ||  !IntegerQ[n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} + \frac{\int \frac{2(ae-bc(1-n))-(bd(2-n)-af(2+n))x^{n/2}}{a+bx^n}\,dx}{2abn}$$

$$= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} + \frac{(ae-bc(1-n))\int \frac{1}{a+bx^n}\,dx}{abn} - \frac{(bd(2-n)-af(2+n))\int \frac{x^{n/2}}{a+\dots}}{2abn}$$

$$= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} - \frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-\dots\right)}{a^2bn(2+n)}$$

Mathematica [A] time = 0.338448, size = 147, normalized size = 0.91

$$\frac{x\left((bc-ae)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{2x^{n/2}(bd-af)\,{}_2F_1\left(2,\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-\frac{bx^n}{a}\right)}{n+2}+ae\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{2afx^{n/2}\,{}_2F_1\left(1,\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-\frac{bx^n}{a}\right)}{n+2}\dots\right)}{a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

```
[Out] (x*((2*a*f*x^(n/2)*Hypergeometric2F1[1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^
n)/a)])/(2 + n) + a*e*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
] + (2*(b*d - a*f)*x^(n/2)*Hypergeometric2F1[2, 1/2 + n^(-1), 3/2 + n^(-1),
 -((b*x^n)/a)])/(2 + n) + (b*c - a*e)*Hypergeometric2F1[2, n^(-1), 1 + n^(-
1), -((b*x^n)/a)]))/(a^2*b)
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2}\left(c+dx^{\frac{n}{2}}+ex^n+fx^{\frac{3n}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bd-af)xx^{\frac{1}{2}n}+(bc-ae)x}{ab^2nx^n+a^2bn}+\int \frac{2\,bc(n-1)+2\,ae+\left(af(n+2)+bd(n-2)\right)x^{\frac{1}{2}n}}{2\left(ab^2nx^n+a^2bn\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="maxim
a")
```

```
[Out] ((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integra
te(1/2*(2*b*c*(n - 1) + 2*a*e + (a*f*(n + 2) + b*d*(n - 2))*x^(1/2*n))/(a*b
^2*n*x^n + a^2*b*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="frica
s")
```

```
[Out] integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n +
 a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="giac"
)
```

```
[Out] integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2, x)
```

3.589 $\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=24

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rubi [A] time = 0.0525376, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1590}

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
,x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Mathematica [A] time = 0.148485, size = 24, normalized size = 1.

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]),x]
```

```
[Out] x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

Maple [A] time = 0.009, size = 21, normalized size = 0.9

$$x\sqrt{bx^2+a}\sqrt{dx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Maxima [A] time = 1.10455, size = 27, normalized size = 1.12

$$\sqrt{bx^2+a}\sqrt{dx^2+c}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Fricas [A] time = 1.61799, size = 47, normalized size = 1.96

$$\sqrt{bx^2+a}\sqrt{dx^2+c}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ac+2adx^2+2bcx^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**(
1/2),x)
```

```
[Out] Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b*x**2)*sqr
t(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,bdx^4+2\,(bc+ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^2
 + c)), x)
```

3.590 $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}} \, dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^
(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + Arc
Tanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rubi [A] time = 0.10257, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1899, 377, 212, 206, 203, 444, 63, 298}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2^
(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + Arc
Tanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

Rule 1899

```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)
*(x_)^(n_))^(q_.), x_Symbol] :> Dist[A, Int[(a + b*x^n)^p*(c + d*x^n)^q, x]
, x] + Dist[B, Int[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b,
c, d, A, B, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{\left(1-x^4\right)\sqrt[4]{1+x^4}}\,dx &= \int \frac{1}{\left(1-x^4\right)\sqrt[4]{1+x^4}}\,dx + \int \frac{x^3}{\left(1-x^4\right)\sqrt[4]{1+x^4}}\,dx \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt[4]{1+x}}\,dx, x, x^4\right) + \operatorname{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + \operatorname{Subst}\left(\int \frac{}{2}\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \sqrt[4]{1+x^4}\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}+}\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.168213, size = 93, normalized size = 0.9

$$\frac{1}{4}x^4F_1\left(1;\frac{1}{4},1;2;-x^4,x^4\right) + \frac{-\log\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right) + \log\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}+1\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

```
[Out] (x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/4 + (2*ArcTan[(2^(1/4)*x)/(1 + x^4)
^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x
^4)^(1/4)])/(4*2^(1/4))
```

Maple [F] time = 0.387, size = 0, normalized size = 0.

$$\int \frac{x^3+1}{-x^4+1}\frac{1}{\sqrt[4]{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

```
[Out] int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3+1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx-\int \frac{x^2}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx-\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

```
[Out] -Integral(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1
)**(1/4) - (x**4 + 1)**(1/4)), x) - Integral(x**2/(x**3*(x**4 + 1)**(1/4) -
 x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) - In
tegral(1/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(
1/4) - (x**4 + 1)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3+1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

3.591 $\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx$

Optimal. Leaf size=28

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.101101, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1898}

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),x
]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Rule 1898

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*x*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(p + 1))/(a*c), x] /; FreeQ[{a, b, c, d, e, g, n, p}, x] && EqQ[n2, 2*n] &
& EqQ[n*(p + 1) + 1, 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx = x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Mathematica [A] time = 0.306663, size = 28, normalized size = 1.

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2
*n)),x]
```

```
[Out] x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.817, size = 0, normalized size = 0.

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

```
[Out] int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bdx^{2\,n} - ac}{(bx^n + a)^{\frac{n+1}{n}}(dx^n + c)^{\frac{n+1}{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="maxima")
```

```
[Out] -integrate((b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n + 1
)/n)), x)
```

Fricas [B] time = 2.00106, size = 128, normalized size = 4.57

$$\frac{bdxx^{2\,n} + acx + (bc + ad)xx^n}{(bx^n + a)^{\frac{n+1}{n}}(dx^n + c)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)/n)*(d*x^n
 + c)^((n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11932, size = 308, normalized size = 11.

$$bdxx^{2\,n}e^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(bx^n+a\right)}{n}-\frac{n\log\left(dx^n+c\right)+\log\left(dx^n+c\right)}{n}\right)} + bcxx^n e^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(bx^n+a\right)}{n}-\frac{n\log\left(dx^n+c\right)+\log\left(dx^n+c\right)}{n}\right)} + adxx^n e^{\left(-\frac{n\log\left(bx^n+a\right)+\log\left(\right.}{n}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, al
gorithm="giac")
```

```
[Out] b*d*x*x^(2*n)*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c)
 + log(d*x^n + c))/n) + b*c*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n
 - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*d*x*x^n*e^(-(n*log(b*x^n + a)
 + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*c*x*e^(-(
n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/
n)
```

3.592 $\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p \left(ac-bdx^{2n}\right) dx$

Optimal. Leaf size=45

$$-\frac{(hx)^{-n(p+1)} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{hn(p+1)}$$

```
[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p)))
)
```

Rubi [A] time = 0.156862, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1849}

$$-\frac{(hx)^{-n(p+1)} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{hn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x]
```

```
[Out] -(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p)))
)
```

Rule 1849

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*(h*x)^(m + 1)*
(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c*h*(m + 1)), x] /; FreeQ[{a, b
, c, d, e, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[m + n*(p + 1) + 1, 0]
&& EqQ[a*c*g*(m + 1) - b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p \left(ac-bdx^{2n}\right) dx = -\frac{(hx)^{-n(1+p)} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn(1+p)}$$

Mathematica [A] time = 0.354939, size = 46, normalized size = 1.02

$$-\frac{(hx)^{n(-p)-n} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{hnp+hn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*
n)),x]
```

```
[Out] -(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n + h*n*p))
```

Maple [C] time = 0.51, size = 138, normalized size = 3.1

$$-\frac{x\left(bd\left(x^{n}\right)^{2}+adx^{n}+bcx^{n}+ac\right)\left(a+bx^{n}\right)^{p}\left(c+dx^{n}\right)^{p}}{n\left(1+p\right)}\mathrm{e}^{-\frac{\left(np+n+1\right)\left(-i\pi\left(\mathrm{csgn}\left(ihx\right)\right)^{3}+i\pi\left(\mathrm{csgn}\left(ihx\right)\right)^{2}\mathrm{csgn}\left(ih\right)+i\pi\left(\mathrm{csgn}\left(ihx\right)\right)^{2}\mathrm{csgn}\left(ix\right)-i\pi\,\mathrm{csgn}\left(ihx\right)\mathrm{cs}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x)
```

```
[Out] -(a+b*x^n)^p*exp(-1/2*(n*p+n+1)*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*csg
n(I*h)+I*Pi*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+2*
ln(h)+2*ln(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*x/n/(1+p)*(c+d*x^n)^p
```

Maxima [A] time = 1.36532, size = 104, normalized size = 2.31

$$-\frac{\left(bdx^{2n}+ac+(bc+ad)x^{n}\right)h^{-np-n-1}e^{\left(-np\log(x)+p\log(bx^{n}+a)+p\log(dx^{n}+c)-n\log(x)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*log(x) + p*
log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1))
```

Fricas [B] time = 2.0071, size = 312, normalized size = 6.93

$$-\frac{\left(bdxx^{2n}e^{\left(-\left(np+n+1\right)\log(h)-\left(np+n+1\right)\log(x)\right)}+acxe^{\left(-\left(np+n+1\right)\log(h)-\left(np+n+1\right)\log(x)\right)}+(bc+ad)xx^{n}e^{\left(-\left(np+n+1\right)\log(h)-\left(np+n+1\right)\mathrm{lo}\right.}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] -(b*d*x*x^(2*n)*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + a*c*x*e^
(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + (b*c + a*d)*x*x^n*e^(-(n*p
 + n + 1)*log(h) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/(n*p
+ n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n)),
x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1456, size = 320, normalized size = 7.11

$$-\frac{(bx^n+a)^p(dx^n+c)^p bdxx^{2n}e^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))}+(bx^n+a)^p(dx^n+c)^p bcxx^n e^{(-np\log(h)-np\log(x)-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] -((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(-n*p*log(h) - n*p*log(x) - n
*log(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*b*c*x*x
^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)) + (
b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h)
 - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*a*c*x*e^(-n*p*
log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)))/(n*p + n)
```

3.593 $\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np}{ac}\right.$

Optimal. Leaf size=31

$$\frac{ex\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{p+1}}{ac}$$

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rubi [A] time = 0.205318, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {1897}

$$\frac{ex\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*c
) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Rule 1897

```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(e*x*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(p + 1))/(a*c), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}
, x] && EqQ[n2, 2*n] && EqQ[a*c*f - e*(b*c + a*d)*(n*(p + 1) + 1), 0] && Eq
Q[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex\left(a+bx^n\right)^{1+p}\left(c+dx^n\right)^{1+p}}{ac}$$

Mathematica [A] time = 0.523959, size = 31, normalized size = 1.

$$\frac{ex\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{p+1}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n
)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

```
[Out] (e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

Maple [A] time = 0.153, size = 52, normalized size = 1.7

$$\frac{xe\left(bd\left(x^n\right)^2+adx^n+bcx^n+ac\right)\left(a+bx^n\right)^p\left(c+dx^n\right)^p}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+2
*n+1)*x^(2*n)/a/c),x)
```

```
[Out] (a+b*x^n)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(c+d*x^n)^p
```

Maxima [A] time = 1.25654, size = 80, normalized size = 2.58

$$\frac{\left(bdexx^{2\,n} + acex + (bce + ade)xx^{n}\right)e^{\left(p\log(bx^{n}+a)+p\log(dx^{n}+c)\right)}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="maxima")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c*e + a*d*e)*x*x^n)*e^(p*log(b*x^n + a) + p
*log(d*x^n + c))/(a*c)
```

Fricas [A] time = 1.79826, size = 115, normalized size = 3.71

$$\frac{\left(bdexx^{2\,n} + acex + (bc + ad)exx^{n}\right)(bx^{n} + a)^{p}(dx^{n} + c)^{p}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(d*x^n + c)
^p/(a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b*d
*e*(2*n*p+2*n+1)*x**(2*n)/a/c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17098, size = 155, normalized size = 5.

$$\frac{(bx^{n} + a)^{p}(dx^{n} + c)^{p}bdxx^{2\,n}e + (bx^{n} + a)^{p}(dx^{n} + c)^{p}bcxx^{n}e + (bx^{n} + a)^{p}(dx^{n} + c)^{p}adxx^{n}e + (bx^{n} + a)^{p}(dx^{n} + c)^{p}acxe}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2
*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e + (b*x^n + a)^p*(d*x^n + c)^p*
b*c*x*x^n*e + (b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e + (b*x^n + a)^p*(d*x^
n + c)^p*a*c*x*e)/(a*c)
```

3.594 $\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1\cdots\right.$

Optimal. Leaf size=45

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*(1 + m))
```

Rubi [A] time = 0.553113, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 86, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.012, Rules used = {1848}

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n*
p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1 + m))
),x]
```

```
[Out] (e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*(1 + m))
```

Rule 1848

```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Sim
p[(e*(h*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(p + 1))/(a*c*h*(m + 1))
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[
a*c*f*(m + 1) - e*(b*c + a*d)*(m + n*(p + 1) + 1), 0] && EqQ[a*c*g*(m + 1)
- b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{e(hx)^{1+m}(a+bx^n)^{1\cdots}}{ach(1+\cdots}$$

Mathematica [A] time = 0.767357, size = 41, normalized size = 0.91

$$\frac{ex(hx)^m(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m +
n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1
 + m))),x]
```

```
[Out] (e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m))
```

Maple [C] time = 0.429, size = 136, normalized size = 3.

$$\frac{xe\left(bd\left(x^{n}\right)^{2}+adx^{n}+bcx^{n}+ac\right)\left(a+bx^{n}\right)^{p}\left(c+dx^{n}\right)^{p}}{ac\left(1+m\right)}\mathrm{e}^{-\frac{m\left(i(\mathrm{csgn}(ihx))^{3}\pi-i\pi\left(\mathrm{csgn}(ihx)\right)^{2}\mathrm{csgn}(ih)-i\pi\left(\mathrm{csgn}(ihx)\right)^{2}\mathrm{csgn}(ix)+i\,\mathrm{csgn}(ihx)\mathrm{csgn}(ih)\mathrm{csgn}(}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+m
)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x)
```

```
[Out] (a+b*x^n)^p*exp(-1/2*m*(I*csgn(I*h*x)^3*Pi-I*Pi*csgn(I*h*x)^2*csgn(I*h)-I*P
i*csgn(I*h*x)^2*csgn(I*x)+I*csgn(I*h*x)*csgn(I*h)*csgn(I*x)*Pi-2*ln(h)-2*ln
(x)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c/(1+m)*(c+d*x^n)^p
```

Maxima [B] time = 1.36721, size = 124, normalized size = 2.76

$$\frac{\left(aceh^{m}xx^{m}+bdeh^{m}xe^{(m\log(x)+2\,n\log(x))}+(bceh^{m}+adeh^{m})xe^{(m\log(x)+n\log(x))}\right)e^{\left(p\log(bx^{n}+a)+p\log(dx^{n}+c)\right)}}{ac(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="maxima")
```

```
[Out] (a*c*e*h^m*x*x^m + b*d*e*h^m*x*e^(m*log(x) + 2*n*log(x)) + (b*c*e*h^m + a*d
*e*h^m)*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + p*log(d*x^n + c))/
(a*c*(m + 1))
```

Fricas [A] time = 1.7337, size = 223, normalized size = 4.96

$$\frac{\left(bdexx^{2\,n}e^{(m\log(h)+m\log(x))}+acexe^{(m\log(h)+m\log(x))}+(bc+ad)exx^{n}e^{(m\log(h)+m\log(x))}\right)(bx^{n}+a)^{p}(dx^{n}+c)^{p}}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="fricas")
```

```
[Out] (b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) + m*log(x))
+ (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/
(a*c*m + a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x
**n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x**(2*n)/a/c/(1+m)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18857, size = 209, normalized size = 4.64

$$\frac{(bx^n+a)^p(dx^n+c)^p bdxx^{2n}e^{(m\log(h)+m\log(x)+1)}+(bx^n+a)^p(dx^n+c)^p bcxx^n e^{(m\log(h)+m\log(x)+1)}+(bx^n+a)^p(dx^n+c)^p adxx}{acm+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/
c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(m*log(h) + m*log(x) + 1) + (b
*x^n + a)^p*(d*x^n + c)^p*b*c*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n +
a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(m*log(h) + m*log(x) + 1) + (b*x^n + a)^p*(d
*x^n + c)^p*a*c*x*e^(m*log(h) + m*log(x) + 1))/(a*c*m + a*c)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
                return "A";
            else
                return "B";
            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C";
        end if
    else # result do not contain complex
          # this assumes optimal do not as well
          if debug then
              print("result do not contain complex, this assumes optimal do
    not as well");
          fi;
          if leaf_count_result<=2*leaf_count_optimal then
             if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
             fi;
             return "A";
          else
             if debug then
                print("leaf_count_result>2*leaf_count_optimal");
             fi;
             return "B";
          end if
    end if
  else #ExpnType(result) > ExpnType(optimal)
    if debug then
       print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```